190F Foundations of Data Science

Spring 2020

Lecture 09

Statistics

Announcements

Statistics

Estimation

Statistical Inference:

Making conclusions based on data in random samples

Example:

Use the data to guess the value of an unknown number

fixed

depends on the random sample

Create an **estimate** of the unknown quantity

Terminology

- **Population:** A collection of individuals
  - All flights out of SFO last summer

- **Variable:** Something that varies in the population
  - airline *(categorical variable)*
  - amount of delay in departure *(quantitative variable)*

- **Sample:** A subset of the population

Why take a sample?

- You want to understand the variable in the population,

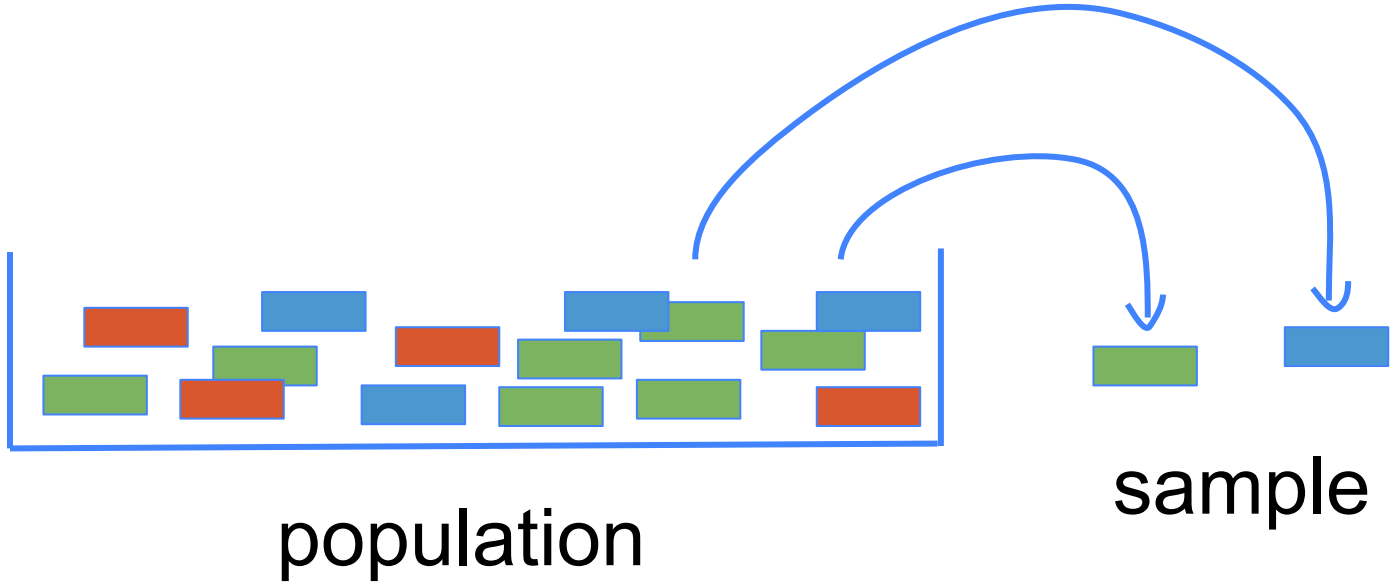
but

- you don’t have the resources to measure the variable on all the individuals in the population,

so

- you just measure it on a subset of them.

“Tickets in a box”

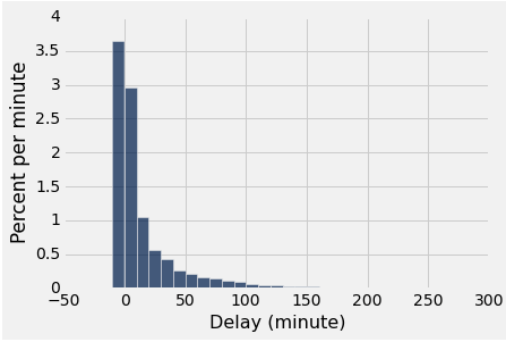


Best way to draw the sample

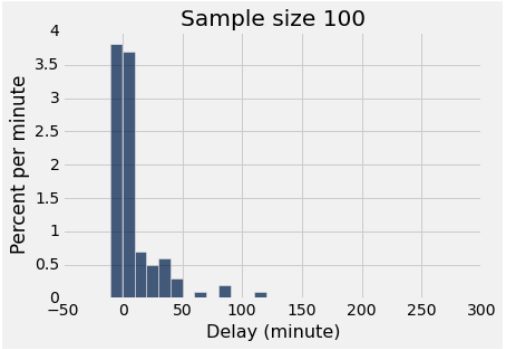
At random!

Two distributions

distribution of the population



empirical distribution of a sample



More terminology

- **Parameter:** A number calculated using the values in the population
  - Median delay among all flights
  - Proportion of voters who are Republican

- **Statistic:** A number calculated using the values in a sample
- A statistic can be used as an **estimate** of a parameter.

