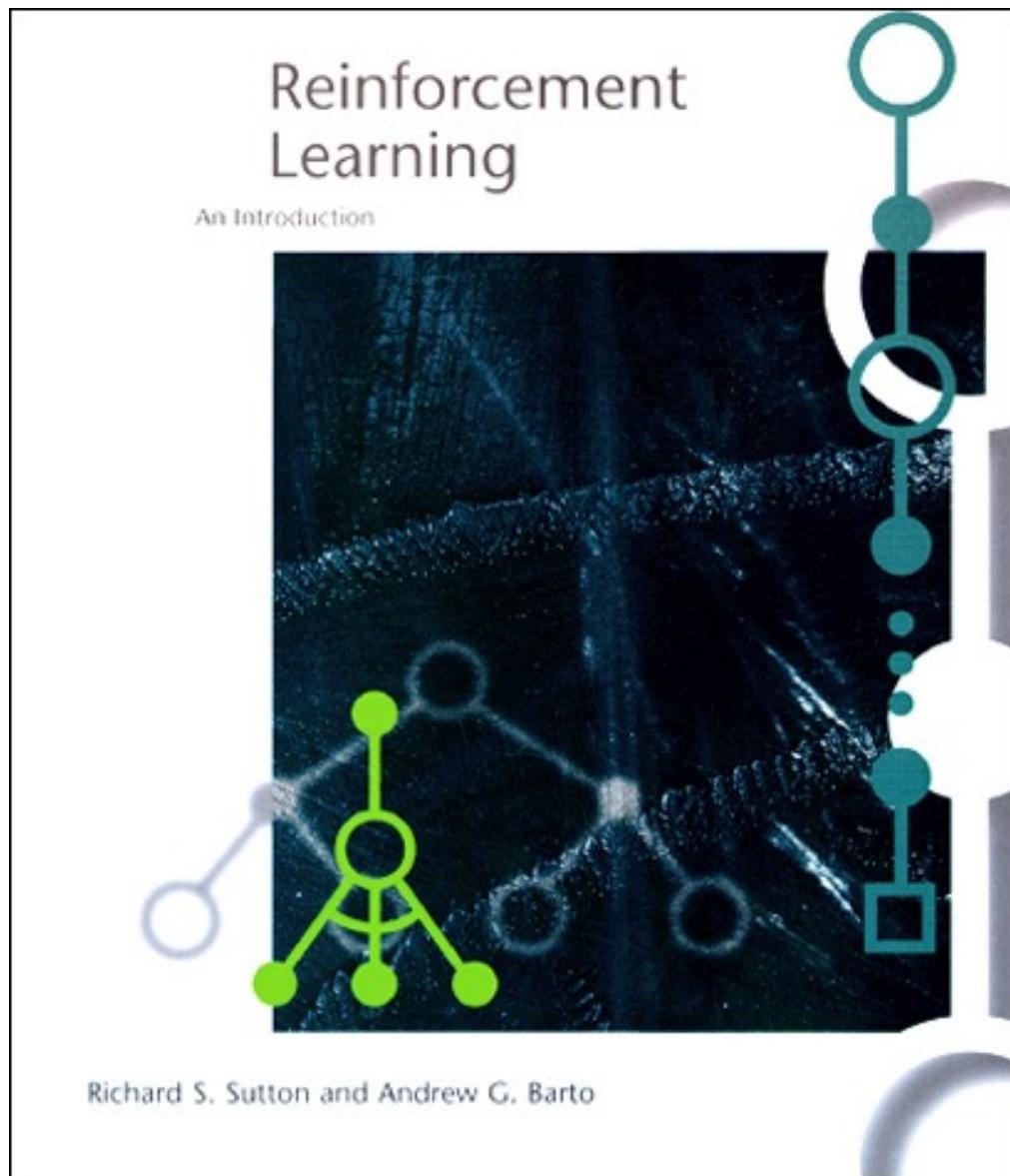


CMSC 478: Reinforcement Learning

There's an entire book!

<http://incompleteideas.net/book/the-book-2nd.html>



The Big Idea

- “Planning”: Find a sequence of steps to accomplish a goal.
 - Given start state, transition model, goal functions...
- This is a kind of **sequential decision making**.
 - Transitions are deterministic.
- What if they are stochastic (probabilistic)?
 - One time in ten, you drop your sock
- Probabilistic Planning: Make a plan that accounts for probability by **carrying it through the plan**.

