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1. Let T be the linear transformation such that:

1 / 1 point

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\3\\-4\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\-1\\-3\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}4\\5\\-11\end{bmatrix}$$

Find its rank.

2

Correct

To find the rank of T, we must decide if the three image vectors are linearly independent or not. Notice that

$$\begin{bmatrix}4\\5\\-11\end{bmatrix} = 2\cdot\begin{bmatrix}1\\3\\-4\end{bmatrix} + \begin{bmatrix}2\\-1\\-3\end{bmatrix}$$

Furthermore, $\begin{bmatrix}1\\3\\-4\end{bmatrix}$ and $\begin{bmatrix}2\\-1\\-3\end{bmatrix}$ are linearly independent. Therefore, the rank is 2. You may also row-reduce the matrix that generates this linear transformation to find this result.

2. Let M be a square matrix.

0.5 / 1 point

Check all that are true.

☑ If M is non-singular, then so is M^{-1}.

Correct
If M is non-singular, then $\det(M) \neq 0$. Since $\det(M^{-1}) = \frac{1}{\det(M)}$, then $\det(M^{-1}) \neq 0$.

☑ If M has size n, then it has n distinct eigenvalues.

This should not be selected
Remember that the eigenvalues of a square matrix is related to its characteristic polynomial. Even though it is true that if M has size n, then its characteristic polynomial has **degree** n, it is not always true that such polynomial has n (real) roots. You may review the lecture on [Eigenvalues and eigenvectors].

☑ The determinant is the area of a parallelogram spanned by M and the vectors $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. Therefore, it is always positive.

This should not be selected
Even though we can interpret the determinant as an area, it is not true that it is always positive. Please review the lecture on [Determinant as an area] to understand what does a negative determinant mean in terms of the area of a parallelogram.

☑ If $\det(M) = 5$, then $\det(M^n) = 5^n$.

Correct
This is a straightforward application of the product rule for determinants:

$$\det(M^n) = \det(M \cdot M \cdots M) = \det(M) \cdot \det(M) \cdots \det(M) = 5^n$$

3. Let

1 / 1 point

$$M = \begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\1 & 1 & 1\end{bmatrix}$$

$$N = \begin{bmatrix}2 & 8 & 7\\4 & 3 & 9\\1 & 9 & 5\end{bmatrix}$$

The value for $\det(M \cdot N)$ is:

0

Correct
Note that the third row of M is the sum of the first two, therefore $\det(M) = 0$. Therefore $\det(M \cdot N) = \det(M) \cdot \det(N)$ = $0 \cdot \det(N) = 0$.

4. What is the span of the following vectors vectors?

1 / 1 point

$$\begin{bmatrix}2\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$$

◉ The entire 3 dimensional space.

○ A plane in a 3 dimensional space

Correct
That is correct. Because the three vectors are linearly independent, then they span all the space. Note that this also makes them a basis.

5. Select all the options that are a basis for the 3D space

0 / 1 point

- [x] $$\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Correct
That is correct, since the three vectors are linearly independent, they form a basis for 3D space.

- [x] $$\begin{bmatrix}2\\3\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\2\end{bmatrix}, \begin{bmatrix}-1\\1\\4\end{bmatrix}$$

Correct
That is correct, since the three vectors are linearly independent, they form a basis for 3D space.

- [x] $$\begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}1\\6\\5\end{bmatrix}$$

This should not be selected
Remember that all three vectors need to be linearly independent. Please take a look at video [Span in Linear Algebra](#).

- [x] $$\begin{bmatrix}2\\3\\1\end{bmatrix}, \begin{bmatrix}0\\2.5\\3.5\end{bmatrix}, \begin{bmatrix}0\\5.5\\4.5\end{bmatrix}$$

Correct
That is correct, since the three vectors are linearly independent, they form a basis for 3D space.

- [x] $$\begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$$

Correct
That is correct, since the three vectors are linearly independent, they form a basis for 3D space.

