190F Foundations of Data Science

Spring 2020

Lecture 11

Error Probabilities and A/B testing

Announcements

Testing a Hypothesis

Step 1: Select Two Hypotheses

- A test chooses between two views of how data were generated: *Null hypothesis* proposes that data were generated at random; *Alternative hypothesis* proposes some effect other than chance

Step 2: Choose a Test Statistic

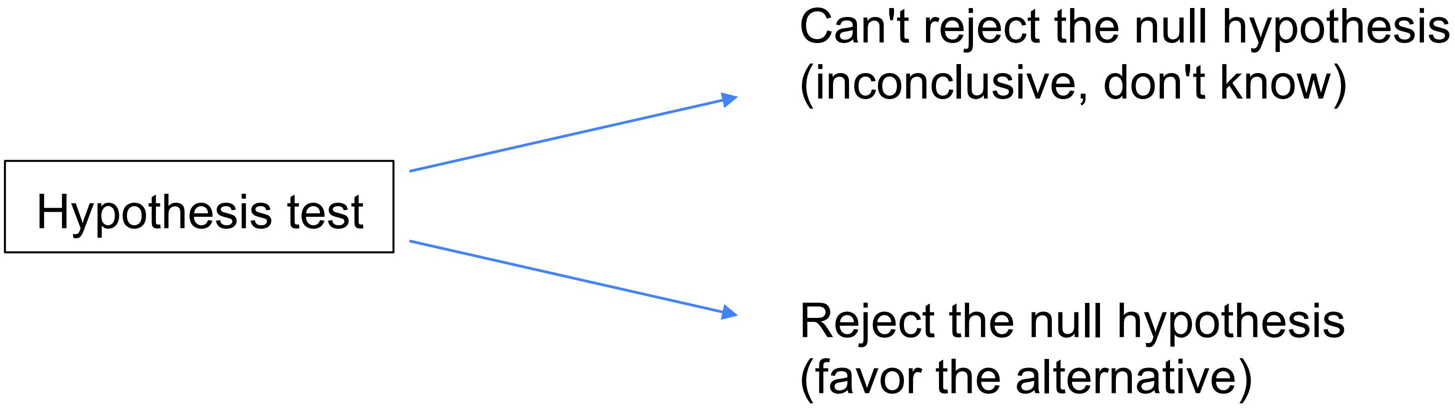
- A value that can be computed from the data

Step 3: Compute What The Null Hypothesis Predicts

- Compute the distribution of the test statistic: what the test statistic might be if the null hypothesis were true.

Step 4: Compare the Prediction to the Observed Data

Conclusions From a Test



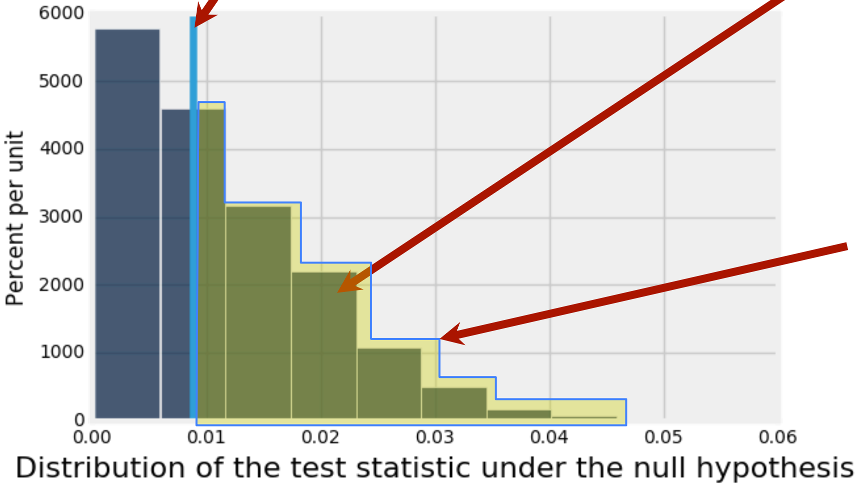
Hypothesis test

Can't reject the null hypothesis
(inconclusive, don't know)

Reject the null hypothesis
(favor the alternative)

Quantifying Conclusions

P(the test statistic would be equal to or more extreme
than the observed test statistic under the null hypothesis)