Probability distribution of a statistic

- Values of a statistic vary because random samples vary
- “Sampling distribution” or “probability distribution” of a statistic consists of:
  - All possible values of the statistic
  - Corresponding probabilities
- Can be hard to calculate
  - Need math
  - Or generate many random samples

Empirical distribution of a statistic

- Empirical distribution of the statistic:
  - Based on simulated values of the statistic
  - Consists of all the observed values of the statistic,
  - and the proportion of times each value appeared

- Good approximation to the probability distribution of the statistic *if the number of repetitions in the simulation is large*.

Why sample at random?

The empirical distribution
of a large random sample
is very likely to be close
to the distribution of the population.

That's
why.

The effect of sample size

- Larger **random** samples are more likely to resemble the population than smaller ones.

- However, if the method of sampling is not random, taking a larger sample isn't necessarily better.
  - You could just end up with a big bad sample.

Simulating a Statistic

- Figure out the code to generate one value of the statistic
- Create an empty array in which you will collect simulated values
- For each repetition of the process:
  - Simulate one value of the statistic
  - Append this value to the collection array
- The array will contain all of the simulated values

(Demo)

Jury Selection

Swain vs. Alabama, 1965

- Talladega County, Alabama
- Robert Swain, black man convicted of crime
- Appeal: one factor was all-white jury
- Only men 21 years or older were allowed to serve
- 26% of this population were black
- Swain’s jury panel consisted of 100 men
- 8 people on the panel were black (8%)

Supreme Court Ruling

- About disparities between the percentages in the eligible population and the jury panel, the Supreme Court wrote:

  "... the overall percentage disparity has been small and reflects no studied attempt to include or exclude a specified number of [blacks]"

- The Supreme Court denied Robert Swain's appeal

Sampling from a Distribution

- Sample at random from a categorical distribution:

```
sample_proportions(sample_size, pop_distribution)
```

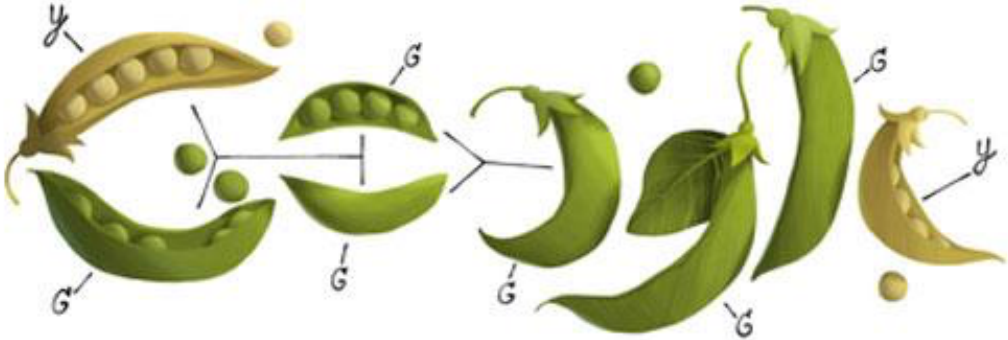
- Samples at random from the population
- Returns an array containing the distribution of the categories in the sample

A Genetic Model

Steps in Assessing a Model

- Come up with a statistic that will help you decide whether the data support the model or an alternative view of the world.
- Simulate the statistic under the assumptions of the model.
- Draw a histogram of the simulated values. This is the model's prediction for how the statistic should come out.
- Compute the observed statistic from the sample in the study.
- Compare this value with the histogram.
- If the two are not consistent, that's evidence against the model.

Gregor Mendel, 1822-1884



A Model

- Pea plants of a particular kind
- Each one has either purple flowers or white flowers

- Mendel's model:
  - Each plant is purple-flowering with chance 75%, regardless of the colors of the other plants

- Question:
  - Is the model good, or not?

Choosing a Statistic

- Start with percent of purple-flowering plants in sample
- If that percent is much larger or much smaller than 75, that is evidence against the model
- ***Distance*** from 75 is the key
- Statistic:

  | sample percent of purple-flowering plants - 75 |
- If the statistic is large, that is evidence against the model

Testing Hypotheses

Choosing One of Two Viewpoints

- Based on data

  - “Chocolate has no effect on cardiac disease.”
  - “Yes, it does.”

  - “This jury panel was selected at random from eligible jurors.”
  - “No, it has too many people with college degrees.”

Estimation

Perfect information

- You want to know how many US voters support a particular policy.
- You could ask everyone. That works.
- But, sometimes we can't afford to do that. So, instead, we could ask some of them, and draw inferences about the general population.

A common scenario

- You have to make a decision based on incomplete information.
- The quality of your decision is affected by
  - the information that you have
  - the information that you don’t have

- So, before making the decision, it is worth examining why and how your information came to be incomplete.

If you have the entire population...

- Formulate a question you want to answer (a parameter of the population).
- Visualize the data (the population).
- Compute the answer.
- Interpret the results, and explain them in language without statistical jargon.

If you don't...

- Formulate a question you want to answer (a parameter of the population).
- Select a method of inference.
- Visualize the data (the sample).
- Calculate the statistic on your sample, then apply the method to estimate the population parameter.
- Interpret the results, and explain them in language without statistical jargon.