190F Foundations of Data Science

Spring 2020

Lecture 13

Confidence Intervals

Percentiles

Computing Percentiles

The 80th percentile is the value in a set that is at least as large as 80% of the elements in the set

For `s = [1, 7, 3, 9, 5]`, `percentile(80, s)` is 7

The 80th percentile is ordered element 4: `(80/100) * 5`

Percentile

Size of set

For a percentile that does not exactly correspond to an element, take the next greater element instead

The `percentile` Function

- The *p*th percentile is the value in a set that is at least as large as *p*% of the elements in the set
- Function in the `datascience` module:

```
percentile(p, values)
```

- `p` is between 0 and 100
- Returns the *p*th percentile of the array

Discussion Question

Which are `True`, when `s = [1, 7, 3, 9, 5]`?

```
percentile(10, s) == 0

percentile(39, s) == percentile(40,s)


percentile(40, s) == percentile(41,s)


percentile(50, s) == 5
```

Estimation (Review)

Inference: Estimation

- How big is an unknown parameter?
- If you have a census (that is, the whole population):
  - Just calculate the parameter and you're done
- If you don't have a census:
  - Take a random sample from the population
  - Use a statistic as an **estimate** of the parameter

Variability of the Estimate

- One sample ➜ One estimate
- But the random sample could have come out differently
- And so the estimate could have been different
- Main question:
  - **How different could the estimate have been?**
- The variability of the estimate tells us something about how accurate the estimate is:

  estimate = parameter + error

Where to Get Another Sample?

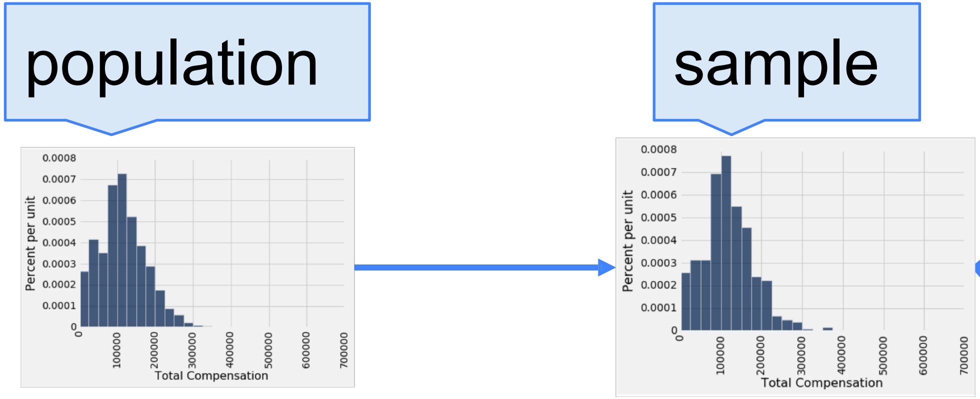
- One sample ➜ One estimate
- To get many values of the estimate, we needed many random samples
- Can’t go back and sample again from the population:
  - No time, no money
- Stuck?

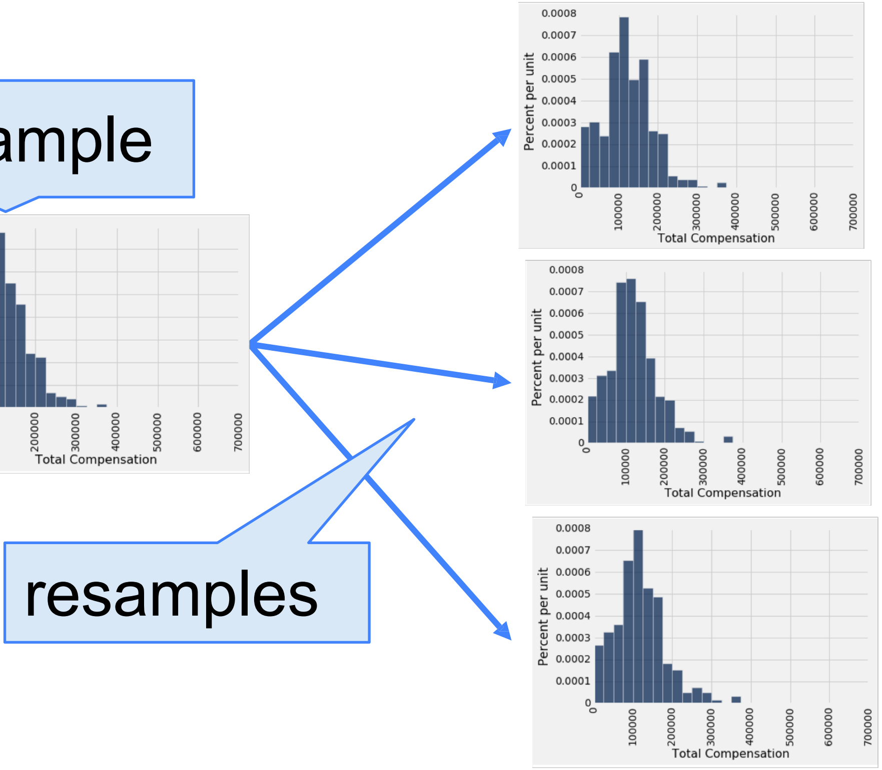
The Bootstrap

The Bootstrap

- A technique for simulating repeated random sampling
- All that we have is the original sample
  - … which is large and random
  - Therefore, it probably resembles the population
- So we sample at random from the original sample!