Evaluating Mendel's pea flower hypothesis

This area is the P-value (approximately)

Statistical Significance

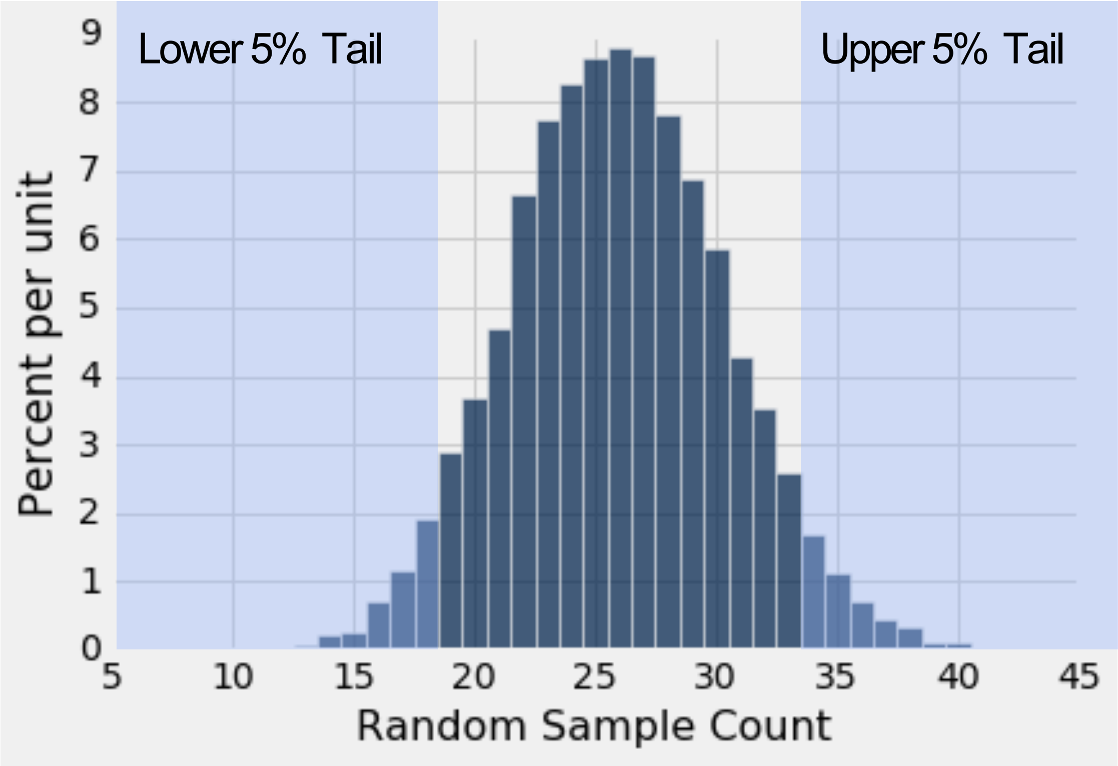
Conventions of Consistency

- **“Inconsistent”:** The test statistic is in the tail of the null distribution.