6. Select the characteristic polynomial for the given matrix.

1 / 1 point

$$M = \begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$$

- ○ $\lambda^3 - 8\lambda + 15$
- ○ $\lambda^2 - 8\lambda - 1$
- ○ $\lambda^2 + 8\lambda + 15$
- ◉ $\lambda^2 - 8\lambda + 15$

Correct

The chacacteristic polynomial is the polynomial corresponding to the following:

$$\det(M - \lambda I)$$

Where

$$\lambda I = \lambda \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Therefore,

$$\det(M - \lambda I) = \det\left(\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = \det\left(\begin{bmatrix} 2-\lambda & 1 \\ -3 & 6-\lambda \end{bmatrix}\right)$$

And

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ -3 & 6-\lambda \end{bmatrix}\right) = (2-\lambda)\cdot(6-\lambda) - (-3)\cdot 1 = \lambda^2 - 8\lambda + 15$$

7. Consider the following matrix:

1 / 1 point

$$M = \begin{bmatrix} 3 & 2 \\ 5 & 8 \end{bmatrix}$$

The covariance matrix related to this matrix is:

Hint: Remember you need to centralize M for each column to first get the matrix denoted in lectures as X, then use the correct formula. You may want to watch again the lecture on <Add proper lecture>.

- ○ $\begin{bmatrix} 36 & 46 \\ 46 & 68 \end{bmatrix}$

- ○ $\begin{bmatrix} 0.5 & -1.5 \\ -1.5 & 4.5 \end{bmatrix}$

- ○ $\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$

- ◉ $\begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix}$

✓ **Correct**

The matrix M may represent a set of two variables, $v_1 = (3, 2)$ and $v_2 = (5, 8)$. Where the first component is the first feature and the second component is the second feature. The covariance matrix is centered at the mean, i.e., every feature must be subtracted by the mean of all values in that feature. Note that each column represent a feature whereas each row represents one point. So the mean must be computed in the **columns**. In this case, the mean for the first column, $\mu_x = \frac{3+5}{2} = 4$ and for the second column $\mu_y = \frac{2+8}{2} = 5$. So, the centralized matrix, X is

$$X = \begin{bmatrix} 3 - \mu_x & 2 - \mu_y \\ 5 - \mu_x & 8 - \mu_y \end{bmatrix} = \begin{bmatrix} 3 - 4 & 2 - 5 \\ 5 - 4 & 8 - 5 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 1 & 3 \end{bmatrix}$$

Therefore,

$$X^T = \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$$

Finally, the covariance matrix, Σ is:

$$\Sigma = \frac{1}{2-1} X^T X = \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix}$$

8. Consider the following matrix

0.5 / 1 point

$$M = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$$

Check all the options that represent the eigenvectors of this matrix.

Hint:

- *The characteristic polynomial for M is given by $(3-\lambda)(1-\lambda)$.*
- *Remember that for each eigenvalue, if there is a non-zero eigenvector related to it, then there are infinitely many more eigenvectors related to the same eigenvalue. In other words, if v is an eigenvector for an eigenvalue λ, then kv is also an eigenvector for the same eigenvalue λ, for any real valued number k.*
- *You may want to watch again the lecture on "Eigenvalues and eigenvectors".*

- [x] $\begin{bmatrix} 0 \\ k \end{bmatrix}$, for any k real.

Correct
You first find the eigenvalues for the matrix M. This is given by the characteristic polynomial, and the hint shows us that the eigenvalues are $\lambda = 3$ and $\lambda = 1$.

For $\lambda = 1$, you must solve

$$\begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1 \begin{bmatrix} x \\ y \end{bmatrix}.$$

This translates to $\begin{cases} 3x = x \\ -2x + y = y \end{cases}$. The first equation implies $x = 0$ and the second equation leads to $y = y$, which is always true, so setting $y = k$, any eigenvector related to this eigenvalue is of the form $\begin{pmatrix} 0 \\ k \end{pmatrix}$.

- [x] $\begin{bmatrix} k \\ k \end{bmatrix}$, for any k real.

This should not be selected
Vectors of such form are not eigenvectors for any eigenvalue of M. Review the lecture on "Eigenvalues and eigenvectors" to get a better undesrstanding on how to solve such problem.

