CMSC 471: Games

KMA Solaiman

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Slides courtesy Tim Finin and Frank Ferarro. Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer,
Some materials adopted from slides by Dan Klein and Pieter Abbeel at UC Berkeley [http://ai.berkeley.edu]

Overview

- Game playing
  - State of the art and resources
  - Framework
- Game trees
  - Minimax
  - Alpha-beta pruning
  - Adding randomness

Why study games?

- Interesting, hard problems requiring minimal "initial structure"
- Clear criteria for success
- Study problems involving {hostile, adversarial, competing} agents and uncertainty of interacting with the natural world
- People have used them to assess their intelligence
- Fun, good, easy to understand, PR potential
- Games often define very large search spaces, e.g. chess 35^{100} nodes in search tree, 10^{40} legal states

Chess early days

- **1948**: Norbert Wiener describes how chess program can work using minimax search with an evaluation function
- **1950**: Claude Shannon publishes Programming a Computer for Playing Chess
- **1951**: Alan Turing develops *on paper* 1st program capable of playing full chess games (Turochamp)
- **1958**: 1st program plays full game on IBM 704 (loses)
- **1962**: Kotok & McCarthy (MIT) 1st program to play credibly
- **1967**: Greenblatt's Mac Hack Six (MIT) defeats a person in regular tournament play

State of the art

- **1979 Backgammon:** BKG (CMU) tops world champ
- **1994 Checkers**: Chinook is the world champion
- **1997 Chess**: IBM Deep Blue beat Gary Kasparov
- **2007 Checkers:** solved (it's a draw)
- **2016 Go**: AlphaGo beat champion Lee Sedol
- **2017 Poker:** CMU's Libratus won $1.5M from 4 top poker players in 3-week challenge in casino
- **20?? Bridge**: Expert bridge programs exist, but no world champions yet

Classical vs. Statistical/Neural Approaches

- We’ll look first at the classical approach used from the 1940s to 2010
- Then at newer statistical approached of which AlphaGo is an example
- These share some techniques

Typical simple case for a game

- **2-person** game, with **alternating moves**
- **Zero-sum**: one player’s loss is the other’s gain
- **Perfect information**: both players have access to complete information about state of game. No information hidden from either player.
- **No chance** (e.g., using dice) involved

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- **No chance** (e.g., using dice) involved
- Examples: Tic-Tac-Toe, Checkers, Chess, Go, Nim, Othello
- But not: Bridge, Solitaire, Backgammon, Poker, Rock-Paper-Scissors, ...

Can we use …

- Uninformed search?
- Heuristic search?
- Local search?
- Constraint based search?

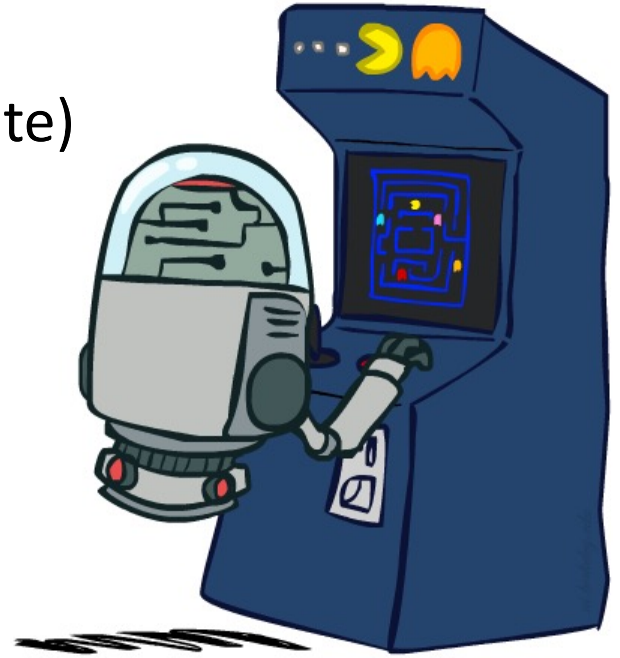
None of these model the fact that we have an **adversary** …

How to play a game

- A way to play such a game is to:
  - Consider all the legal moves you can make
  - Compute new position resulting from each move
  - Evaluate each to determine which is best
  - Make that move
  - Wait for your opponent to move and repeat
- Key problems are:
  - Representing the "board" (i.e., game state)
  - Generating all legal next boards
  - Evaluating a position

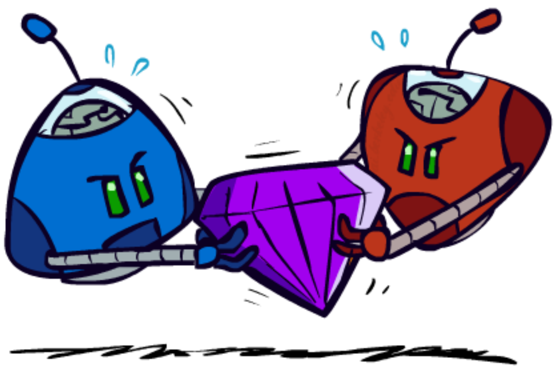
Deterministic Games

- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: $SxA \rightarrow S$
  - Terminal Test: $S \rightarrow \{t,f\}$
  - Terminal Utilities: $SxP \rightarrow R$



- Solution for a player is a policy: $S \rightarrow A$

Zero-Sum Games



- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - **Lets us think of a single value that one maximizes and the other minimizes**
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Evaluation function

- **Evaluation function** or **static evaluator** used to evaluate the "goodness" of a game position

  Contrast with heuristic search, where evaluation function estimates **cost** from start node to goal passing through given node

- Zero-sum assumption permits **single function to describe goodness of board for both players**
  - **f(n) >> 0**: position n good for me; bad for you
  - **f(n) << 0**: position n bad for me; good for you
  - **f(n) near 0**: position n is a neutral position
  - **f(n) = +infinity**: win for me
  - **f(n) = -infinity**: win for you

Evaluation function examples

- **For Tic-Tac-Toe**

  f(n) = [# my open 3lengths] - [# your open 3lengths]

  Where 3length is complete row, column or diagonal that has no opponent marks

- **Alan Turing's function for chess**
  - **f(n) = w(n)/b(n)** where w(n) = sum of point value of white's pieces and b(n) = sum of black's
  - Traditional piece values: pawn:1; knight:3; bishop:3; rook:5; queen:9

Evaluation function examples

- Most evaluation functions specified as a weighted sum of positive features

  $f(n) = w_1 * feat_1(n) + w_2 * feat_2(n) + ... + w_n * feat_k(n)$

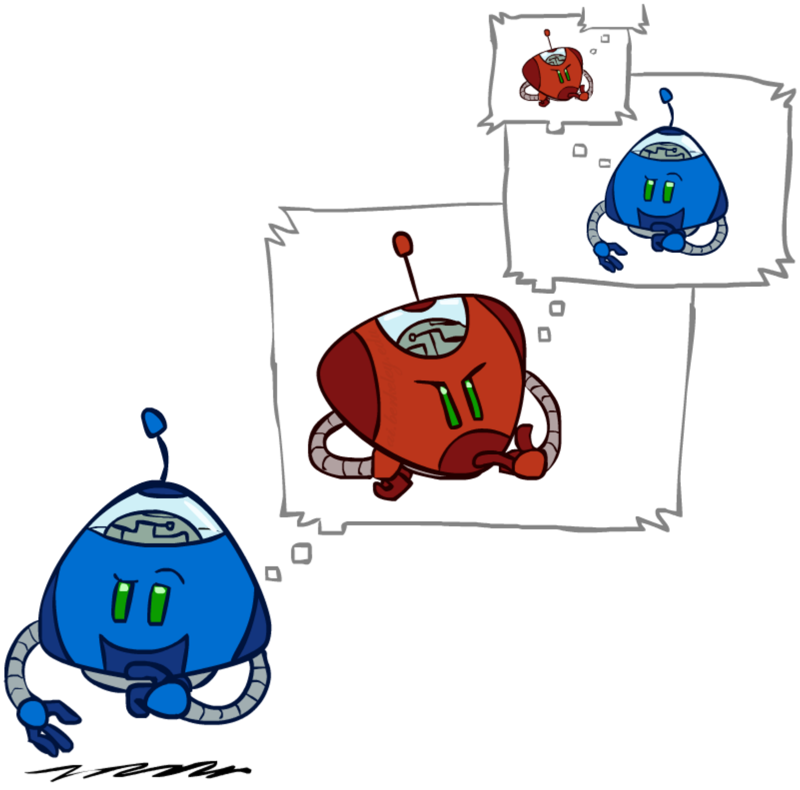
- Example chess features are piece count, piece values, piece placement, squares controlled, etc.
- IBM's chess program Deep Blue (circa 1996) had **>8K features** in its evaluation function

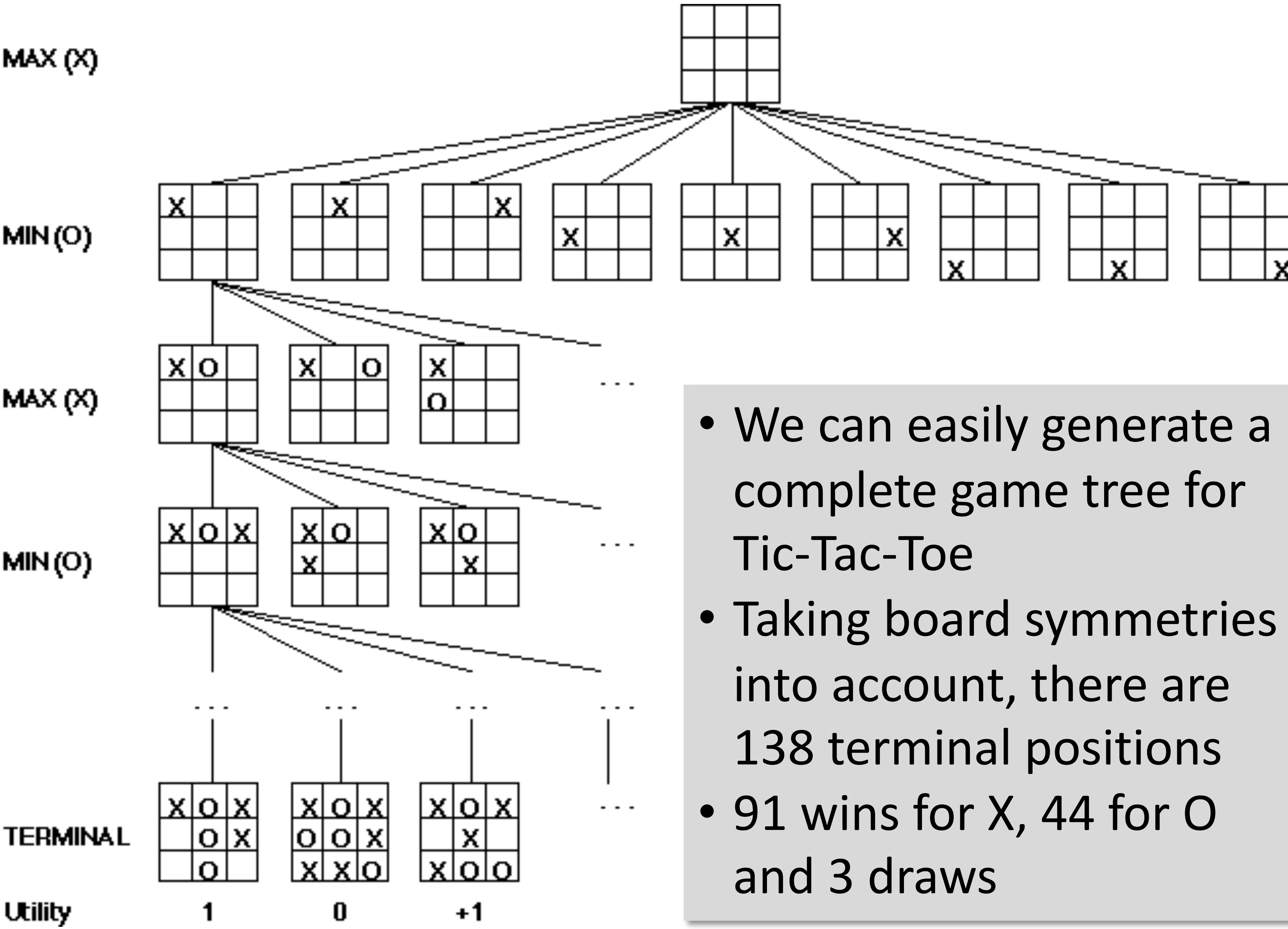
But, that's not how people play

- People also use *look ahead*

  i.e., enumerate actions, consider opponent's possible responses, REPEAT
- Producing a *complete* **game tree** is only possible for simple games
- So, generate a partial game tree for some number of plys
  - Move = each player takes a turn
  - Ply = one player's turn
- What do we do with the game tree?

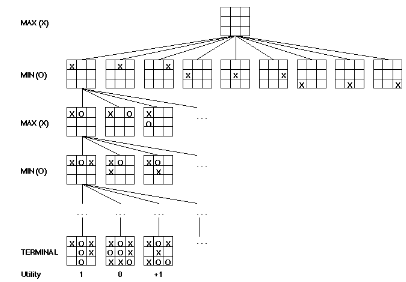
Adversarial Search





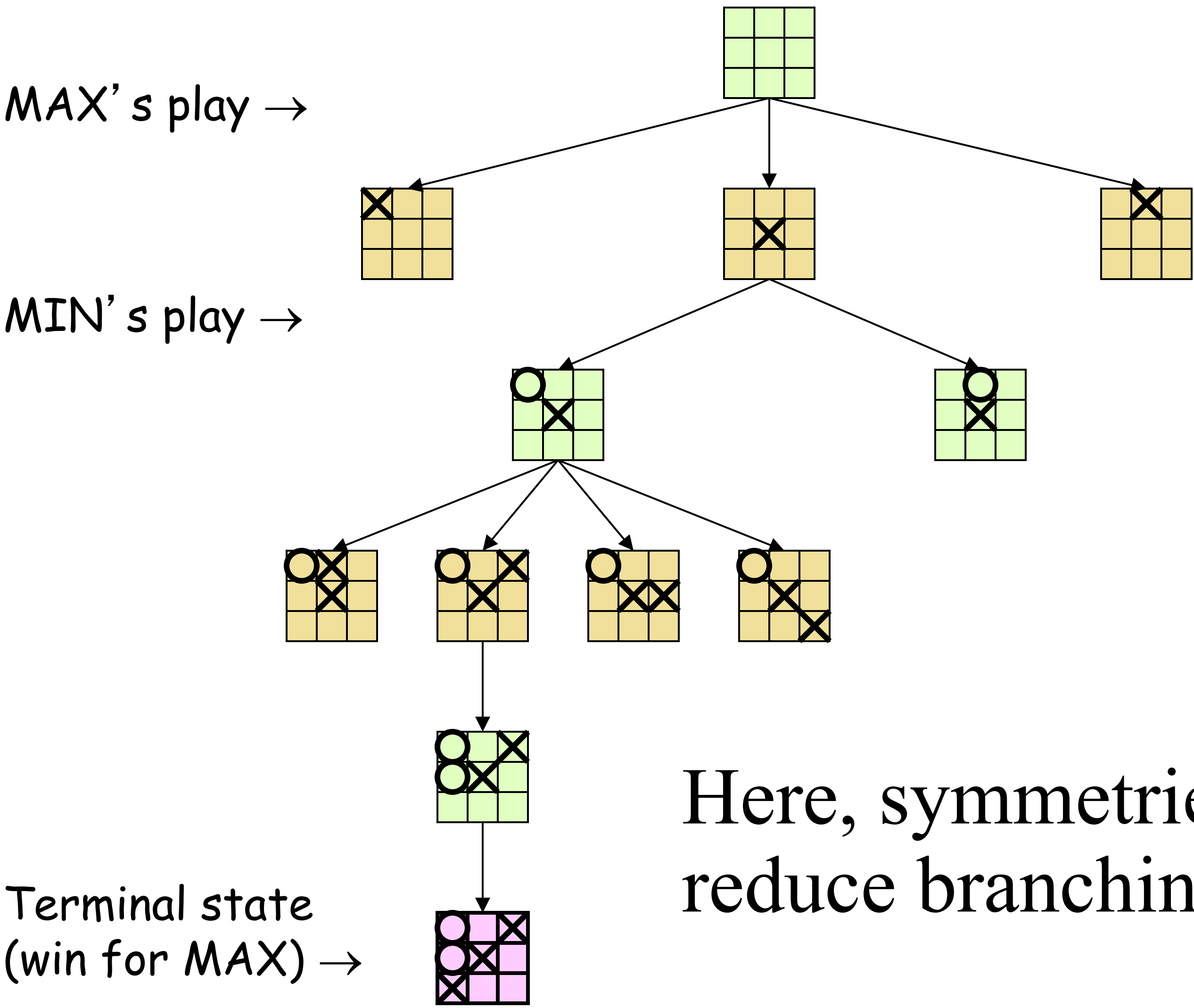
- We can easily generate a complete game tree for Tic-Tac-Toe
- Taking board symmetries into account, there are 138 terminal positions
- 91 wins for X, 44 for O and 3 draws

Game trees



- Problem spaces for typical games are trees
- Root node is current board configuration; player must decide best single move to make next
- **Static evaluator function** rates board position **f(board):**real, > 0 for me; < 0 for opponent
- Arcs represent possible legal moves for a player
- If **my turn** to move, then root is labeled a "**MAX**" node; otherwise it's a "**MIN**" node
- Each tree level's nodes are all MAX or all MIN; nodes at level i are of opposite kind from those at level i+1

Game Tree for Tic-Tac-Toe



Here, symmetries are used to reduce branching factor

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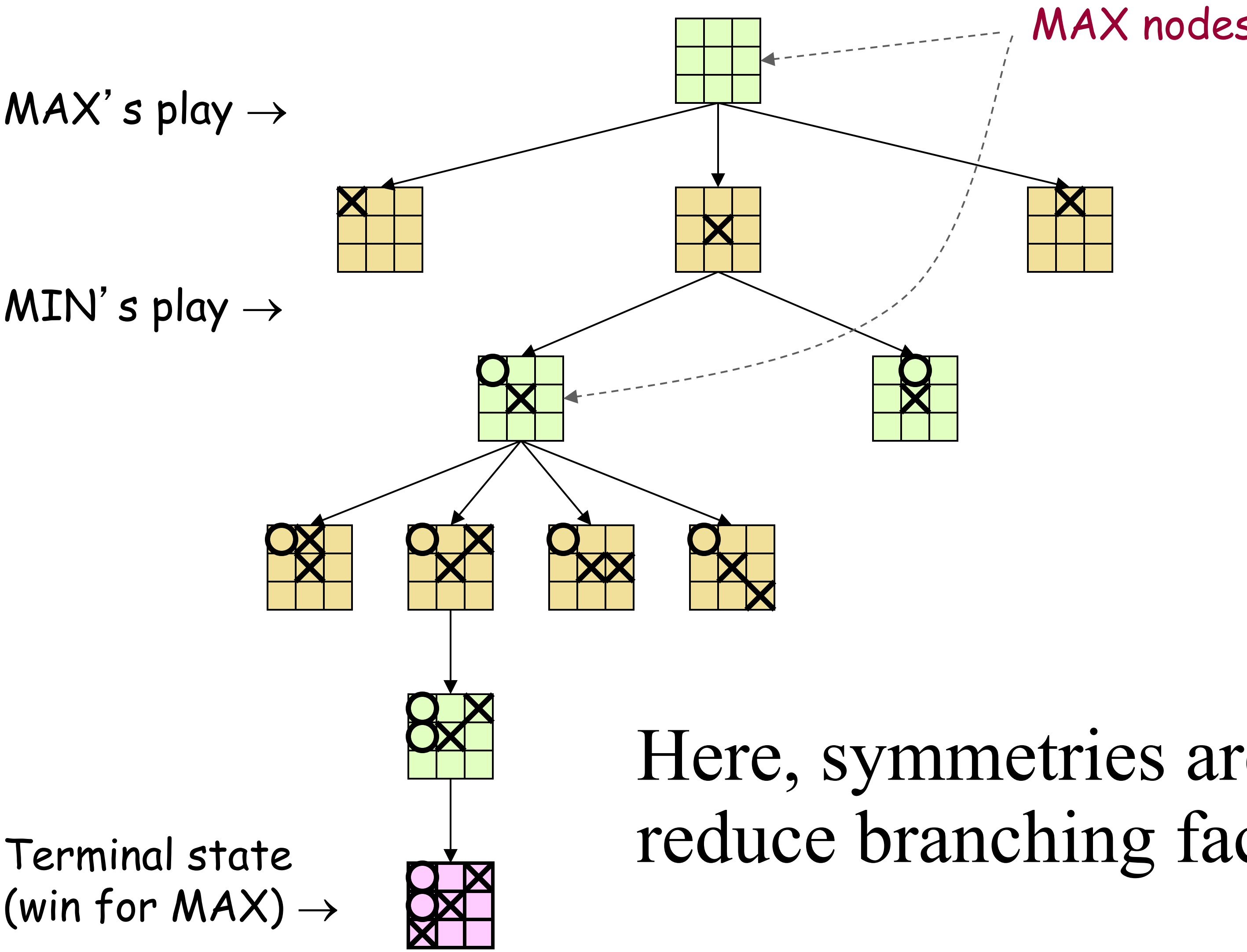

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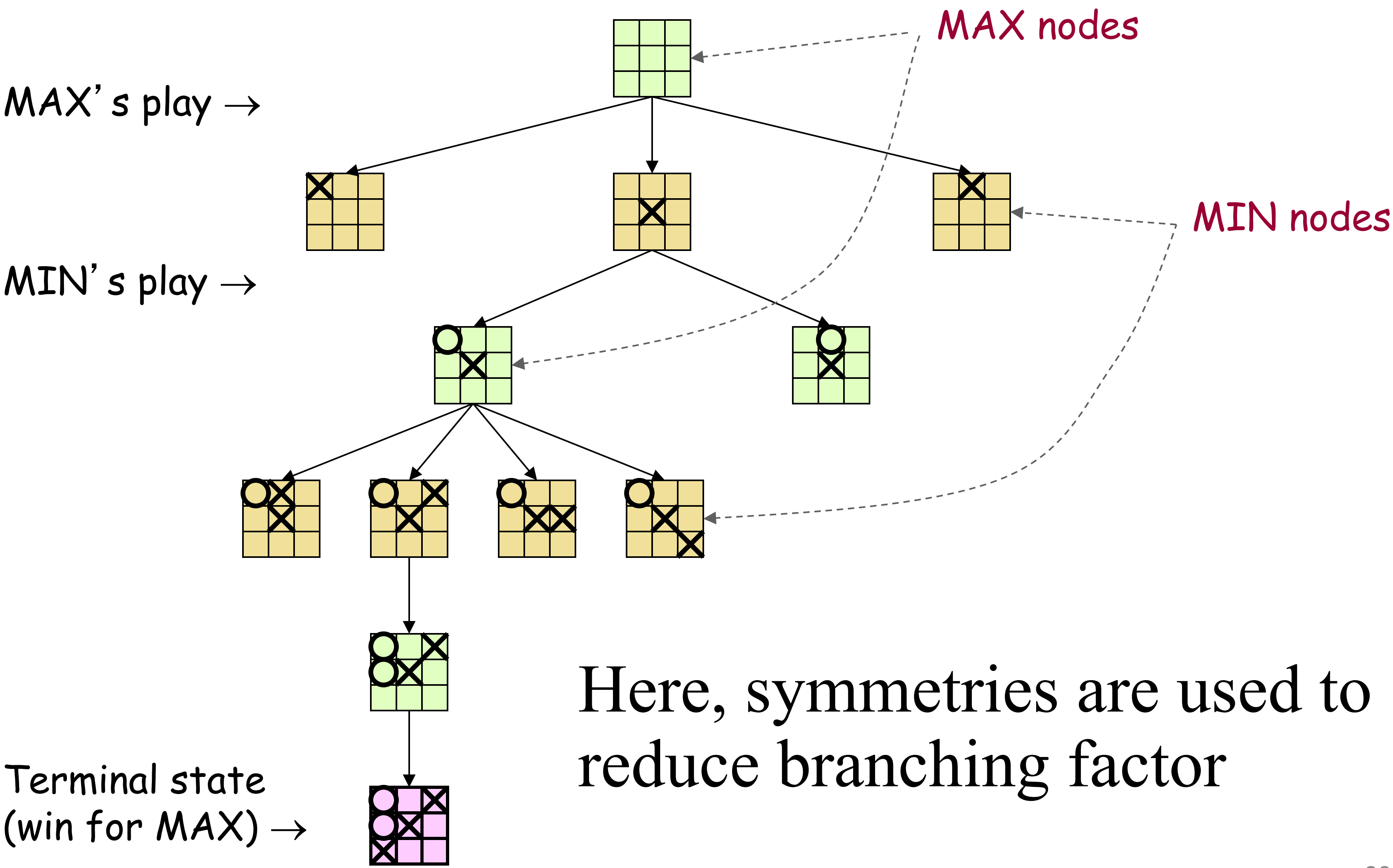

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Single-Agent Trees

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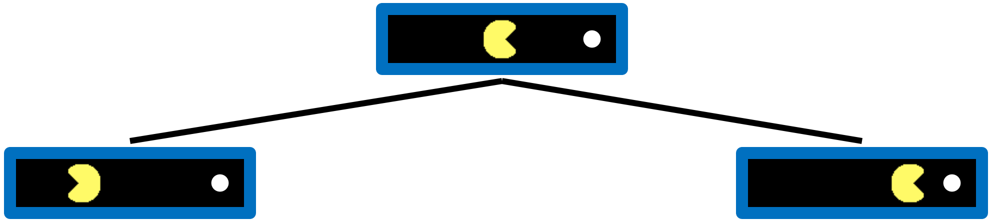

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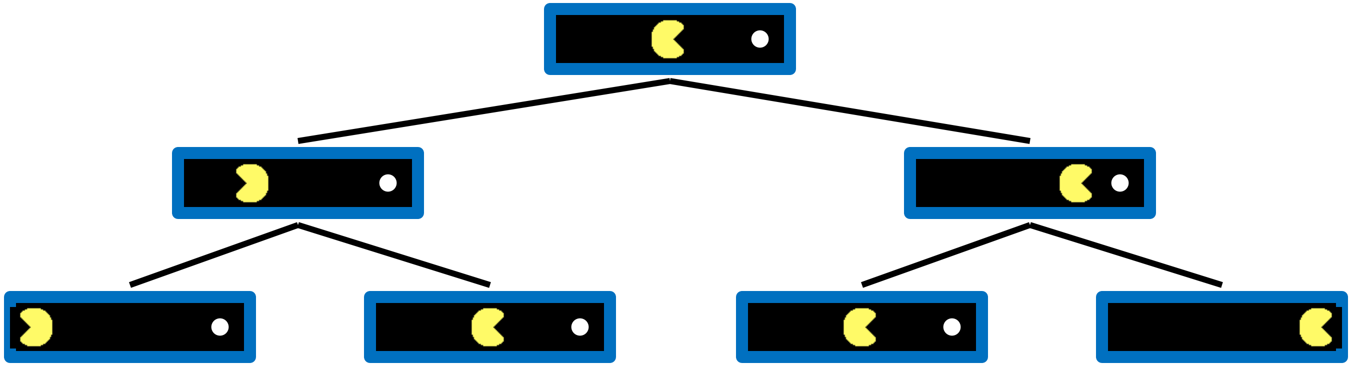

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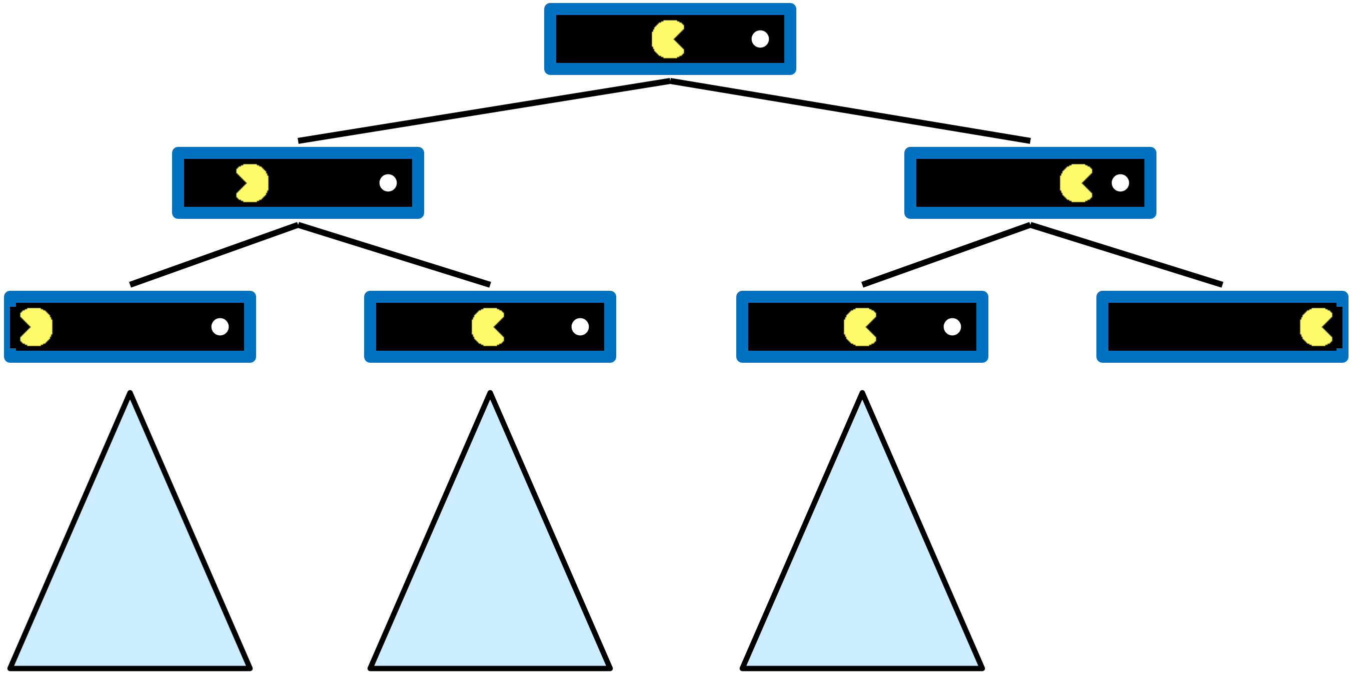

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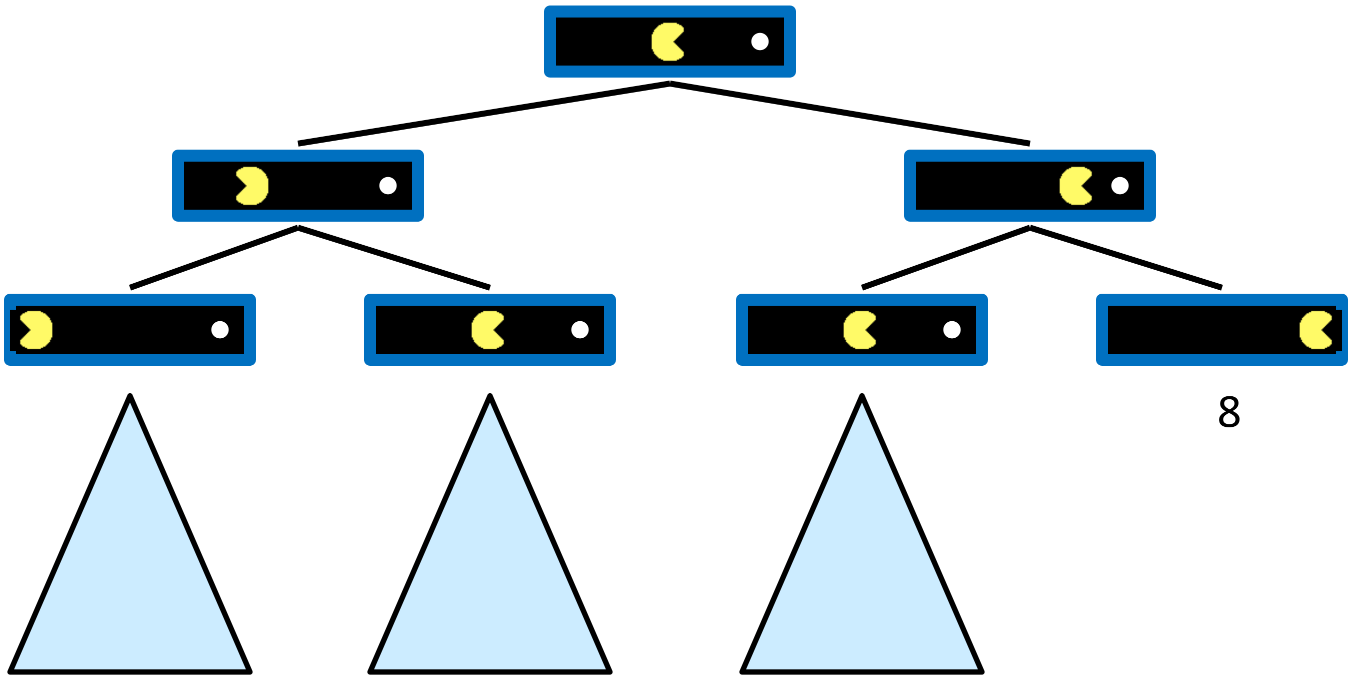