Review: Formalizing Agents

- Given:
 - A state space S
 - A set of actions a_1, \dots, a_k including their results
 - Reward value at the end of each trial (series of action) (may be positive or negative)
- Output:
 - A **mapping from states to actions**

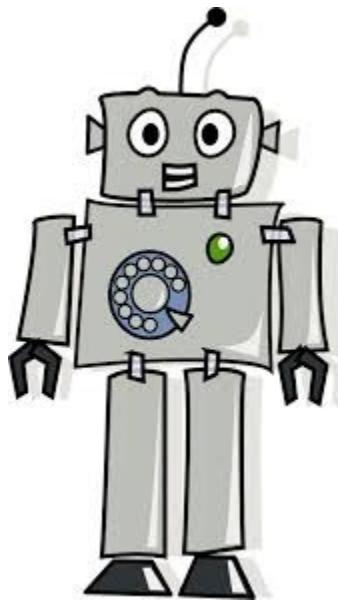
Review: Formalizing Agents

- Given:
 - A state space S
 - A set of actions a_1, \dots, a_k including their results
 - Reward value at the end of each trial (series of action) (may be positive or negative)
- Output:
 - A **mapping from states to actions**
 - Which is a **policy**, π

Reinforcement Learning

- We often have an agent which has a **task** to perform
 - It takes some actions in the world
 - At some later point, gets feedback on how well it did
 - The agent performs the same task repeatedly
- This problem is called **reinforcement learning**:
 - The agent gets positive reinforcement for tasks done well
 - And gets negative reinforcement for tasks done poorly
 - Must somehow figure out which actions to take next time