Why the Bootstrap Works





All of these look pretty similar, most likely.

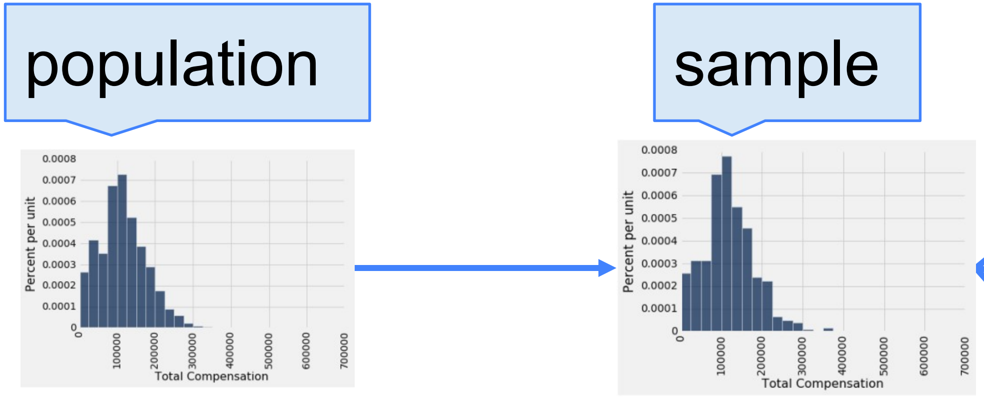
Key to Resampling

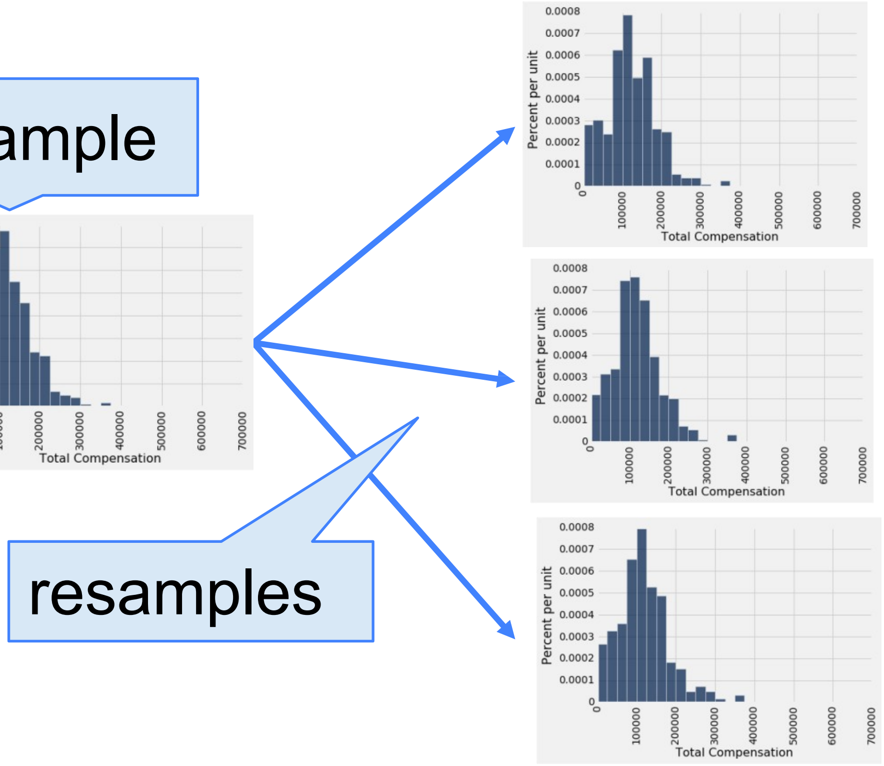
- From the original sample,
  - draw at random
  - with replacement
  - as many values as the original sample contained

- The size of the new sample has to be the same as the original one, so that the two estimates are comparable

(Demo)

Why the Bootstrap Works





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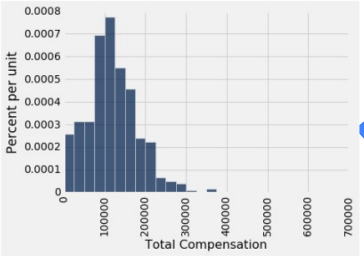
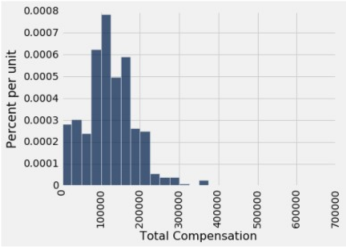
Inference Using the Bootstrap

