190F Foundations of Data Science

Spring 2020

Lecture 17

Correlation & Linear Regression

Prediction

Guessing the Future

- Based on incomplete information

- One way of making predictions:
  - To predict an outcome for an individual,
  - find others who are like that individual
  - and whose outcomes you know.
  - Use those outcomes as the basis of your prediction.

Association

Two Numerical Variables

- Trend
  - Positive association
  - Negative association
- Pattern
  - Any discernible “shape” in the scatter
  - Linear
  - Non-linear

Visualize, then quantify

(Demo)

Correlation Coefficient

Definition of *r*

Correlation Coefficient (*r*) =

average of	product of	x in standard units	and	y in standard units

Measures how clustered the scatter is around a straight line

The Correlation Coefficient *r*

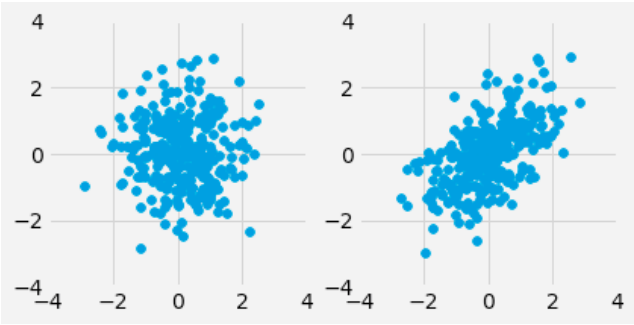
- Measures **linear** association
- Based on standard units
- $-1 \le r \le 1$
  - $r = 1$: scatter is perfect straight line sloping up
  - $r = -1$: scatter is perfect straight line sloping down
- $r = 0$: No linear association; *uncorrelated*

(Demo)

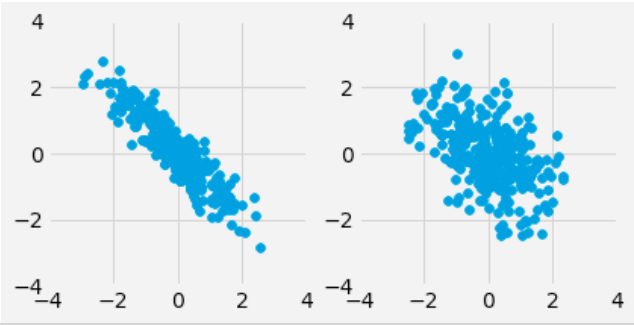
Discussion Question

For each pair, which one will have a higher value of r?

a)



b)



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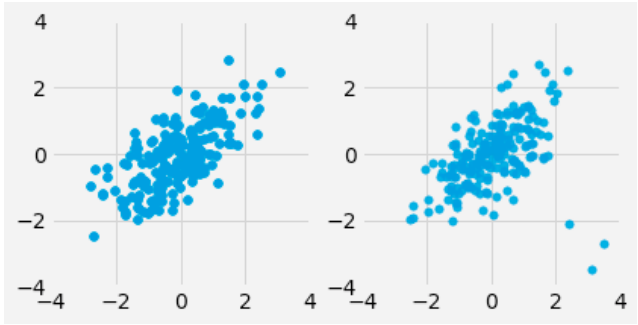

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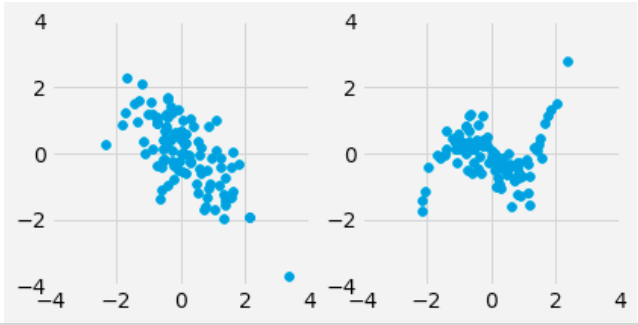


Properties of Correlation

Properties of *r*

- *r* is a pure number, with no units
- *r* is not affected by changing units of measurement
- *r* is not affected by switching the horizontal and vertical axes