Reinforcement Learning

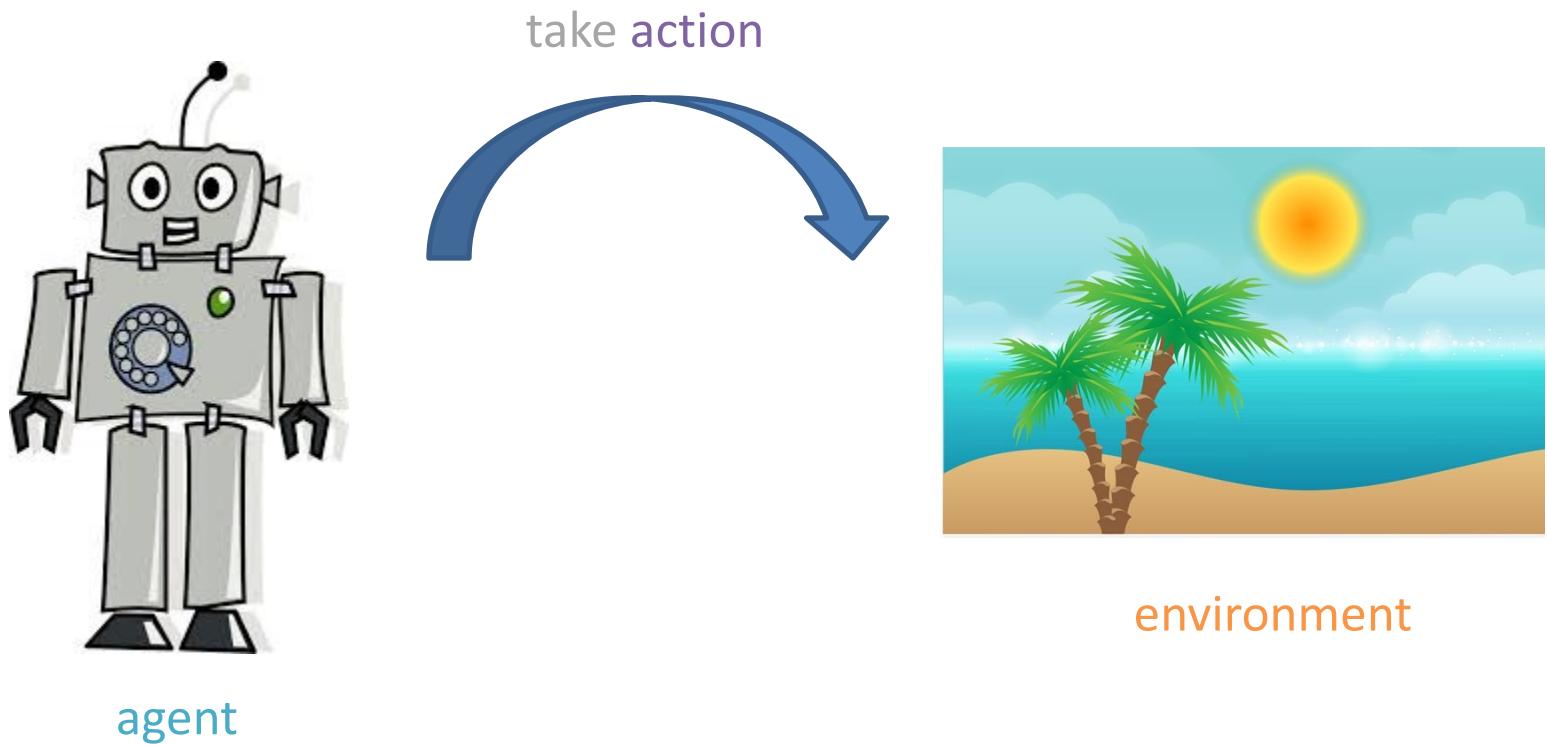


agent



environment

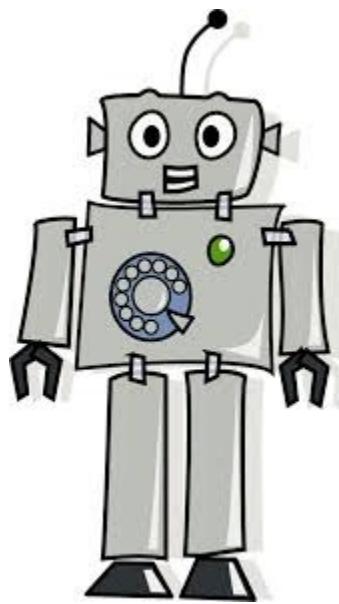
Reinforcement Learning



Reinforcement Learning



Reinforcement Learning



agent

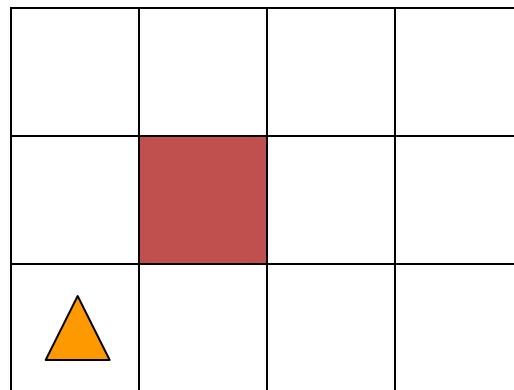


get new state
and/or reward



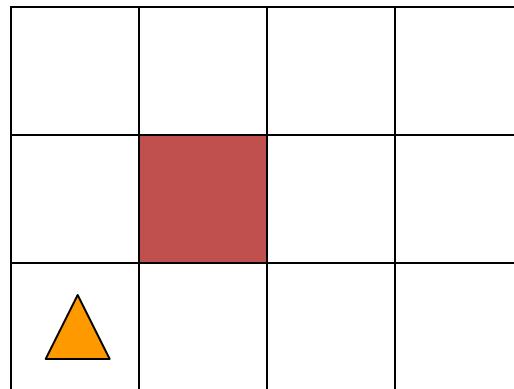
environment

Simple Robot Navigation Problem



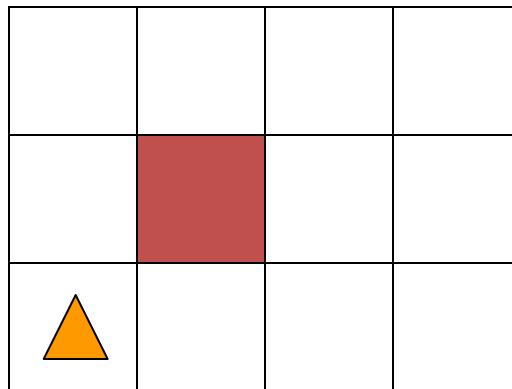
- In each state, the possible actions are **U**, **D**, **R**, and **L**

