# The History of Prime Numbers: The Unsolved Puzzle

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**Abstract**

In mathematics, prime numbers are the foundational elements of integers, but they still hold their mystery by being chaotic and unpredictable. The questions about prime numbers were studied throughout the history of many generations, and it is waiting to be solved, or some may say, proved. This paper explores the history of prime numbers, how prime numbers became the prime numbers we know today, from ancient civilizations to modern applications like cryptography. This paper explores the history of primes by beginning with the earliest times that prime numbers appeared in history as primes in Babylonia, then Egypt. Then the paper continues with exploring the first proofs about prime numbers, which were done by the Ancient Greek Mathematicians, Euclid and Eratosthenes. Then the developments in Islamic Mathematicians like Thabit Ibn-Qurra and Ibn al-Haytham. The main focus of the paper is the modern period of number theory after the 17th Century. It starts with the early contributions from French Mathematicians, Marin Mersenne and Pierre de Fermat, whose ideas noted the important first steps towards modern number theory. Then continues with major developments leading towards the famous Prime Number Theorem with contributions from Euler and Gauss. Finally, the paper addresses the German Mathematician, Riemann, and his Zeta function, which uncovered one of the most famous unsolved problems in mathematics, the Riemann Hypothesis, which still remains unproven to this day. The prime numbers cover many essential areas in our lives today. Prime numbers can be seen in many areas, like computer science algorithms, cryptography, and signal processing. The paper's main purpose is to show how a simple question, what are prime numbers, can bring many different generations of mathematicians together for thousands of years.

# Contents

# 1 Introduction

Prime numbers are the foundations for all the integers, and they are also fundamental for number theory we know today. They are still one of the biggest unsolved problems in the field of mathematics because of their chaotic distribution and unpredictability. A prime number is a positive integer bigger than 1 that is only divisible by 1 and itself.[18] From this definition, one can think they should be simple enough to be solved and to be found, but even if they seem simple, their distribution is not as simple as the definition. When the prime numbers get larger, their distribution become more unpredictable and chaotic, and mathematicians tried to find a formula for their distribution for many years. It is still trying to be solved.

When we think of prime numbers, we think about the modern mathematicians or the ancient Greeks, but we can see that long before the Greek mathematicians and proof, ancient civilizations like Babylonia and Egypt were working with prime numbers without naming them explicitly. The first proof we see with prime numbers is the famous proof of Euclid, that there are infinitely many prime numbers.[16] Another very notable work was done by Eratosthenes, the Sieve of Eratosthenes, the first algorithm to find the prime numbers up to a certain integer.[21] These two major breakthroughs were the stepping stone for number theory we know today, and how modern mathematicians studied prime numbers going forward.

The prime numbers were studied by many different cultures and civilizations. During the Golden Age, many Islamic mathematicians like Thabit Ibn-Qurra and Ibn al-Haytham worked with primes and tried to come up with a theorem that can find all the prime numbers. Around the same time, the development of mathematics in Europe was very limited, and the Islamic mathematicians expanded and kept the field of number theory active until the Modern European mathematicians. Their contributions influenced the European mathematicians to work on Number Theory and prime numbers.

The main focus of this paper is the modern period of Number Theory around 17th Century Europe, by very famous mathematicians like Fermat, Euler, and Riemann. Their work is based on the ancient civilizations, also other fellow mathematicians of their time. The modern era of number theory starts with French Mathematicians Marin Mersenne and Pierre de Fermat. Their work on prime numbers and number theory influenced other mathematicians to continue their work on prime numbers. Then comes Leonard Euler, whose work is based on Fermat's theorems, and his work on the Zeta function led to the final theorem for understanding how primes are distributed. The German Mathematician Bernhard Riemann continued working on Number Theory and Euler's Zeta function and developed his own theorem that gave us the famous Riemann Hypothesis that is waiting to be proven. To understand how we got to the Riemann Zeta function, it is important that we understand how the knowledge of mathematics has been built on itself for thousands of years.