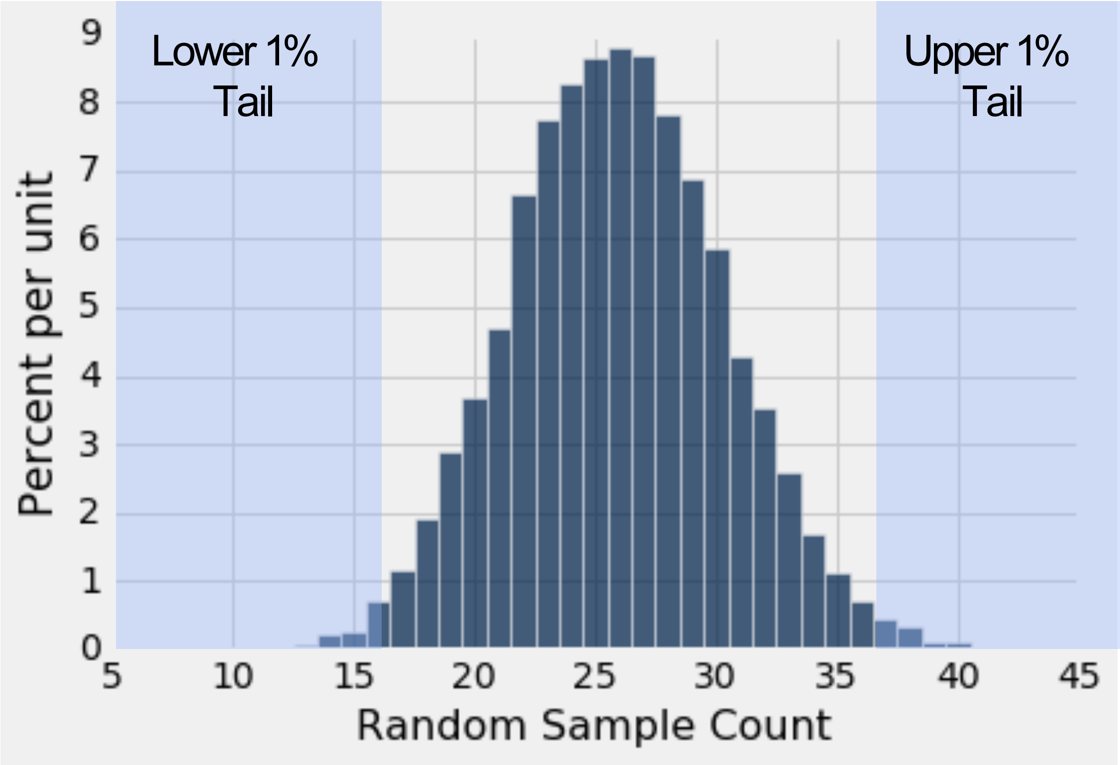
- **“In the tail,” first convention:**
  - The area in the upper (or lower) tail is less than 5%.
  - The result is “statistically significant.”

- **“In the tail,” second convention:**
  - The area in the upper (or lower) tail is less than 1%.
  - The result is “highly statistically significant.”

Tail Areas


Lower 5% Tail
Upper 5% Tail
Percent per unit
0
1
2
3
4
5
6
7
8
9
5
10
15
20
25
30
35
40
45
Random Sample Count

Tail Areas


Lower 1%
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Random Sample Count

Conventions About Inconsistency

- Which tail do you look at?

- The tail that corresponds to values of the statistic that favor the alternative.

- **This is why you generally don’t want a statistic where *both tails* indicate support for the alternative hypothesis.**