Probabilistic Transition Model



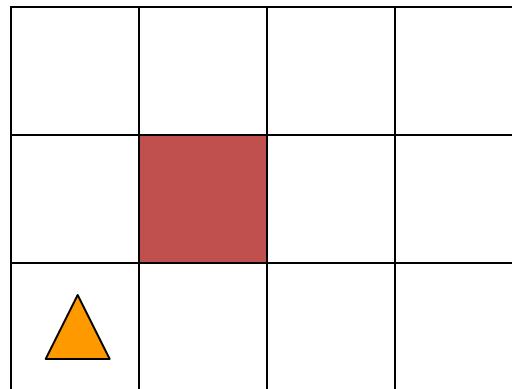
- In each state, the possible actions are **U**, **D**, **R**, and **L**
- The effect of **U** is as follows (**transition model**):
 - With probability 0.8, the robot moves up one square (if the robot is already in the top row, then it does not move)

Probabilistic Transition Model



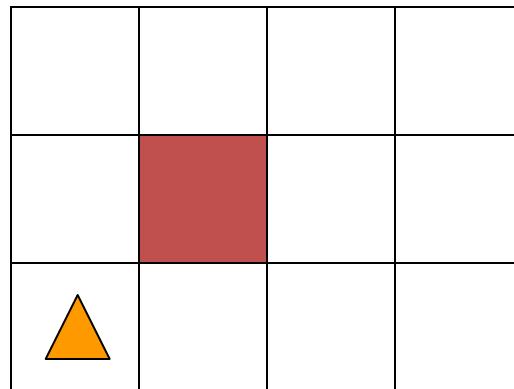
- In each state, the possible actions are U, D, R, and L
- The effect of U is as follows (transition model):
 - With probability 0.8, the robot moves up one square (if the robot is already in the top row, then it does not move)
 - With probability 0.1, the robot moves right one square (if the robot is already in the rightmost row, then it does not move)

Probabilistic Transition Model



- In each state, the possible actions are U, D, R, and L
- The effect of U is as follows (transition model):
 - With probability 0.8, the robot moves up one square (if the robot is already in the top row, then it does not move)
 - With probability 0.1, the robot moves right one square (if the robot is already in the rightmost row, then it does not move)
 - With probability 0.1, the robot moves left one square (if the robot is already in the leftmost row, then it does not move)

Probabilistic Transition Model



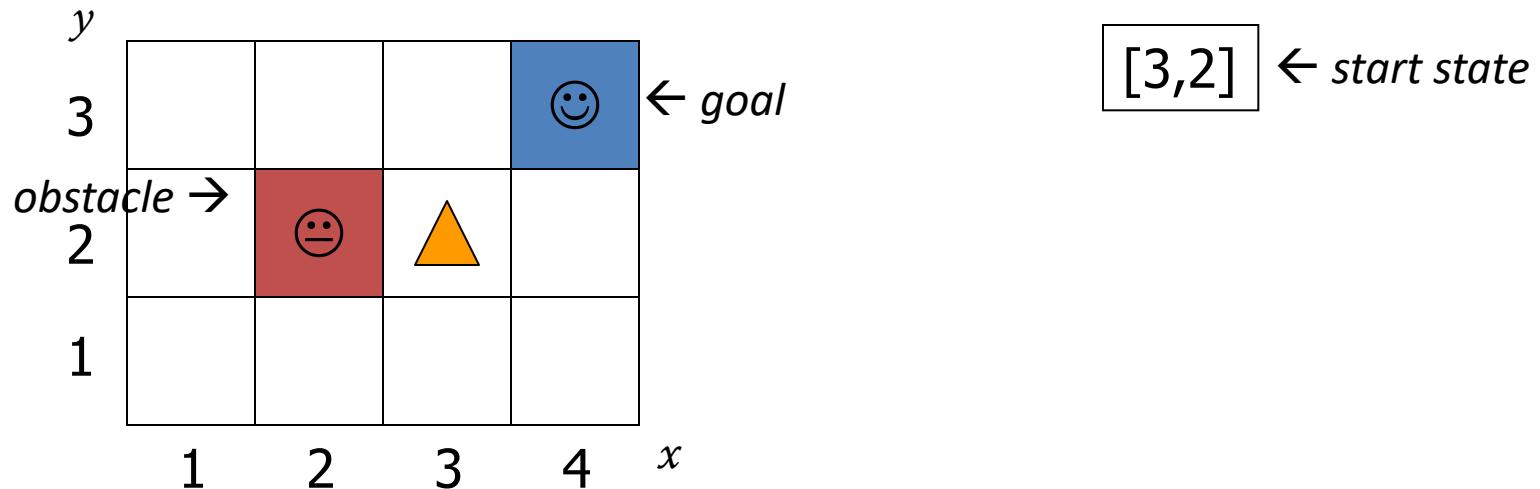
- In each state, the possible actions are U, D, R, and L
- The effect of U is as follows (transition model):
 - With probability 0.8, the robot moves up one square (if the robot is already in the top row, then it does not move)
 - With probability 0.1, the robot moves right one square (if the robot is already in the rightmost row, then it does not move)
 - With probability 0.1, the robot moves left one square (if the robot is already in the leftmost row, then it does not move)
- D, R, and L have similar probabilistic effects