Interpreting *r*

Watch out for:

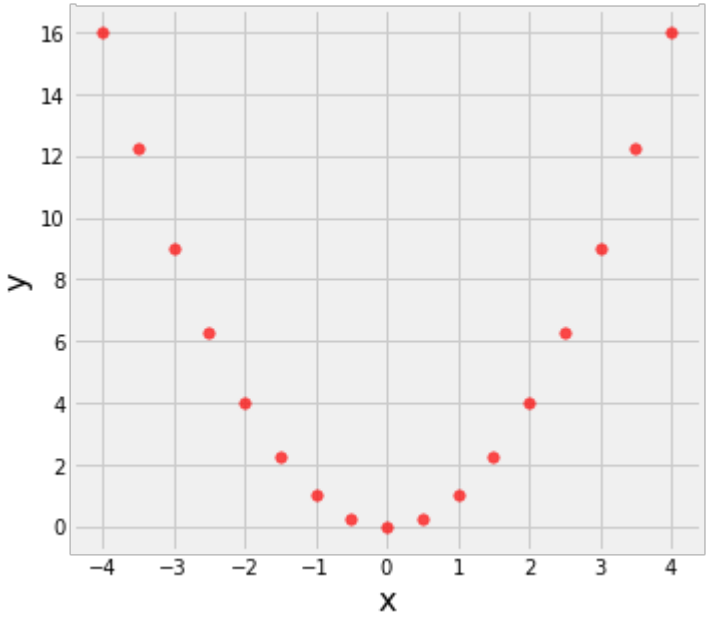
- Jumping to conclusions about causality
- Non-linearity
- Outliers
- Ecological correlations, based on aggregates or averaged data

Interpreting *r*

Don't jump to conclusions about causality

Interpreting *r*

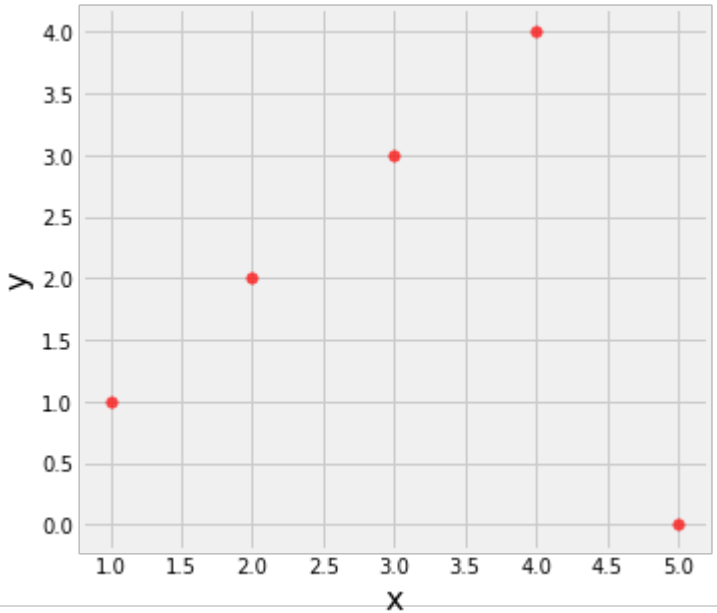
Watch out for non-linearity.



r = 0.0

Interpreting *r*

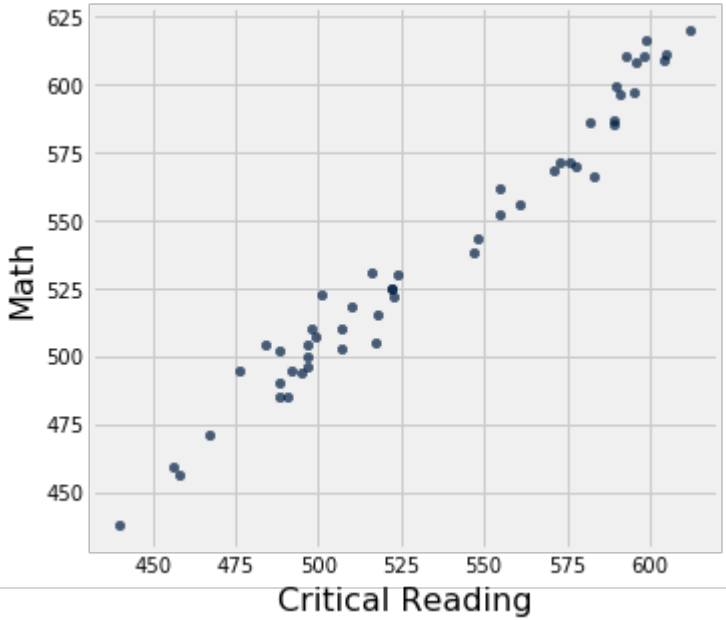
Watch out for outliers.