# 2 First Artifacts of Prime Numbers

## 2.1 Ishango Bone

Figure 1: The Ishango Bone with carved markings [17]

The history of primes starts earlier than we can estimate, dated about 25,000 years ago in Africa around the Nile, later buried by a volcanic eruption. Researchers found the second oldest mathematical artifact.[27] The Ishango bone is an artifact that has some carved markings that are believed to have been used for counting. However, some researchers believe that the markings actually represent prime numbers because they are grouped on the bone as prime numbers, but some think that they represent calendar days.[2]

## 2.2 Plimpton 322 (Babylonia)

Sometime around 1,800 BCE in Babylonia, modern day Iraq, was a clay tablet that had Babylonian numbers in a table.[12]

Figure 2: Babylonian Clay Tablet Plimpton 322 [24]

In the early 1940s, Otto Neugebauer, a historian of ancient science, found out that the entries represented Pythagorean triples, so the numbers in triples were they were relatively prime to each other and gave the integer solution to $a^2 + b^2 = c^2$.[1]

## 2.3 Rhind Papyrus (Egypt)

The Rhind Papyrus is an ancient Egyptian document written around 1650 BCE, a mathematical sheet. It is known to be written by a scribe named Ahmes. This papyrus shows some calculations of fractions, specifically a table of fractions in the form of $2/n$.[23]

Figure 3: The Rhind Papyrus from Egypt [26]

In these calculations, we can see that the prime numbers are treated different than the others. This shows us that in Ancient Egypt, they had the idea of divisibility and primes, even if they did not explicitly named as prime numbers. From these artifacts, we can say that even if there is no proof or definitions, they were already working with modern concepts in mathematics in ancient civilizations.

# 3 Ancient Greeks

## 3.1 Euclid of Alexandria

The first and one of the most influential contributions came from the famous Euclid of Alexandria around 300 BCE. He is known for his work on Geometry, as known as The Elements of Euclid.[6] While he was known as the 'Father of Geometry' he also worked on Number Theory, and his way of giving deductive proofs enabled him to give us the first proof about prime numbers, there are infinitely many primes. His proof goes on like this and follows contradiction: Assume there is a finite set of primes $P = \{P_1, P_2, ..., P_n\}$. If we define a number $N = (P_1 \cdot P_2 \cdot ... \cdot P_n) + 1$, $N$ should either be a prime or divisible by a prime. By the construction $N$ is not divisible by any $P_i$. Thus, $N$ is either a prime itself or it is divisible by a prime that is not in the set $P$. Thus, by contradiction, there should be infinitely many primes.[13] What Euclid has shown has influenced the field of Number Theory and other mathematicians for many years, and Euclid's way of handling mathematical knowledge with systematic proofs has changed the future of how others studied mathematics.

### 3.2 Eratosthenes of Alexandria

Another important figure in early Number Theory was Eratosthenes of Alexandria, who lived around 200 BCE. He was an astronomer, scientist, writer, and poet also the head librarian of Alexandria and was famously known for measuring the Earth's circumference.[5] His work on prime numbers was the first occurrence of an algorithm, the Sieve of Eratosthenes. His sieve can find all the prime numbers up to a given integer $N$ starting from 2. The sieve begins with writing all of the integers starting from 2 to the given integer. Then the method follows an iterative pattern of crossing out the multiples. We start from 2, since 2 is only divisible by 1 and itself, we keep 2 as prime, then cross out every multiple of 2. Then we take 3, since 3 is also prime, we keep 3 and cross out all the multiples of 3. Since 4 is already crossed out from multiples of 2, we skip. So we skip the crossed out integers and cross out every multiple of primes until we reach the $\sqrt{N}$ since every composite integer less than $N$ must have a prime factor less than $\sqrt{N}$.[21] This algorithm is a systematic and practical way of finding primes, and also the first actual work we know that has tried to find prime numbers in a mathematical way. The Sieve of Eratosthenes can seen explicit, but it is still being used by many computer scientists and it has opened up a new way of computation, the algorithms.

### 3.3 Nichomachus of Gerasa

Nichomachus of Gerasa, lived around 100 CE, was a Neo-Pythagorean philosopher and mathematician. He wrote one of the earliest books only about Number Theory, Introduction to Arithmetic.[10] He classified numbers into different types, including primes, and followed a systematic way of proof. He did not work on primes himself, but he wrote the earlier proofs done by other mathematicians such as Eratosthenes, and his work played an important role in systematizing number theory and influenced other mathematicians to work on prime numbers.