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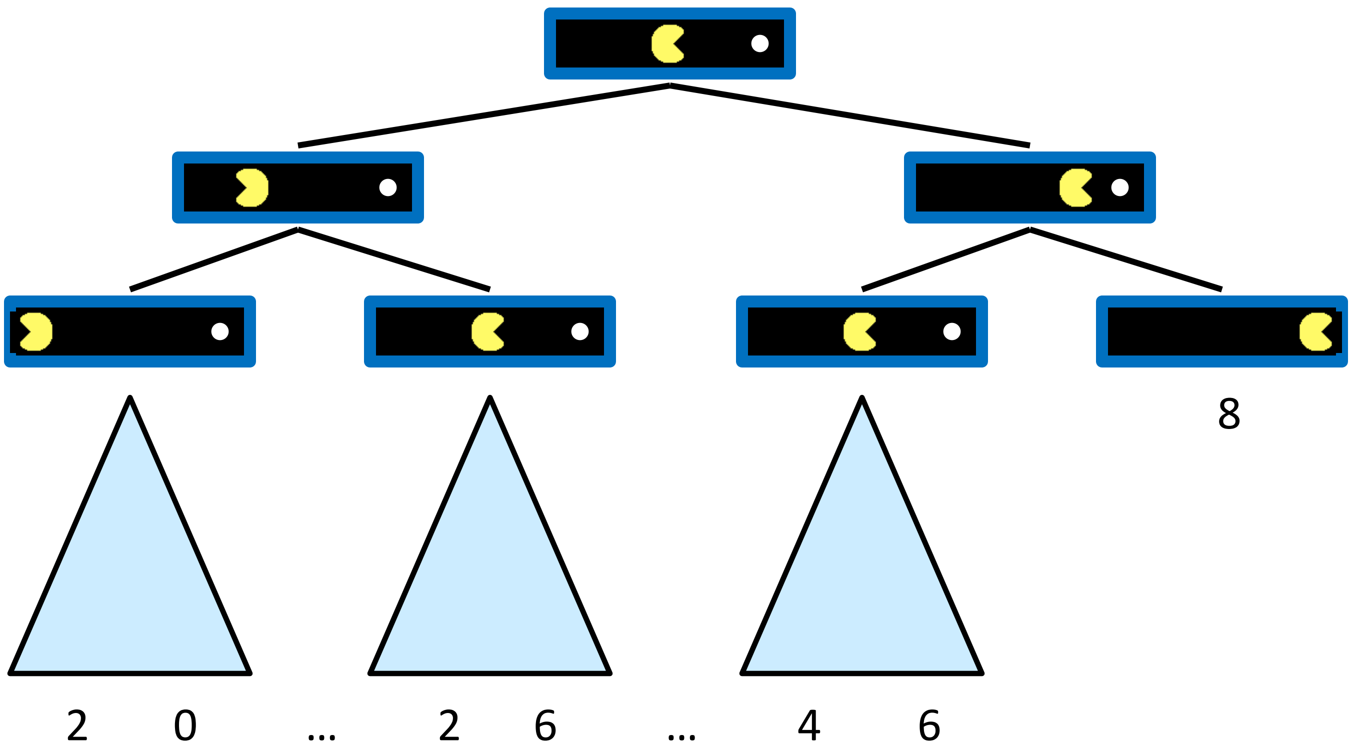
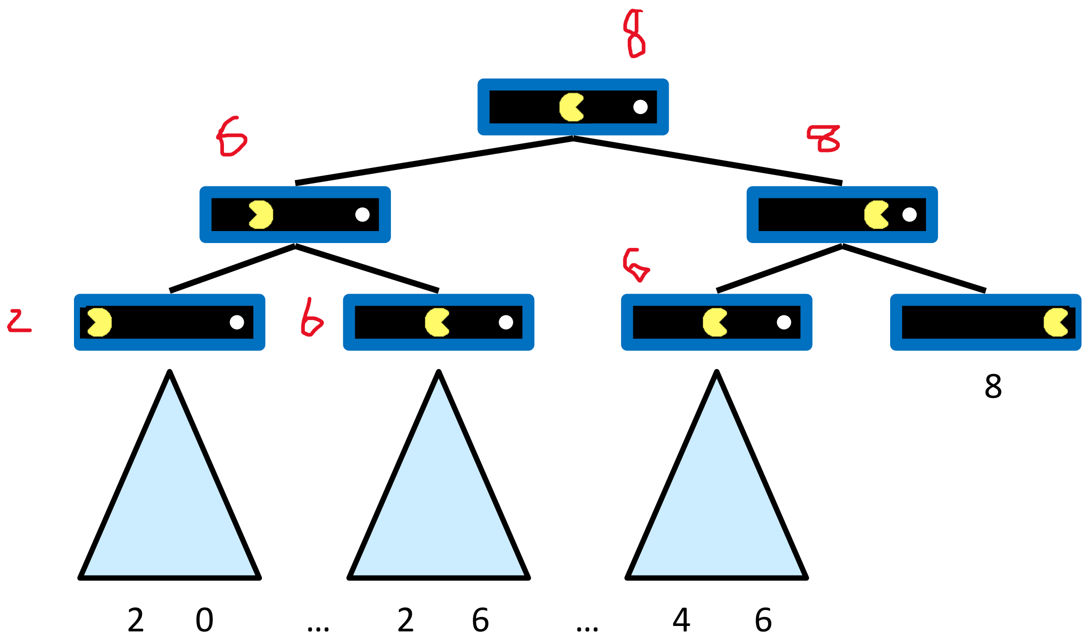
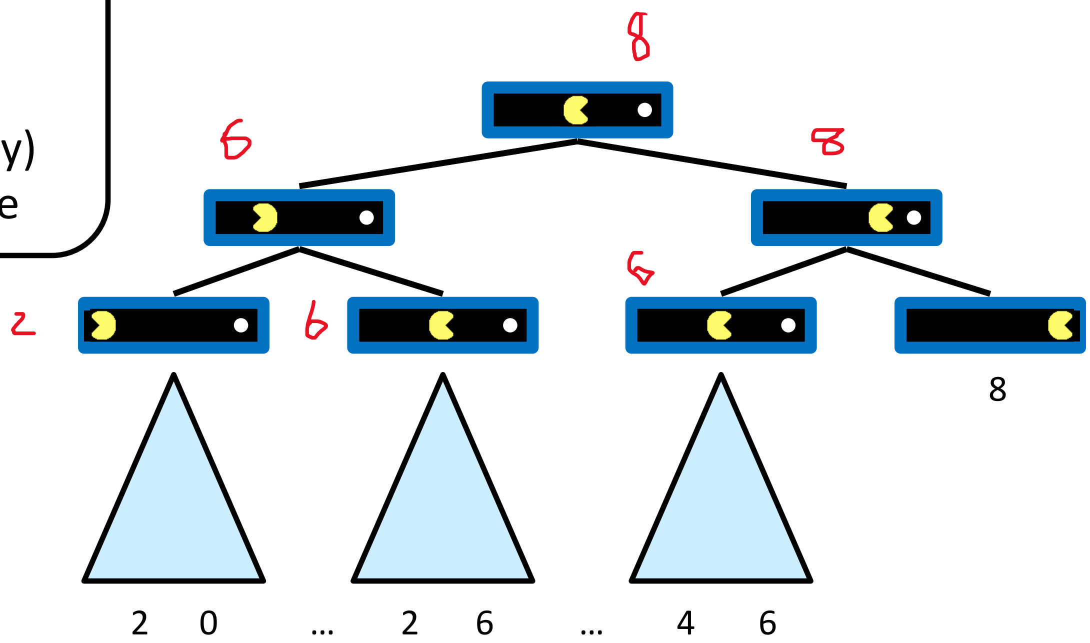


Value of a State



Value of a State

Value of a state:
The best achievable outcome (utility) from that state



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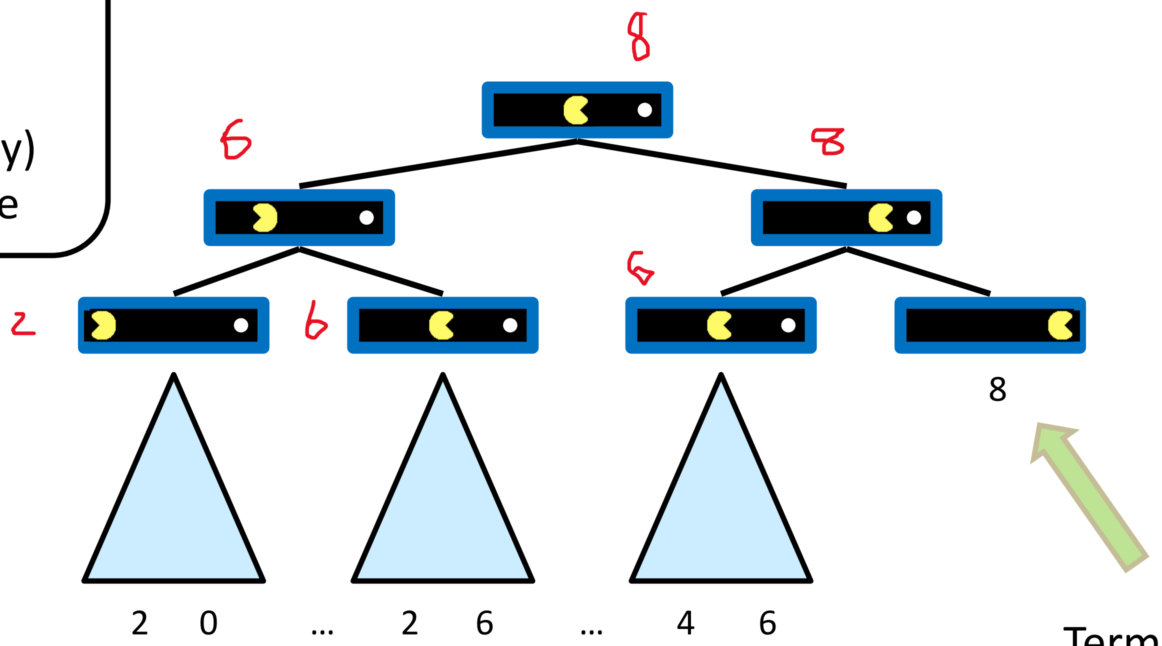

Terminal States:

$$V(s) = \text{known}$$

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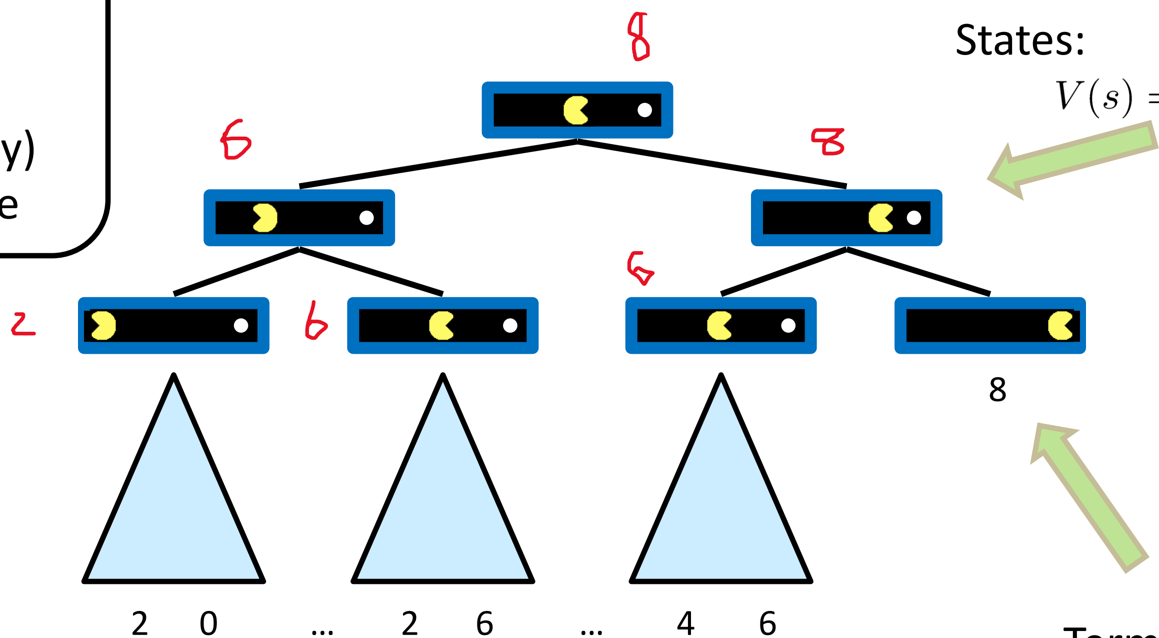
The best achievable outcome (utility) from that state

Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



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Adversarial Game Trees

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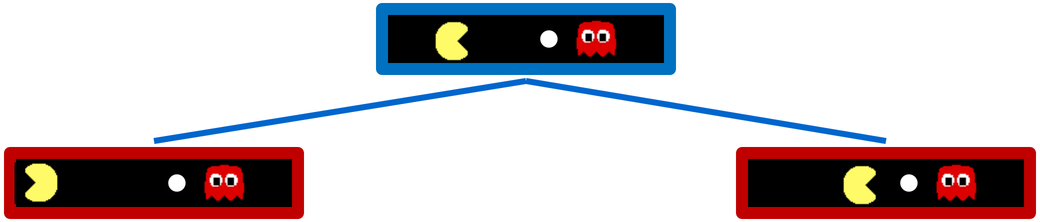

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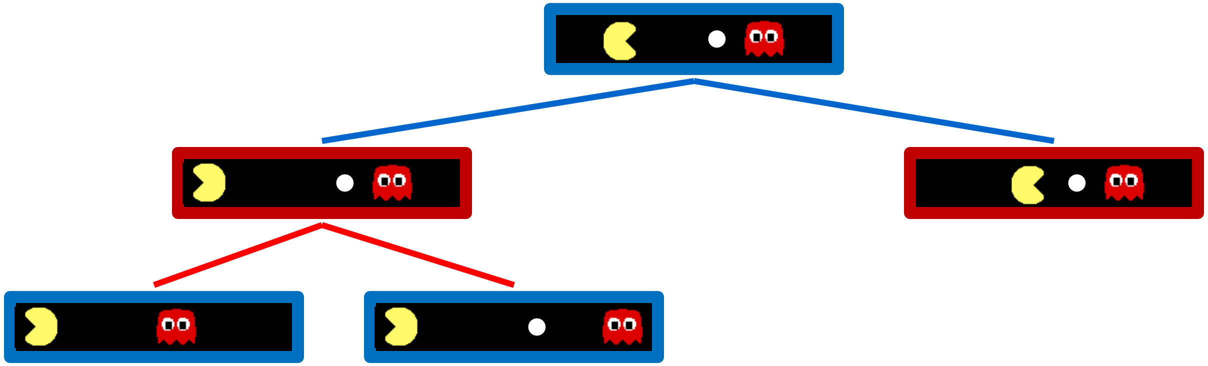

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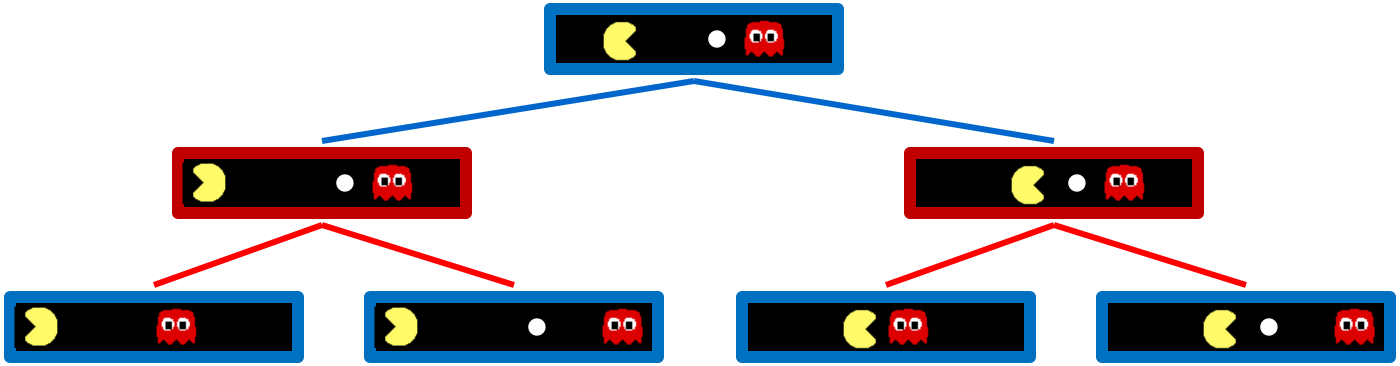

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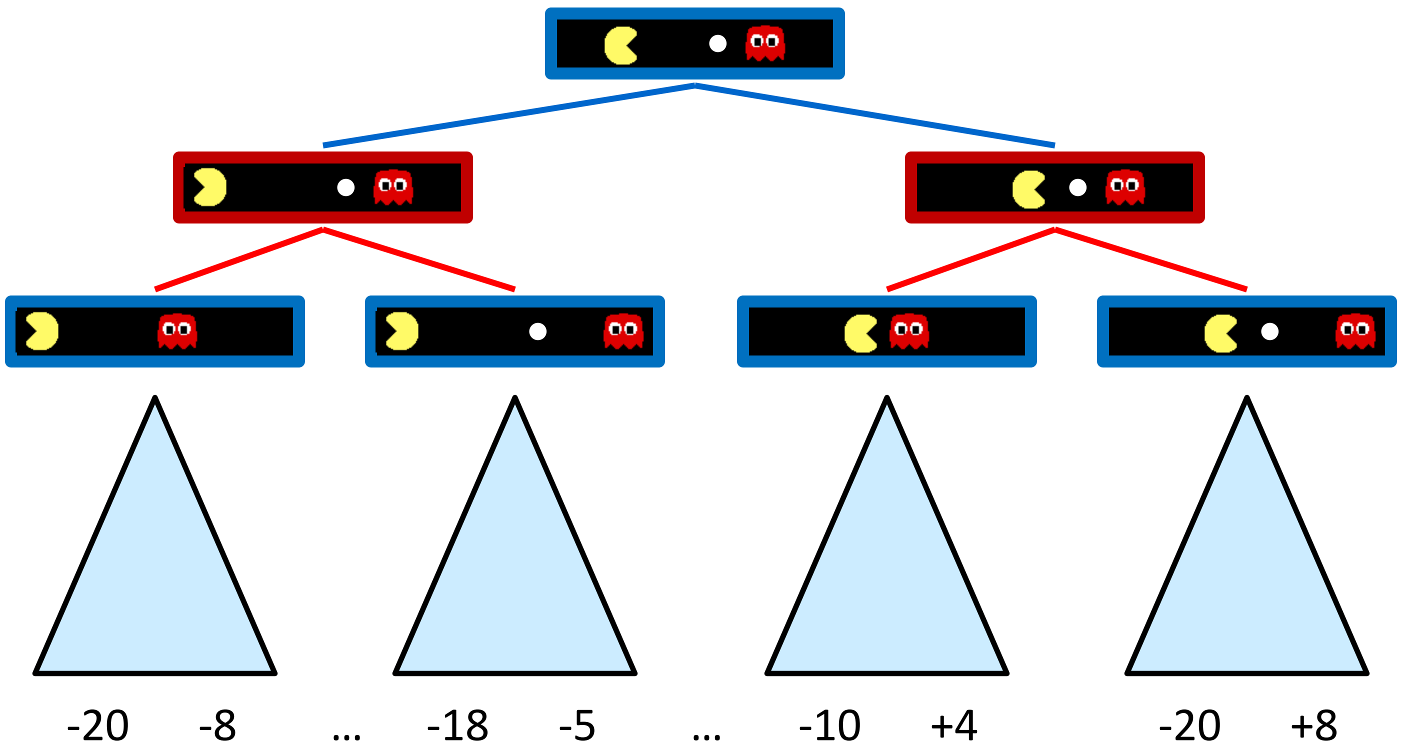
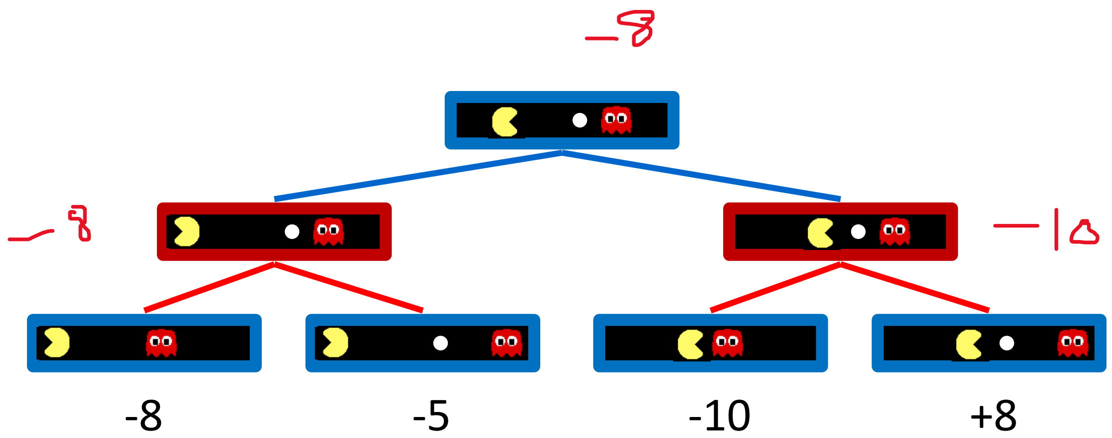


Minimax Values



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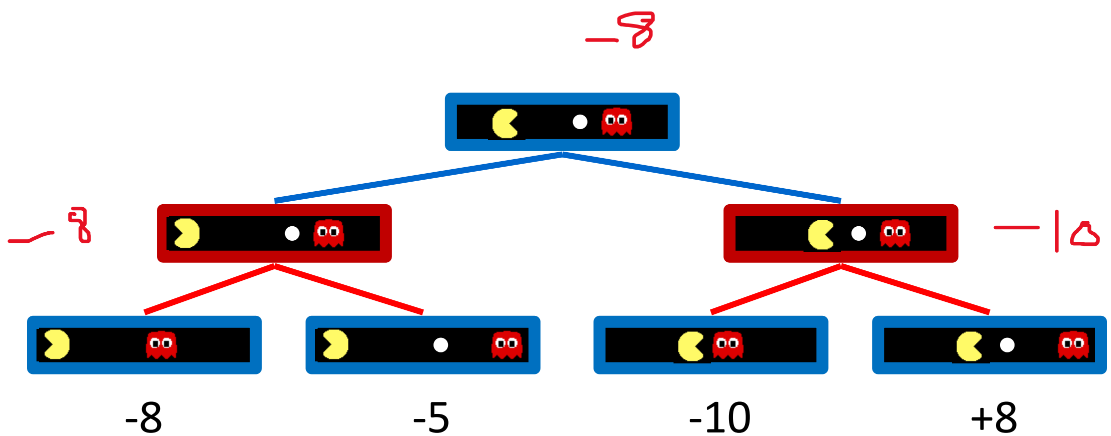

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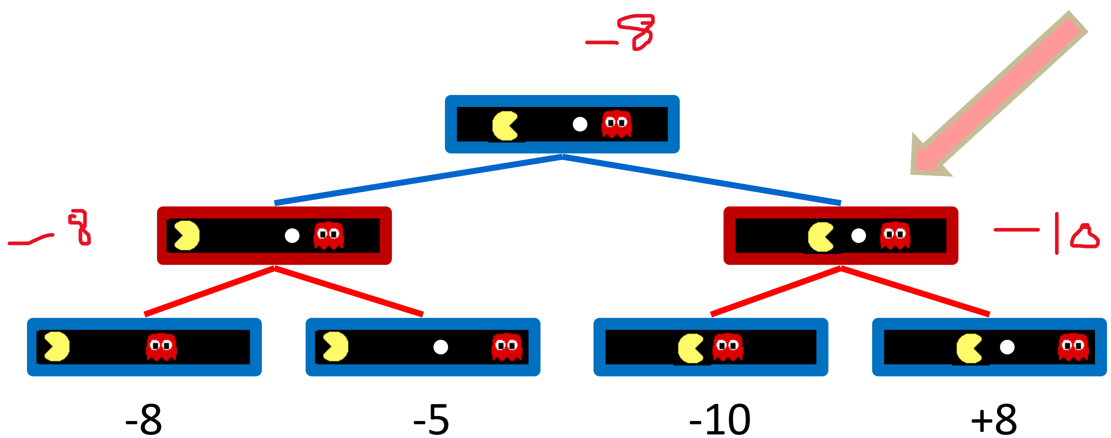
Minimax Values

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



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Minimax Values

States Under Agent's Control:

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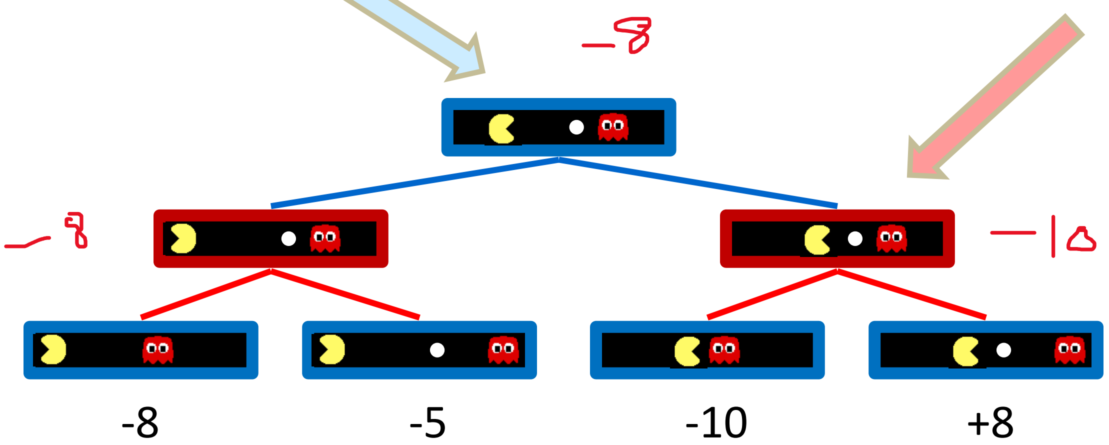

Terminal States:

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Minimax Algorithm

1. Create MAX node with current board configuration
2. *Expand nodes to some* ***depth*** *(a.k.a.* ***plys****) of* ***lookahead*** *in game*
3. Apply evaluation function at each **leaf** node
4. ***Back up*** values for each non-leaf node until value is computed for the root node
   – At MIN nodes: value is **minimum** of children's values
   – At MAX nodes: value is **maximum** of children's values
5. Choose move to child node whose backed-up value determined value at root

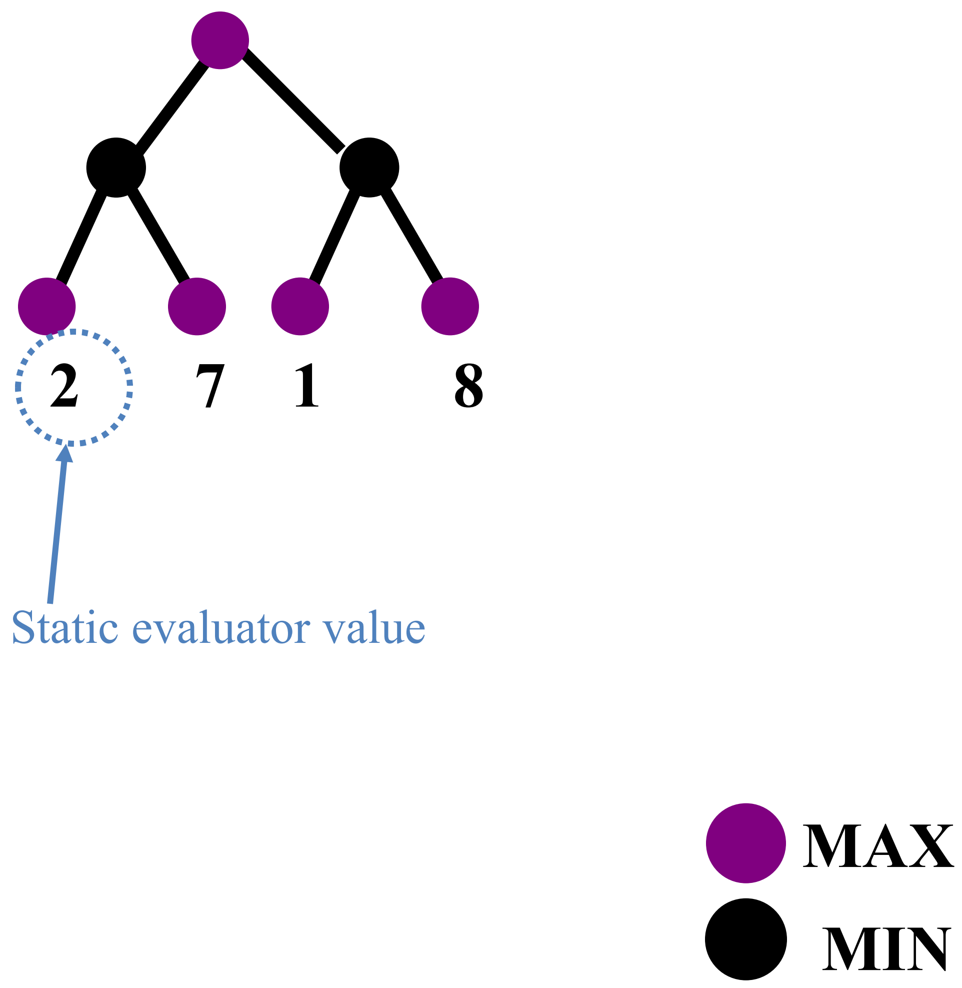
Minimax theorem

- Intuition: assume your opponent is at least as smart as you and play accordingly
  - If she's not, you can only do better!
- Von Neumann, J: *Zur Theorie der Gesellschafts-spiele* Math. Annalen. **100** (1928) 295-320

For every 2-person, 0-sum game with finite strategies, there is a value V and a mixed strategy for each player, such that (a) given player 2's strategy, best payoff possible for player 1 is V, and (b) given player 1's strategy, best payoff possible for player 2 is –V.

- You can think of this as:
  - Minimizing your maximum possible loss
  - Maximizing your minimum possible gain

Minimax Algorithm



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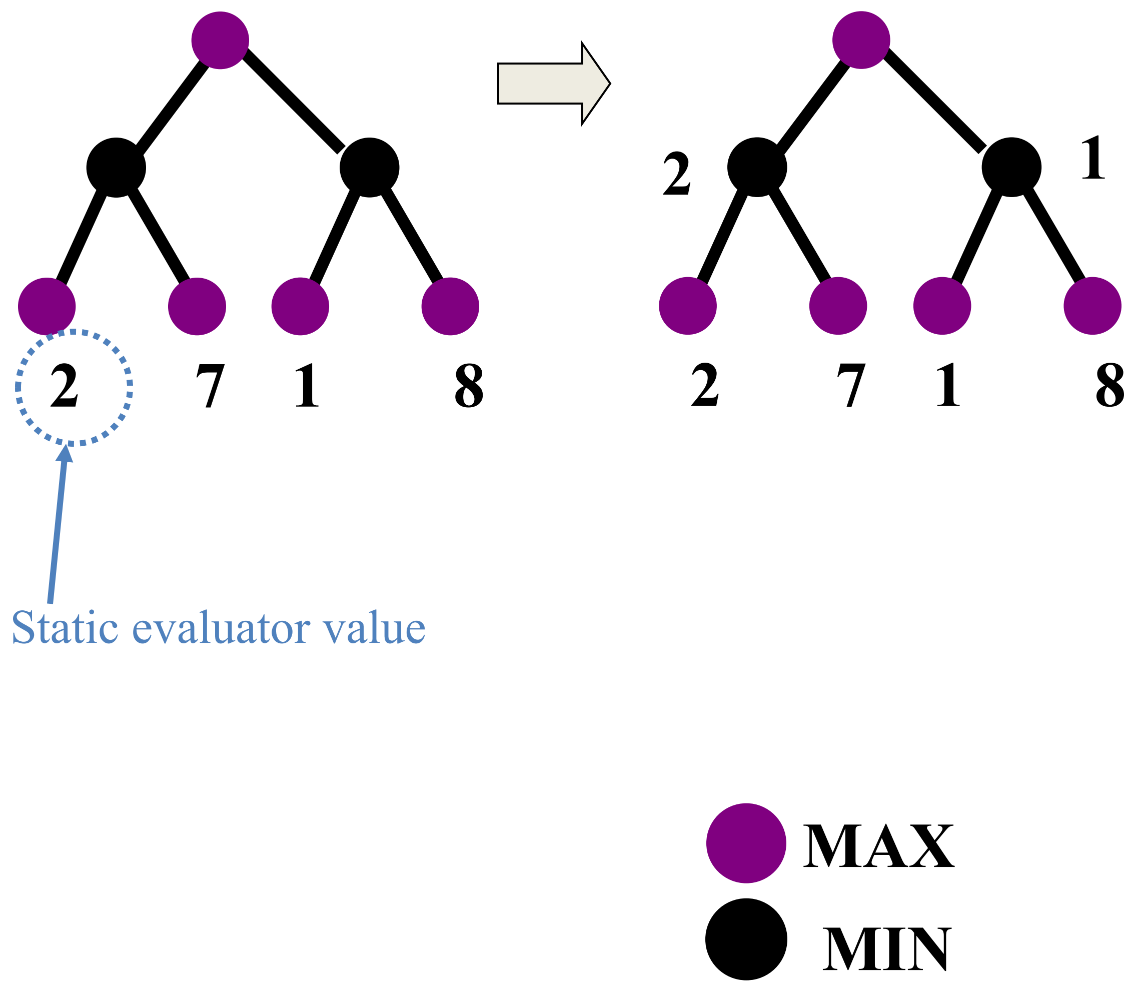

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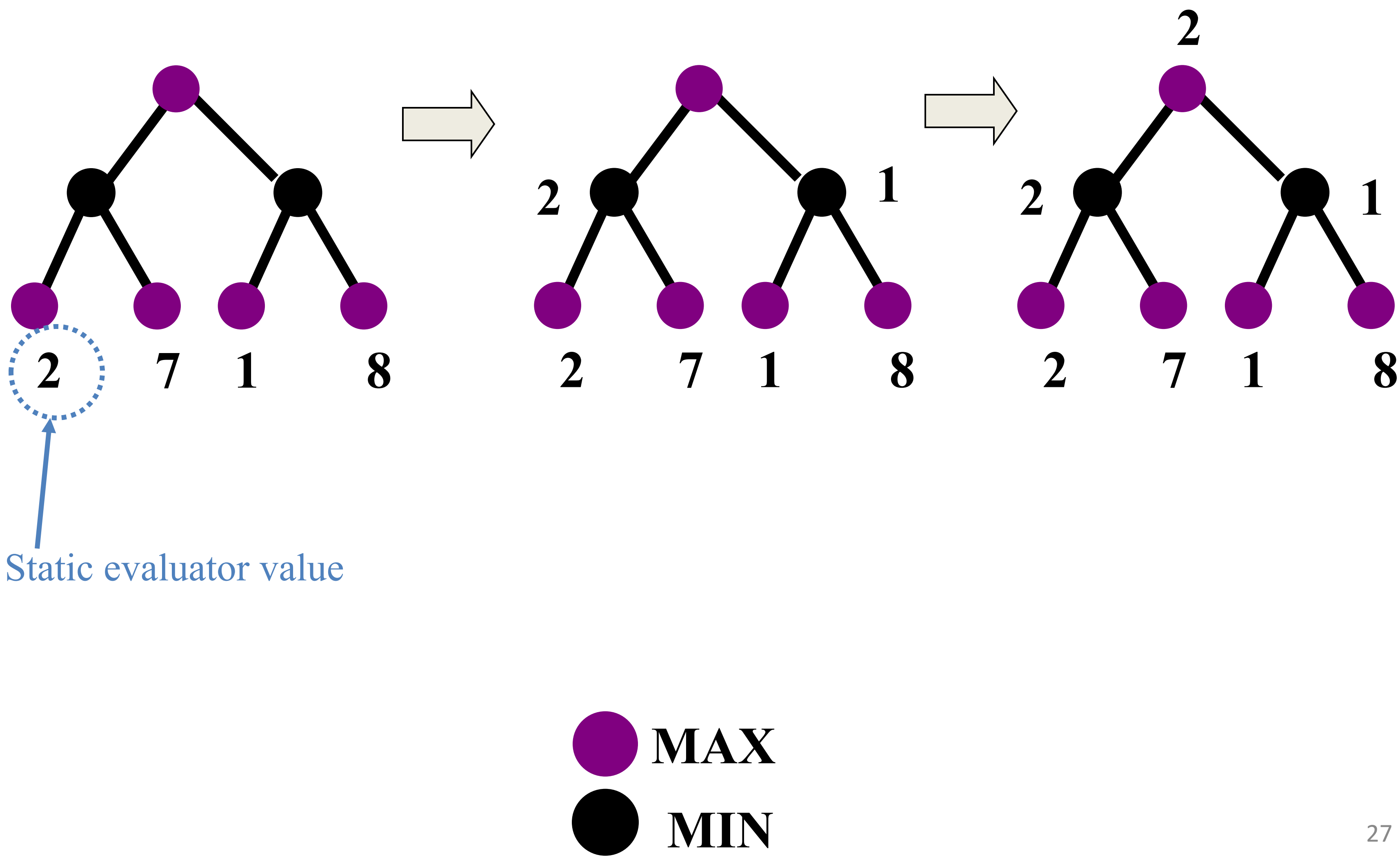

