190F Foundations of Data Science

Fall 2018

Lecture 12

Chance

Announcements

Probability

Outcomes and Events

- **Outcome Space:** Set of possible outcomes (mutually exclusive and exhaustive). Examples:
  - Coin flip: {heads, tails}
  - Dice rolls: {1,2,3,4,5,6}

- **Event:** A subset of outcomes. Examples:
  - Coin flip: Coin comes up heads
  - Dice roll: Value is 4 or greater

Probability

- Describes how likely different *events* are
- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (or 100%)
  - Chance of event that is certain
- Probability calculus
  - Rules for calculating and manipulating probabilities

Compliment Rule

- Suppose the probability of event A is P(A)=p.
- Define the event notA to contain all of the outcomes in the outcome space that are not part of the event A.
- Then P(notA) = 1-P(A) = 1-p.

Multiplication Rule

Gives the chance that two events *A* and *B* both happen is

P(*A* happens) x P(*B* happens given that *A* happened)

P(*B* happens) x P(*A* happens given that *B* happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- If events A and B are *independent* then the chance that two events *A* and *B* both happen is P(*A*) x P(B).

Addition Rule

If event *A* can happen in *exactly one* of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

Event Probabilities

Assuming all outcomes are *equally likely*, the probability of an event A is:

$$P(A) = \frac{\text{number of outcomes in the set A}}{\text{number of outcomes in outcome space}}$$

Example: At Least One Head

- Question: What is the probability of at least one head in three tosses of a fair coin?
  - P(at least one head in 3 tosses) = 1 - P(TTT)
  - P(T) = ½
  - P(TTT) = (½) x (½) x (½) = ⅛
  - P(at least one head) = 1 - P(TTT) = ⅞ = 87.5%
- In 10 tosses:
  - $1 - (½)^{10}$
  - 99.9%