## 4 Islamic Mathematicians

### 4.1 Thabit Ibn-Qurra

Around 9th Century we come across a scholar from Baghdad, Thabit Ibn-Qurra. He was well known for his work in mathematics. He translated and work on several other documents like Euclid's Elements and enables Islamic mathematicians to gain the knowledge from earlier studies.[23] He made many contributions to number theory, especially with work on prime numbers. He studied perfect and amicable numbers,two natural numbers $m$ and $n$ are amicable if $\sigma(m) = \sigma(n) = m + n$, where $\sigma(m)$ denotes the sum of all the divisors of $m$. If $m = n$, then $m$ is perfect.[15] He discovered formulas to generate amicable numbers with specific condition, where some components should be prime. He also introduced Thabit Primes in the form of $3 \cdot 2^n - 1$ where $n$ is a nonnegative integer.[25] He found several primes in this form but this formula does not work for all integers. The largest known Thabit Prime has 3.4 million digit, which is very advanced for the time he discovered Thabit primes.

### 4.2 Ibn al-Haytham

Around 10th Century we come across another mathematician and scientist from Bashra, Ibn al-Haytham also known as Alhazen in the West. He was known to be the earliest users of the scientific method in his studies. His most known work was about principles of optics and he also worked on finding volumes of paraboloids and areas crescent-shaped figures.[7] Aside from these, he also contributed to the number theory. He worked on prime numbers and he discovered an earlier version of Wilson's Theorem. He said if you take any prime number $p$, then the sum $1 \cdot 2 \cdot ... \cdot (p-1) + 1$ is divisible by $p$ and if we divide it by the any number between 1 and $(p-1)$ we get the unit. This formula corresponds to the modern day notation of $(p-1)! = -1(\text{mod } p)$.[19] This theorem is today known as the Wilson's Theorem but some think that it was actually stated and proven by Ibn al-Haytham. His work on number theory has influenced many other Islamic mathematician to continue working on prime numbers and gave insight on other problems in number theory.

## 4.3 Kamal al-Din al-Farisi

Kamal al-Din al-Farisi was another Islamic mathematician and scientist that also worked with optics and number analysis. He contributed to number theory by working on amicable numbers, and refined the work of Thabit Ibn-Qurran's work on amicable numbers. His new formula for amicable numbers are for $n > 1$, let $p_n = 3 \cdot 2^n - 1$ and $q_n = 9 \cdot 2^{2n-1} - 1$. If $p_{n-1}, p_n$ and $q_n$ are prime numbers, then $a = 2^n p_n P_{n-1}$ and $b = 2^n q_n$ are amicable numbers.[23] With this approach, he factorized numbers into their primes and this approach is now known to be the Fundamental theorem of Arithmetic.

# 5 Modern Time Mathematicians

## 5.1 17th Century

### 5.1.1 Marine Mersenne

Marin Mersenne was a French theologian, philosopher and mathematician. He played a very important role in the modern mathematics especially in number theory. He held many meetings in Paris with fellow scientists which then became the foundations of French Academy of Sciences.[9] His best known work for number theory is the Mersenne Primes in the form of $2^n - 1$ where n is nonnegative integer. he found out that if this number is prime then $n$ also is a prime. However, he could not prove this and found out that this does not hold for $n = 11$. He was able to find 9 primes in this form including $2^{217} - 1$ which is a 36 digit prime number.[23]

### 5.1.2 Pierre de Fermat

Pierre de Fermat is an another very important French mathematician in modern mathematics and Number Theory. He is even called the founder of the modern theory of numbers.[11] He worked in analytic geometry with another very influential mathematician Descartes. His work on tangents and min max points later became the differential calculus.[11] But he was known for his work in Number Theory, the Fermat primes and Fermat's Little Theorem. The Fermat primes are the numbers in the form of $2^{2^n} + 1$ where $n$ is a nonnegative integer. Later, Euler disproved Fermat Primes for $n = 5$ however Fermat never said Fermat Primes actually hold for every integer.[23]

His most famous work was Fermat's Little Theorem which states: given any prime $p$ and any whole number $n$, the number $n^p - n$ is exactly divisible by $p$. In notation this gives us $n^p \equiv n \pmod p$ where $n \in Z$ and $p \in$ primes. If we divide both sides by $n$, we get the final theorem: $n^{p-1} \equiv 1 (\text{mod } p)$.[3] This formula later became the cornerstone of modular arithmetic and number theory and many other mathematicians studied number theory based on this theorem.