Markov Property

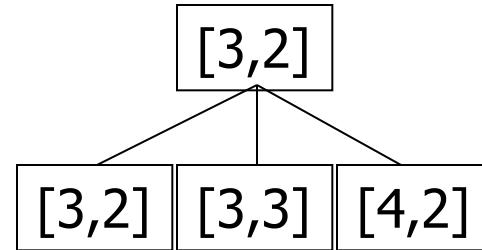
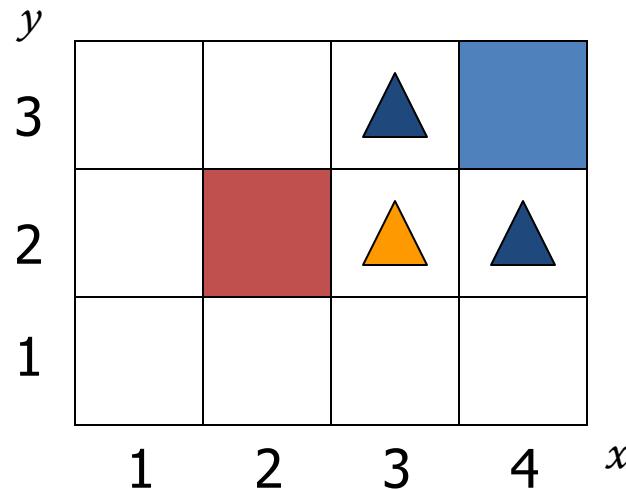
The transition properties depend only on the current state, not on the previous history (how that state was reached)

Markov assumption generally: current state only ever depends on previous state (or finite set of previous states).

Sequence of Actions

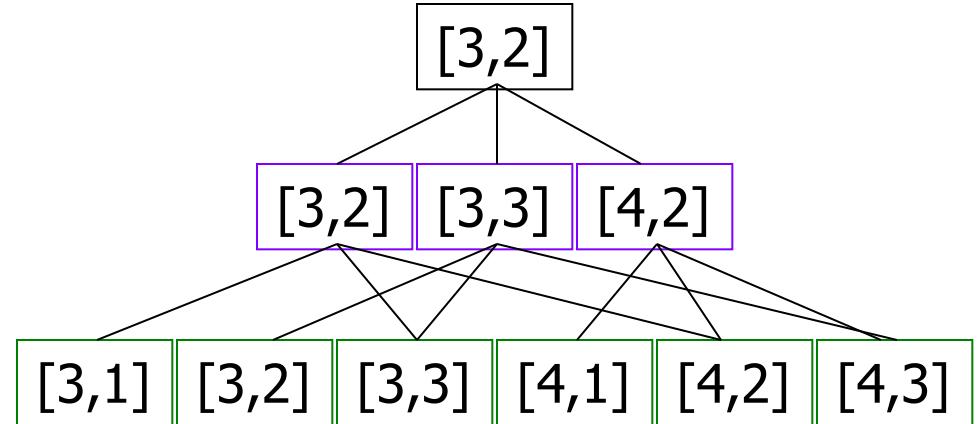
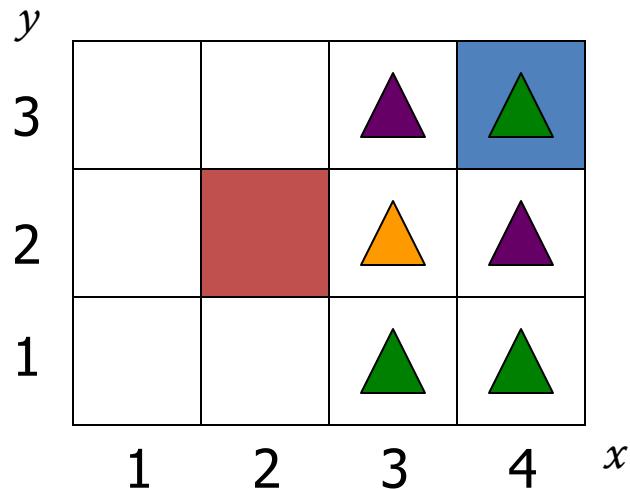