r = 0.0

Interpreting *r*

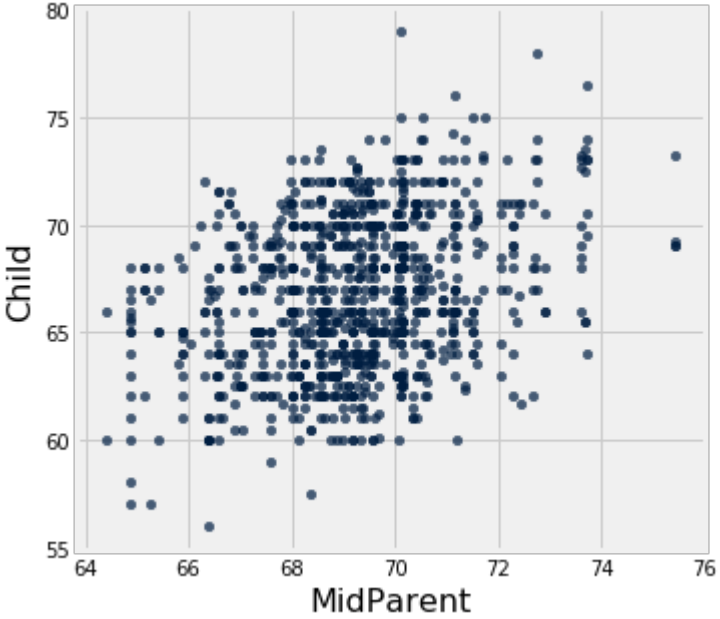
Watch out for ecological correlations, based on aggregates or averaged data.



