🎓 Lecture 1 — Logic (Discrete Mathematics)

🌍 Big Picture: What's Discrete Math?

Think of Discrete Mathematics as the **mathematics of clarity** — where everything happens step by step, count by count.
No smooth curves, no infinite blur — just **crisp jumps**, **logical decisions**, and **digital thinking**.

If **Continuous Math** is like water flowing in a river,
Discrete Math is like the **stones** in that river — you move from one stone to another. Step by step.

That's how computers think.
That's why this subject exists.

🎯 Course Objectives — What You'll Learn

By the end of this course, you'll be able to:

1. Express ideas in the **language of logic**, like lawyers who never get confused.
2. Test whether an argument is **valid or just sounds smart**.
3. Play with **sets** — the building blocks of data.
4. Understand **relations** and **functions** — how data points connect.
5. Build recursive definitions — like defining "yourself" in terms of "your past self."
6. Prove formulas using **mathematical induction**.
7. Prove statements in both **direct and indirect** ways.

8. Calculate probabilities — from simple luck to conditional chances.

9. Use **combinatorics** to count possibilities — how many ways something can happen.

10. Understand **graphs and trees** — the skeletons of networks and algorithms.

11. See how all this connects to **Computer Science** — the logic behind every “if,” “for,” and “while” loop.

🧠 Core Idea: LOGIC

> “Logic is what separates clear thinking from confusion.”

Logic is the study of **how to think clearly** —
how to **know** whether something follows from something else.

It’s not about memorizing rules — it’s about **understanding how truth moves**.

💬 SIMPLE STATEMENT (Proposition)

A **statement** (or **proposition**) is any sentence that is either **true** or **false**, but not both.

Examples:

- 2 + 2 = 4 ✅ (True)
- It is Sunday today ❓ (Depends on the day, but still a valid statement — it’s either T or F.)

But this is **not** a statement:

- “Close the door.” (That’s a command, not true or false.)
- “x > 2” (We don’t know what x is — incomplete thought.)

So, a statement = **a clear claim about the world**.

Truth values are written as:

- **T** → True
- **F** → False

🌿 Examples

Sentence	Statement?	Truth Value
Grass is green.	✅ Yes	T
4 + 2 = 6	✅ Yes	T
4 + 2 = 7	✅ Yes	F
x > 2	❌ No	—
Close the door.	❌ No	—

🔗 COMPOUND STATEMENTS

Simple statements can combine to form **compound** ones —
like atoms joining into molecules.

We join them using **Logical Connectives** — the glue of logic.

⚙️ LOGICAL CONNECTIVES (The Building Blocks)

Connective	Symbol	Meaning	Example
NOT	~p	Negation (“not p”)	~p = “It is not hot”
AND	p ∧ q	Conjunction	“It is hot **and** sunny”
OR	p ∨ q	Disjunction	“It is hot **or** sunny”

IF…THEN	p → q	Conditional	“If it is hot, then it is sunny”
IF AND ONLY IF	p ↔ q	Biconditional	“It is hot **if and only if** it is sunny”

🧩 Example (Symbolic Representation)

Let
p = “Islamabad is the capital of Pakistan”
q = “17 is divisible by 3”

Then:

Symbol	English
p ∧ q	Islamabad is the capital of Pakistan **and** 17 is divisible by 3.
p ∨ q	Islamabad is the capital of Pakistan **or** 17 is divisible by 3.
~p	Islamabad is **not** the capital of Pakistan.

🧠 English ↔ Symbolic Translation

Let
p = “It is hot”
q = “It is sunny”

Sentence	Symbol
It is **not** hot.	~p
It is hot **and** sunny.	p ∧ q
It is hot **or** sunny.	p ∨ q
It is **not** hot but sunny.	~p ∧ q
It is **neither** hot nor sunny.	~p ∧ ~q

💪 Another Example

Let
h = “Zia is healthy”
w = “Zia is wealthy”
s = “Zia is wise”

English Sentence	Symbolic Form
Zia is healthy and wealthy but not wise.	$(h \land w) \land \sim s$
Zia is not wealthy but he is healthy and wise.	$\sim w \land (h \land s)$
Zia is neither healthy, wealthy nor wise.	$\sim h \land \sim w \land \sim s$

🔄 Translating Back to English

Let
m = “Ali is good in Mathematics”
c = “Ali is a Computer Science student”

Symbol	English
$\sim c$	Ali is **not** a Computer Science student.
$c \lor m$	Ali is a CS student **or** good in Math.
$m \land \sim c$	Ali is good in Math but **not** a CS student.

🧮 TRUTH TABLE — The Heart of Logic

A **Truth Table** shows how the truth of a compound statement depends on its parts.

It’s like testing every possible combination of truth.

🔹 Negation (~p)

p	~p
T	F

F	T

Negation just **flips** the truth.

🔹 Conjunction (p ∧ q)

p	**q**	**p ∧ q**
T	T	T
T	F	F
F	T	F
F	F	F

“AND” only works if **both** are true.

🔹 Disjunction (p ∨ q)

p	**q**	**p ∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

“OR” works if **at least one** is true.

🧩 Shortcut to Remember

- For **AND (∧)** → Only **T + T = T**
- For **OR (∨)** → Only **F + F = F**

🏁 Summary

By now, you should be able to:

✅ Define what a statement is
✅ Identify valid and invalid statements
✅ Build compound statements using logical connectives
✅ Represent them symbolically
✅ Construct and read truth tables

💡 Closing Thought

Logic isn't just about symbols —
it's about **clean thinking** in a noisy world.

When you learn logic, you learn how to **see truth through fog**,
how to **debug your thoughts** the way a programmer debugs code.

So from today — every time you hear a claim, don't just nod.
Ask yourself:

> "Is that logically true, or just emotionally loud?"

Welcome to **Discrete Mathematics** —
the art of **thinking like a computer and reasoning like a philosopher.**