## 5.2 18th Century

### 5.2.1 Leonhard Euler

Leonhard Euler was one of the most influential mathematician in 18th Century to this day. He was a Swiss mathematician and physicist. He made many contributions to geometry, calculus, mechanics, and number theory and also demonstrated useful applications of mathematics in technology and public affairs.[8] He was known for being an algorist because he was simplifying mathematics with algorithms and he got more interested in mathematics after loosing his eyesight.[23]

He worked on number theory by connecting algebra and analysis and one of his works on number theory was proving Fermat's Little theorem and he came up with a more generalized version: Euler's $\varphi(m)$ function. With hid

$\varphi$ function he tried to find the number of relatively prime numbers between 1 and that number. When $m$ is prime then we have $\varphi(m) = m - 1$ by definition but for general case we need to find the multiples of a prime $p$ that an divide $m$, then the remaining ones are the relatively primes of $m$. This gives us the formula of $\varphi(m)$:

$$\varphi(m) = m \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdot ... \cdot \left(1 - \frac{1}{p_t}\right) \tag{1}$$

where $p_1, p_2, ..., p_t$ are the all prime factors of $m$.[23] The result of this formula would give us the number of relatively prime numbers from 1 to $m - 1$.

By using Fermat's Little Theorem and hid $\varphi(m)$ function Euler came up with another Function that is key to today's prime number knowledge and number theory. Based on the other functions he derived Euler's Zeta function which links all integers through prime numbers by the infinite sum. He represented the infinite sum through the primes and this function will play a curial role in the future when Riemann discovered the Prime Number Theorem. The Euler's Zeta function follows this logic. We start with the Zeta function which is the infinite sum of fractions where each denominator is a positive integer raised to the power of s, for $s > 1$:

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{2}$$

Then we remove all multipliers of 2 from the $\zeta(s)$:

$$(1 - 2^{-s})\zeta(s) = \left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots\right) - \left(\frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + \cdots\right) = 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \cdots \tag{3}$$

Then continue with removing all the multipliers of 3:

$$\left(1 - 3^{-s}\right)\left(1 - 2^{-s}\right)\zeta(s) = 1 + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \cdots \tag{4}$$

When this steps are continues by removing all primes to infinity, all denominators except 1 will be removed since all numbers are divisible by some prime. The final Zeta function becomes:

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)\zeta(s) = \prod_{p \text{ prime}} \frac{p^s - 1}{p^s}\zeta(s) = 1 \tag{5}$$

After solving for $\zeta(s)$,

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}} \tag{6}$$

Since the Zeta function also equals to the sum of the fractions:

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}} = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{7}$$

This shows the link between the sum of the inverse of all numbers and the prime numbers.[23] This formula is the key step towards the famous Riemann's Zeta function and the Riemann Hypothesis, since Euler's Zeta function works only for $s > 1$, Riemann expanded the Zeta function to work for $s < 1$.

### 5.2.2 Carl Frederick Gauss

Gauss was one of the greatest mathematicians from Germany and he works on many different areas including number theory, geometry, and probability theory. He was raised around mathematicians and scientists and this inspired him to work on mathematics from an early age. He was known to be anxious about judgment about his work among other mathematicians, thus he published a small amount of his work when he was alive. Most of his work became known after his death.[4] His work on prime numbers seem small but has an huge impact in mathematics world. His most known work in number theory about primes if the Prime Number Theorem (PNT). After working with the prime distributions, he discovered that the distributions of prime numbers are following a logarithmic pattern. Then he came up with the approximation for counting how many primes there are until a certain point. His approximation is

$$\pi(a) \sim \frac{a}{ln(a)} \tag{8}$$

This Prime Number Theorem can approximate how many prime numbers there are up until $a$.[23] His approximation for $a = 1,000$ is $\pi(1,000) = 144.77$ where the actual count is 168. This approximation's accuracy increases when $a$ increases thus gives us a great approximation for larger values.