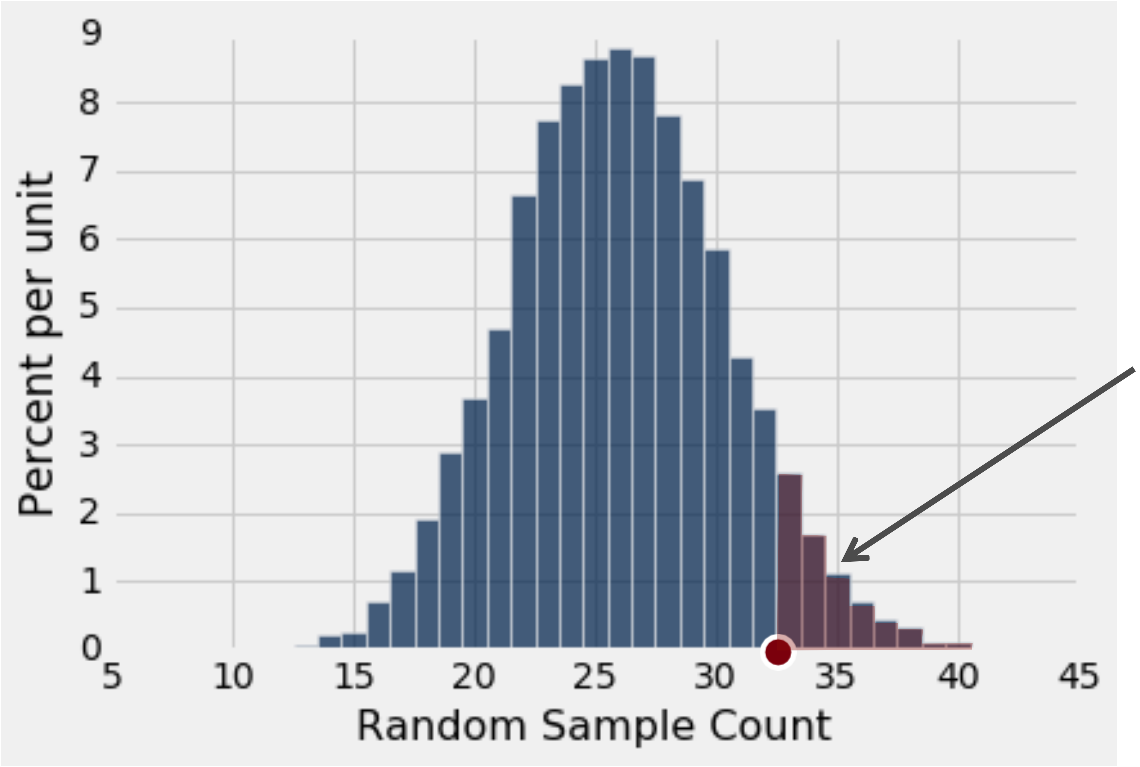
Definition of the *P*-value

Formal name: **observed significance level**

The *P*-value is the chance,

- under the null hypothesis,
- that the test statistic
- is equal to the value that was observed in the data
- or is even *further* in the direction of the alternative.

Tail Areas



The probability associated with **this area** is the (approximate) P-value for the observed statistic

Error probability of a test

Can the Conclusion be Wrong?

Yes.

	Null is true	Alternative is true
Test rejects the null	❌	✅
Test doesn't reject the null	✅	?

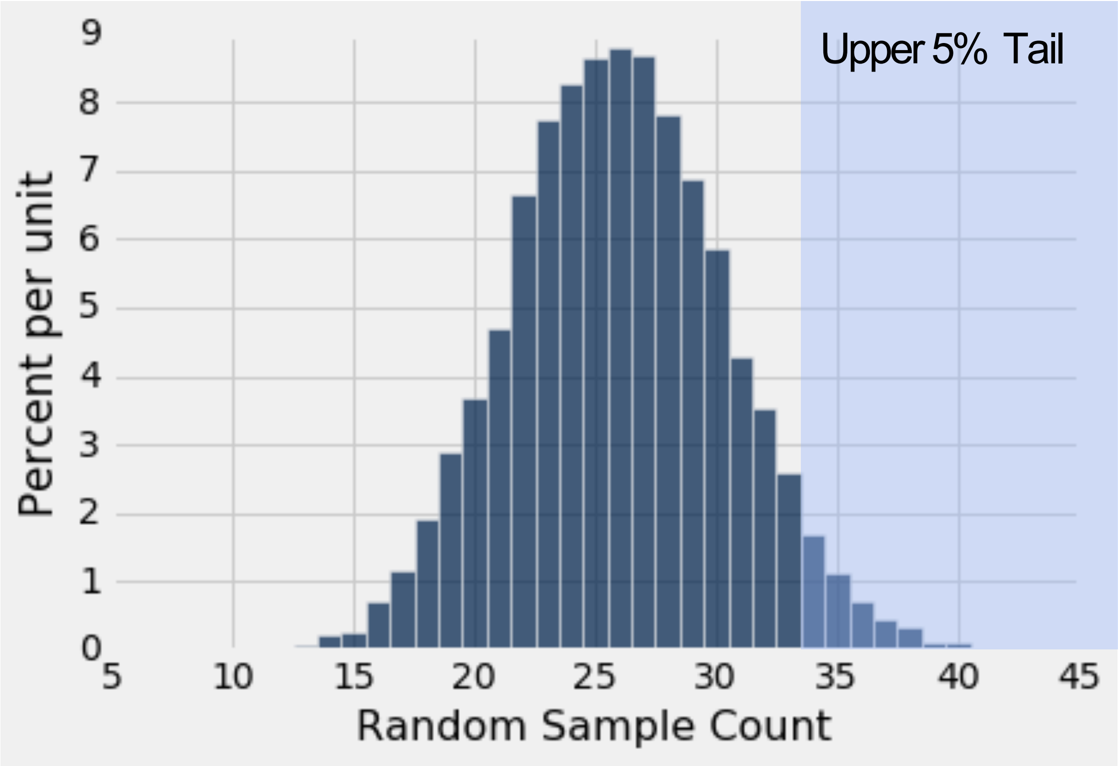
An Error Probability

- The cutoff for the P-value is an error probability.