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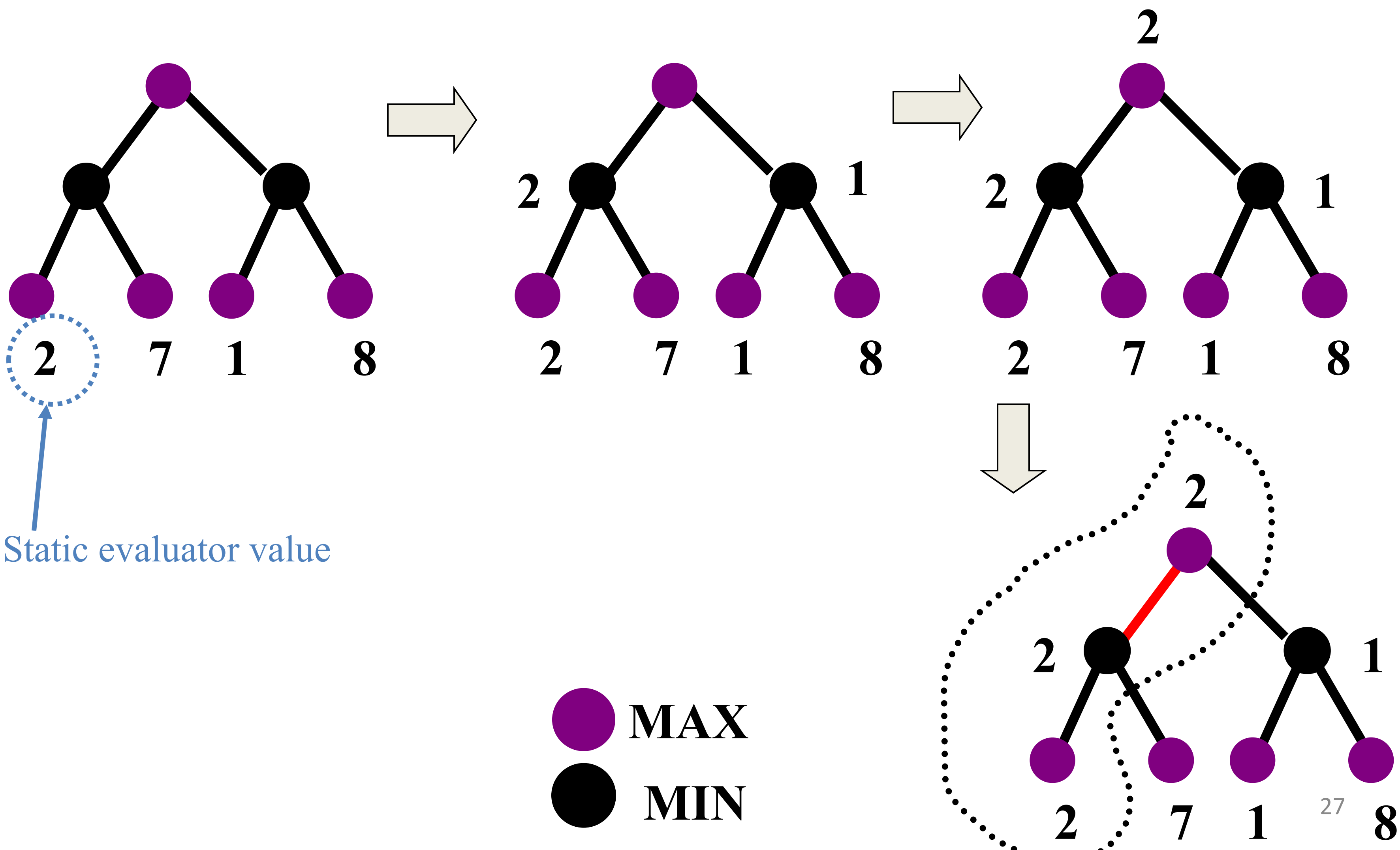

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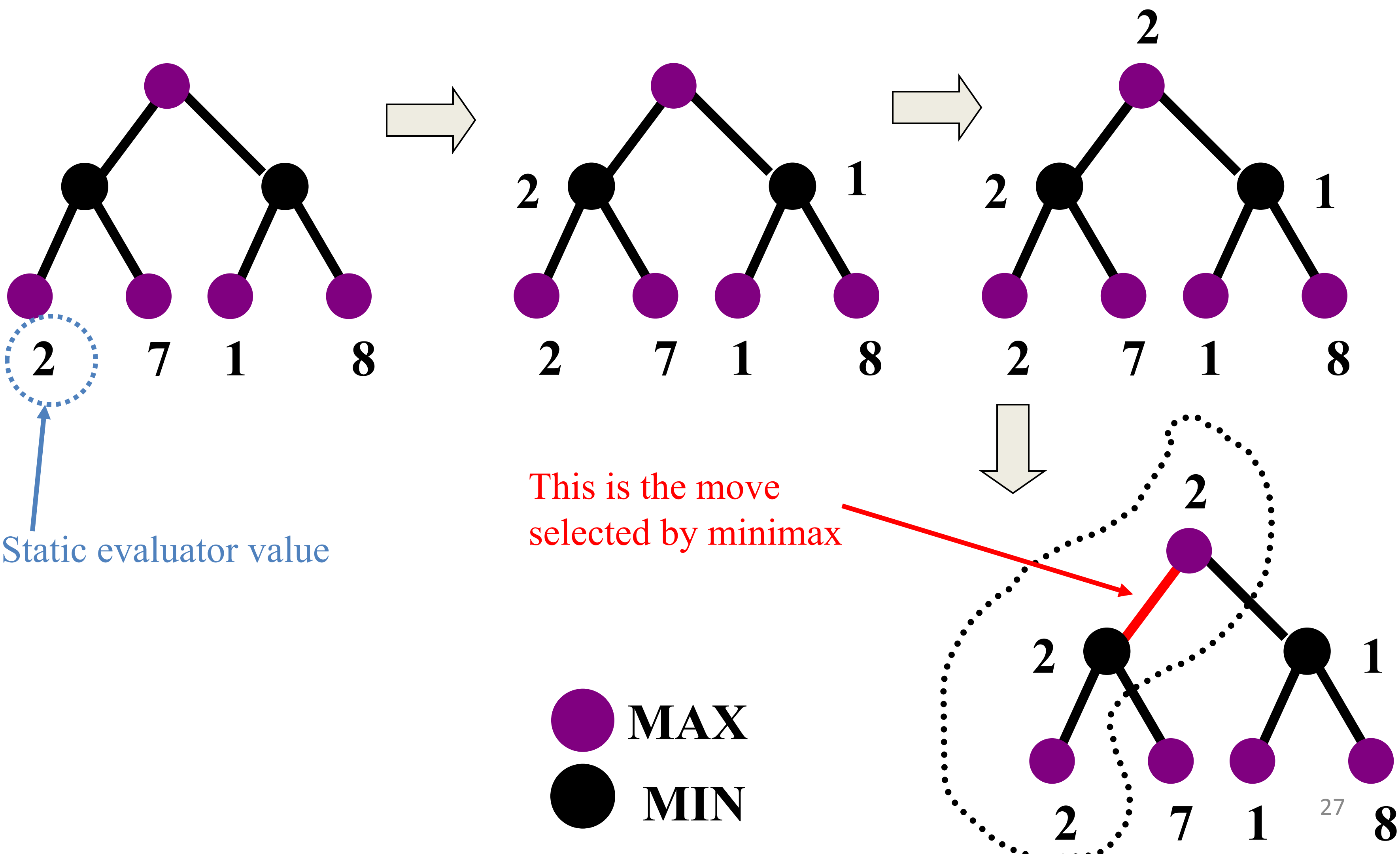
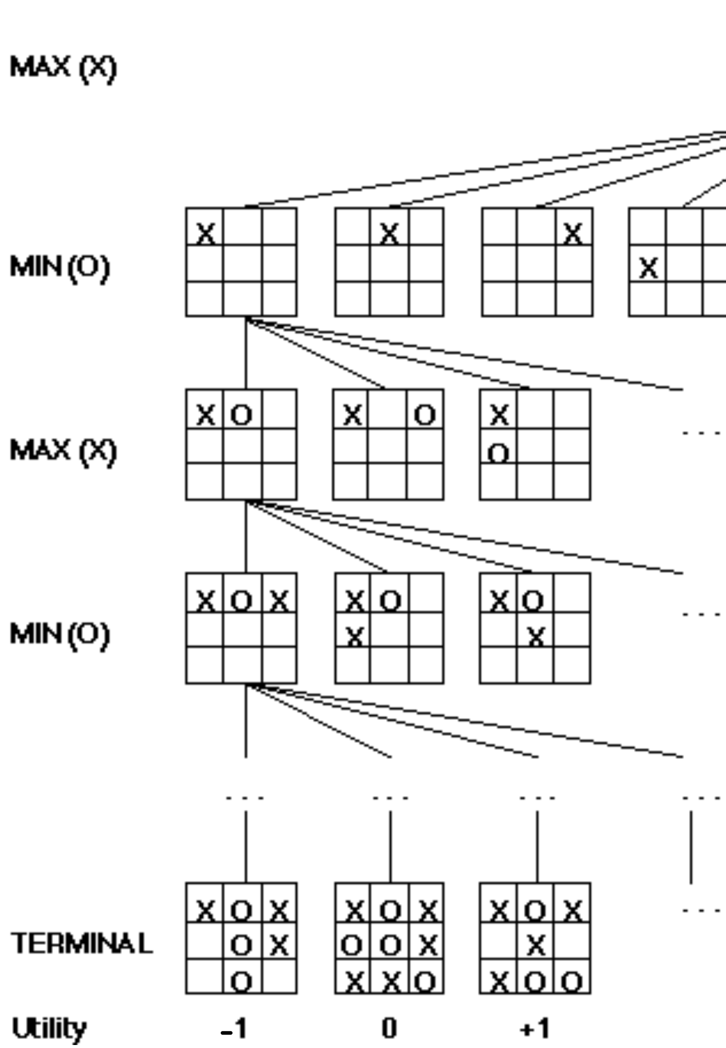


Partial Game Tree for Tic-Tac-Toe



f(n)=+1 if position win for X

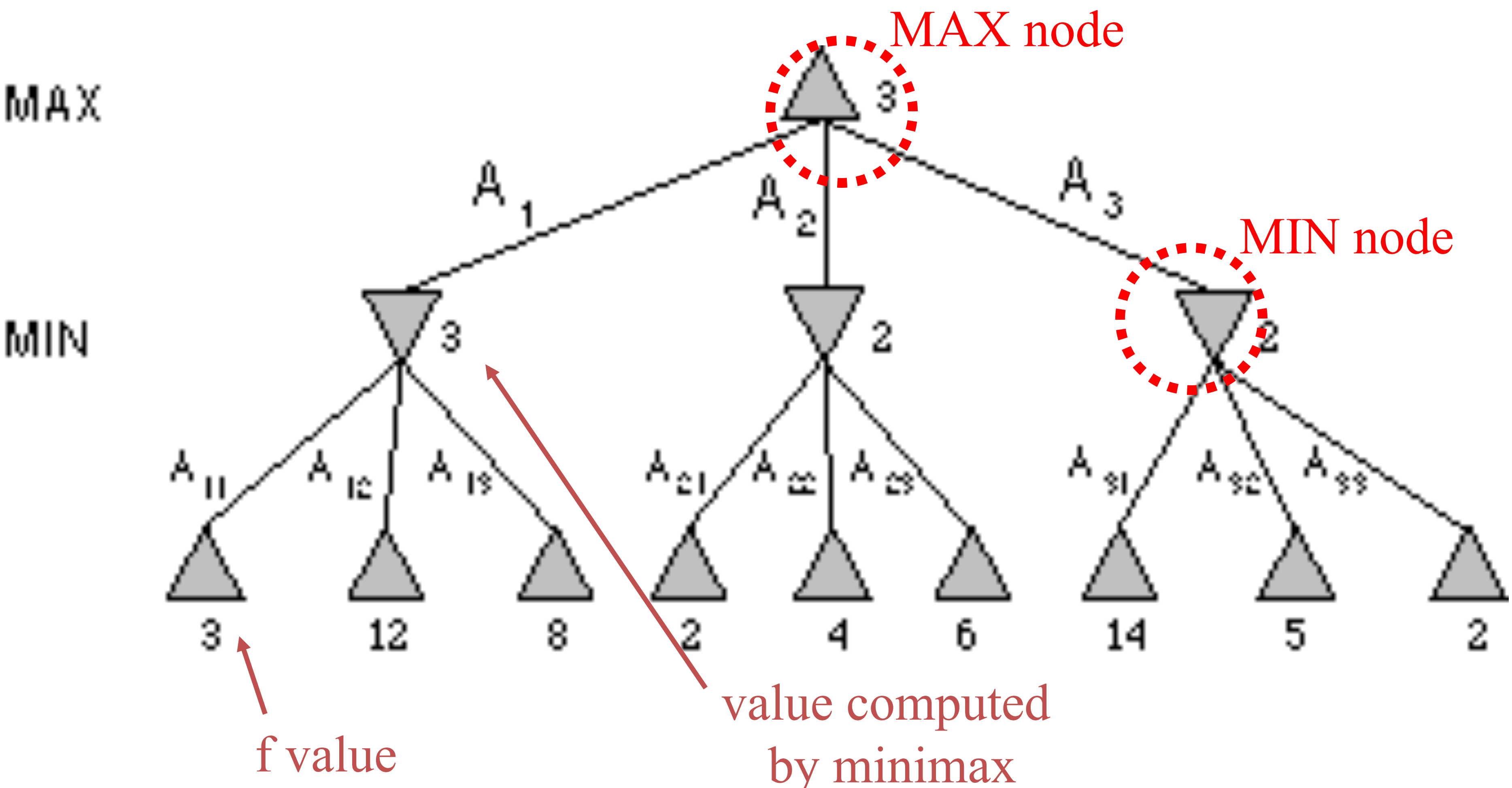
f(n)=-1 if position win for O

f(n)=0 if position a draw

Why backed-up values?

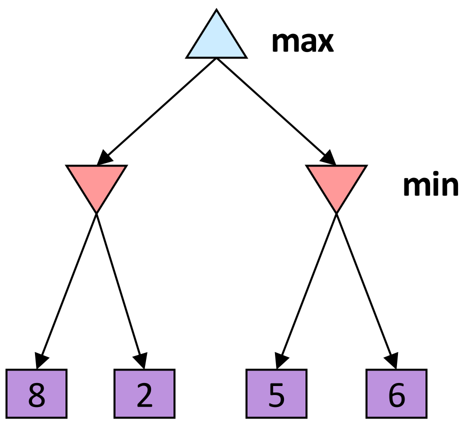
- Why not just use a good static evaluator metric on immediate children
- **Intuition:** if metric is good, doing look ahead and backing up values with Minimax should be better
- Non-leaf node N's backed-up value is value of best state MAX can reach at depth **h** if MIN plays *well*
  - "plays well": same criterion as MAX applies to itself
- If e is good, then backed-up value is better estimate of STATE(N) goodness than e(STATE(N))
- Use lookahead horizon **h** because time to choose move is limited

Minimax Tree



Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



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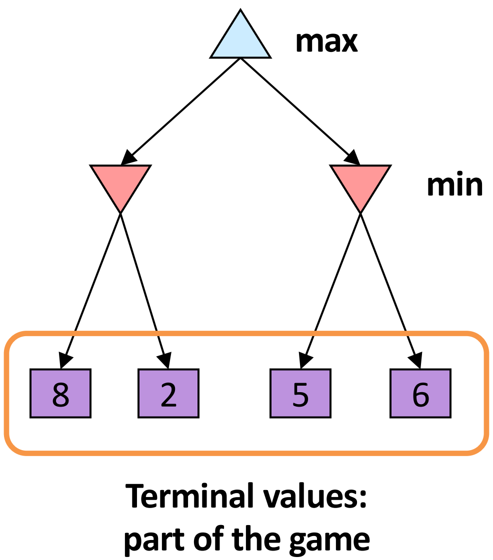

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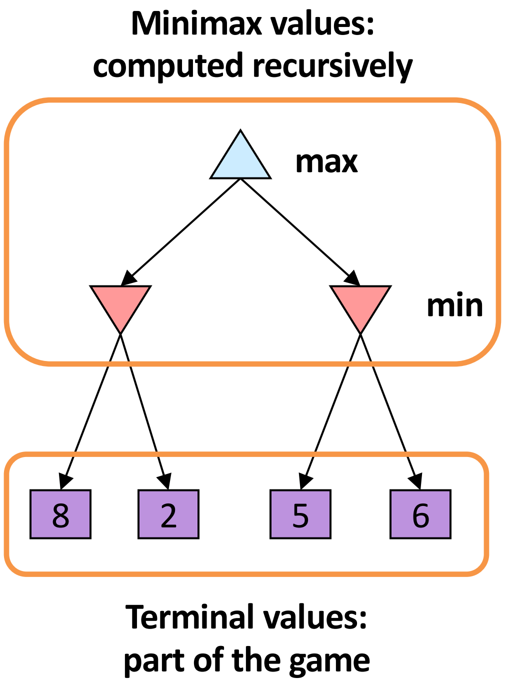

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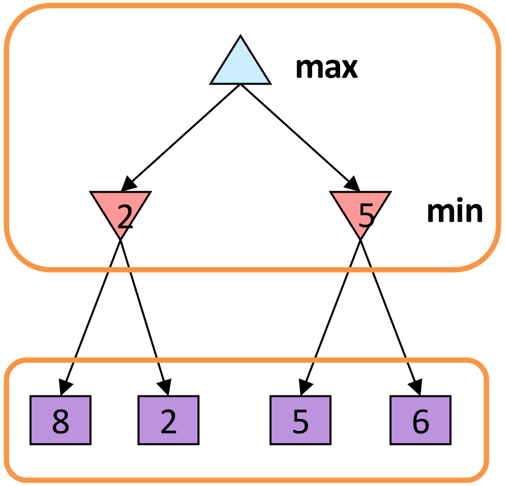
Minimax values: computed recursively



Terminal values: part of the game

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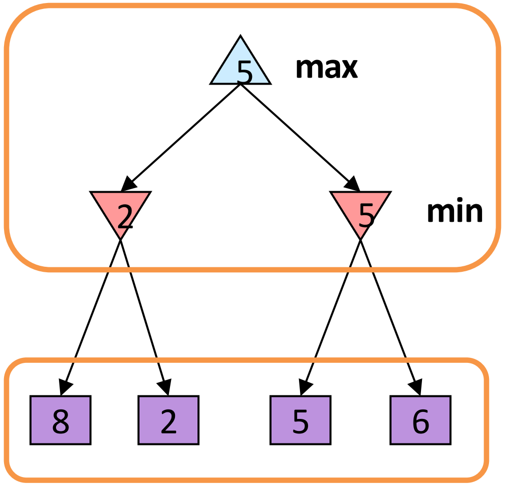

Terminal values: part of the game

Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
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$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

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```
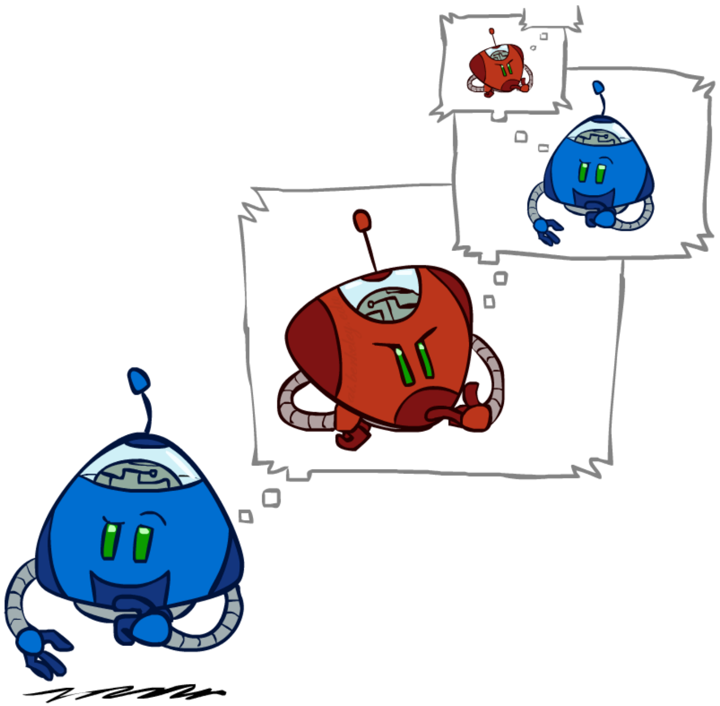

Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Resource Limits

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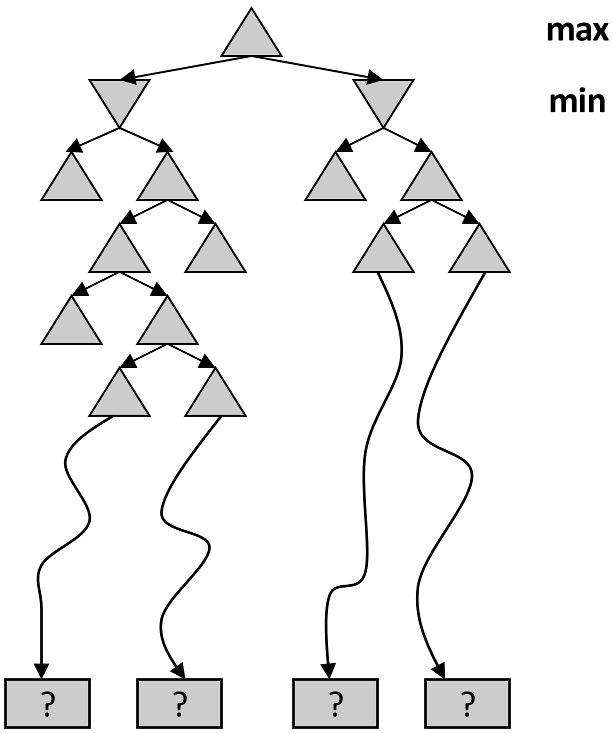
- Problem: In realistic games, cannot search to leaves!



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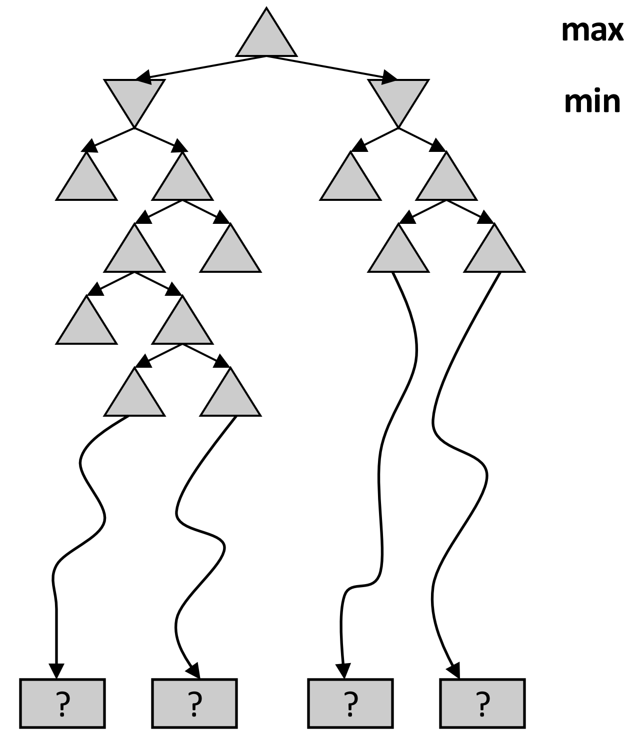
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions



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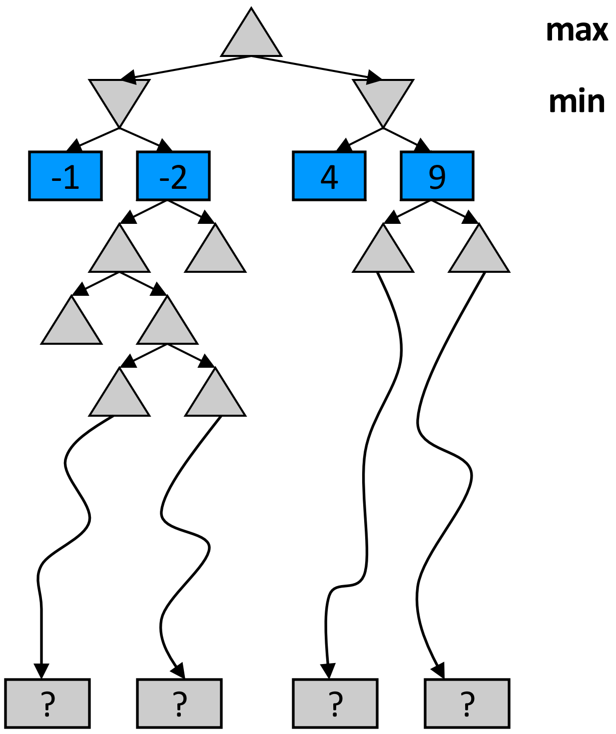

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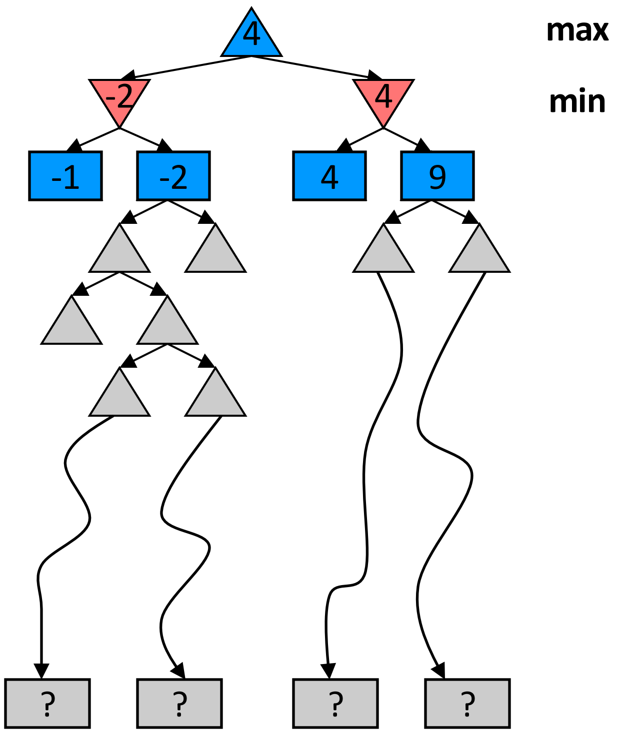

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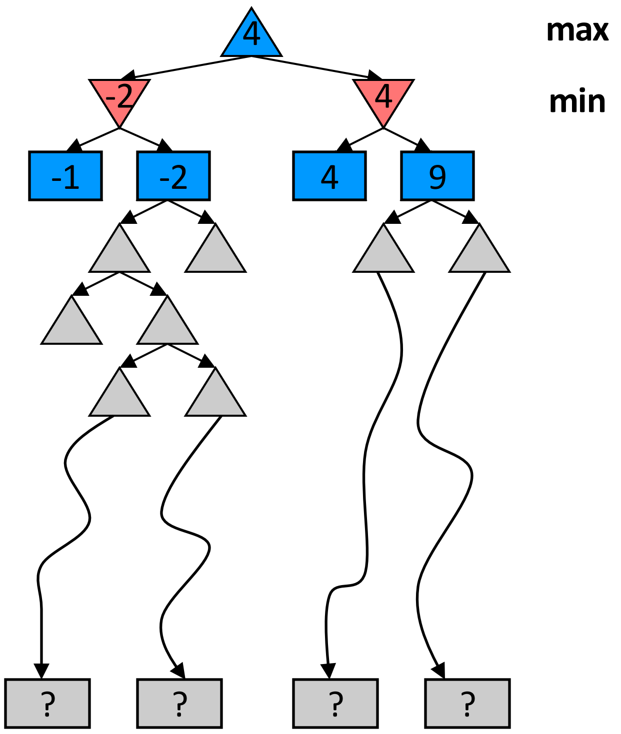
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program



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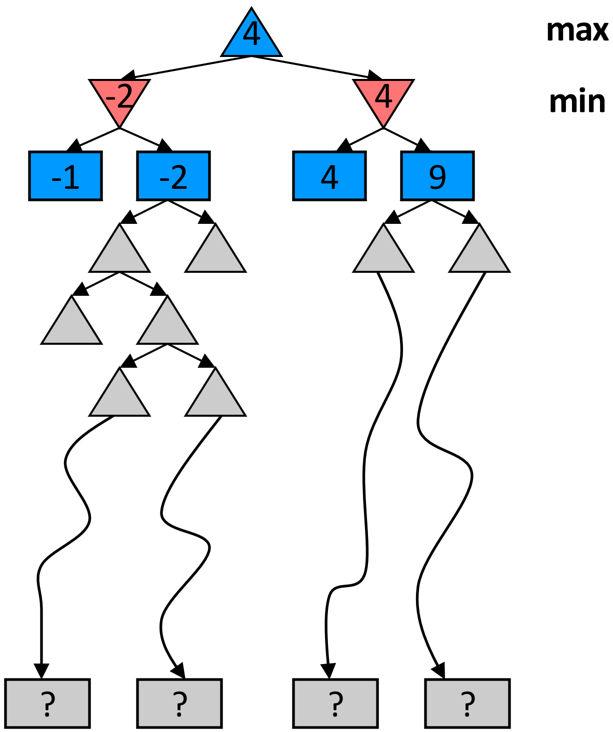
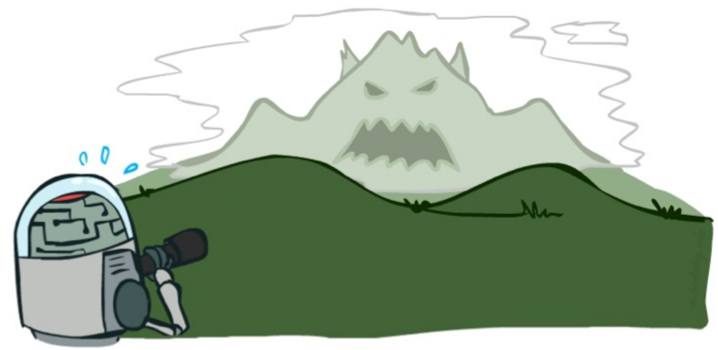
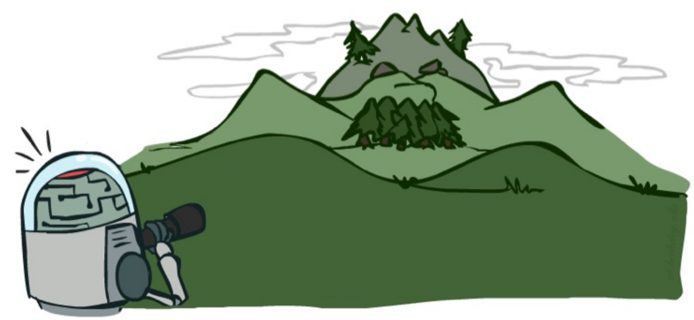
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use iterative deepening for an anytime algorithm



Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

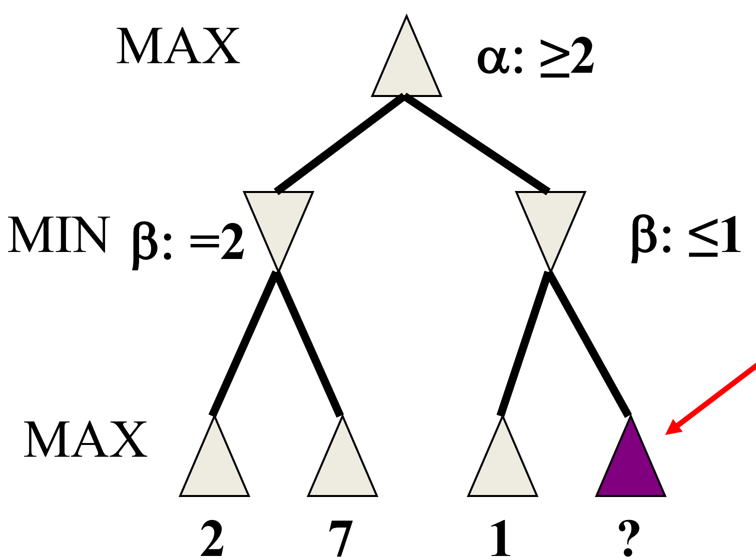




Is that all there is to simple games?