## 5.3 19th Century

### 5.3.1 Bernhard Riemann

Riemann was a very important figure in mathematics and almost all of his contributions are very well known. He was an German Mathematician, he was interested in mathematics from an early age but he wanted to follow his father's footsteps and started university for Theology. He then changed majors and started studying under Gauss, after a year he changed schools because he did not enjoyed Gauss's teaching but eventually came back to do his doctorate under Gauss.[14] Some say he went back to study with Gauss to finish what Gauss started.

His work on Number Theory especially in prime numbers is a very well known because it is still not proven or disproven. He wanted to expand the Euler's Zeta function for the other part which is $s < 1$. And he derived a Zeta function that can work for both parts. He first looked at the Zeros of the function which are $\zeta(z) = 0$, and he got 2 types of zeros; the trivial zeros where they lie at the negative integers $s = -2, -4, -6, ...$ and the non-trivial zeros which all lie on the complex plane and all of them on $\Re(s) = \frac{1}{2}$. The non-trivial zeros being on the same line on $\Re(s) = \frac{1}{2}$ is the famous Riemann Hypothesis that is still needs to be proven.[22] With these zeros we derived a new Zeta function:

$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z)\zeta(1-z) \tag{9}$$

where, $\zeta(z)$ denotes the Zeta function extended to the complex plane, $s < 1$, the factor $2^s$ and the term $\pi^{s-1}$ are scaling constants, $2^s$ creates the symmetry by the line $\Re(s) = \frac{1}{2}$ and $\pi^{s-1}$ extends the zeta function to complex plane. The sine term, $\sin\left(\frac{\pi s}{2}\right)$, in the function gives the trivial zeros and introduces symmetry. The function $\Gamma(1-s)$ is the Gamma function, which generalizes the factorial function to complex numbers and the other Zeta function, $\zeta(1-s)$, represents the value of the zeta function reflected across the vertical line $\Re(s) = \frac{1}{2}$.[22]

The Riemann's Zeta function gives the number of primes until a certain number perfectly, each term in this new Zeta functions adds another feature that is crucial to represent a very chaotic and unpredictable distribution of primes. If we try to visualize it by drawing the actual Prime Counting Function, every time we see a prime number, $p$, in the graph we are going to step up $\log(p)$ amount:

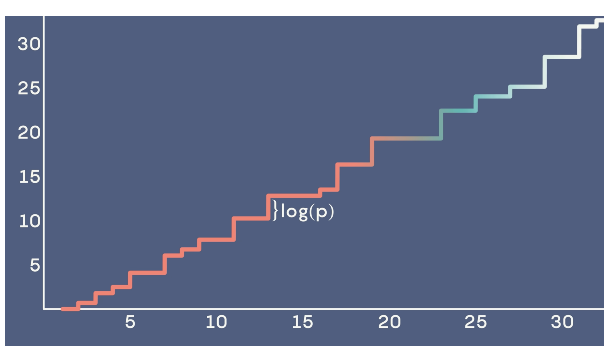


Figure 4: The Prime Counting Function [20]

Then if we add a straight line that matches the distribution with no adjustment:

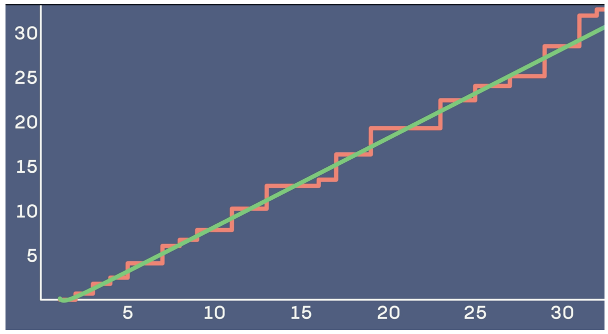


Figure 5: The Prime Counting Function with a straight line no adjustment [20]