- [x] $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

This should not be selected
Note that this is just one vector. As mentioned in the hint, if there is a non-zero eigenvector for an eigenvalue, then there are infinitely many, so the answer must contain a parameter. Review the video to apply this step by step "Eigenvalues and eigenvectors".

- [x] $\begin{bmatrix} k \\ -k \end{bmatrix}$, for any k real.

Correct
You first find the eigenvalues for the matrix M. This is given by the characteristic polynomial, and the hint shows us that the eigenvalues are $\lambda = 3$ and $\lambda = 1$.

For $\lambda = 3$, you must solve

$$\begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}.$$

This translates to $\begin{cases} 3x = 3x \\ -2x + y = 3y \end{cases}$. The first equation is always true, the second equation leads to $x = -y$. So if $x = k$, then $y = -k$. Thus any eigenvector is of $\begin{bmatrix} k \\ -k \end{bmatrix}$.

9. Suppose you have the following dataset

	Size (m^2)	No. Bedrooms	No. Bathrooms
House 1	70	2	2
House 2	110	4	2

1 / 1 point

Which matrix is the $X - \mu$ matrix, used in the covariance matrix computation? The matrix $X - \mu$ is defined in the lecture Covariance Matrix. Remember that the covariance matrix is defined by $\Sigma = \frac{1}{n-1}(X-\mu)^T(X-\mu)$.

- (selected)
$$X - \mu = \begin{bmatrix} -20 & -1 & 0 \\ 20 & 1 & 0 \end{bmatrix}$$

-
$$X - \mu = \begin{bmatrix} 70 & 110 \\ 2 & 4 \\ 2 & 2 \end{bmatrix}$$

-
$$X - \mu = \begin{bmatrix} 70 & 2 & 2 \\ 110 & 4 & 2 \end{bmatrix}$$

-
$$X - \mu = \begin{bmatrix} -20 & 20 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$$

Correct

The matrix that represents the dataset is

$$M = \begin{bmatrix} 70 & 2 & 2 \\ 110 & 4 & 2 \end{bmatrix}$$

By computing the mean for each **column,** you get that

$$\mu_x = 90, \mu_y = 3, \mu_z = 2$$

Therefore,

$$X - \mu = \begin{bmatrix} 70-80 & 2-3 & 2-2 \\ 110-80 & 4-3 & 2-2 \end{bmatrix} = \begin{bmatrix} -20 & -1 & 0 \\ 20 & 1 & 0 \end{bmatrix}$$

10. For the dataset from question 9, what are the eigenvalues of the covariance matrix?

1 / 1 point

- (selected) $\lambda_1 = 802,\ \lambda_2 = 0,\ \lambda_3 = 0$
- $\lambda_1 = 0,\ \lambda_2 = 0$
- $\lambda_1 = 17027,\ \lambda_2 = 0,\ \lambda_3 = 0$

Correct

That is correct. From the previous question, you know that $X - \mu = \begin{bmatrix} -20 & -1 & 0 \\ 20 & 1 & 0 \end{bmatrix}$, so that the covariance matrix is

$$C = \frac{1}{n-1}(X - \mu^T)(X - \mu) = \begin{bmatrix} 800 & 40 & 0 \\ 40 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore,

$$\det(C - \lambda I) = \begin{bmatrix} 800 - \lambda & 40 & 0 \\ 40 & -2 - \lambda & 0 \\ 0 & 0 & 0 - \lambda \end{bmatrix}$$

and

$$\det(C - \lambda I) = \begin{matrix} (800 - \lambda) \cdot (2 - \lambda) \cdot (0 - \lambda) + 40 \cdot 0 \cdot 0 + 40 \cdot 0 \cdot 0 - \\ 0 \cdot (2 - \lambda) \cdot 0 - (800 - \lambda) \cdot 0 \cdot 0 - 40 \cdot 40 \cdot (0 - \lambda) \end{matrix}$$

Solving for $\det(C - \lambda I) = 0$ you get the desired eigenvalues.