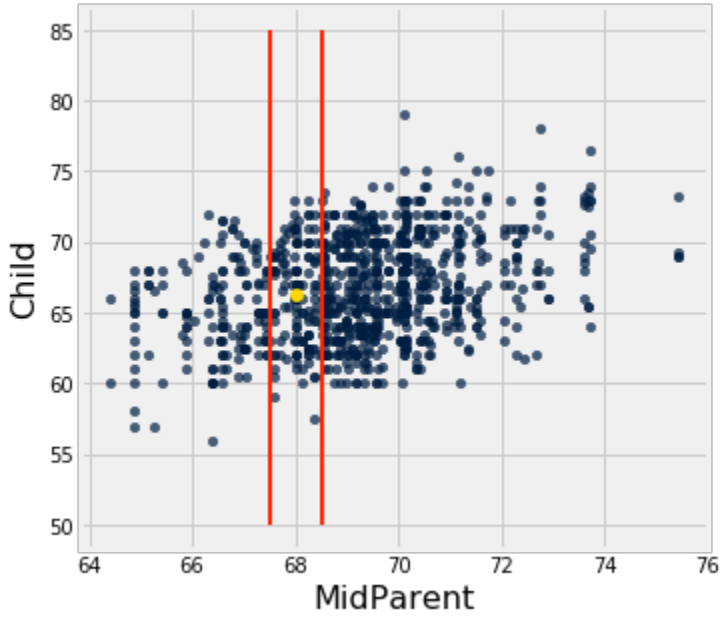
r = 0.98

Prediction

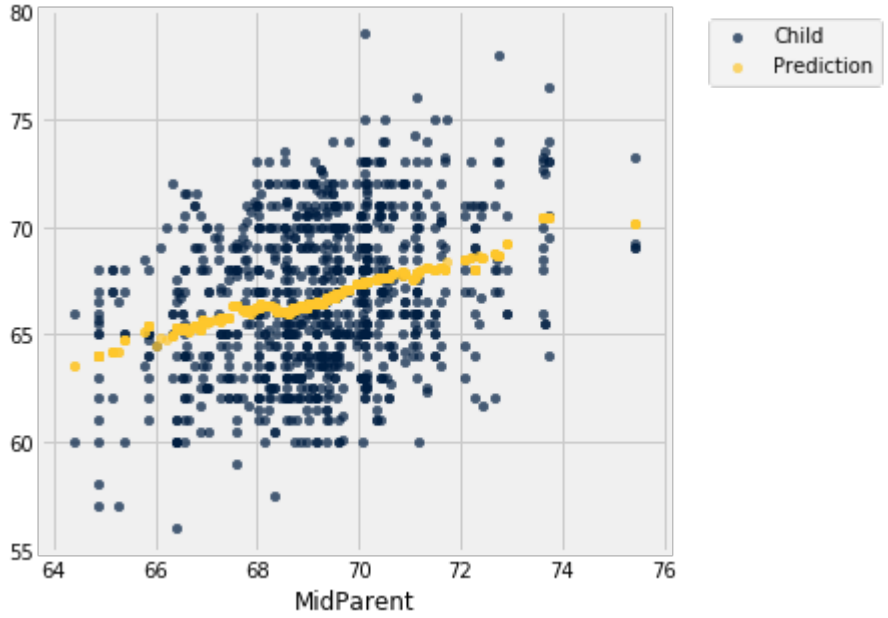
Galton's Heights



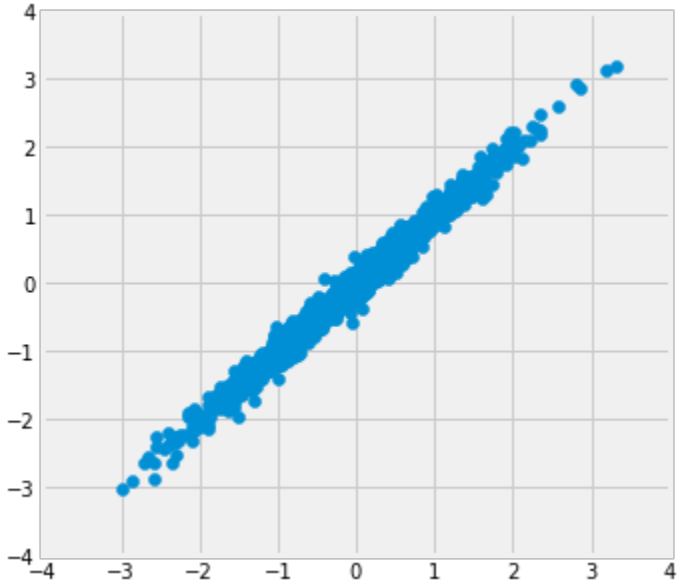
Galton's Heights



Galton's Heights

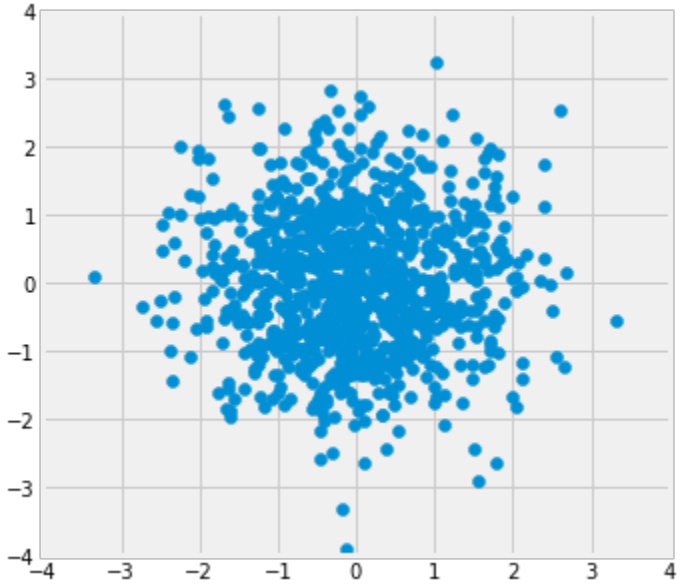


Where is the prediction line?