When the Riemann's zeta function adjustments are added slowly, the line starts to get more wavy, these come from the non-trivial zeros:

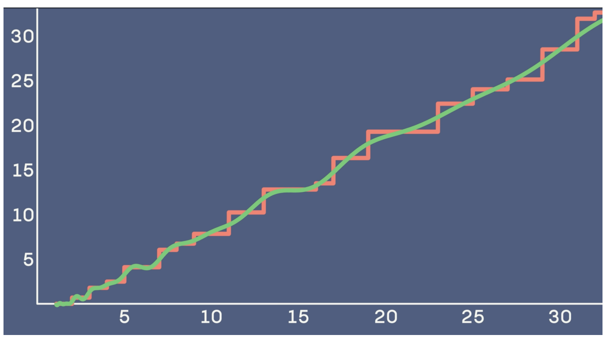


Figure 6: The Prime Counting Function with added one zeta zero [20]

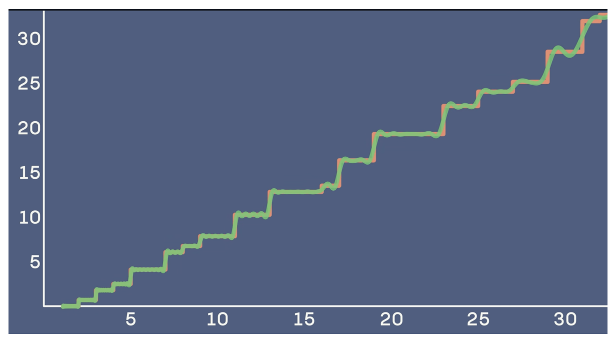


Figure 7: The Prime Counting Function with added 60 zeta zeros [20]

With the Riemann's contributions to prime numbers and Number Theory, the irregular distribution of primes can be understood a little further. However, the Riemann Hypothesis still remains unproven despite much effort and makes it one of the most interesting problem in mathematics.

# 6 Conclusion

Prime numbers have remained one of the most fascinating topic in mathematics due to their simple definition yet chaotic behavior. The topic of prime numbers started way before Number Theory or modern mathematics, it started from thousands of years ago with artifacts such as the Ishango Bone and ancient Egyptian and Babylonian records. Prime numbers were tried to be solved by the rigorous proofs of Euclid and the algorithmic approach of Eratosthenes, and it shaped the development of filed of Number Theory. The prime problem was trying to be solved with many different civilizations, Islamic mathematicians such as Thabit Ibn-Qurra, Ibn al-Haytham, and Kamal al-Din al-Farisi and their contributions with algebra, they preserved and expanded earlier knowledge and allowed it to continue evolving. All of these contributions helped shape the modern day mathematics and Number Theory.

The most of the studies about prime numbers were done in the modern era and in Europe by many important mathematicians around 17th century. It started with mathematicians such as Marin Mersenne, Pierre de Fermat, and Leonhard Euler. Their work introduced new theorems that shaped all of the Number theory and prime numbers and lead to bigger discoveries about prime numbers couple of centuries later by Riemann and Gauss. The Gauss's observations on prime distribution and the Prime Number Theorem provided the first accurate approximation for counting primes. Then the work of Bernhard Riemann marked a major step by improving Euler's Zeta function to connect prime numbers with complex analysis. His ideas about the zeros of the Zeta function led to accurately representing the prime distribution and the Riemann Hypothesis, which remains one of the most important unsolved problems in mathematics.

Today, prime numbers play a crucial role in our world even if we do not know it. Their most important part is in the modern technology. They are fundamental to cryptographic systems such as RSA encryption. With relying on the larger prime numbers are very hard to compute, RSA encryption technique protects our digital data from communication to online banking. Prime numbers are also widely used in computer science algorithms, including hashing and random number generation, as well as in signal processing and error-correcting codes. Even with many mathematicians and studies, the prime numbers are still remains an unsolved puzzle to this day and the questions about prime numbers are waiting to be solved.

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