Alpha-beta pruning

- Improve performance of the minimax algorithm through **alpha-beta pruning**
- *"If you have an idea that is surely bad, don't take the time to see how truly awful it is"* -Pat Winston (MIT)



- We don't need to compute the value at this node
- No matter what it is, it can't affect value of the root node

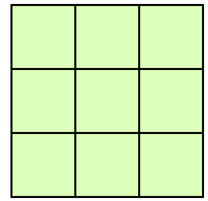
Alpha-beta pruning

- Traverse search tree in depth-first order
- At **MAX** node n, **alpha(n)** = max value found so far

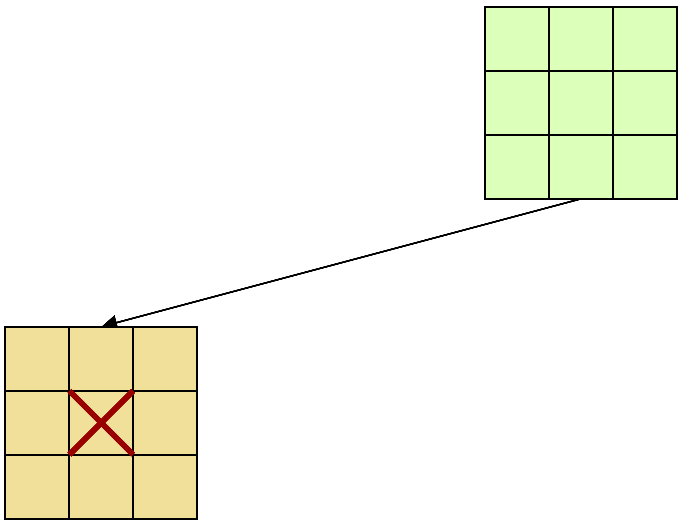
  Alpha values start at -∞ and only increase
- At **MIN** node n, **beta(n)** = min value found so far

  Beta values start at +∞ and only decrease
- **Beta cutoff**: stop search below MAX node N (i.e., don't examine more descendants) if alpha(N) >= beta(i) for some MIN node ancestor i of N
- **Alpha cutoff:** stop search below MIN node N if beta(N)<=alpha(i) for a MAX node anceastor i of N

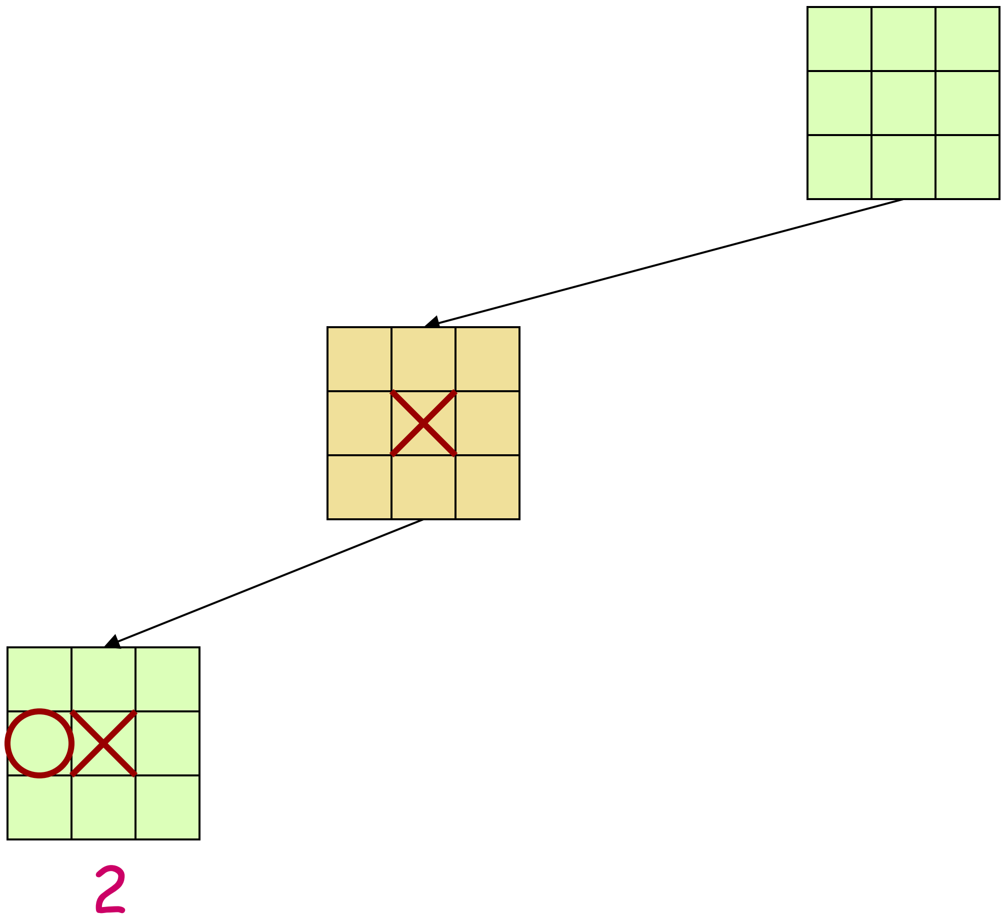
Alpha-Beta Tic-Tac-Toe Example



Alpha-Beta Tic-Tac-Toe Example



Alpha-Beta Tic-Tac-Toe Example



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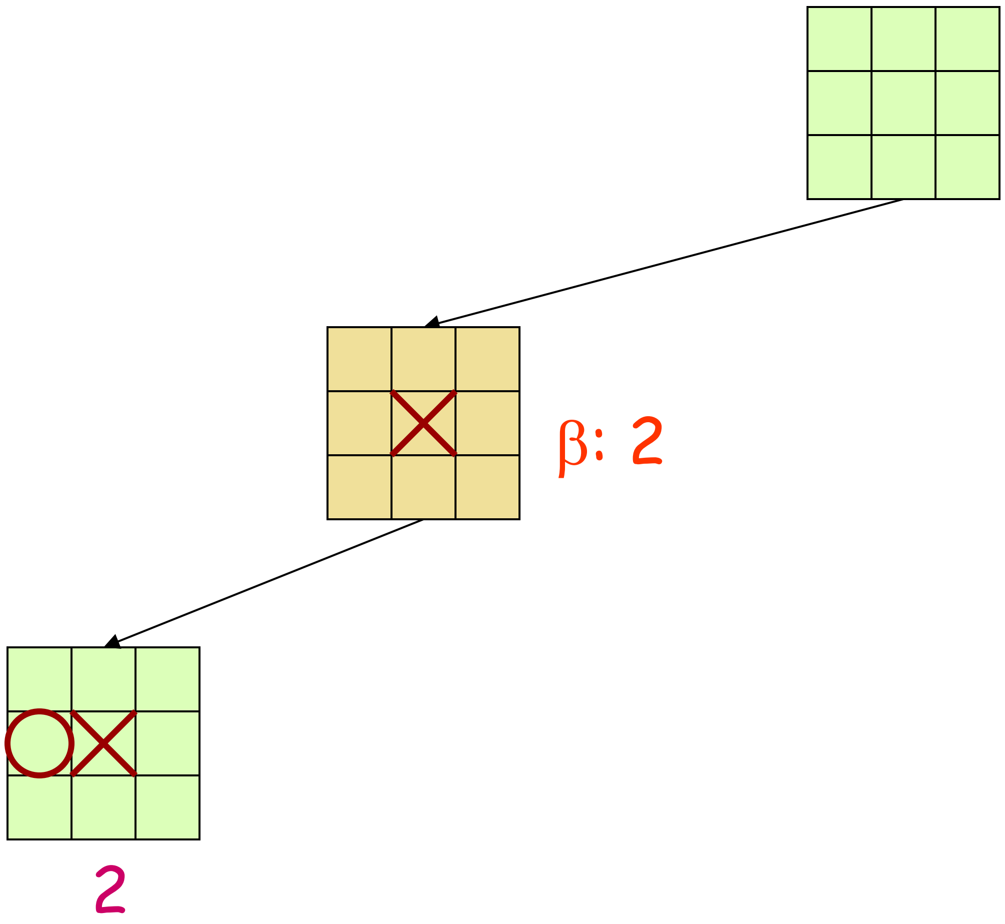

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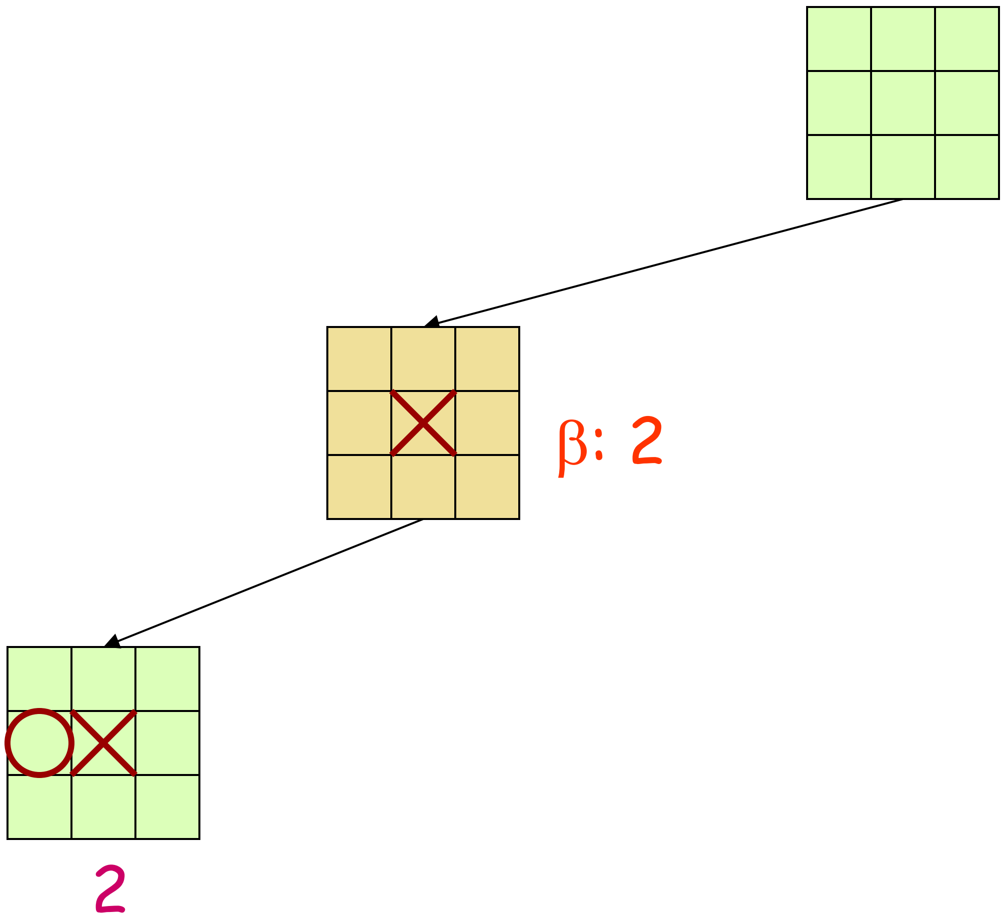
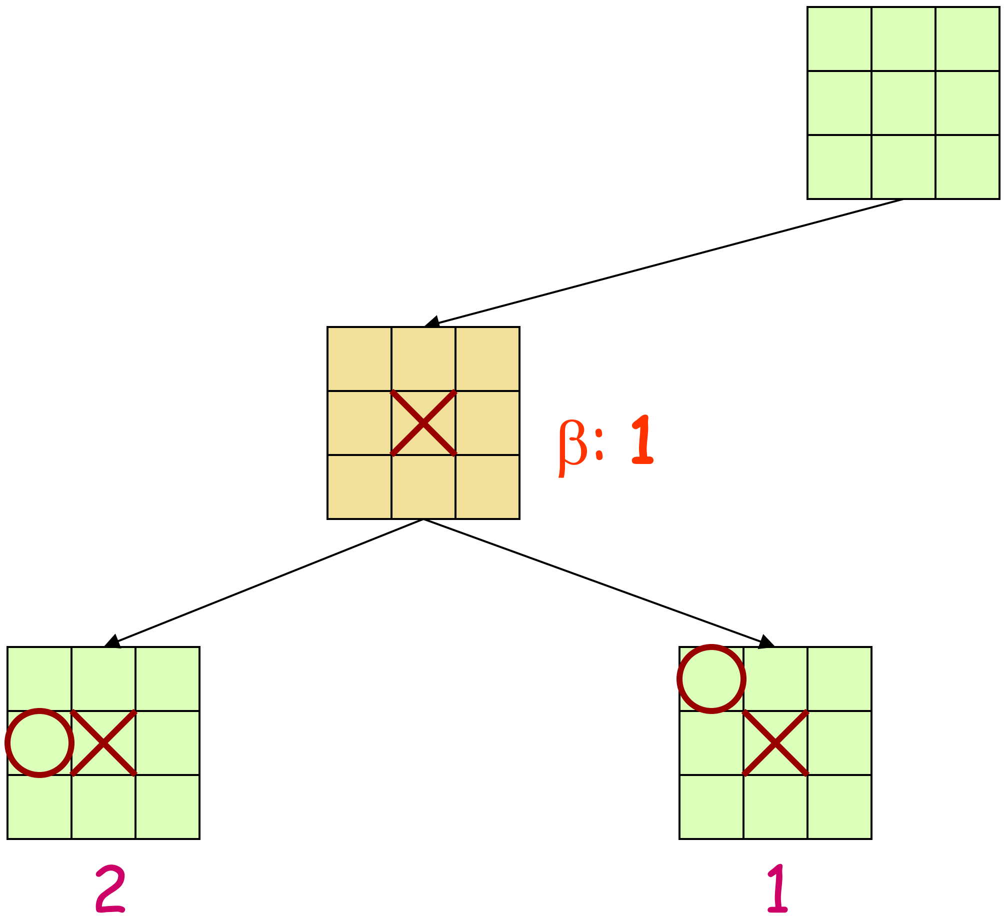


Beta value of a MIN node is **upper** bound on final backed-up value; it can never increase

Alpha-Beta Tic-Tac-Toe Example



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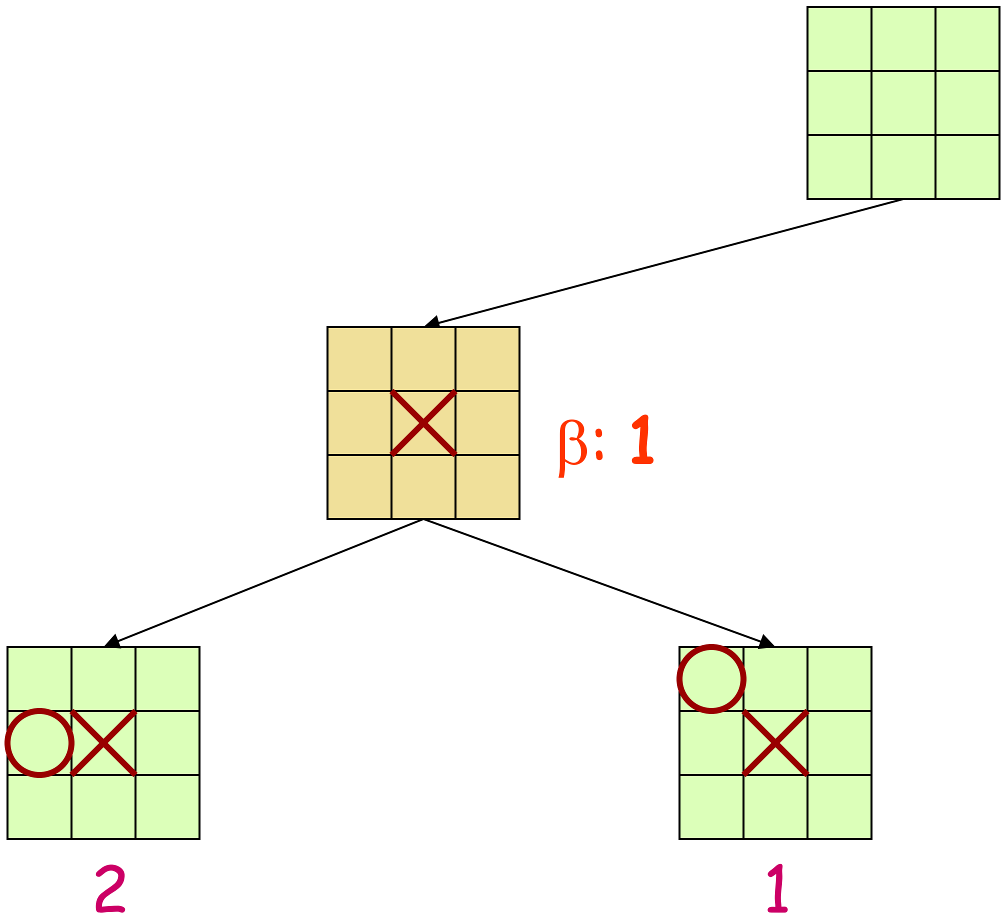

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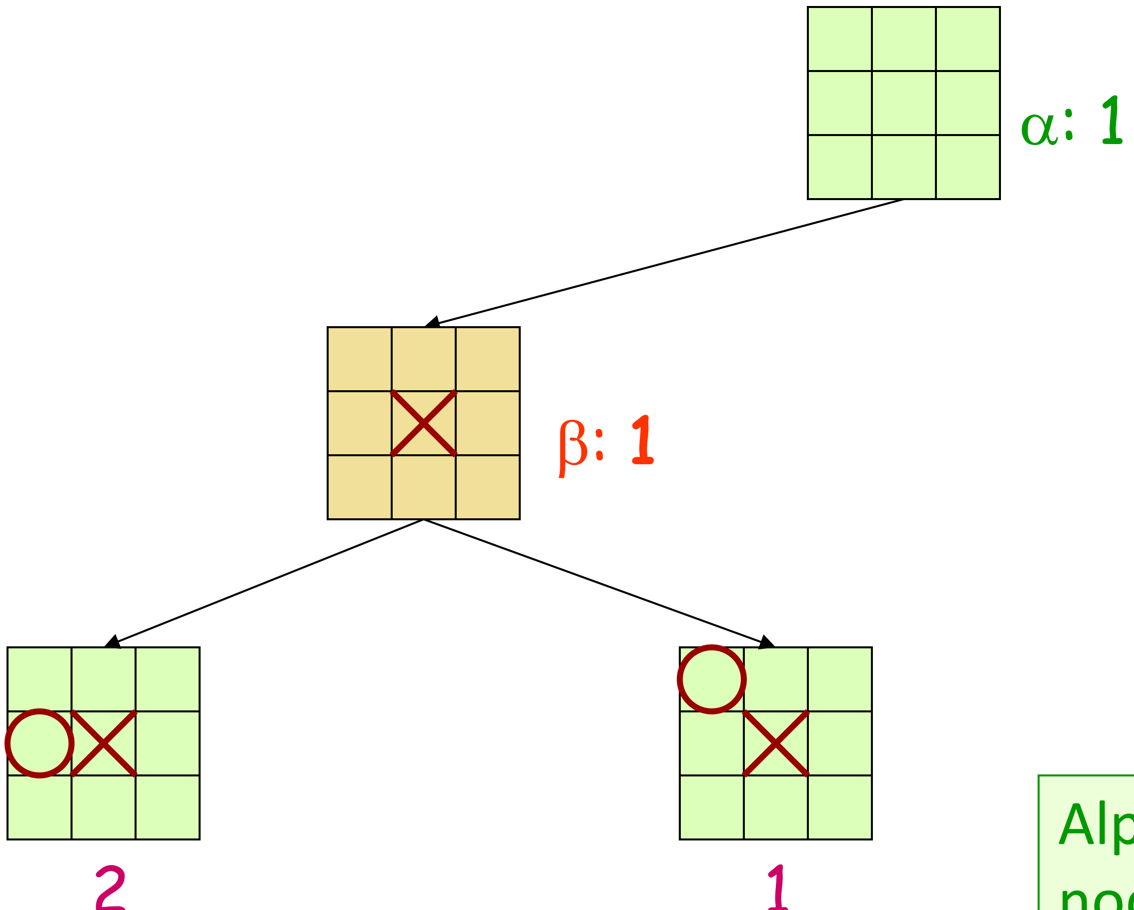
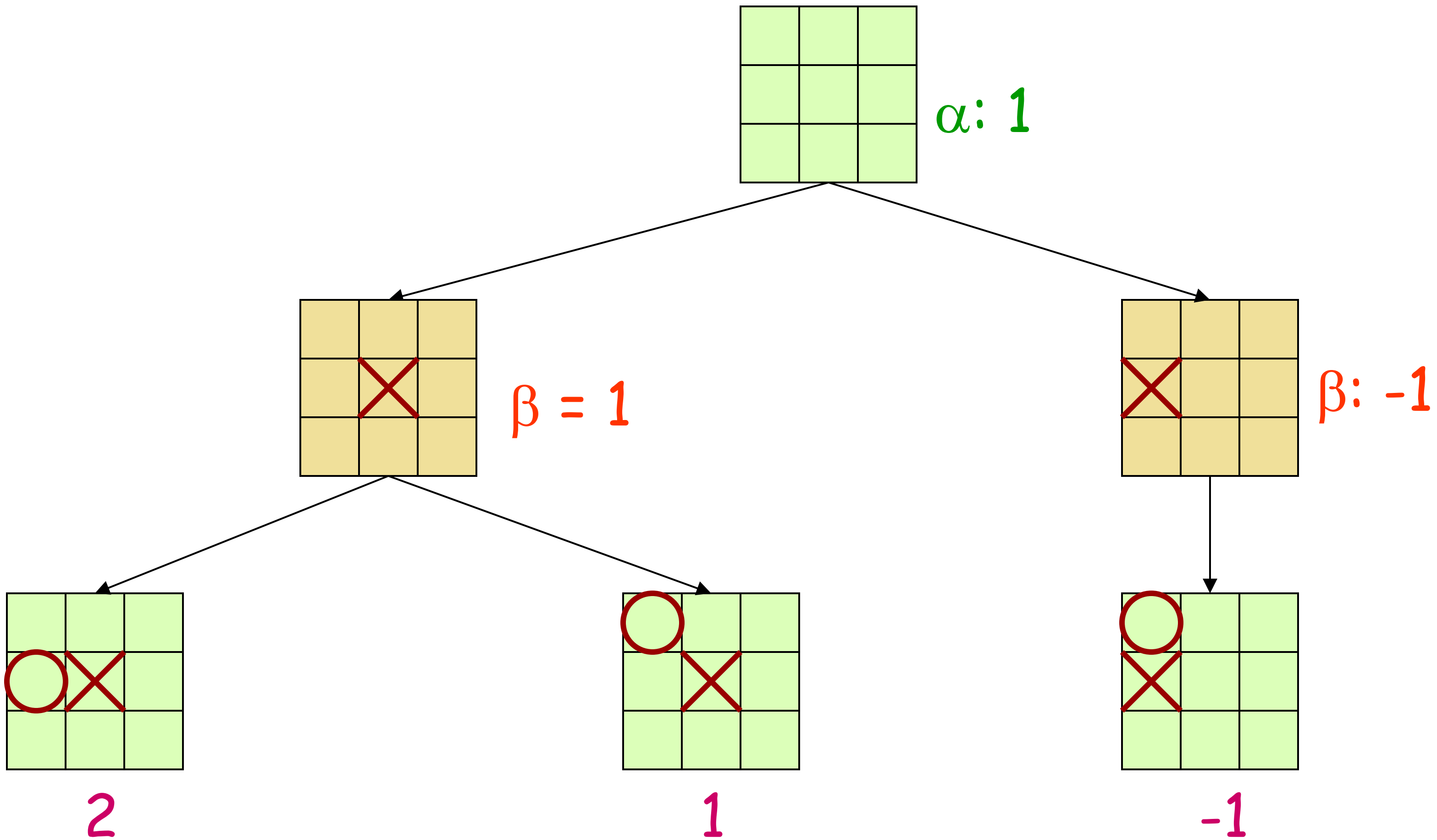


Alpha value of MAX node is **lower** bound on final backed-up value; it can never decrease

Alpha-Beta Tic-Tac-Toe Example



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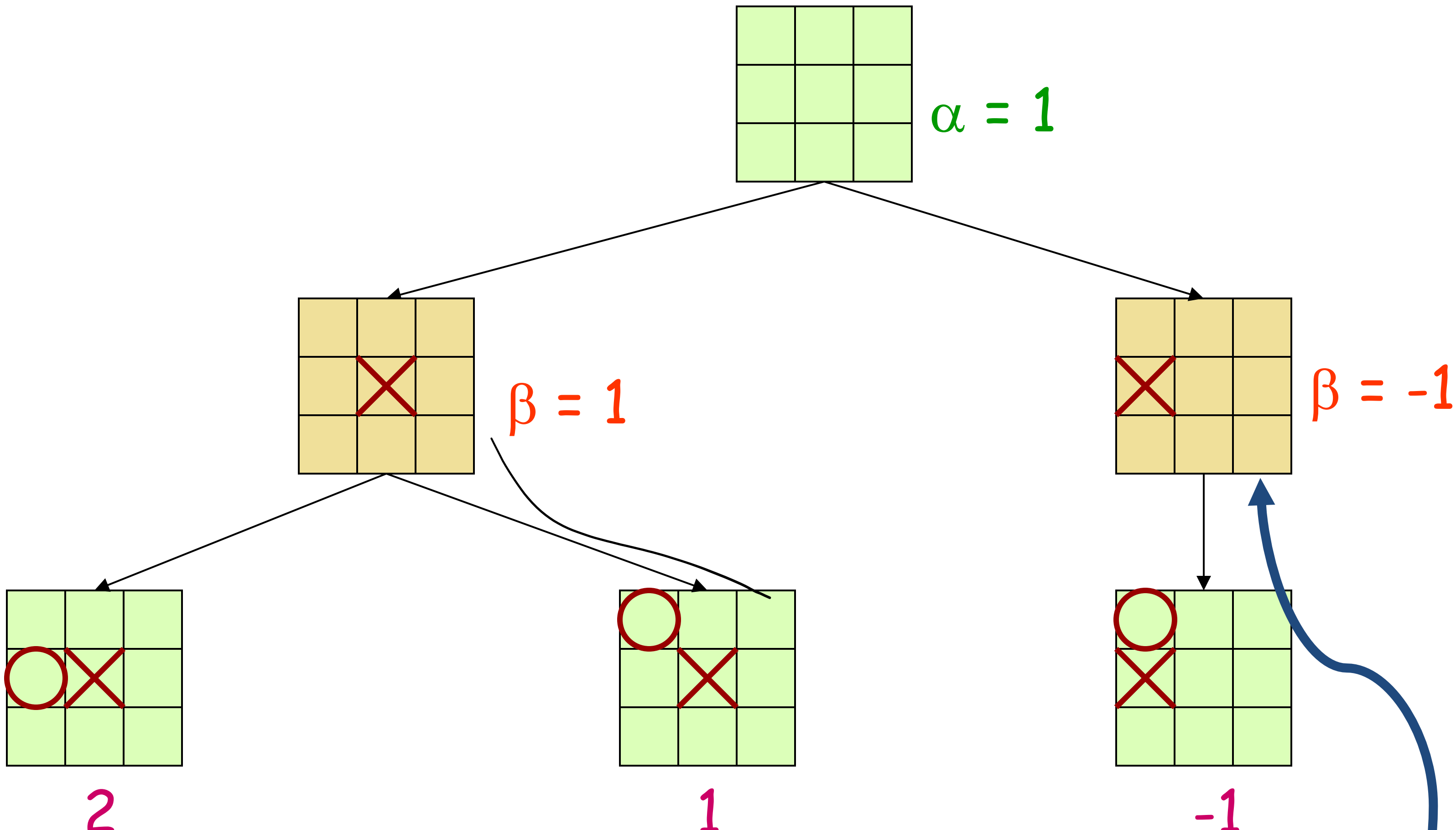


Discontinue search below a MIN node whose beta value ≤ alpha value of one of its MAX ancestors

Alpha-Beta Implementation

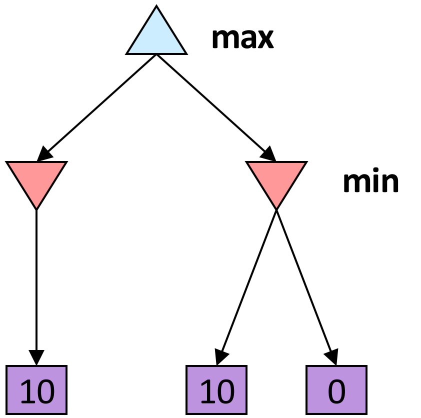
α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

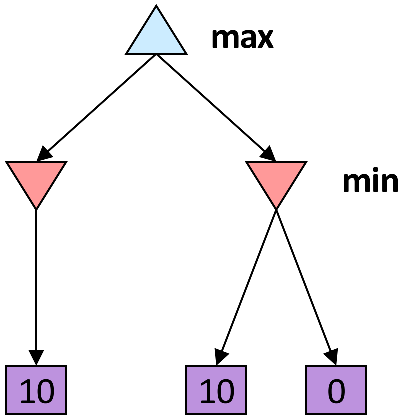
Alpha-Beta Pruning Properties

- This pruning has no effect on minimax value computed for the root!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection

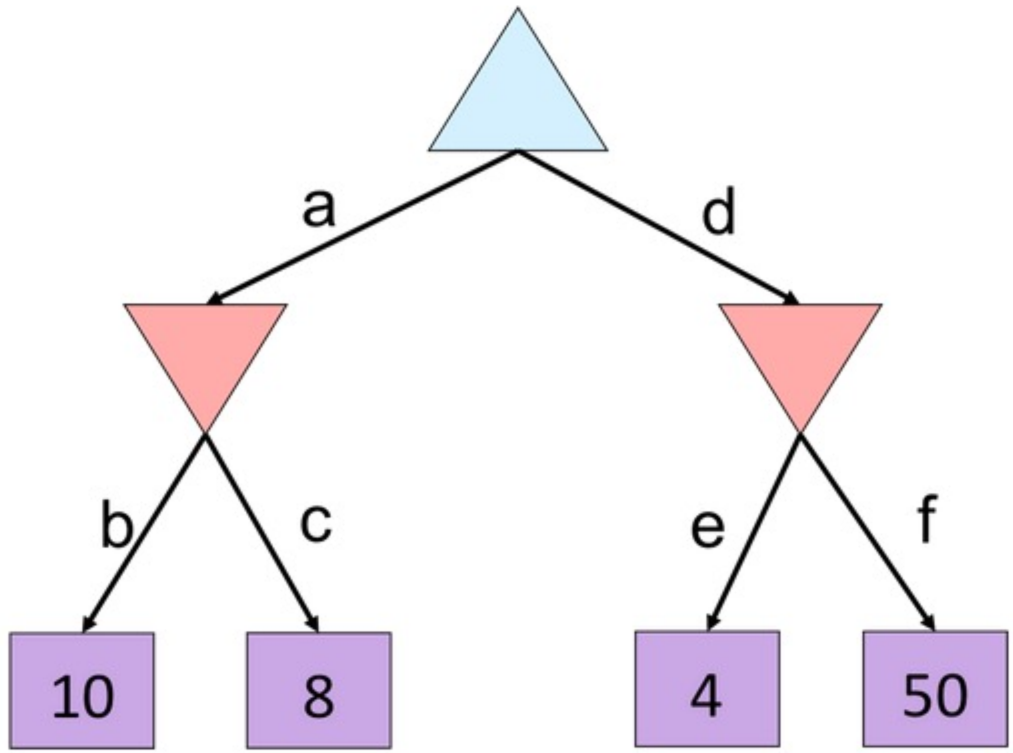


Alpha-Beta Pruning Properties

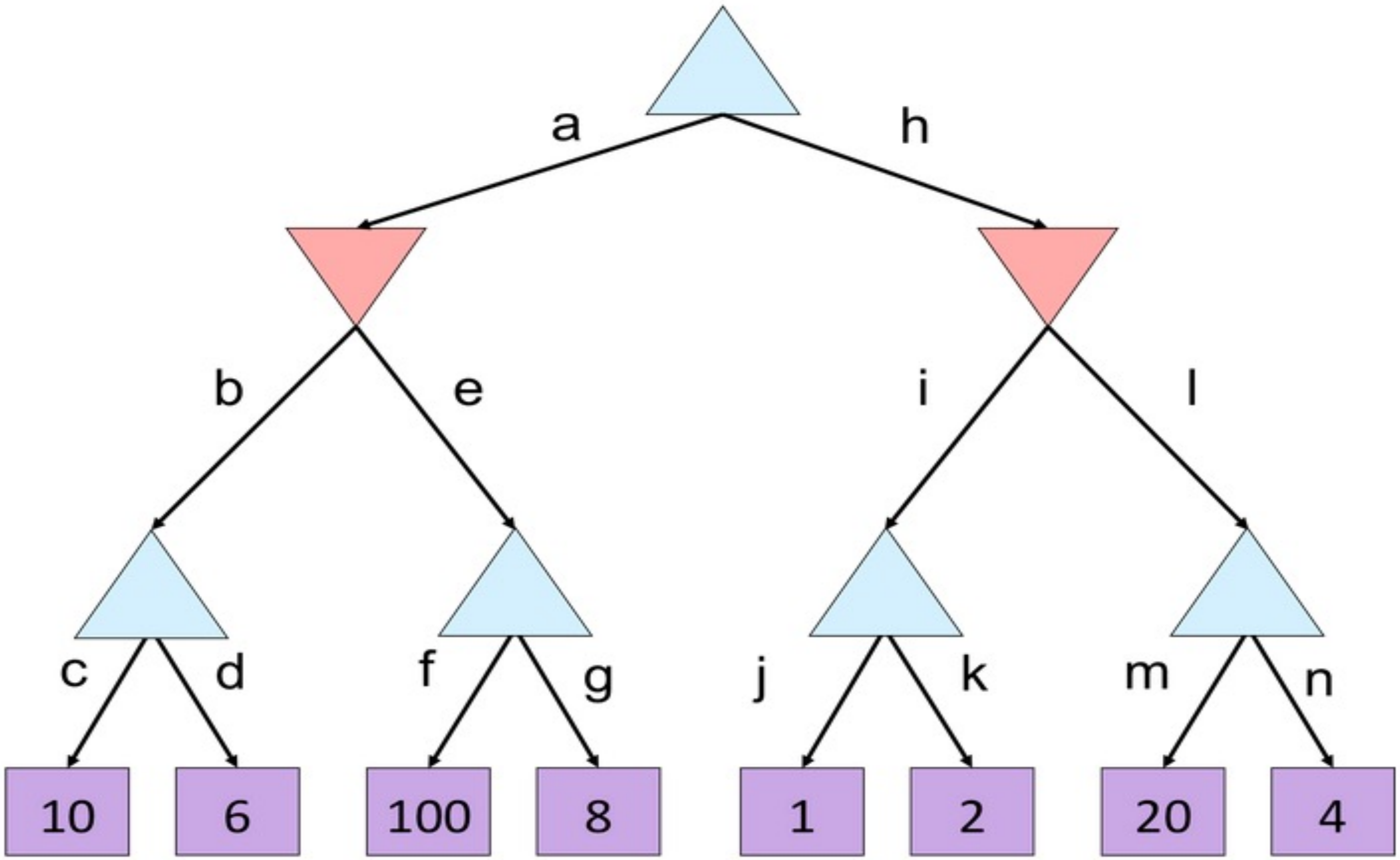
- This pruning has no effect on minimax value computed for the root!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection
- Good child ordering improves effectiveness of pruning
- With "perfect ordering":
  - **Time complexity drops to $O(b^{m/2})$**
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…
- This is a simple example of metareasoning (computing about what to compute)



Alpha-Beta Quiz

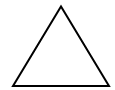


Alpha-Beta Quiz 2



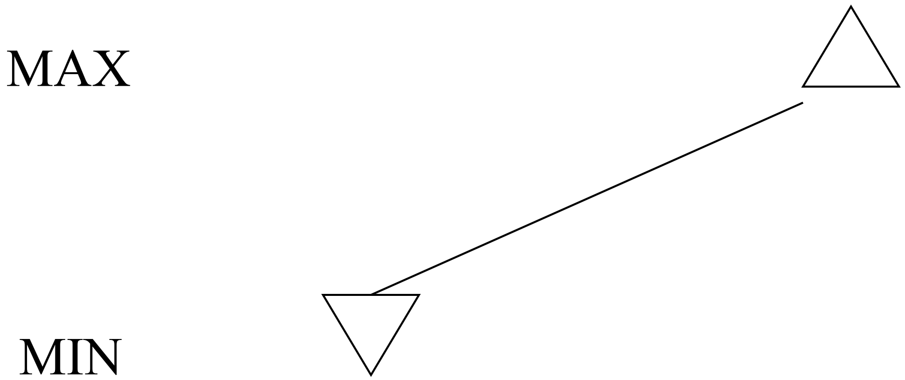
Another alpha-beta example

MAX



MIN

Another alpha-beta example



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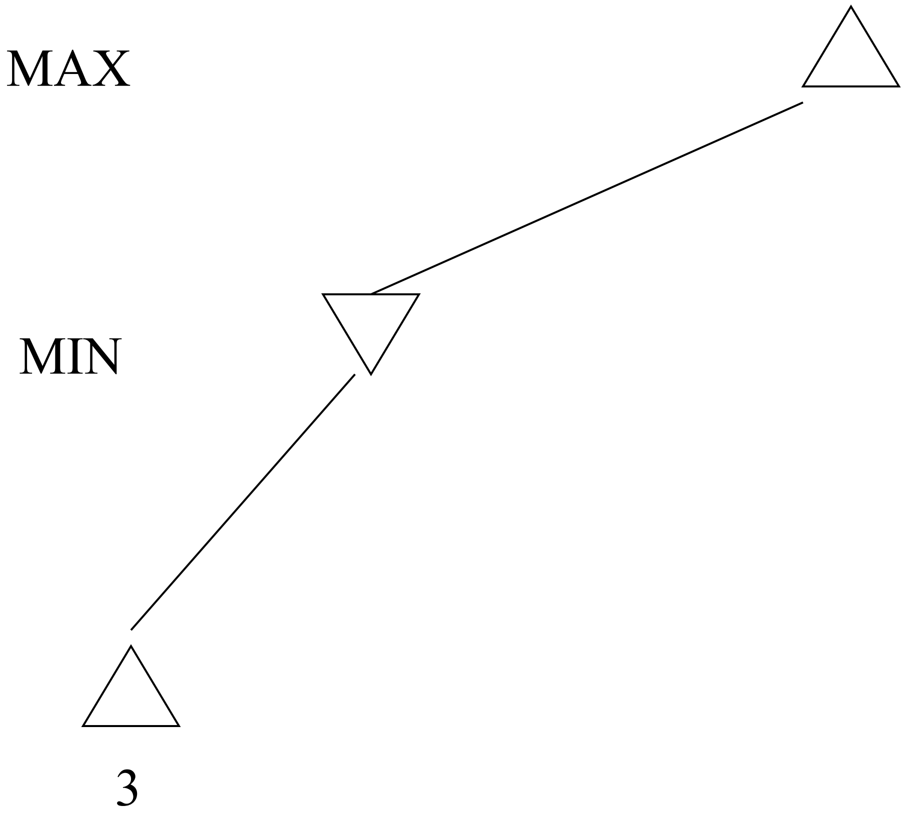