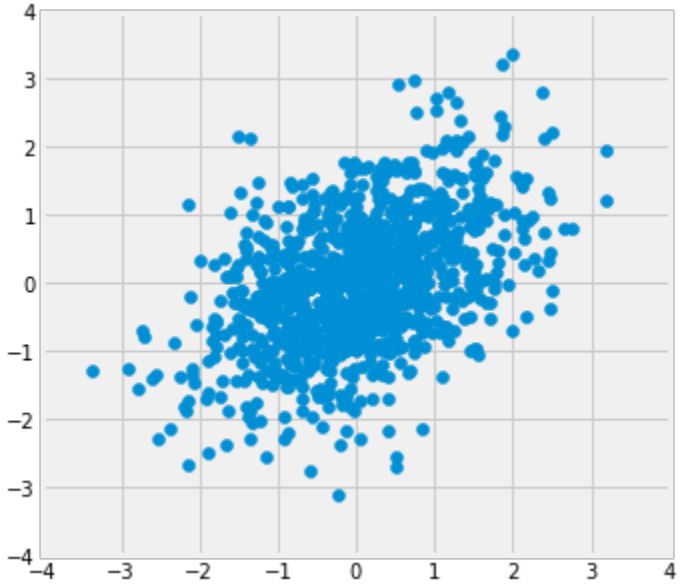
r = 0.99

Where is the prediction line?



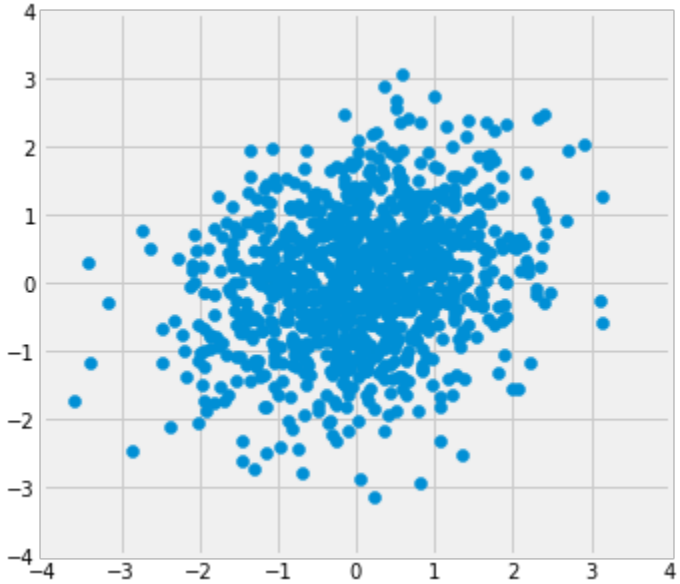
r = 0.0

Where is the prediction line?



r = 0.5

Where is the prediction line?



r = 0.2

Nearest Neighbor Regression

A method for prediction:

- Group each x with a representative x value (rounding)
- Average the corresponding y values for each group

For each representative x value, the corresponding prediction is the average of the y values in the group.

Graph these predictions.

If the association between x and y is linear, then points in the graph of averages tend to fall on the regression line.

