Congratulations! You passed!

Grade received 87.50% **Latest Submission Grade** 87.50% **To pass** 78% or higher

Go to next item

1. Consider the following system of equations in two variables.

1 / 1 point

$$\begin{cases} x + 3y = 15 \\ 3x + 12y = 3 \end{cases}$$

Check all the options that are **true, given the system above.**

- [x] $4x + 15y = 18.$

Correct
This equation is the sum of both equations in the system!

- [x] $2x + 6y = 30.$

Correct
This equation is the first equation from the system, multiplied by 2, i.e., $2x + 6y = 30$ is equivalent to $2 \cdot (x + 3y) = 2 \cdot 15.$

- [x] $y = -14.$

Correct
This can be obtained by dividing the second equation by 3 and subtracting it by the first one:

$$\begin{cases} x + 3y = 15 \\ 3x + 12y = 3 \end{cases}$$

Dividing the second equation by 3, yields the following equivalent system:

$$\begin{cases} x + 3y = 15 \\ x + 4y = 1 \end{cases}$$

Now, subtracting the second equation and by the first one you get that $y = -14$ bu canceling out the variable x.

- [ ] $x + 3y = 0.$

2. Consider the following system of equations in two variables.

1 / 1 point

$$\begin{cases} 2x + y = 5 \\ 4x + 2y = 10 \end{cases}$$

Check all the options that are **true, given the system above.**

- [ ] The system has no solution.
- [x] The system has infinitely many solutions.

Correct
One equation is a multiple of another. Note that the second equation is 2 times the first one. This makes the system redundant, because there is only one piece of information behind both equations.

- [x] $x = 0$ and $y = 5$ is a solution for this system.

Correct
You can always verify if some proposed solution is indeed a solution for any system by simply replacing the values and checking that such values satisfy **every equation in the system.**

- [ ] The solution for this system has 0 degrees of freedom.

3. Consider the following system of equations.

0 / 1 point

$$\begin{cases} x + 2y + 3z = 10 \\ 2x + 6y + 12z = 4 \\ 4x - 8y + 4z = 8 \end{cases}$$

The value for z is:

Hint: You may use the Elimination Method, discussed in lecture [Solving system of equations with more variables.](#)

4

Incorrect
This is the solution for y! Please provide the solution for z. You may review the lecture on [*Solving system of equations with more variables.*](#)

4. Consider the following matrix:

1 / 1 point

$$\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

Its rank is:

- (●) 1
- ( ) 2
- ( ) 0

Correct
Converting the matrix into a linear system of equations in the following way:

$$\begin{cases} 3x + y = 0 \\ 6x + 2y = 0 \end{cases}$$

You see that the second equation is two times the first one, so the solution is all pairs (x, y) such that $3x + y = 0$, this is a line in the plane, therefore it has dimension 1. As you saw in the lecture [The rank of a matrix,](#) this means that the matrix has rank 1 (in this case the 1 comes from $1 = 2 - 1$, where 2 is the matrix size.

5. Check all matrices that are in row echelon form.

1 / 1 point

- [x] $$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

✓ Correct
The matrix satisfies the row echelon form definition:

1. Below any non-zero number in the diagonal there are only zeros.
2. Before the first non-zero element in each row, there are are only zeros.

- [ ] $$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- [ ] $$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

- [x] $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

✓ Correct
The matrix satisfies the row echelon form definition:

1. Below any non-zero number in the diagonal there are only zeros.
2. Before the first non-zero element in each row, there are are only zeros.

6. Check all the options that are a row echelon form of the following matrix.

1 / 1 point

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix}$$

- [x] $$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

✓ Correct
This could be obtained by **row operations.** One set of steps to get the correct matrix is, denoting r_1,r_2 and r_3, respectivelly rows number $1, 2$ and 3:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{r_3 = r_3 - (r_1 + r_2)} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{r_2 = 2 \cdot r_1 - r_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

Finally, to turn the pivots into 1, there is one more step, divide the each row for its **pivot** value. This is not necessary for r_1, since its pivot is already 1. Thus:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{r_2 = \frac{r_2}{5}, r_3 = \frac{r_3}{3}} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

In case you want to review the content, please go back the the lectures on [Row-echelon form in general] and [Reduced row-echelon form].

☑ $$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

Correct

This could be obtained by **row operations.** One set of steps to get the correct matrix is, denoting r_1,r_2 and r_3, respectivelly rows number $1, 2$ and 3:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{r_3 = r_3 - (r_1 + r_2)} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{r_2 = 2 \cdot r_1 - r_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

In case you want to review the content, please go back the the lectures on Row-echelon form in general and Reduced row-echelon form.

☐ $$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

☐ $$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \end{bmatrix}$$

7. Compute the rank of the following matrix:

1 / 1 point

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 3 & 4 & 6 \end{bmatrix}$$

2

Correct

Initially, note that the third row is the sum of the first two, so the row operation row 3 = row 3 - (row 1 + row 2) makes the third row e row of zeros. Therefore, the the resulting matrix is

$$\begin{bmatrix} 2 & 1 & 5 \end{bmatrix}$$

8. Let M be a 2×2 matrix. Check all sentences that are true.

1 / 1 point

☐ Replacing one row by the sum of the two rows of the matrix does not affect singularity, but it does affect the determinant value.

☐ Multiplying a row by a non-zero real number does not affect its **determinant**.

☑ Swapping its rows change the determinant sign.

Correct

Correct! As you've seen in the lecture Row operations that preserve singularity, swapping its rows invert the determinant calculation, therefore its sign will change! This is in fact true for **an arbitrary** $n \times n$ matrix! Swapping two rows will change its determinant sign!

☑ Multiplying a row by a non-zero real number does not affect its **singularity**.

Correct

The singularity is related to the matrix determinant. If M has non-zero determinant, then multiplying any row by a fixed non-zero value will only scale the determinant by that same factor.