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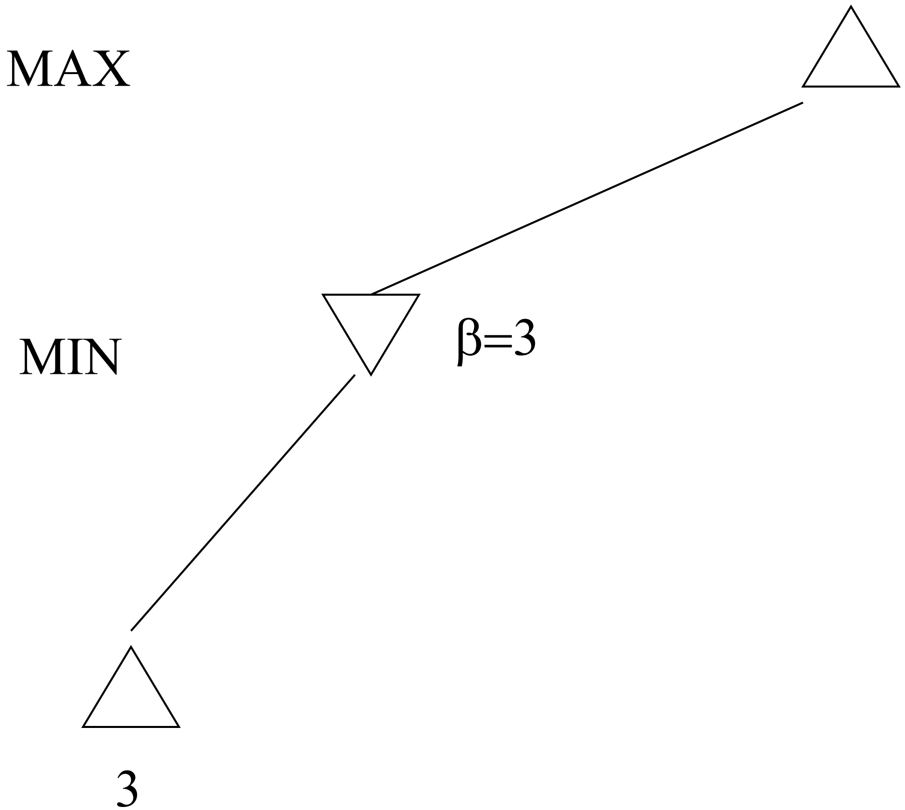

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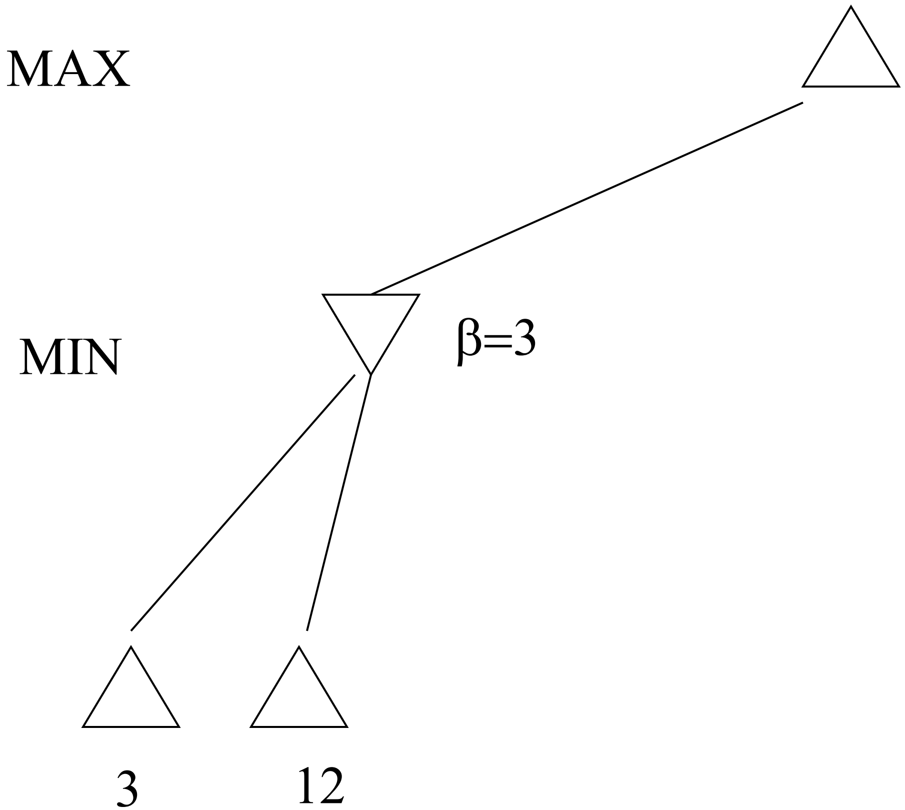

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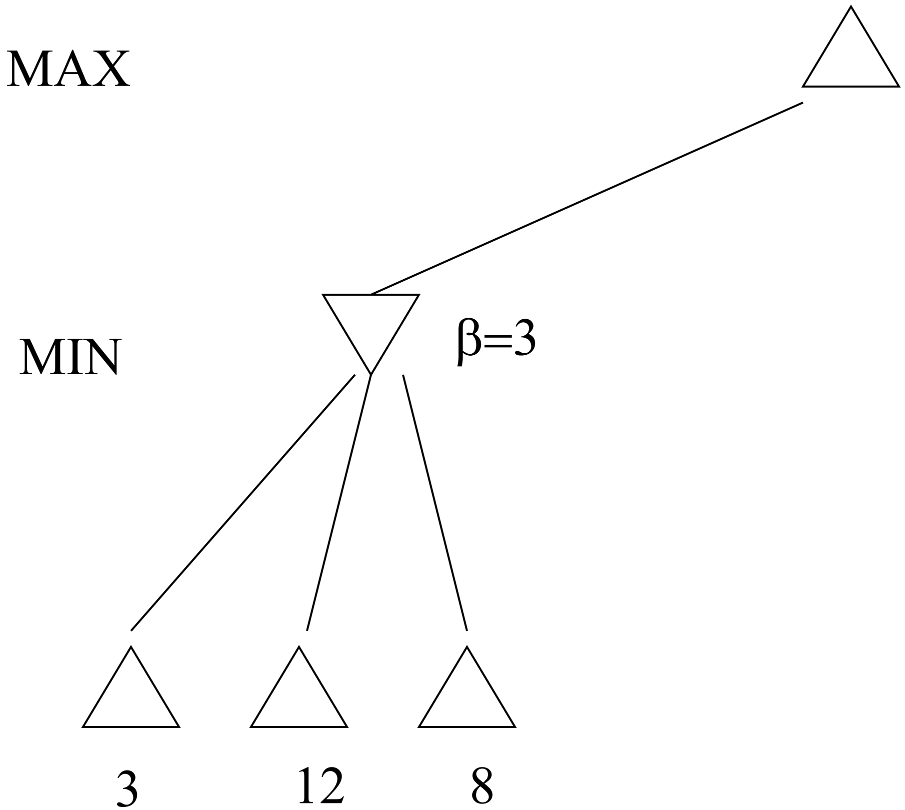

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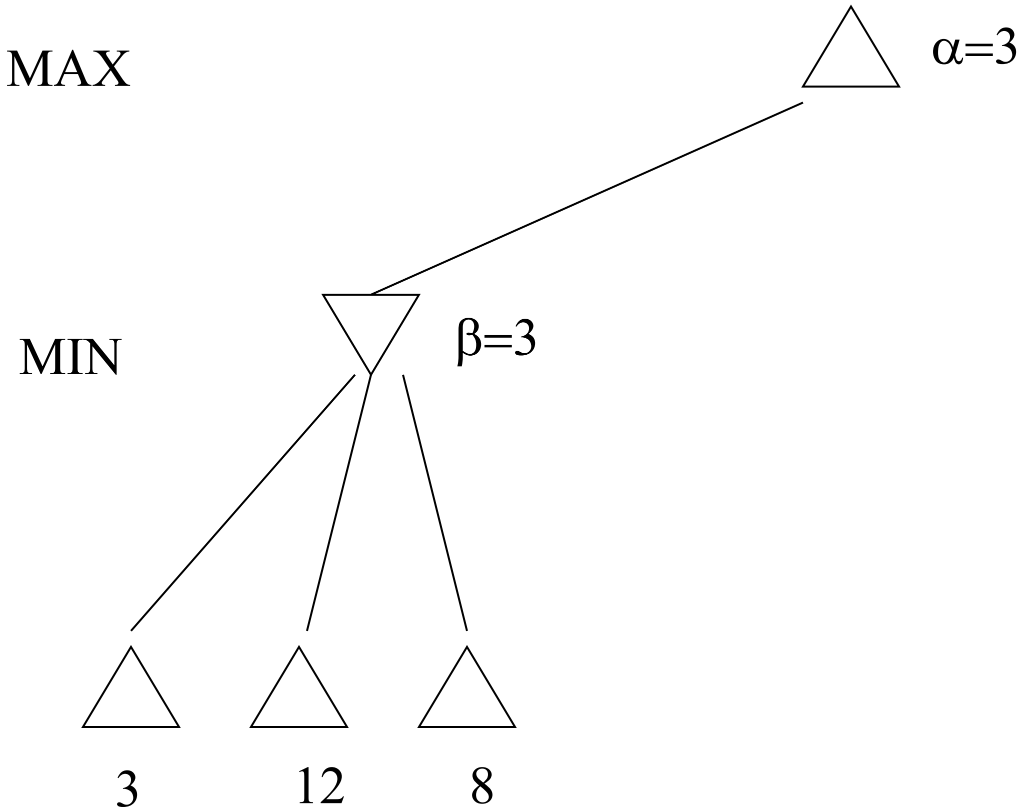

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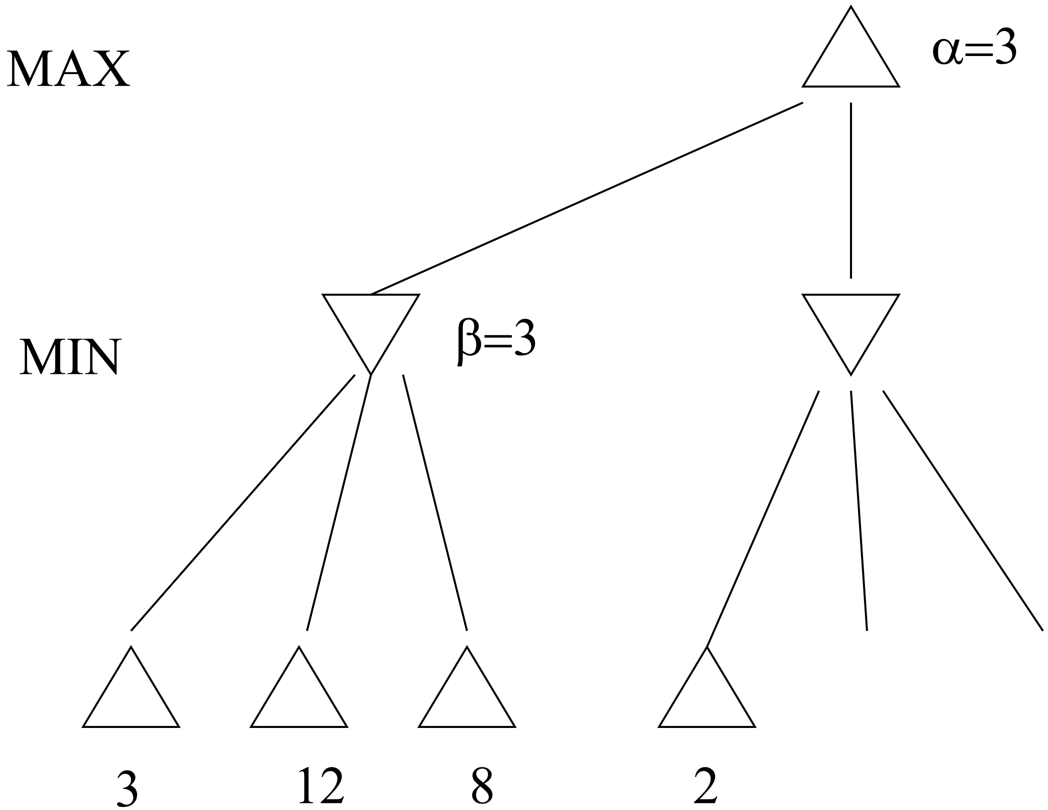

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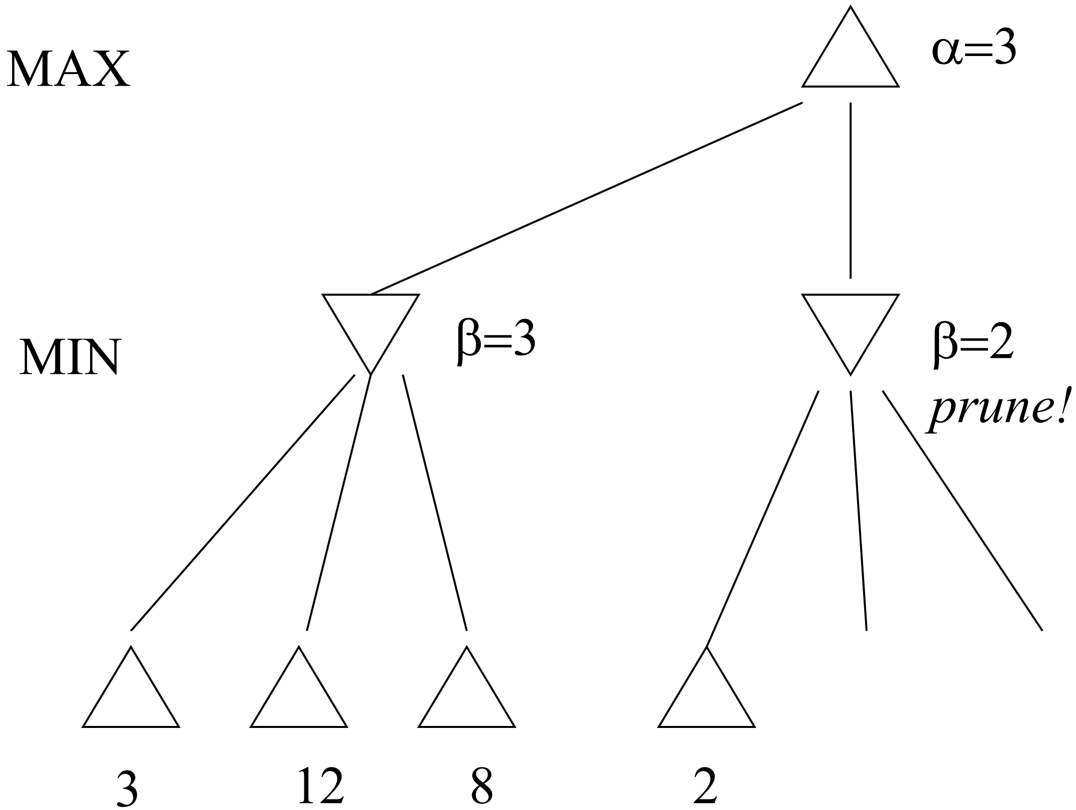

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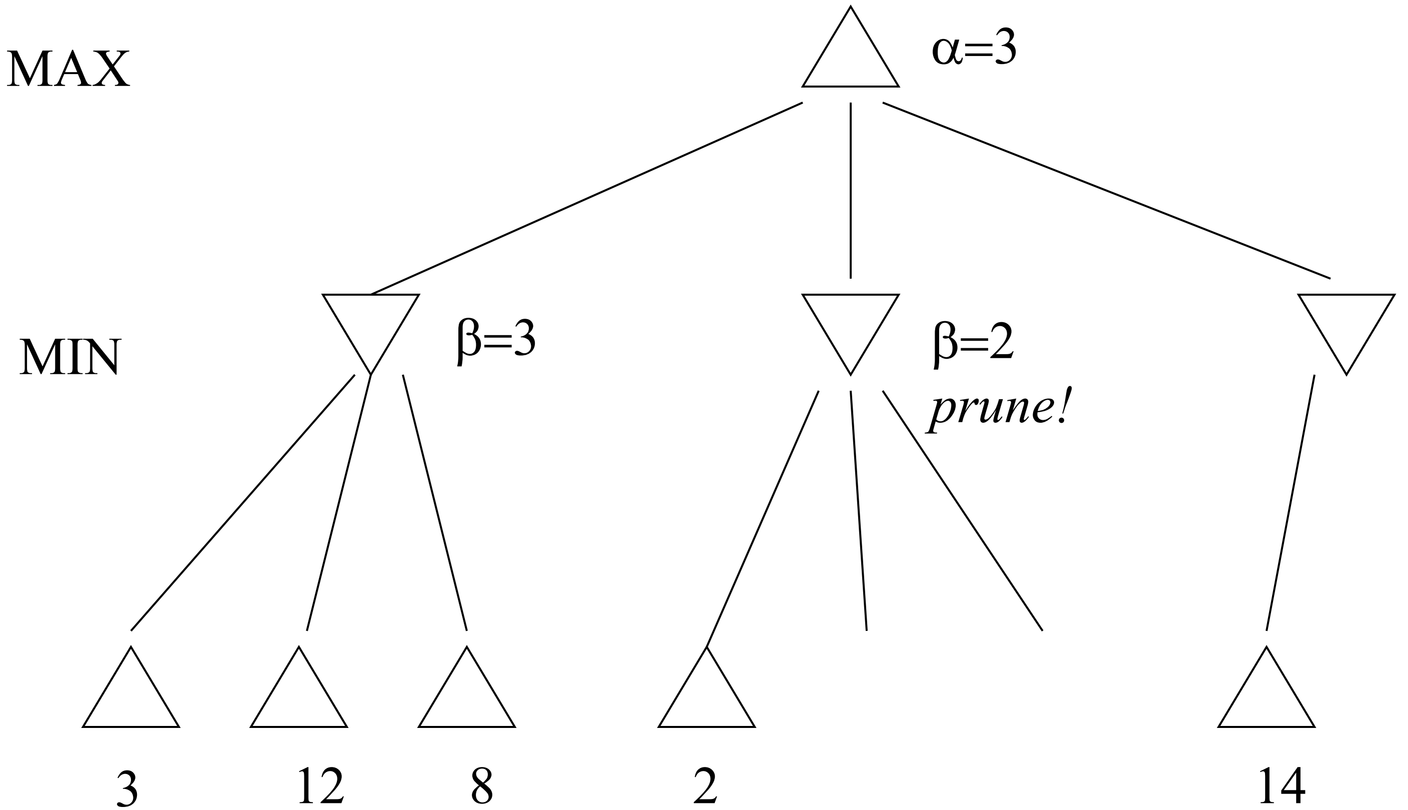

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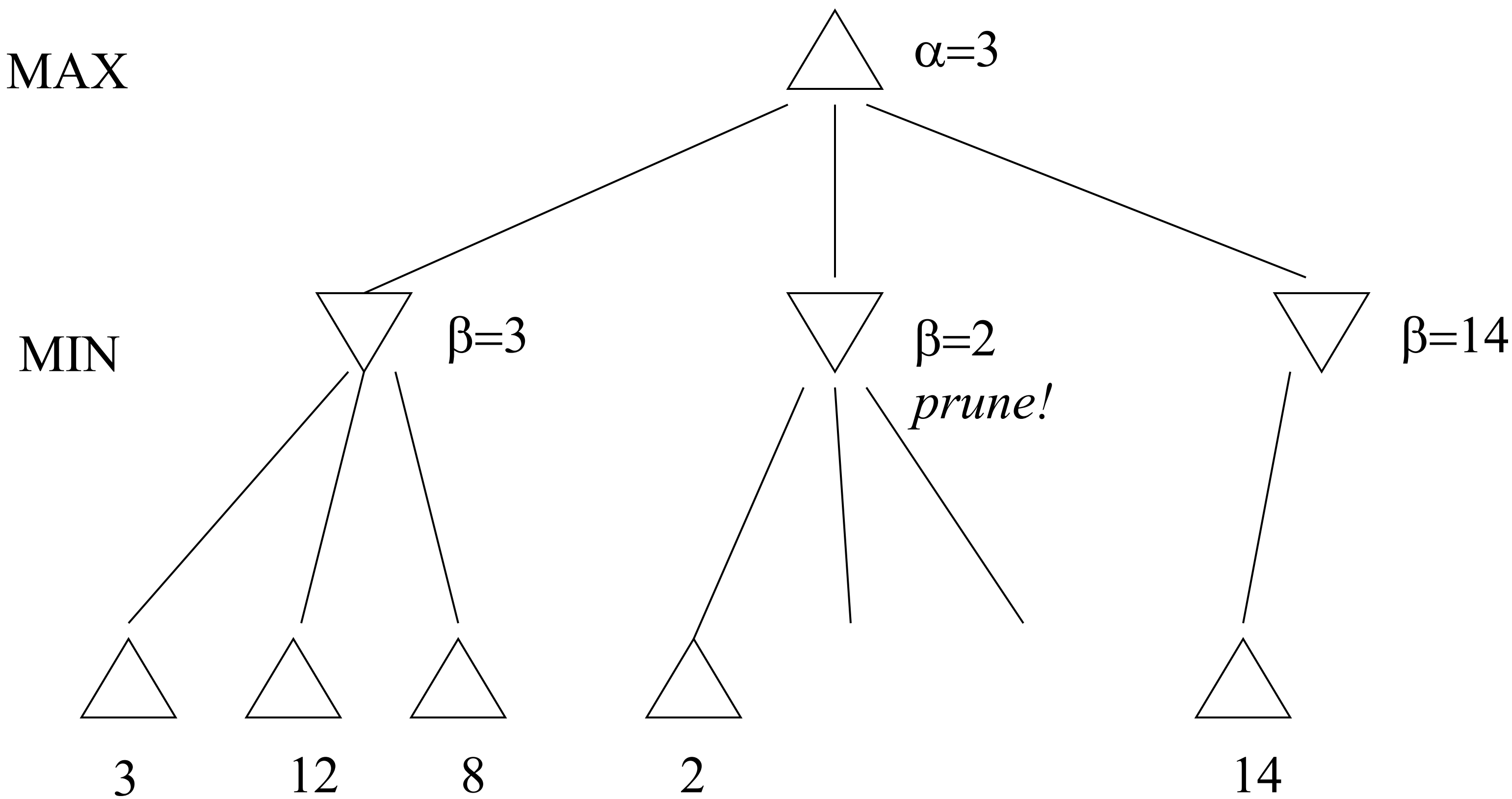

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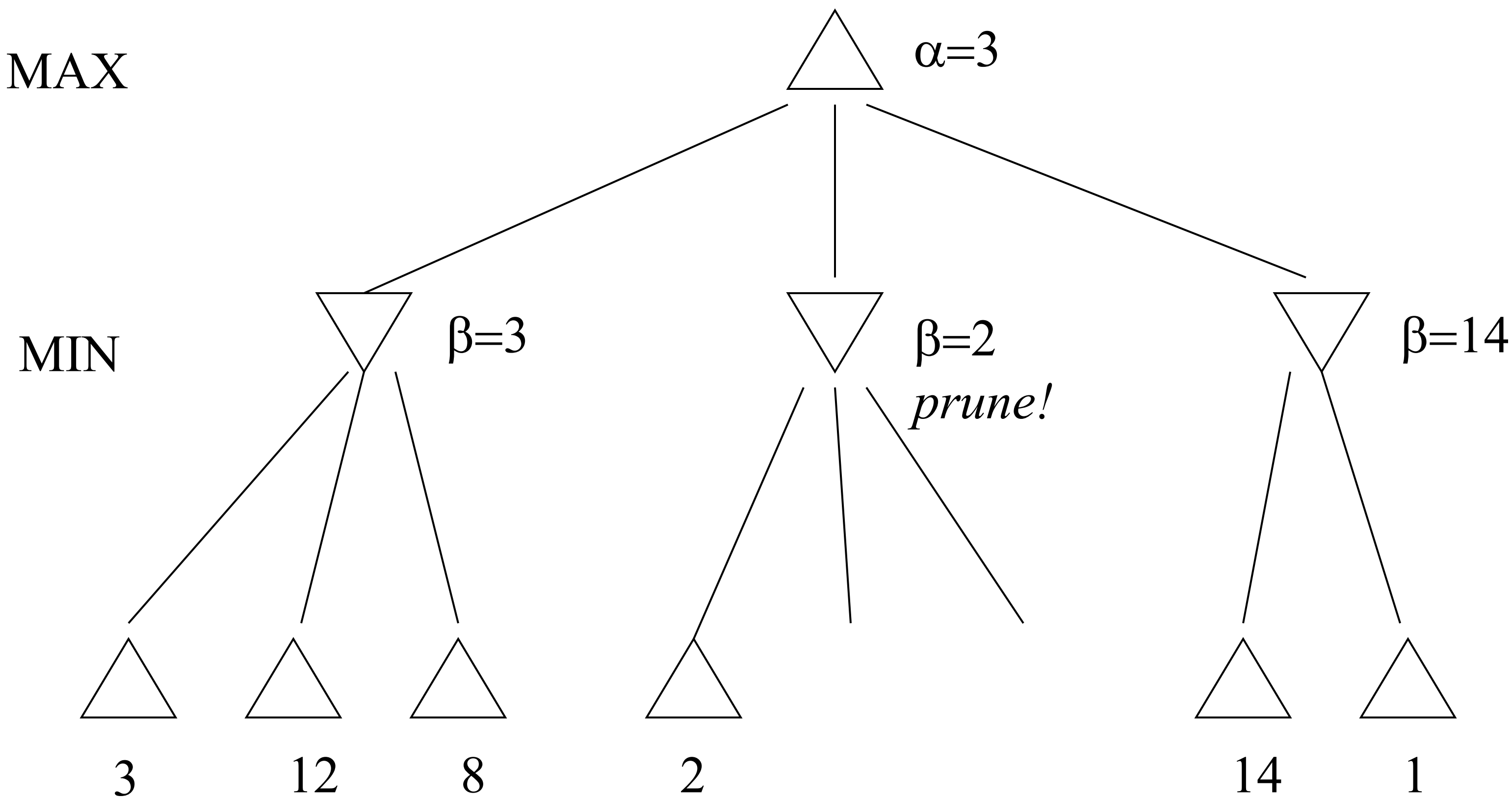

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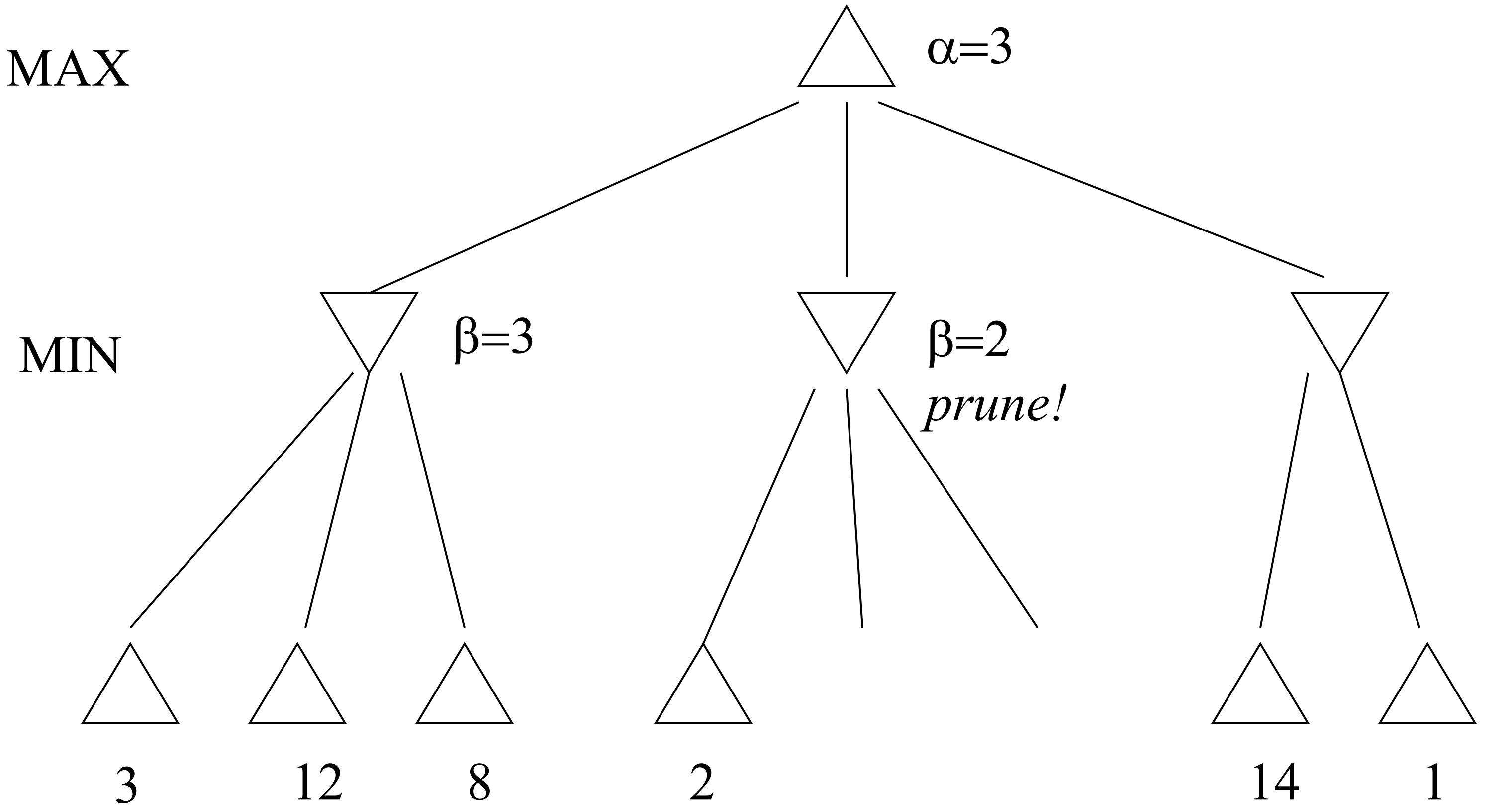

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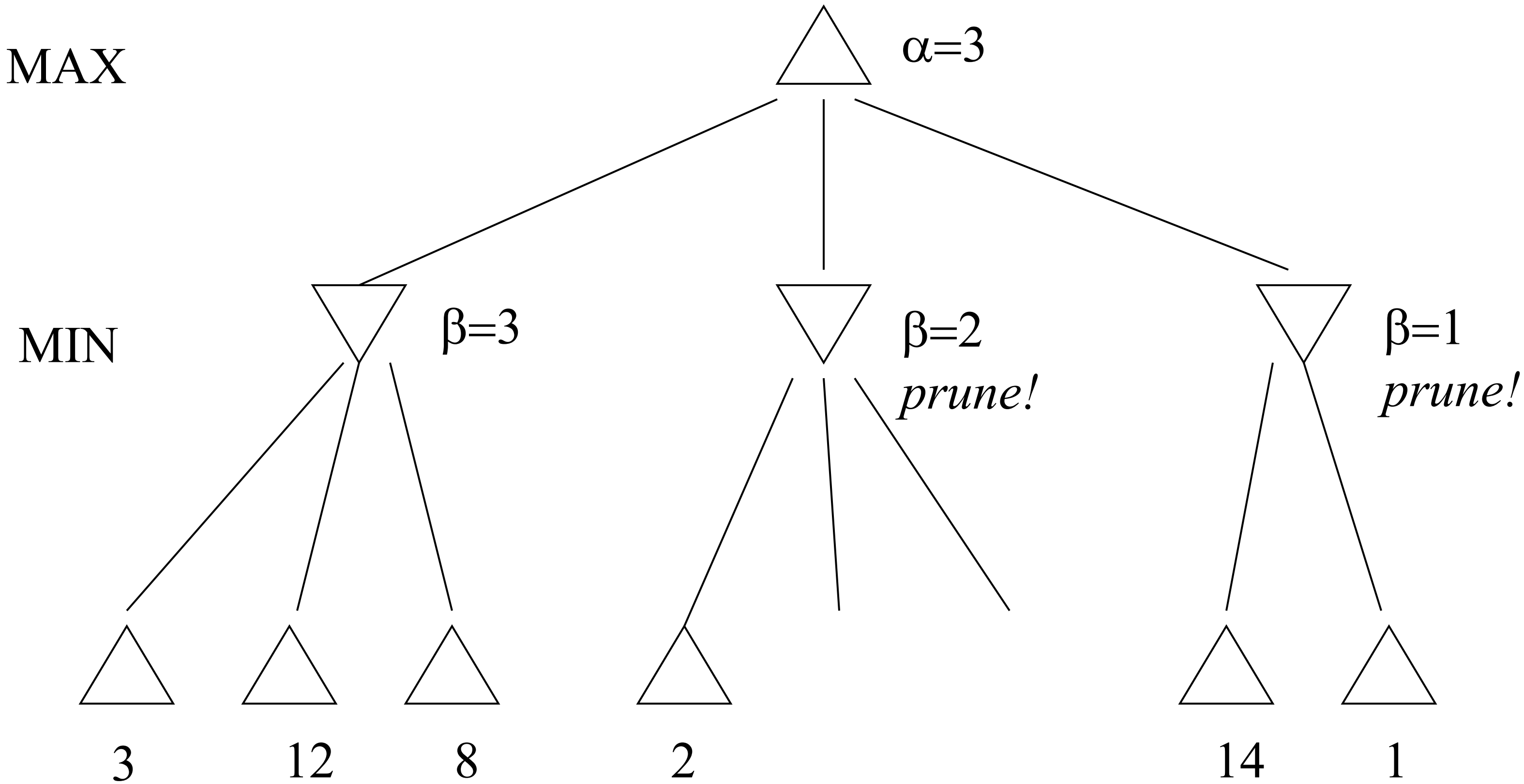

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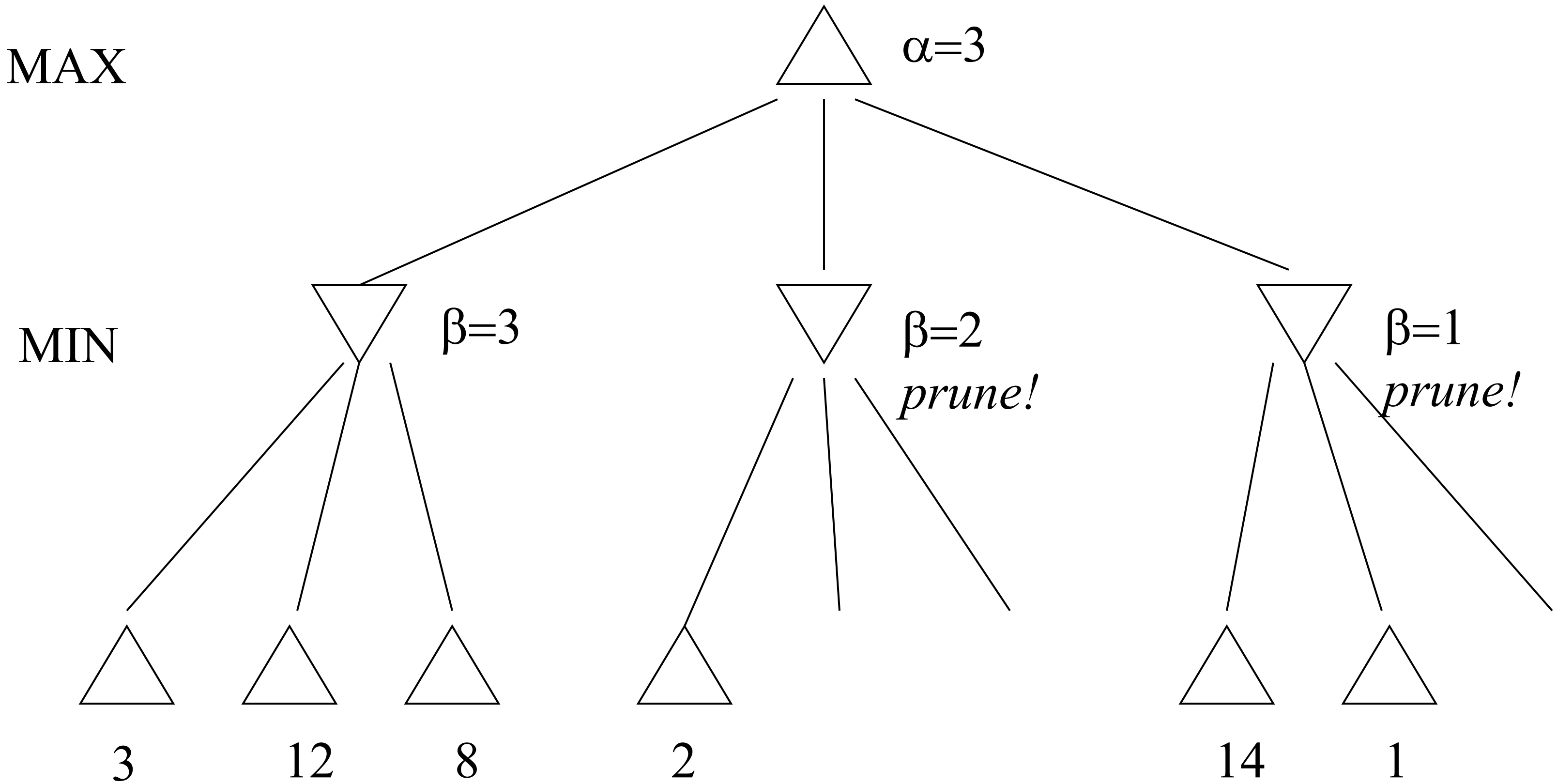