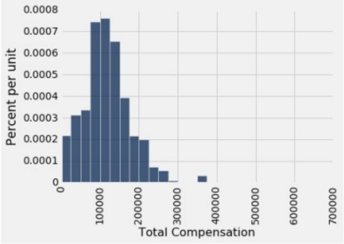
population

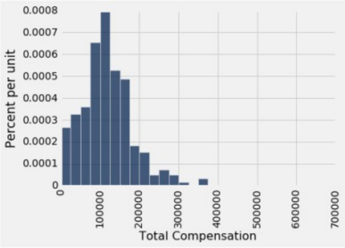
?

sample



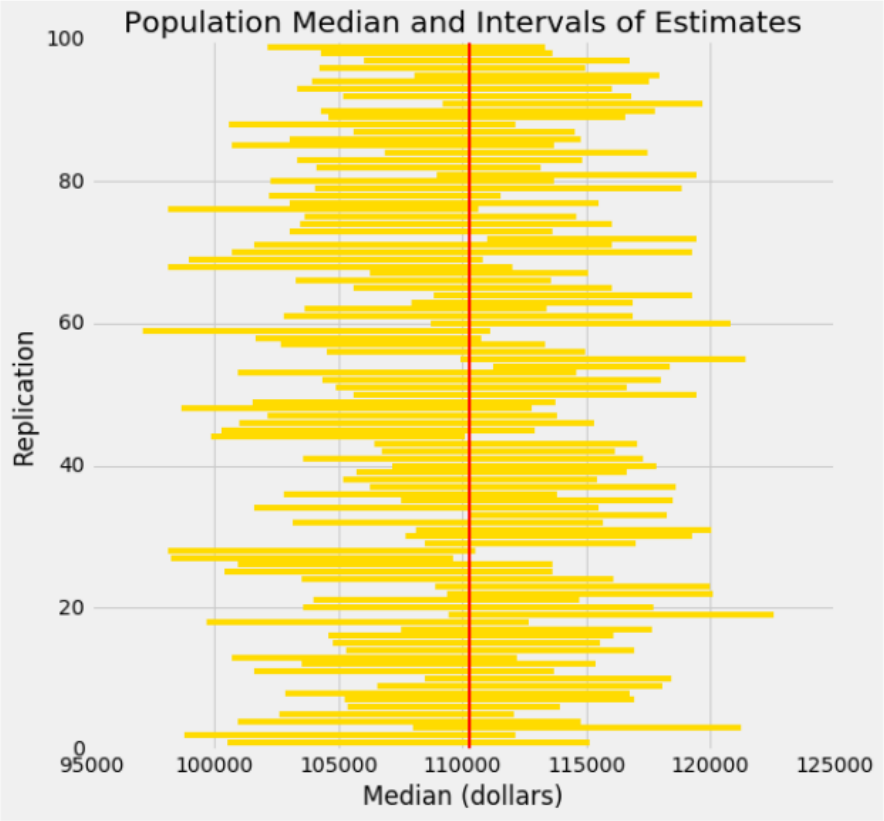






resamples

All of these look pretty similar, most likely.



Each line here is a confidence interval from a fresh sample from the population

95% Confidence Interval

- Interval of **estimates of a parameter**
- Based on random sampling
- 95% is called the confidence level
  - Could be any percent between 0 and 100
  - Bigger means wider intervals
- The **confidence is in the process** that generated the interval:
  - It generates a “good” interval about 95% of the time.

(Demo)

Use Methods Appropriately

Can You Use a CI Like This?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- About 95% of the mothers in the population were between 26.9 years and 27.6 years old.

Answer: False. We're estimating that their **average age** is in this interval 95% of the time.

Is This What a CI Means?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- There is a 0.95 probability that the average age of mothers in the population is in the range (26.9,27.6) years.

Answer: False. It's not a probability. It's either true or false that the average age of mothers is in the range (26.9, 27.6)

When *Not* to Use The Bootstrap

- If you're trying to estimate very high or very low percentiles, or min and max
- If you're trying to estimate any parameter that's greatly affected by rare elements of the population
- If the probability distribution of your statistic is not roughly bell shaped (the shape of the empirical distribution will be a clue)
- If the original sample is very small

(Demo)

Confidence Interval Tests

95% Confidence Interval

- Interval of **estimates of a parameter**
- Based on random sampling
- 95% is called the confidence level
  - Could be any percent between 0 and 100
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- The **confidence is in the process** that generated the interval:
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(Demo)

Using a CI for Testing

- Null hypothesis: **Population mean = *x***
- Alternative hypothesis: **Population mean ≠ *x***
- Cutoff for P-value: *p*%
- Method:
  - Construct a (100-*p*)% confidence interval for the population average
  - If *x* is not in the interval, reject the null
  - If *x* is in the interval, can't reject the null