- If:
    - your **cutoff is 5%**
    - and the **null hypothesis happens to be true**
    - (but you don’t know that)

- then there is about a **5% chance** that **your test will reject the null hypothesis anyway**.

Tail Areas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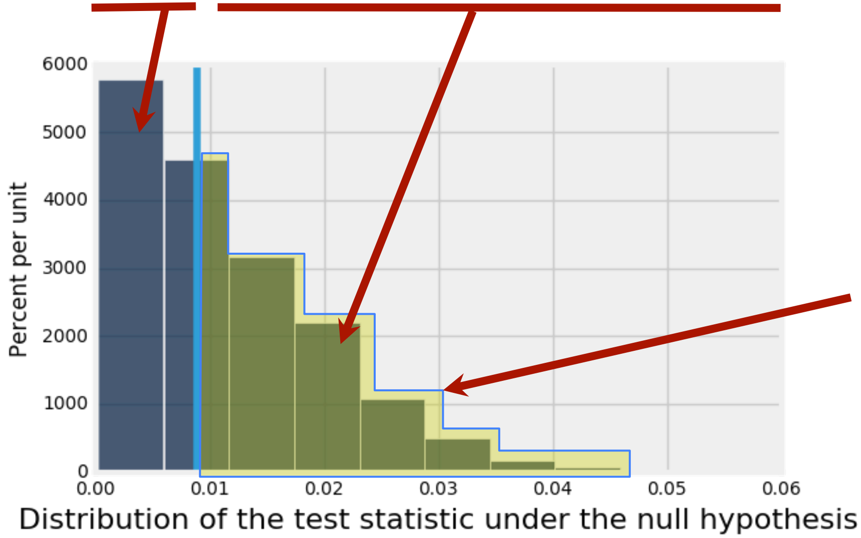
Random Sample Count

Setting a Cutoff

Let's draw a cutoff point for where we'll reject the null.



Won't reject the null

Will reject the null

Evaluating Mendel's pea flower hypothesis

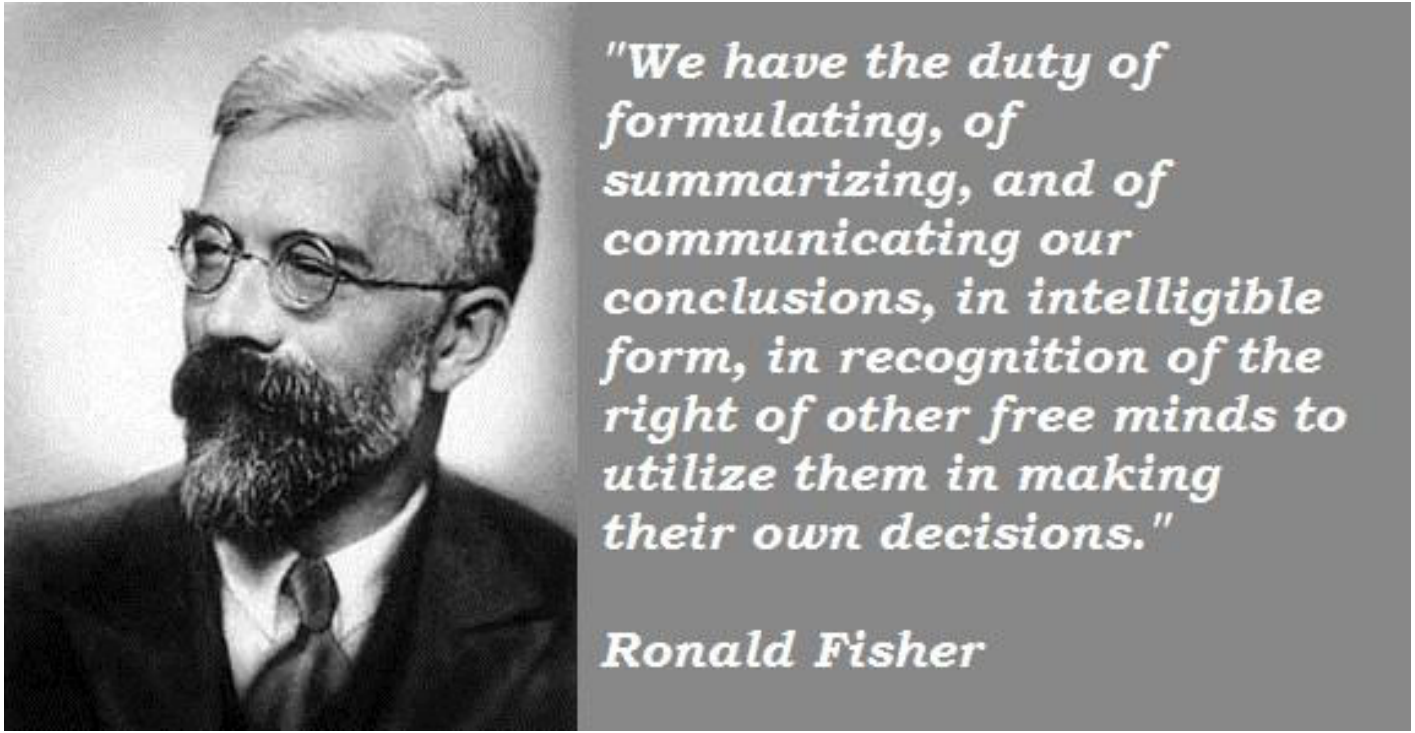
This area is the probability of wrongly rejecting the null