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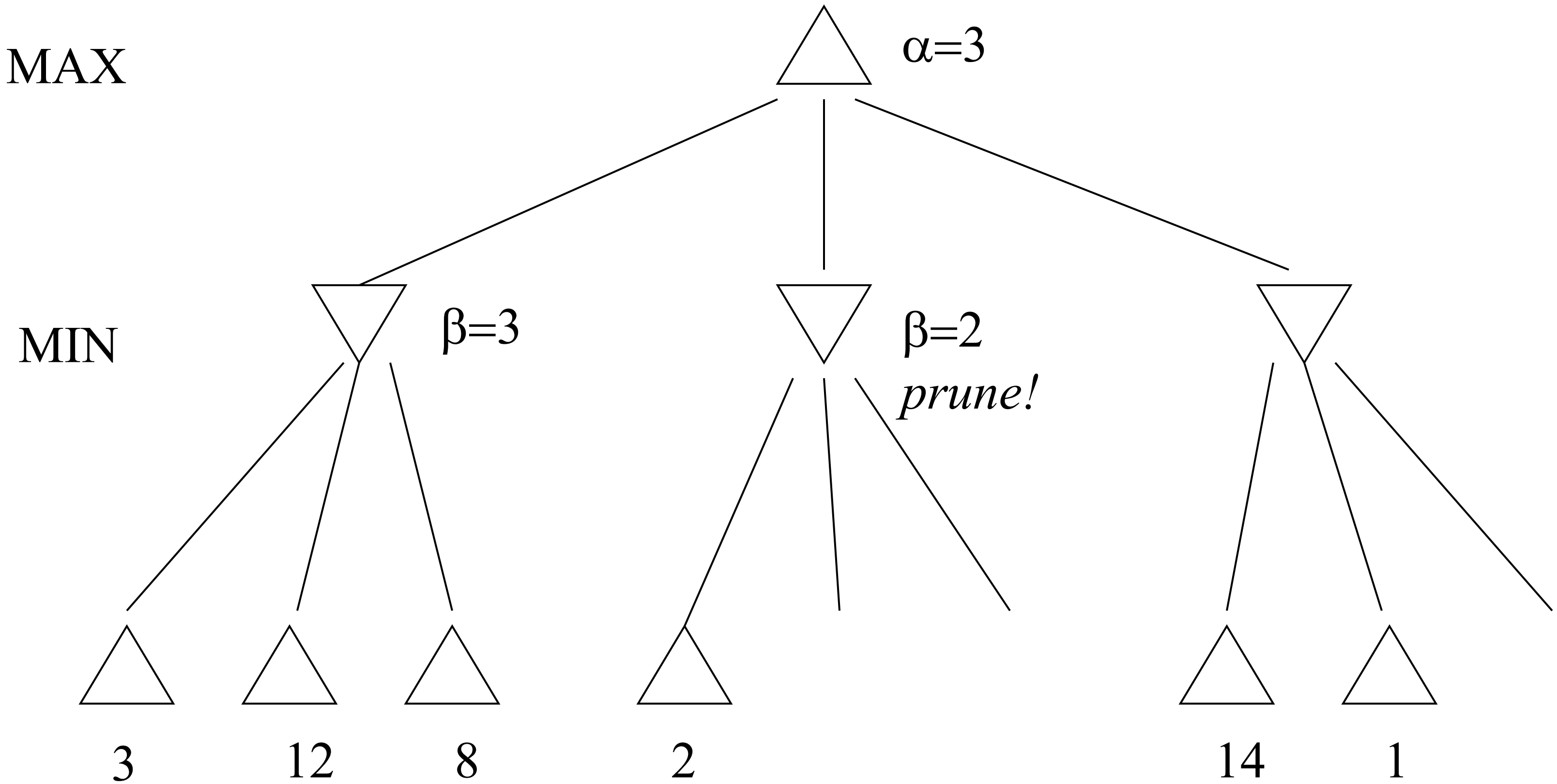
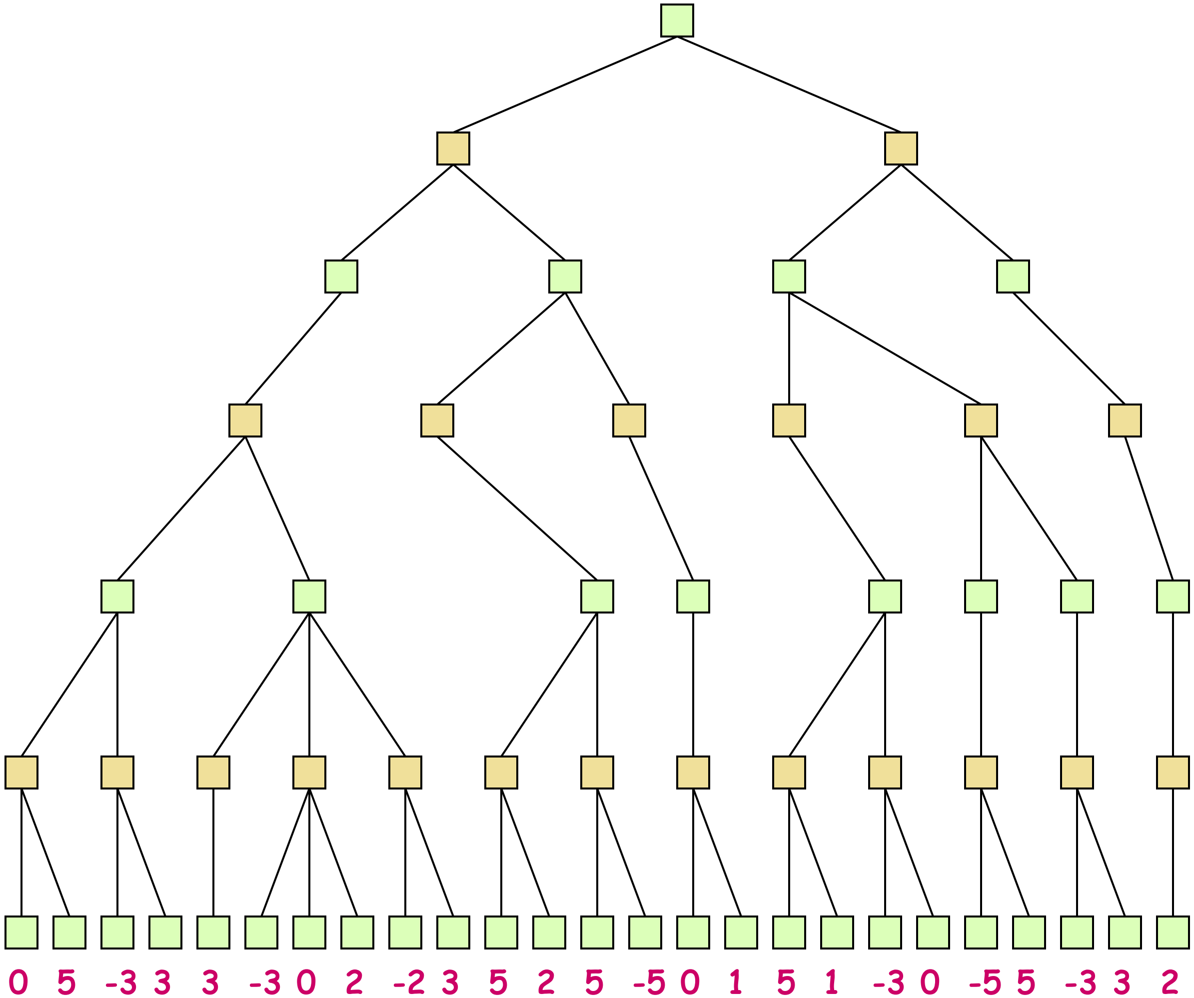
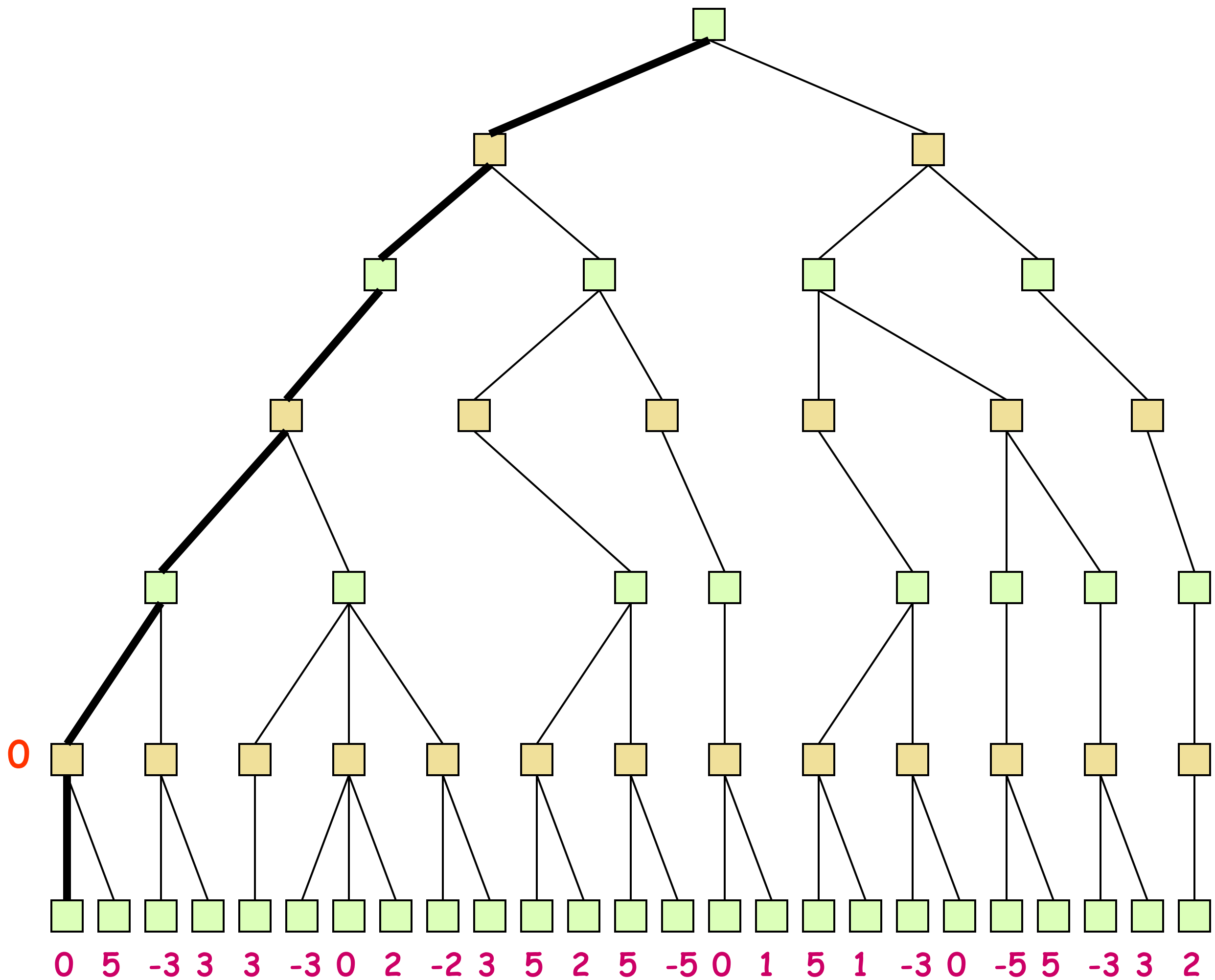


Alpha-Beta Tic-Tac-Toe Example 2





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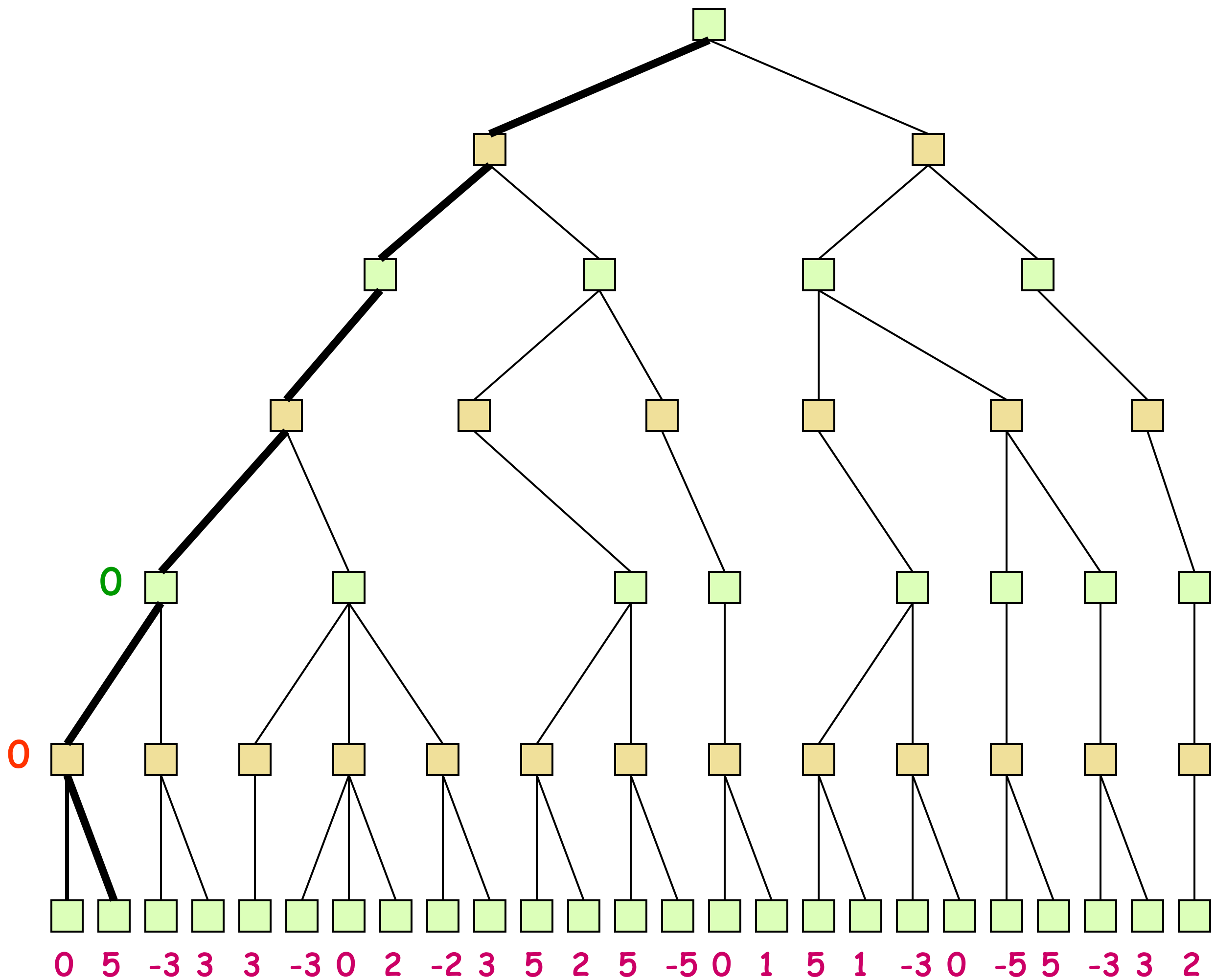
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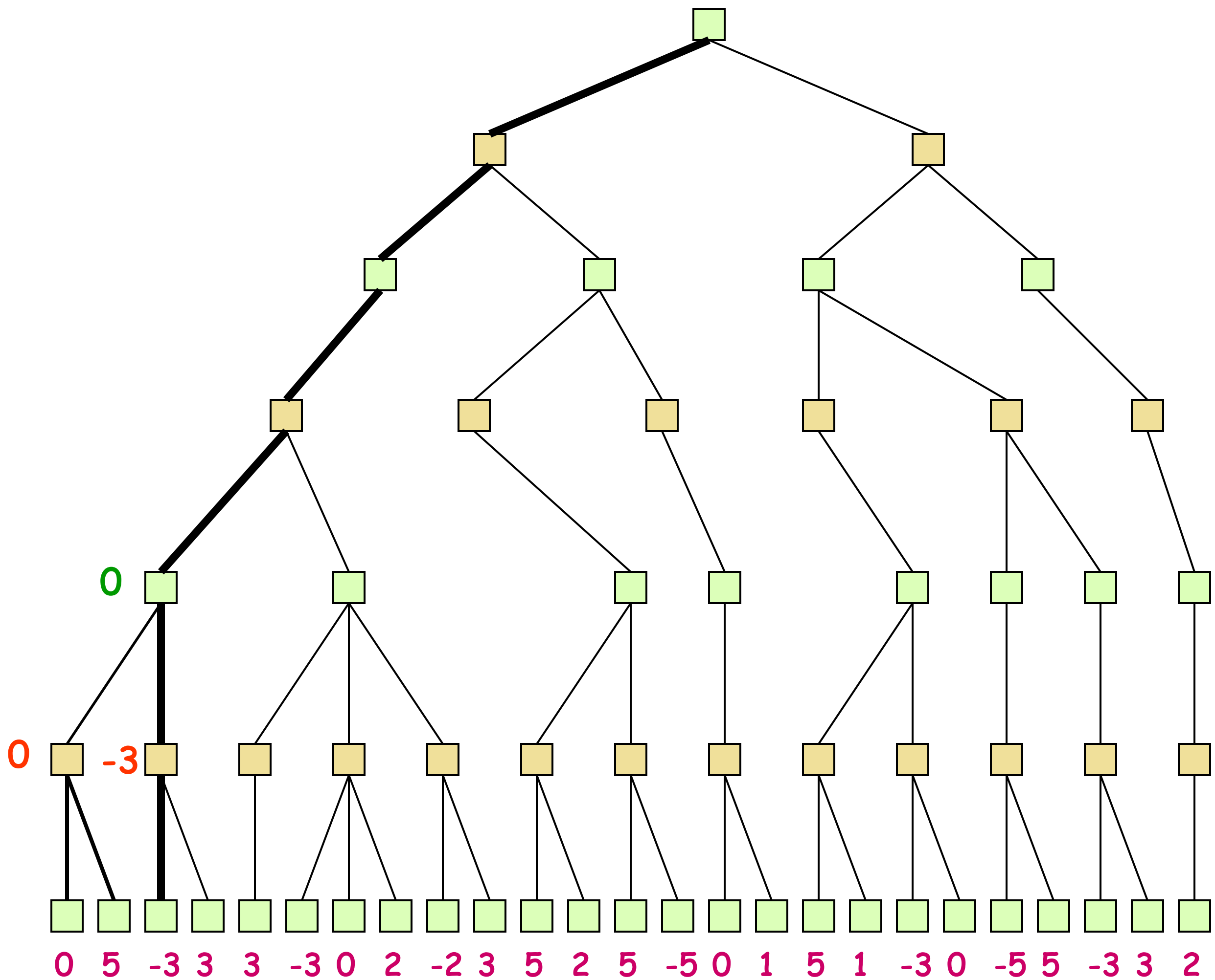