Regression to the Mean

A statement about x and y pairs

- Measured in *standard units*
- Describing the deviation of x from 0 (the average of x's)
- And the deviation of y from 0 (the average of y's)

On average, y deviates from 0 less than x deviates from 0

Regression Line

$$y_{(\text{su})} = r \times x_{(\text{su})}$$

Correlation

Not true for all points — a statement about averages

Linear Regression

(Demo)

Slope & Intercept

Regression Line Equation

In original units, the regression line has this equation:

$$\frac{\text{estimate of } y \ - \ \text{average of } y}{\text{SD of } y} = r \times \frac{\text{the given } x \ - \ \text{average of } x}{\text{SD of } x}$$

y in standard units

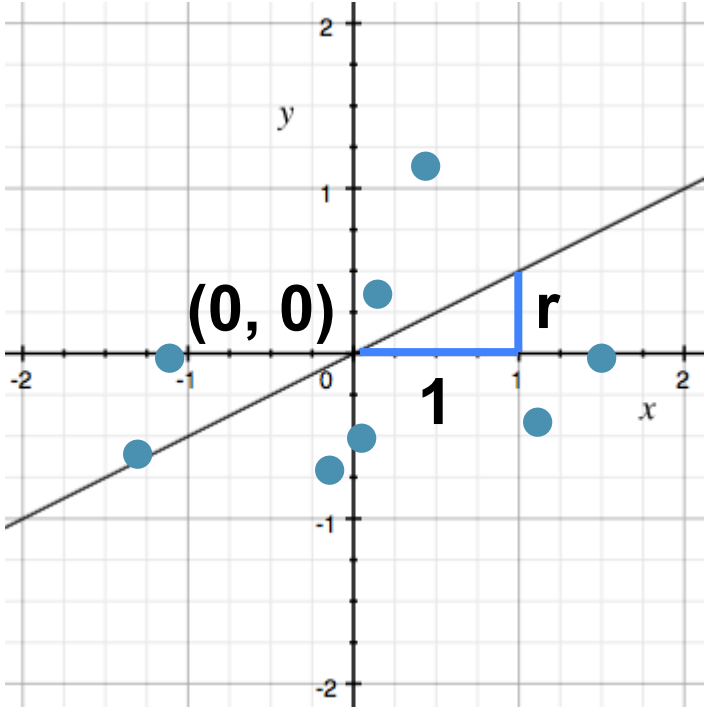
x in standard units

Lines can be expressed by *slope* & *intercept*

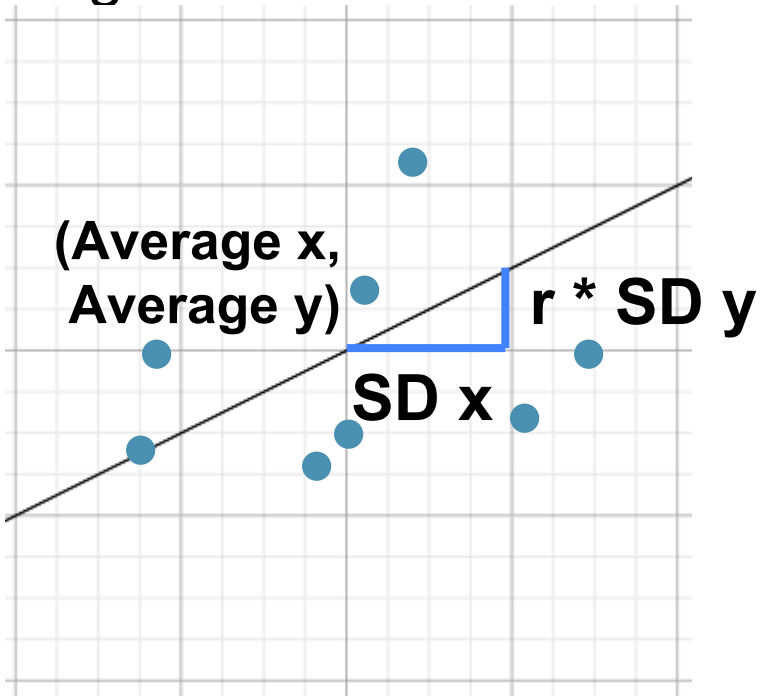
$$y = \text{slope} \times x + \text{intercept}$$

Regression Line

Standard Units



Original Units



Slope and Intercept

estimate of y = slope * x + intercept

$$\textbf{slope of the regression line} = r \cdot \frac{\text{SD of } y}{\text{SD of } x}$$

$$\textbf{intercept of the regression line} = \text{average of } y - \text{slope} \cdot \text{average of } x$$

(Demo)