How Much Risk To Accept?

- **First convention:**
  - Accept a 5% risk of wrongly rejecting the null.
  - The result is “statistically significant.”
- **Second convention:**
  - Accept a 1% risk of wrongly rejecting the null.
  - The result is “highly statistically significant.”

Origin of the conventions

Sir Ronald Fisher, 1890-1962



"We have the duty of formulating, of summarizing, and of communicating our conclusions, in intelligible form, in recognition of the right of other free minds to utilize them in making their own decisions."

Ronald Fisher

Sir Ronald Fisher, 1925

“It is convenient to take this point [5%] as a limit in judging whether a deviation is to be considered significant or not.”

— *Statistical Methods for Research Workers*

Sir Ronald Fisher, 1926

“If one in twenty does not seem high enough odds, we may, if we prefer it, draw the line at one in fifty (the 2 percent point), or one in a hundred (the 1 percent point). Personally, the author prefers to set a low standard of significance at the 5 percent point …”

A/B testing

Comparing Two Samples

- Compare values of sampled individuals in Group A with values of sampled individuals in Group B.

- Question: Do the two sets of values come from the same underlying distribution?

- Answering this question by performing a statistical test is called **A/B testing**.

(Demo)

The Groups and the Question

- Random sample of mothers of newborns. Compare:

  (A) Birth weights of babies born to mothers who smoked during pregnancy

  (B) Birth weights of babies born to mothers who did not smoke during pregnancy

- Question: Could the differences be due to chance alone?

Hypotheses

- Null:
  - In the population, the distributions of the birth weights of the babies in the two groups are the same. (They are different in the sample just due to chance.)
- Alternative:
  - In the population, the babies of the mothers who smoked weighed less, on average, than the babies of the non-smokers.

Test Statistic

- Group A: smokers
- Group B: non-smokers

- Statistic: Difference between average weights

  Group B average - Group A average

- Large values of this statistic favor the alternative

Simulating Under the Null

Non-smoker

120 oz

Non-smoker

113 oz

Smoker

128 oz

Non-smoker

136 oz

…

Smoker

108 oz

Simulating Under the Null

Smoker

120 oz

Non-smoker

113 oz

Non-smoker

128 oz

Smoker

136 oz

...

Non-smoker

108 oz

Simulating Under the Null

- If the null is true, all rearrangements of the birth weights among the two groups are equally likely
- Plan:
  - Shuffle all the birth weights
  - Assign some to "Group A" and the rest to "Group B", maintaining the two sample sizes
  - Find the difference between the averages of the two shuffled groups
  - Repeat

(Demo)