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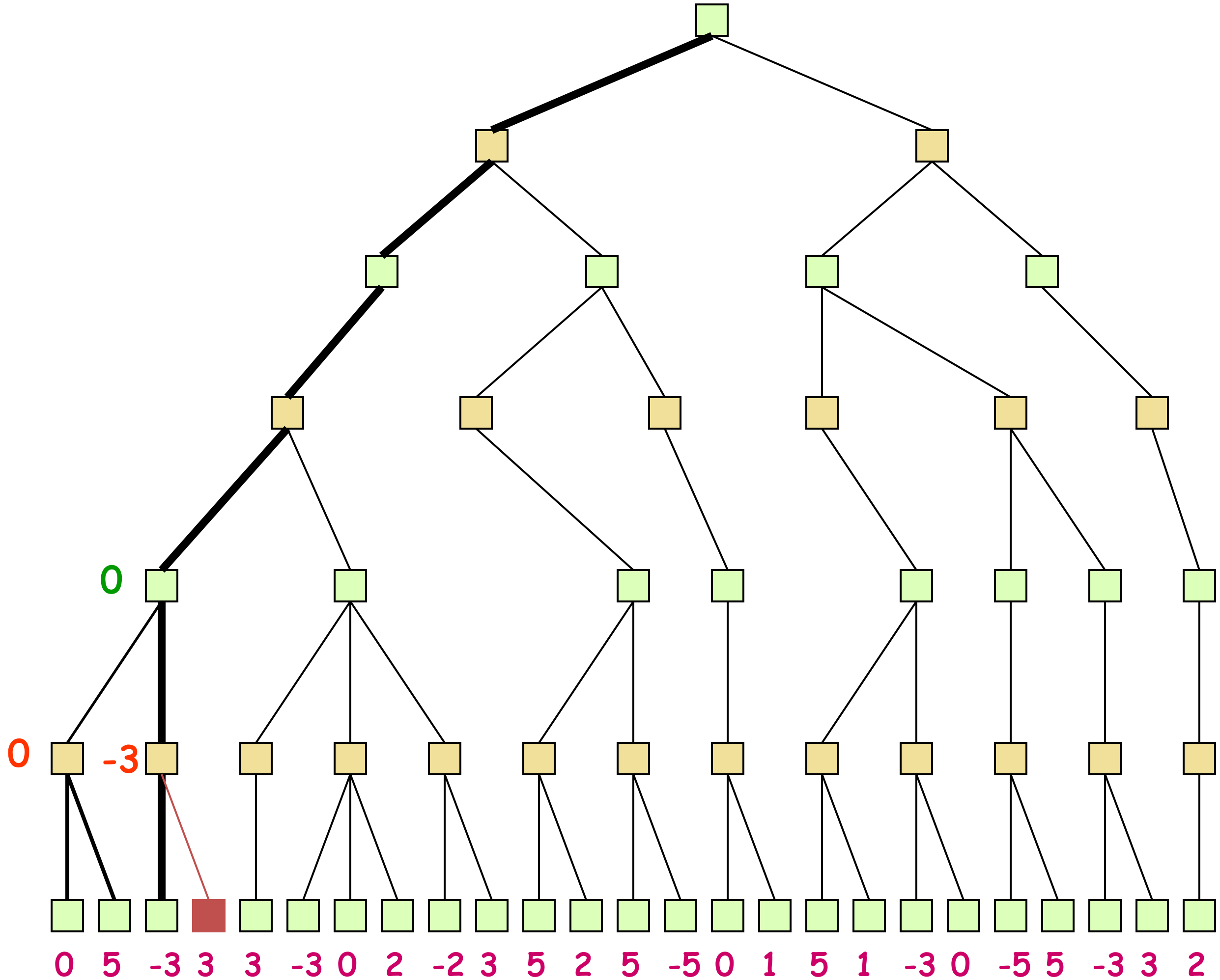
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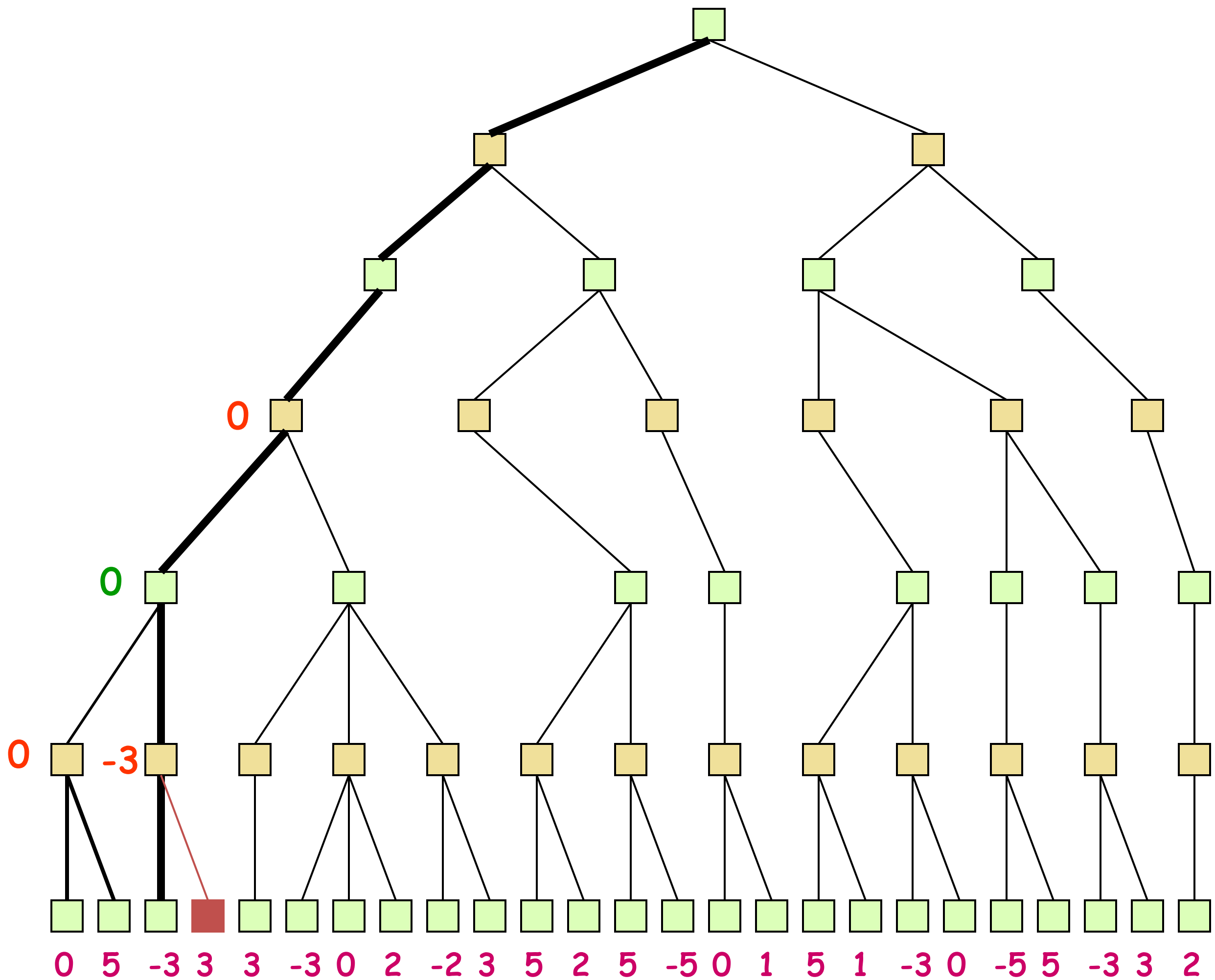
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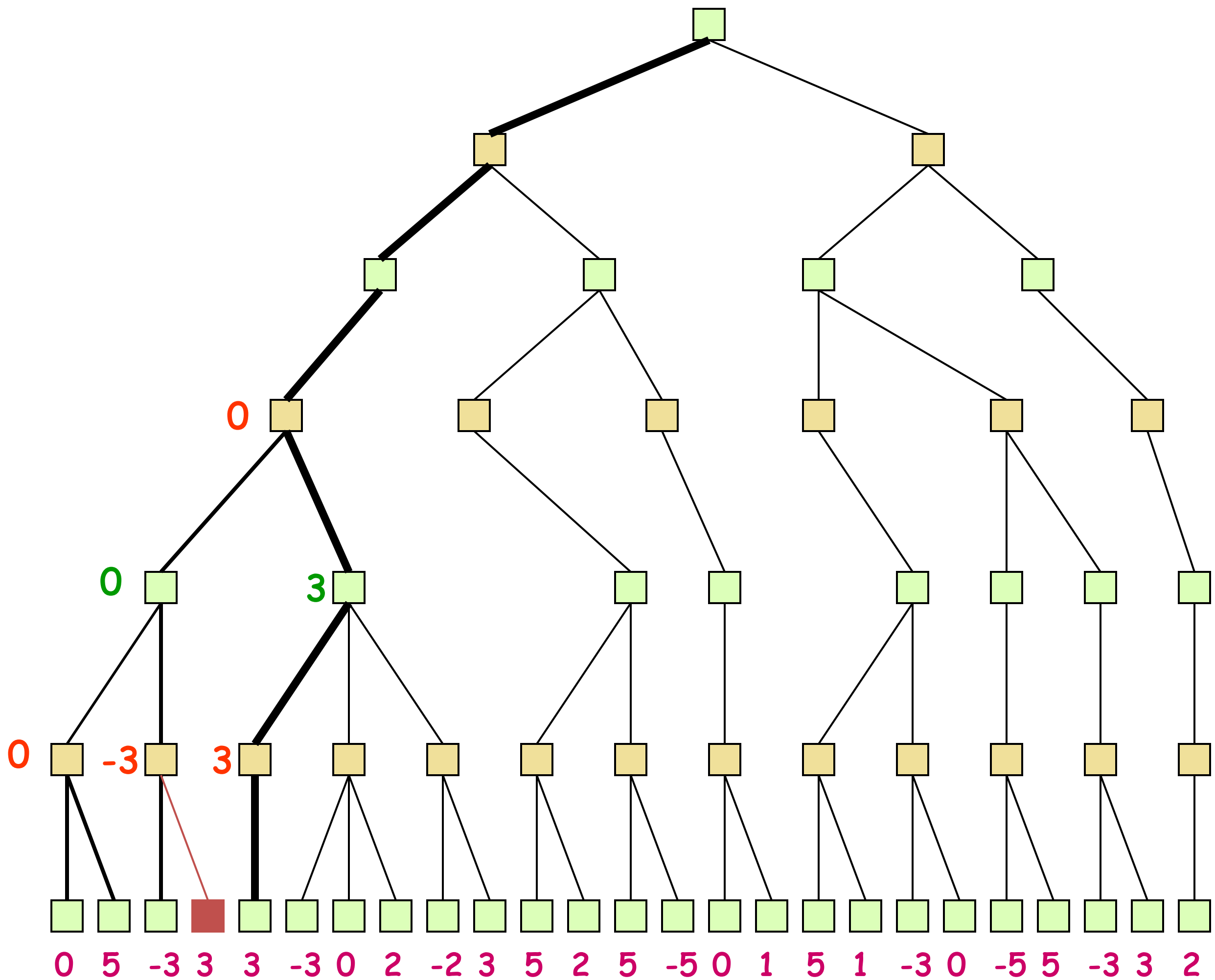
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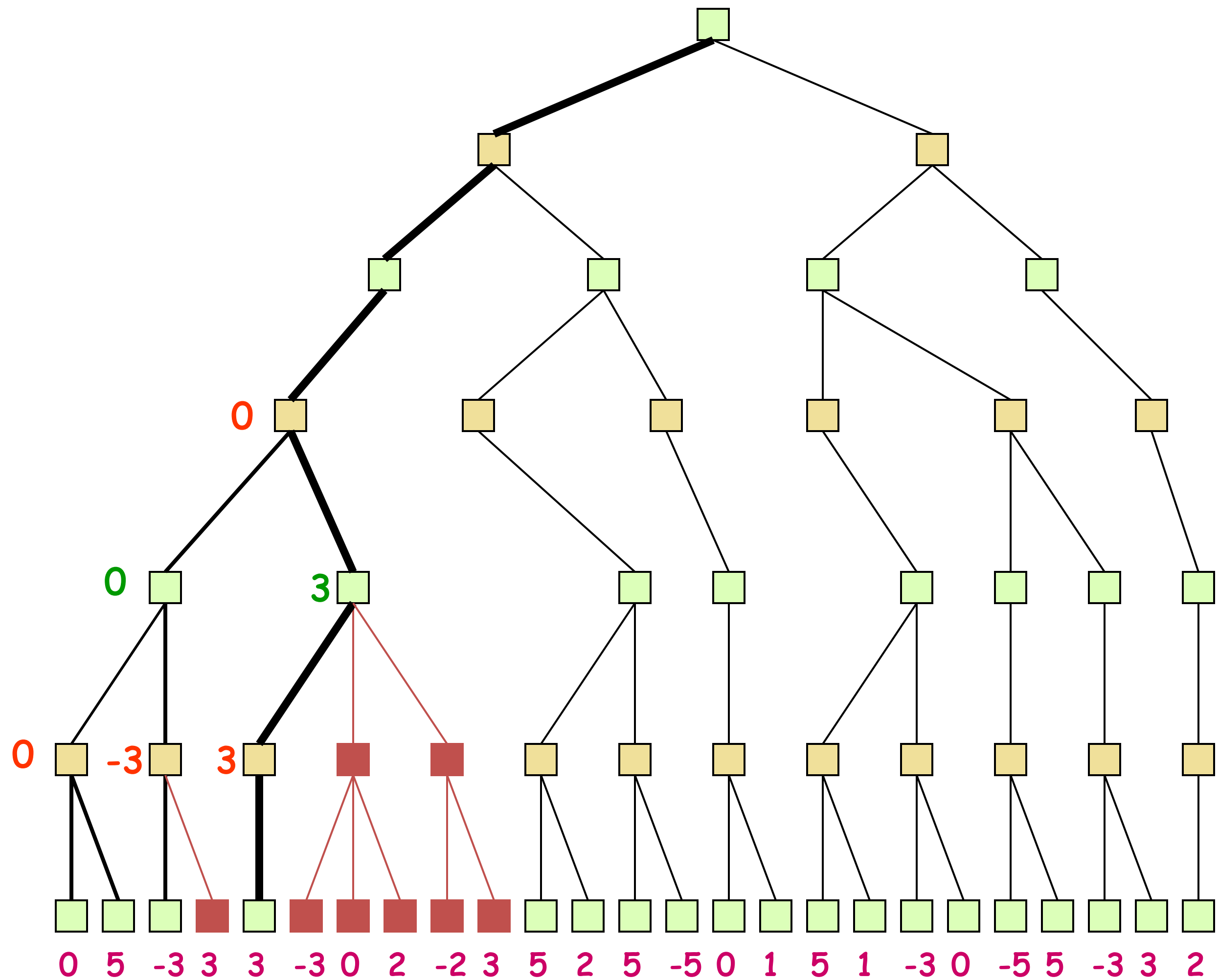
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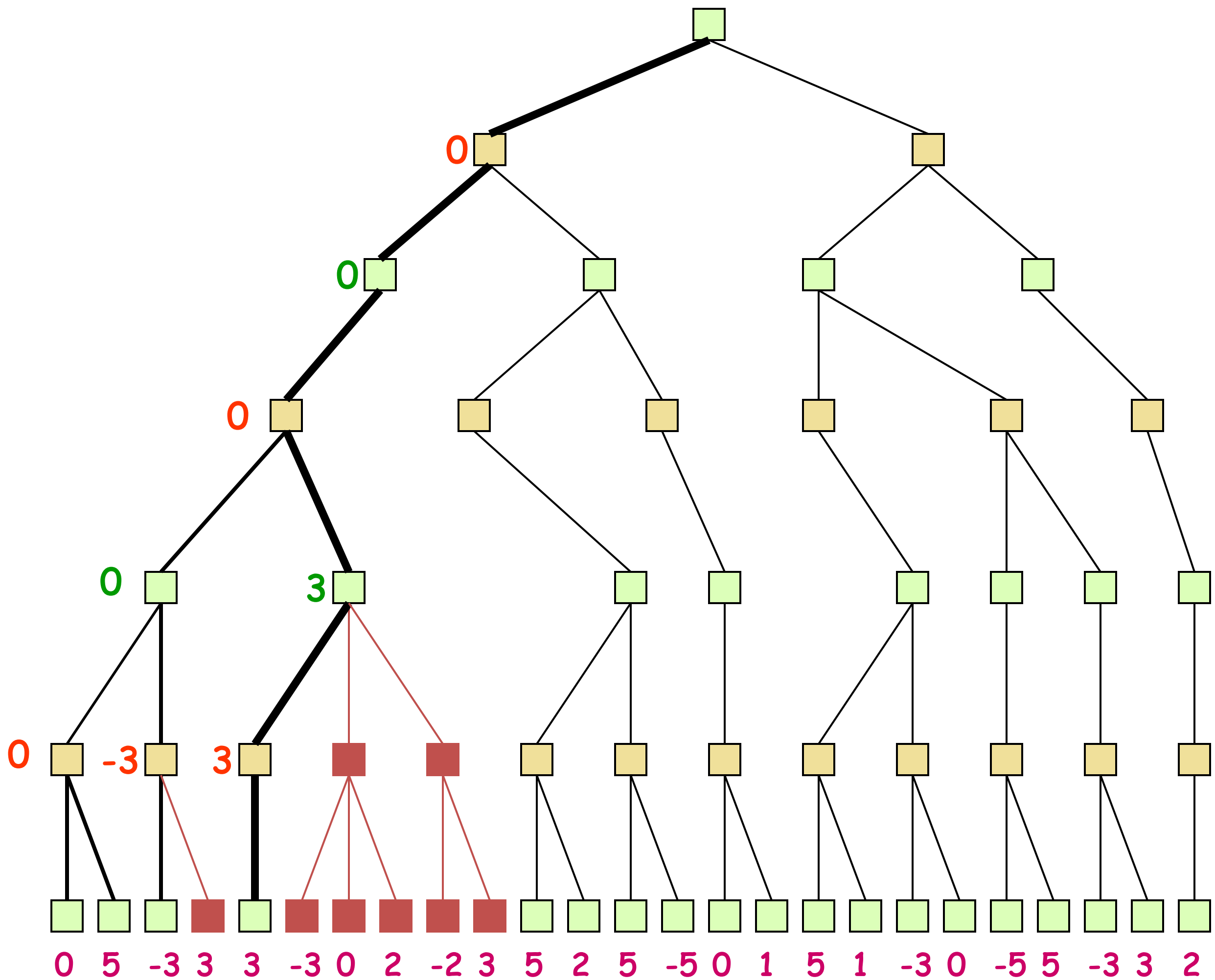
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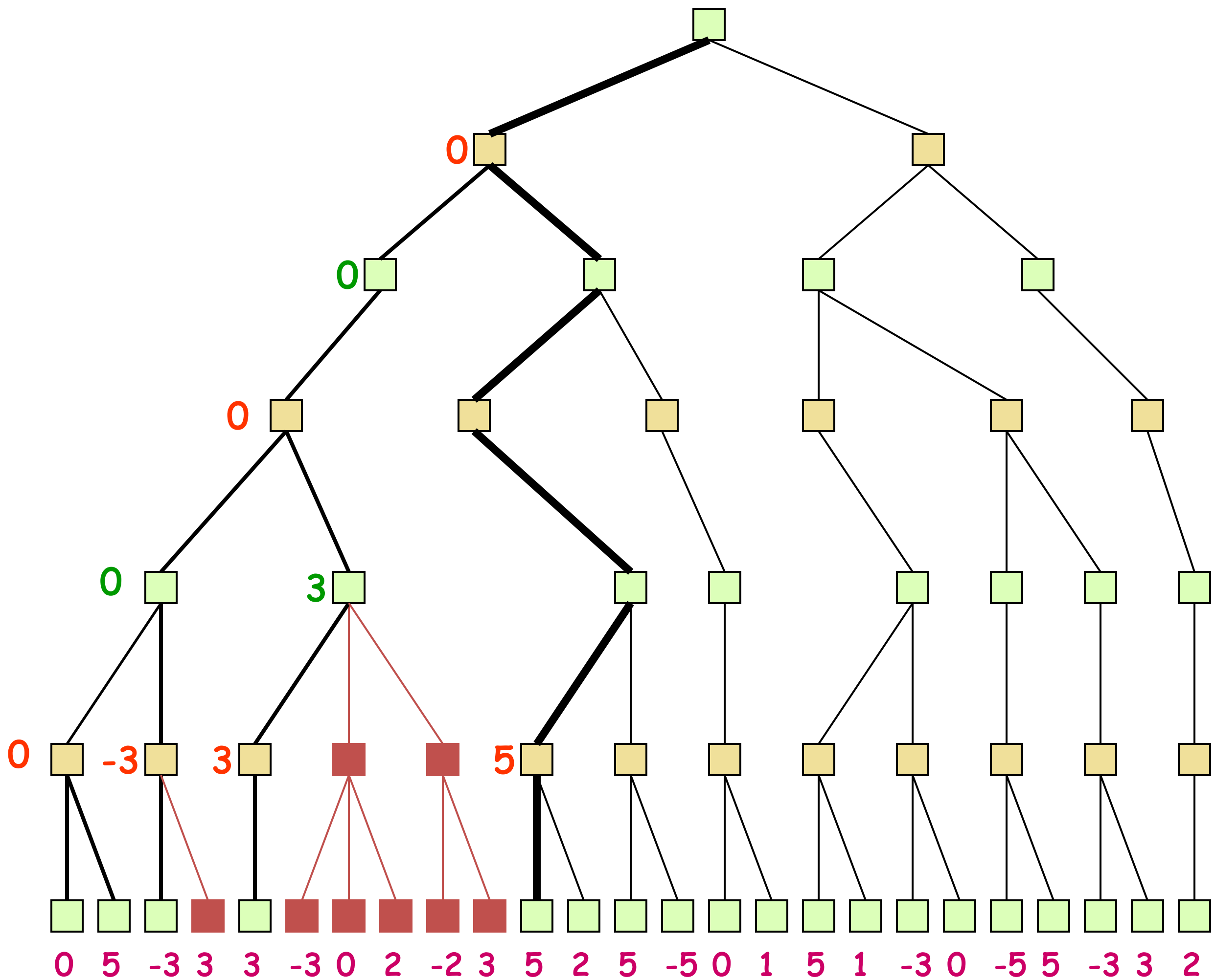
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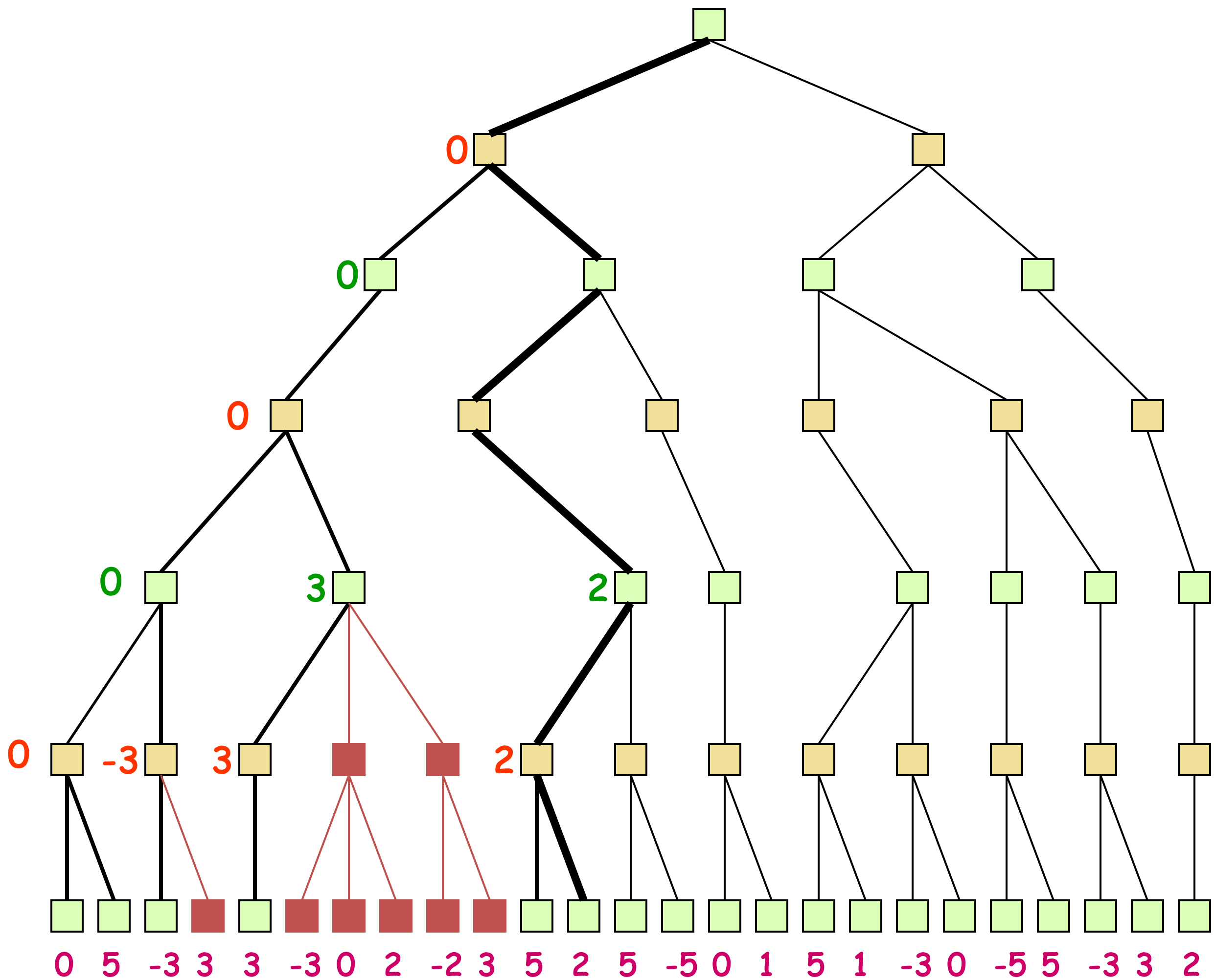
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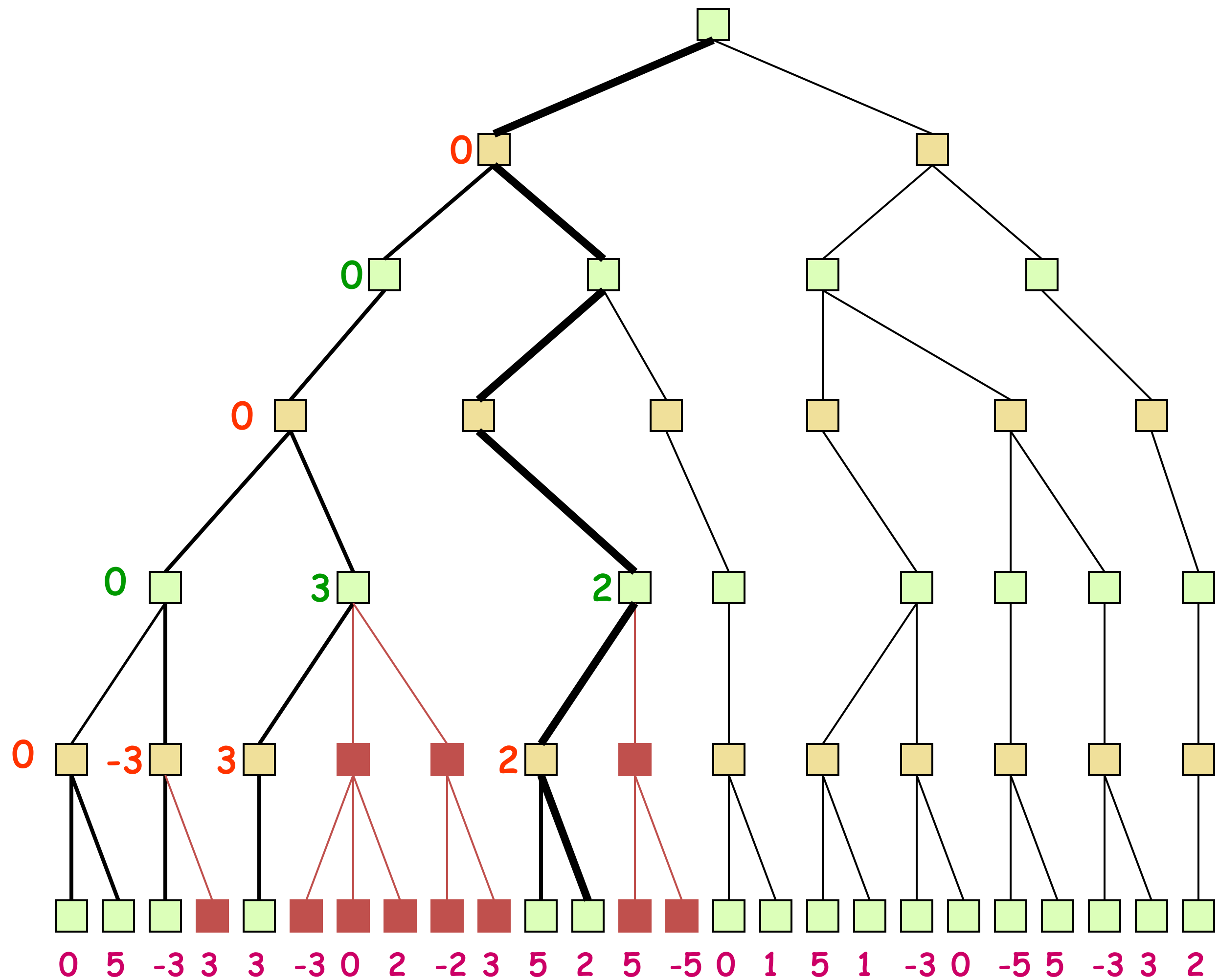
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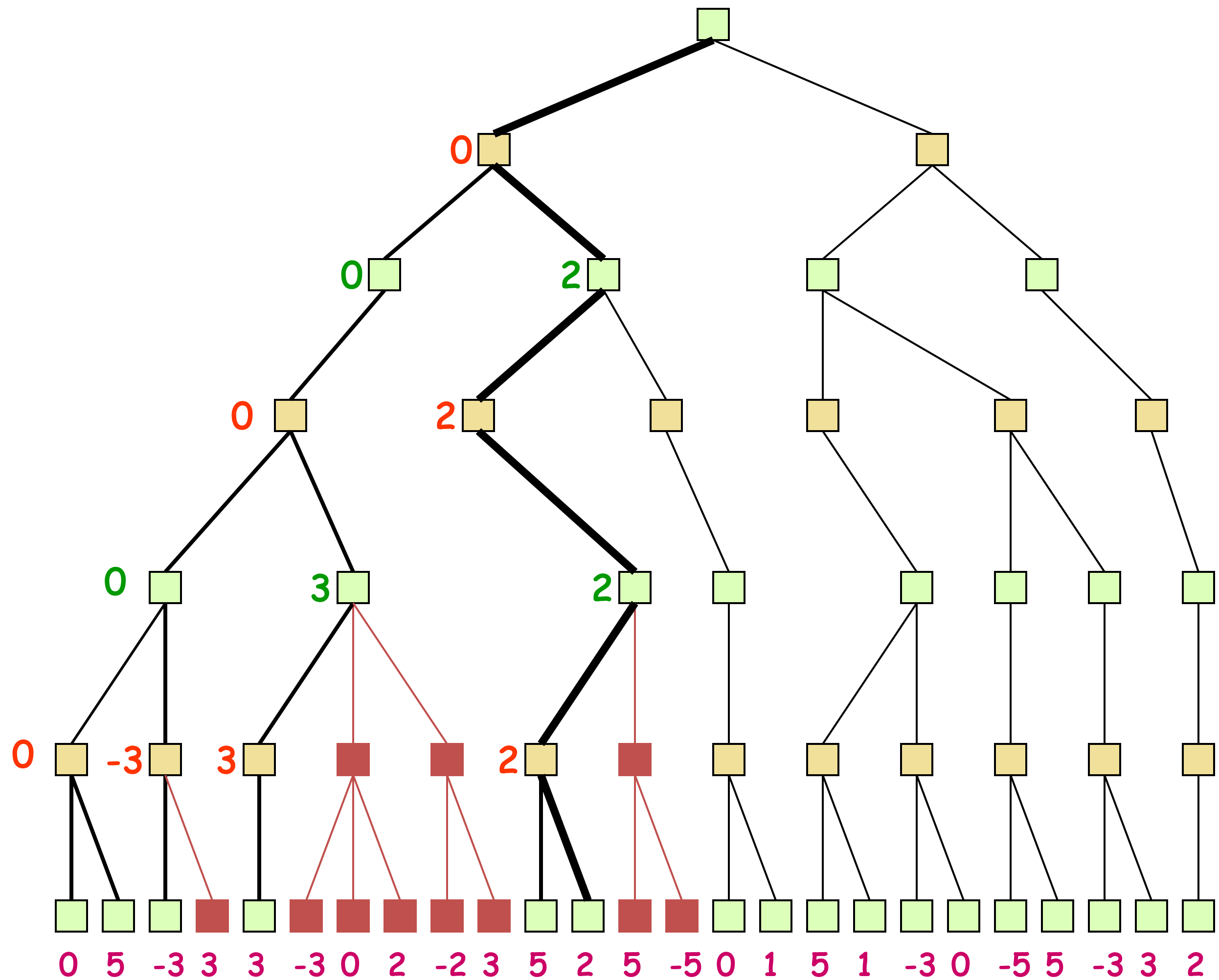
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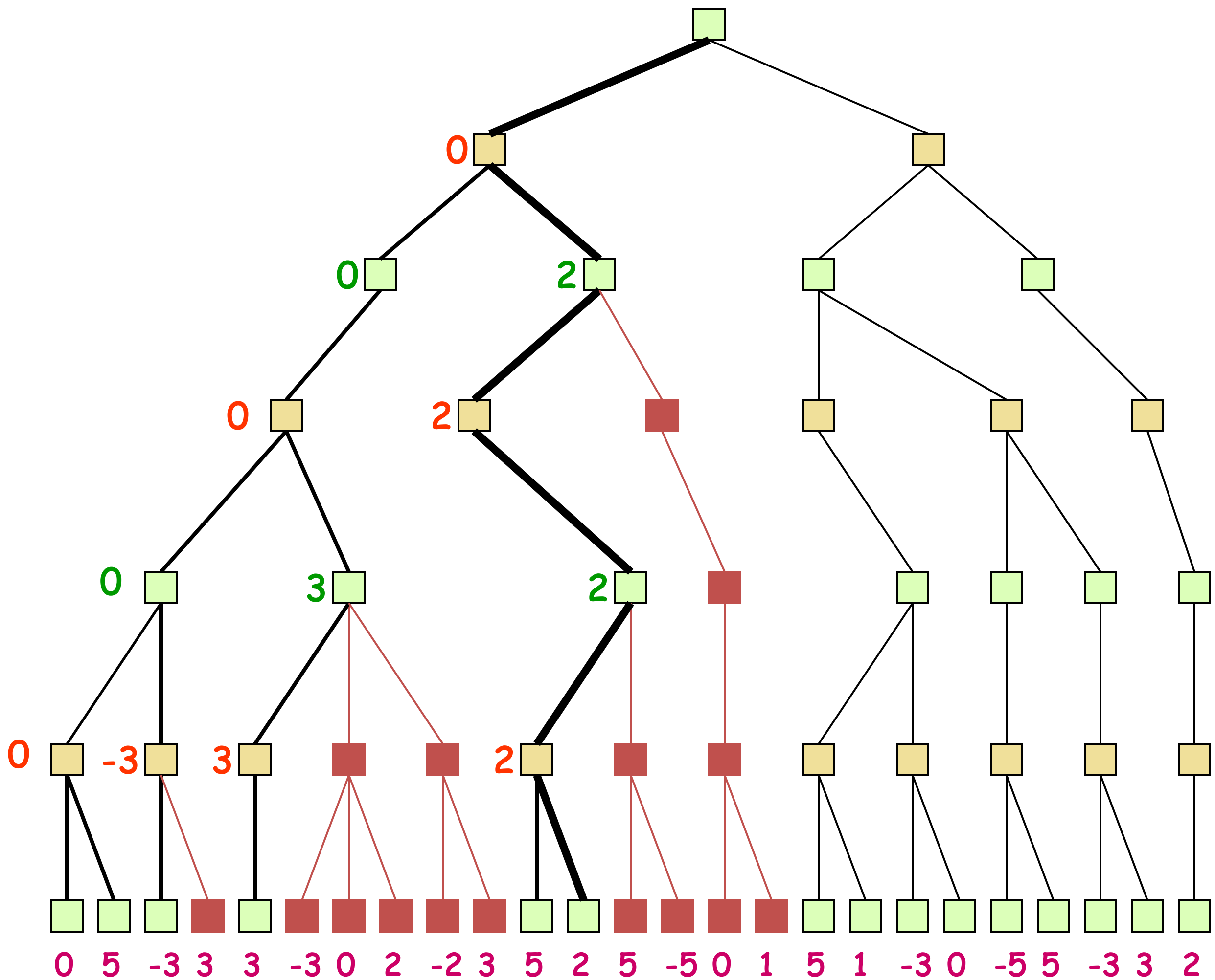
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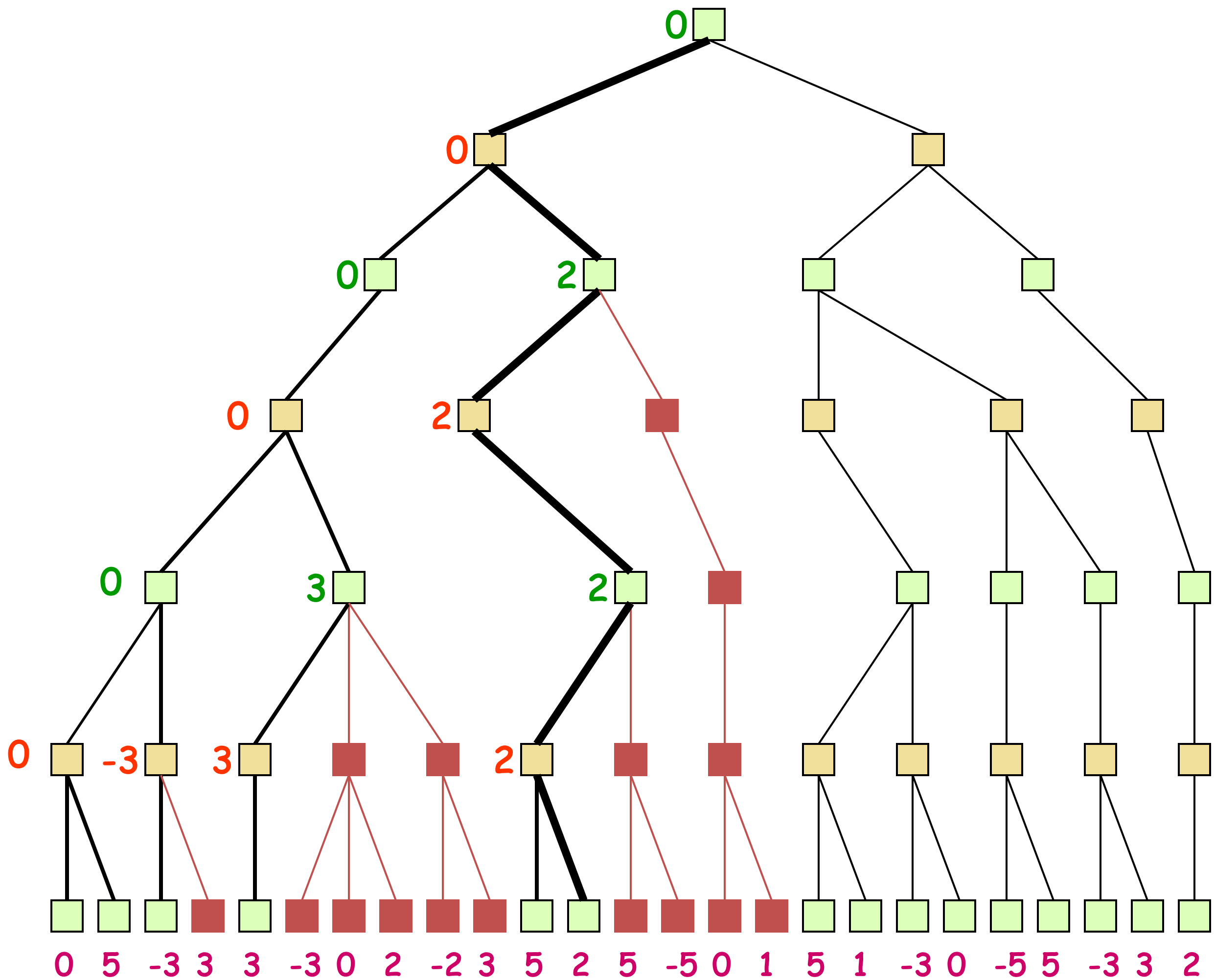

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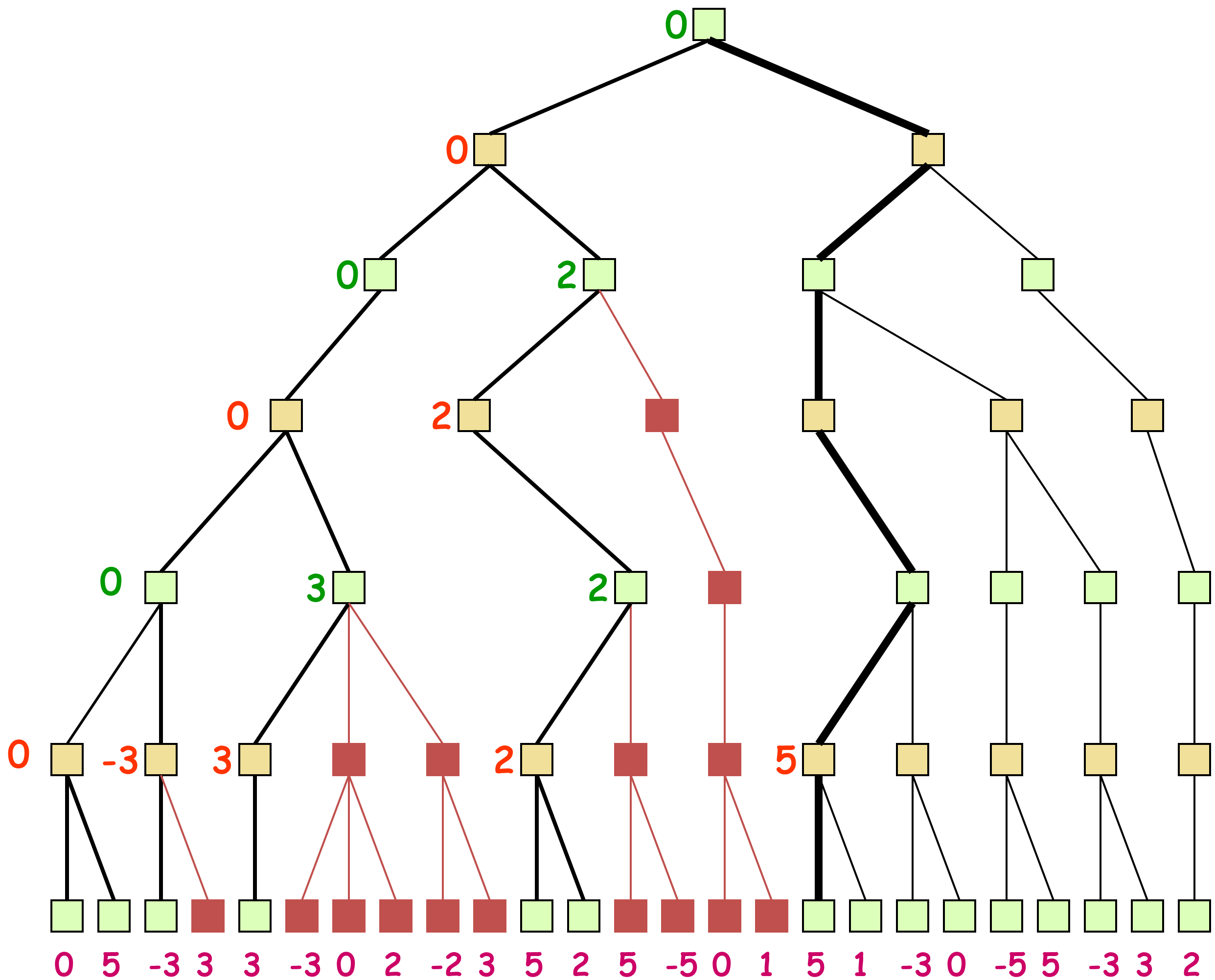
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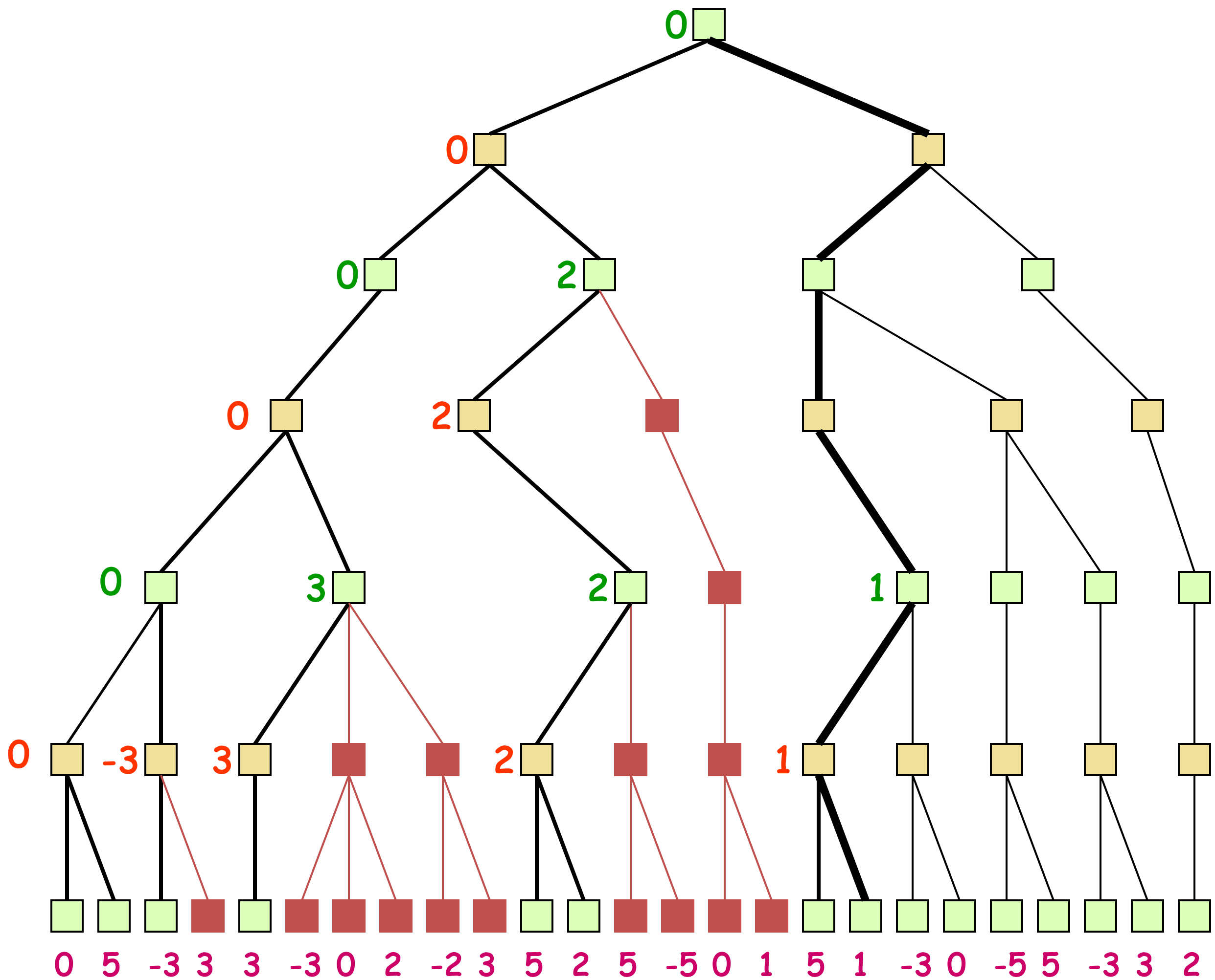
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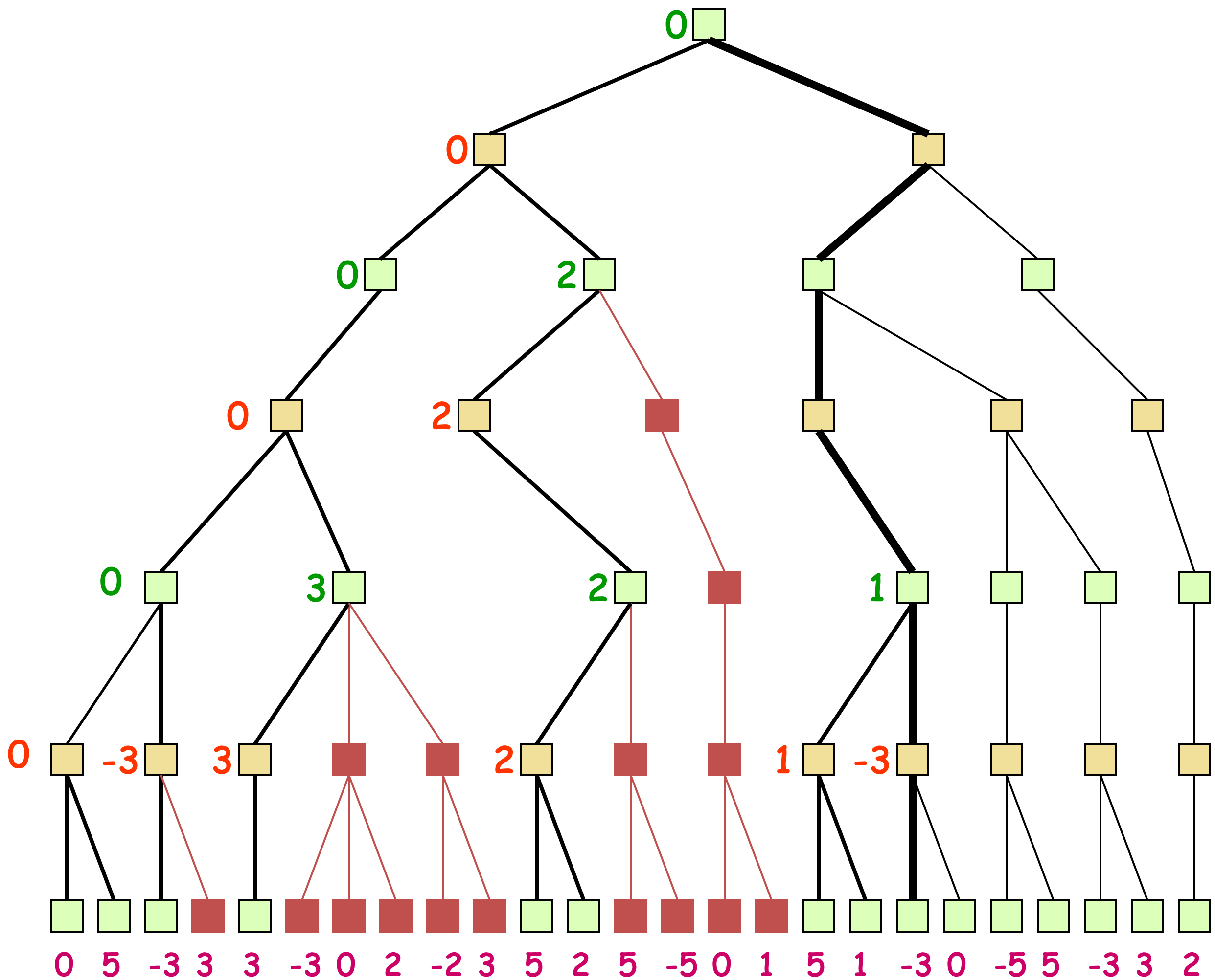
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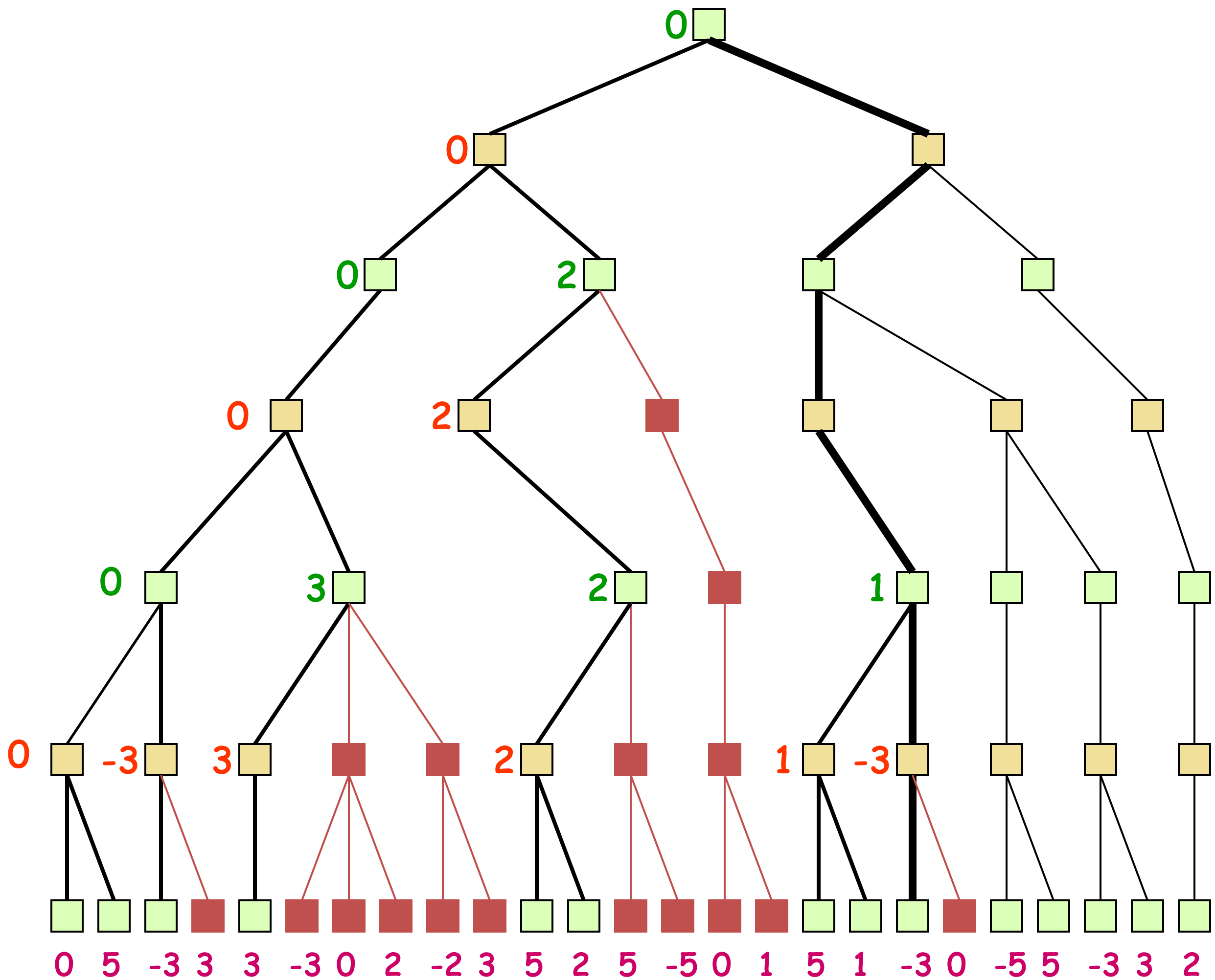
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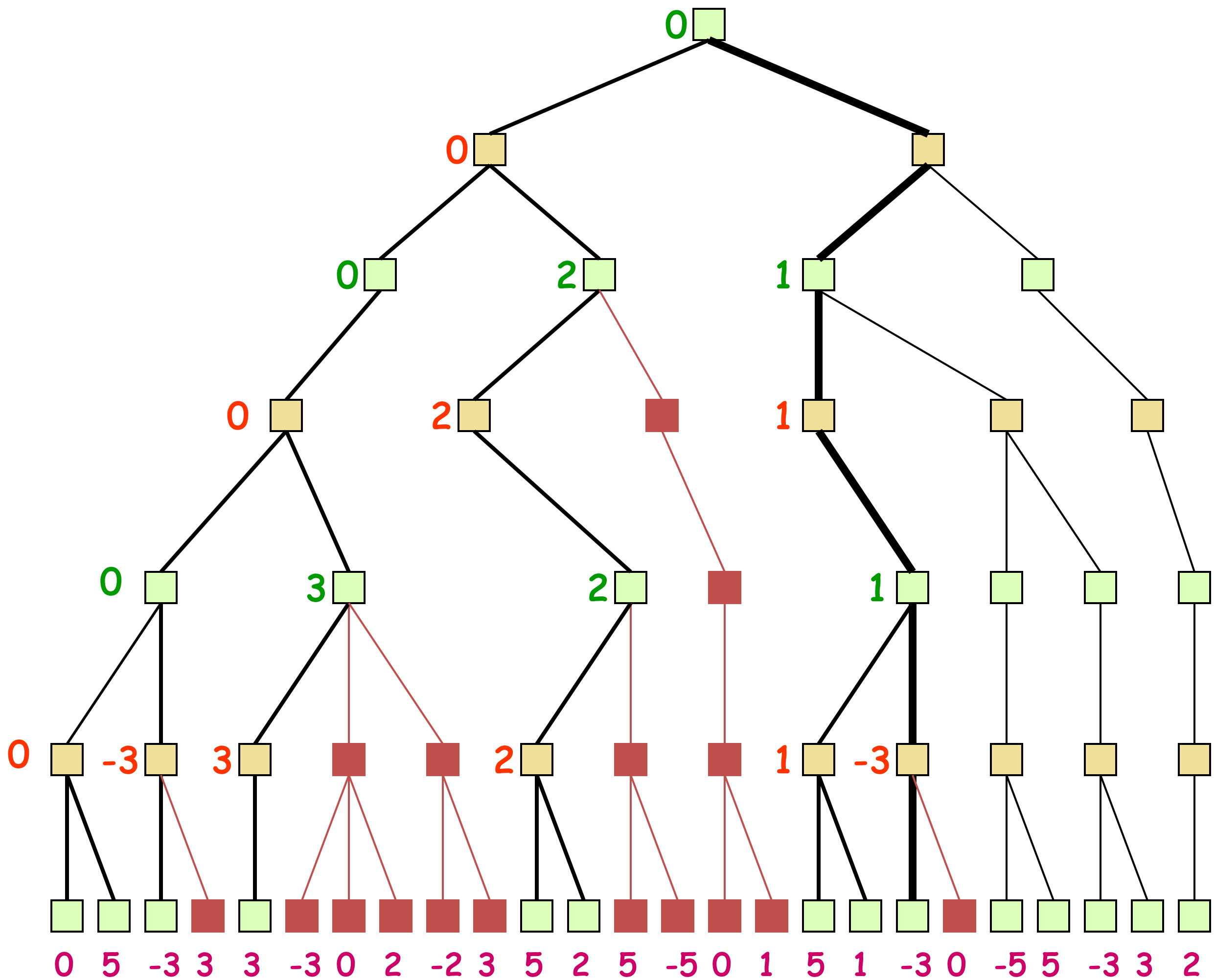
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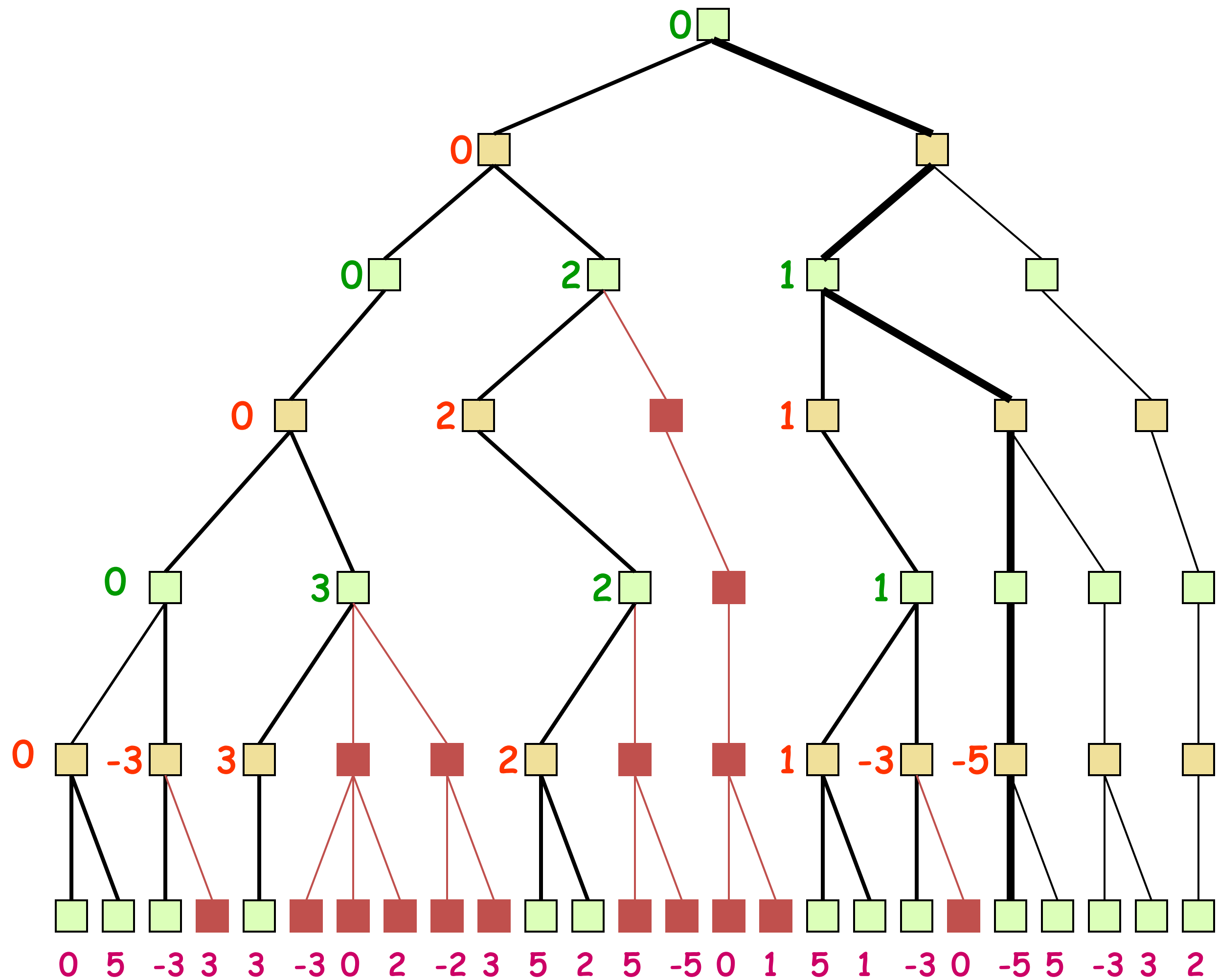
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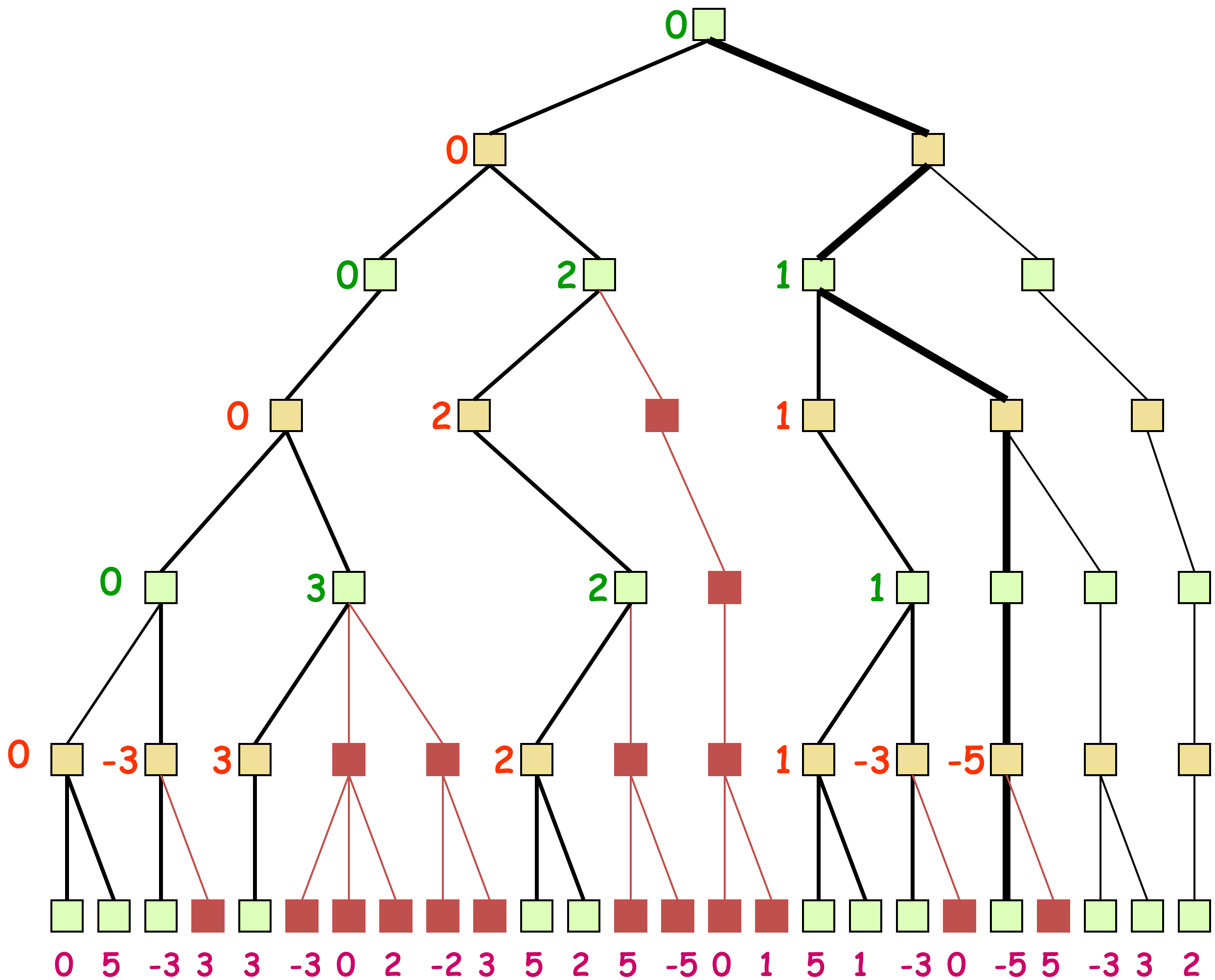
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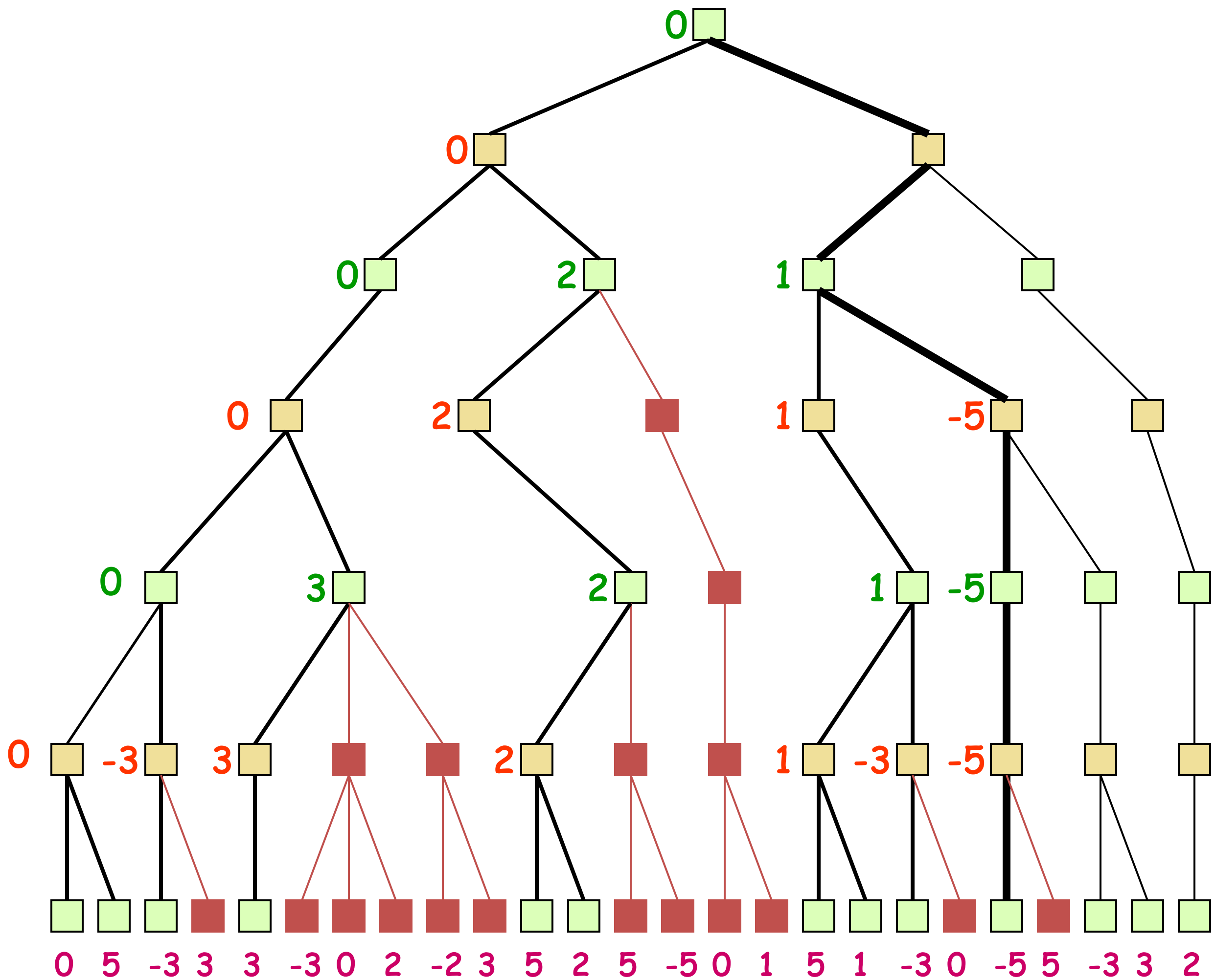
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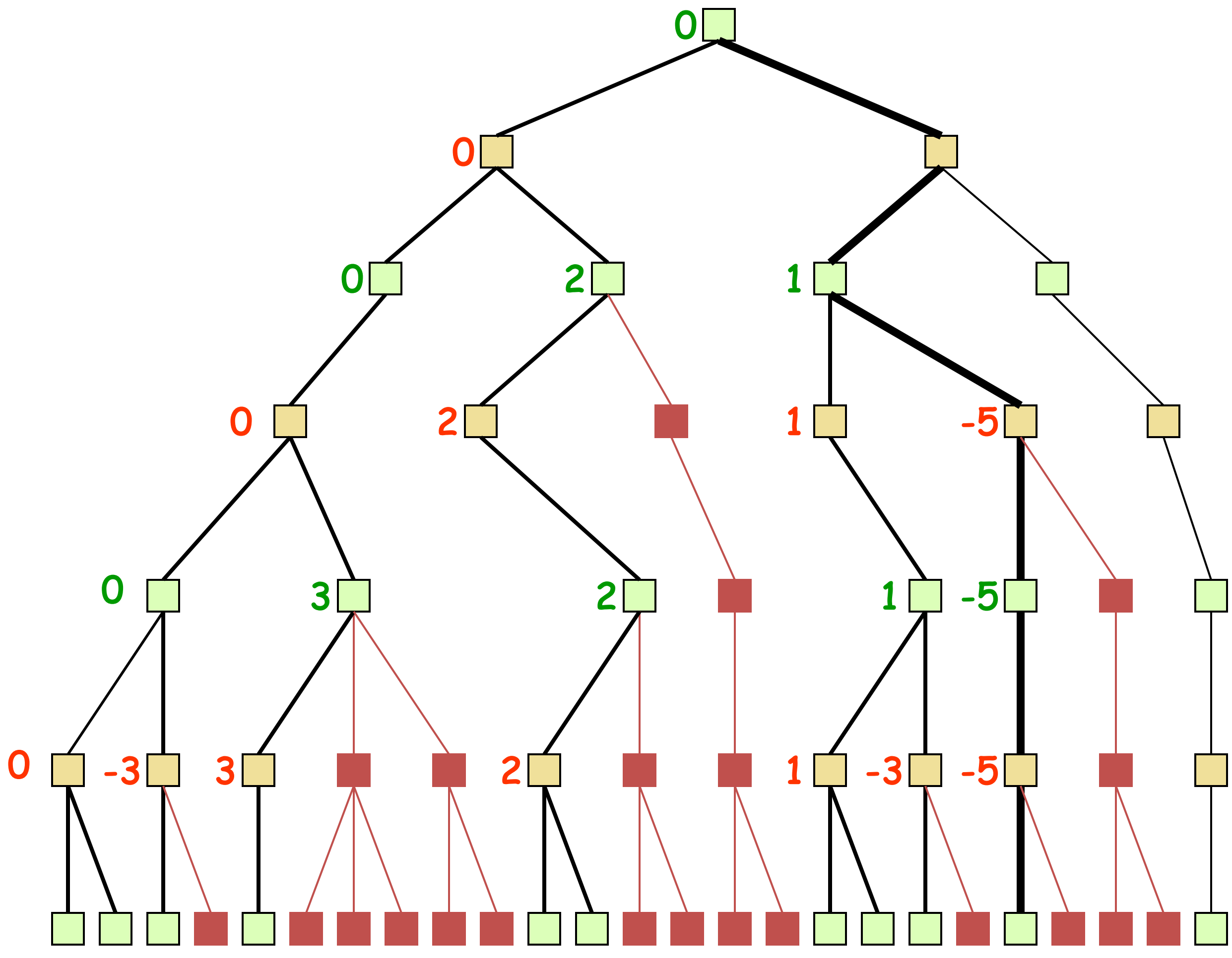
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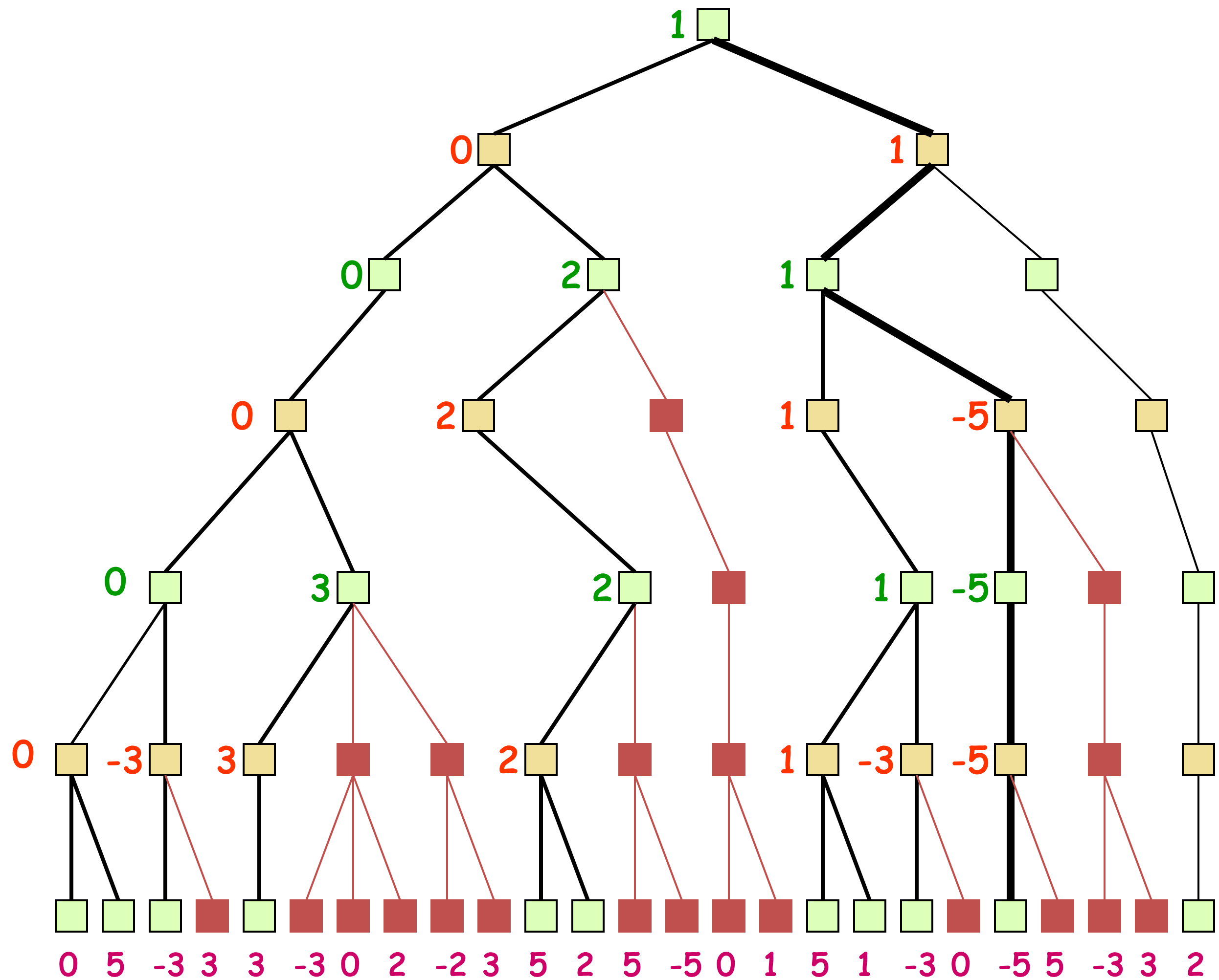
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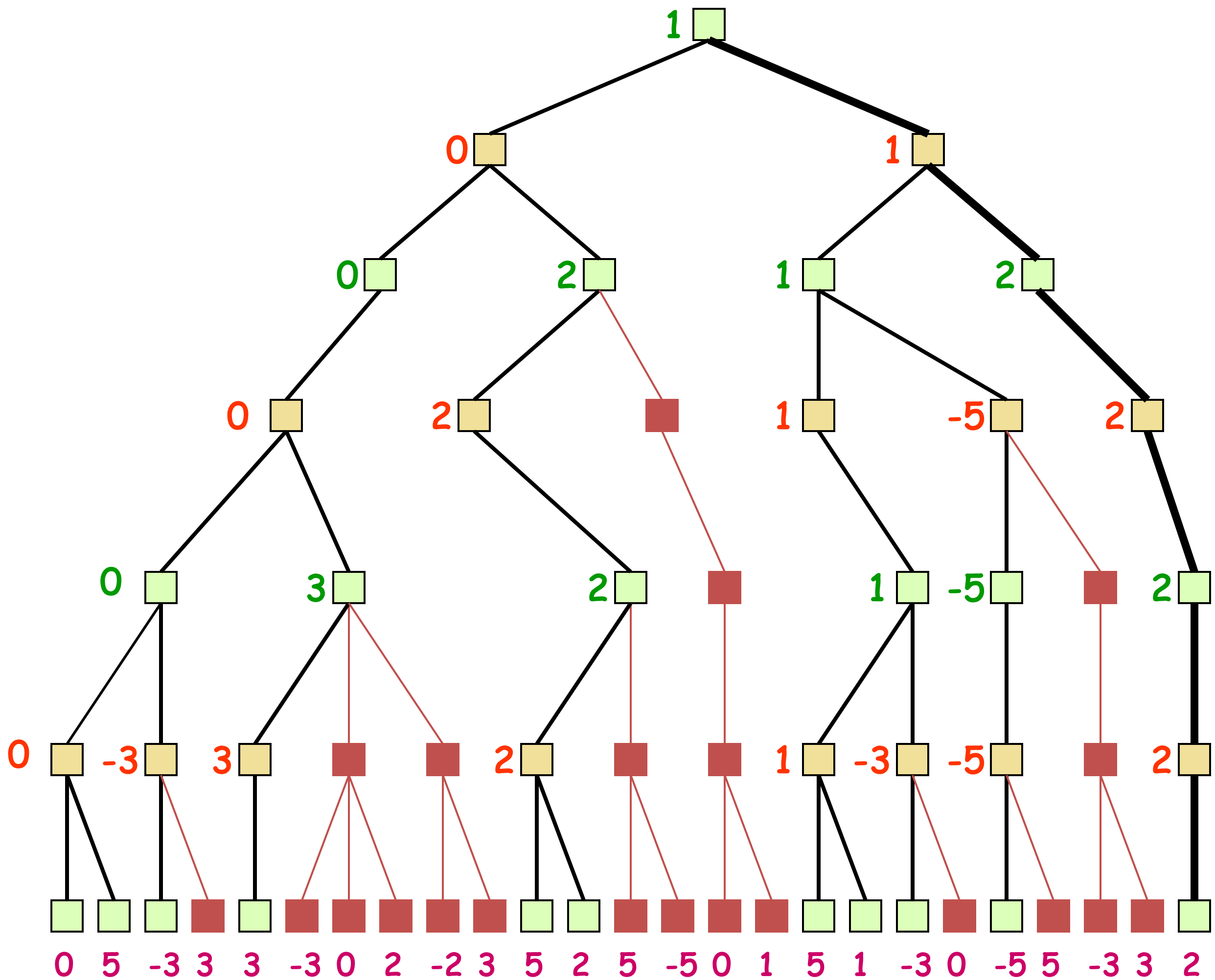
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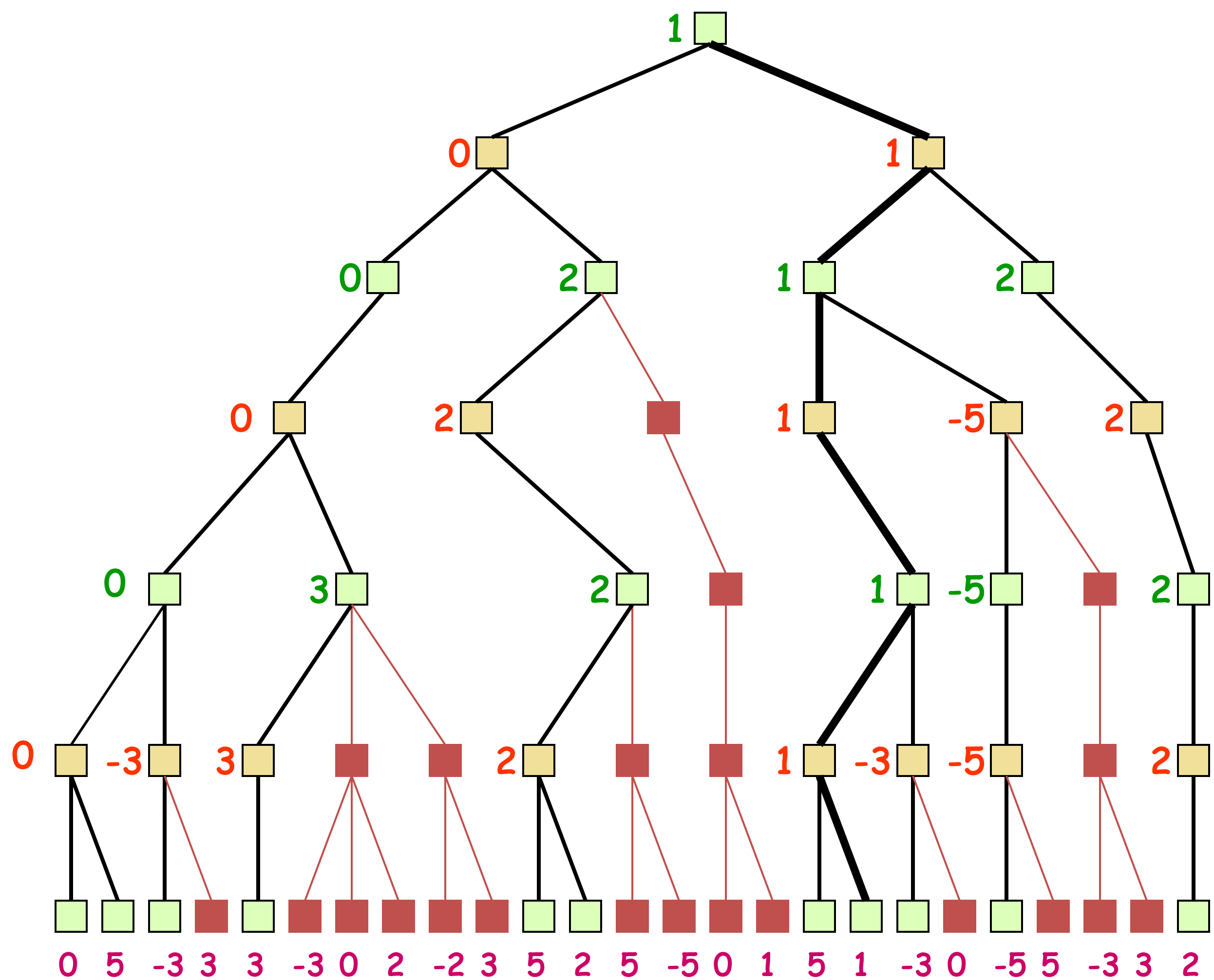
With alpha-beta we avoided computing a static evaluation metric for 14 of the 25 leaf nodes



Many other improvements

- **Adaptive horizon** + **iterative deepening**
- **Extended search**: retain k>1 best paths (not just one) extend tree at greater depth below their leaf nodes to help dealing with “horizon effect”
- **Singular extension**: If move is obviously better than others in node at horizon h, expand it
- Use **transposition tables** to deal with repeated states