Sequence of Actions



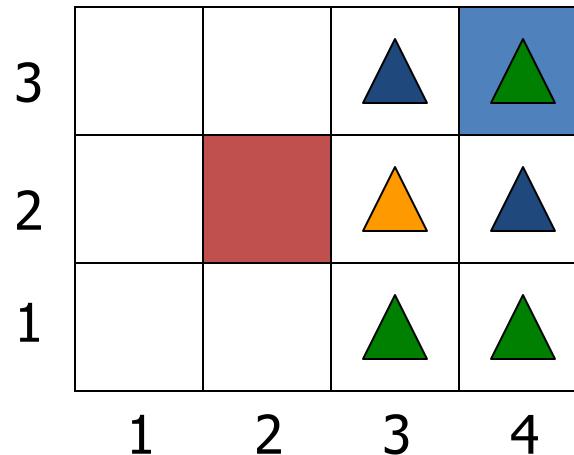
- Planned sequence of actions: (U, R)
- U is executed

Histories



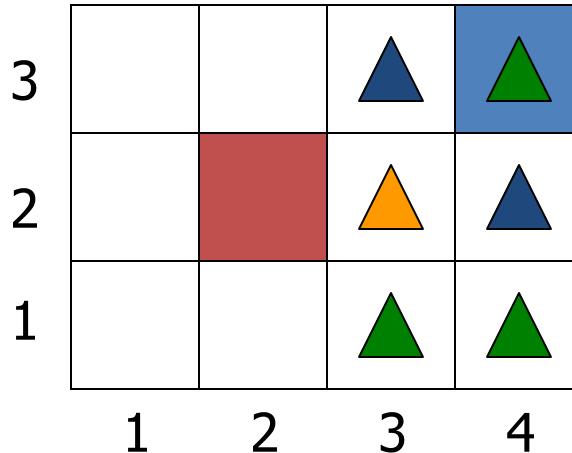
- Planned sequence of actions: (U, R)
- U has been executed
- R is executed
- 9 possible sequences of states – called **histories**
- 6 possible final states for the robot!

Probability of Reaching the Goal



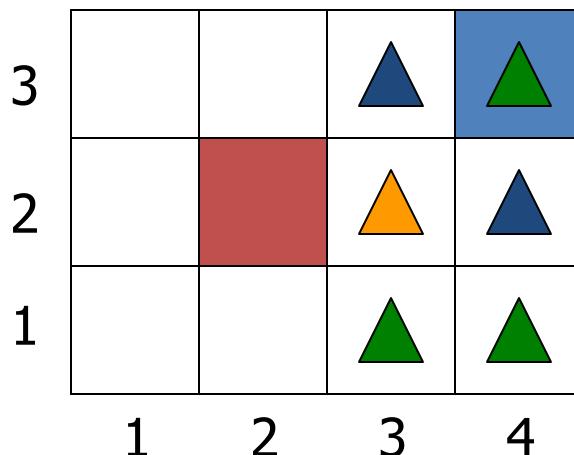
- $\mathbf{P}([4,3] \mid (U,R).[3,2]) =$
 $\mathbf{P}([4,3] \mid R.[3,3]) \times \mathbf{P}([3,3] \mid U.[3,2])$
 $+ \mathbf{P}([4,3] \mid R.[4,2]) \times \mathbf{P}([4,2] \mid U.[3,2])$

Probability of Reaching the Goal



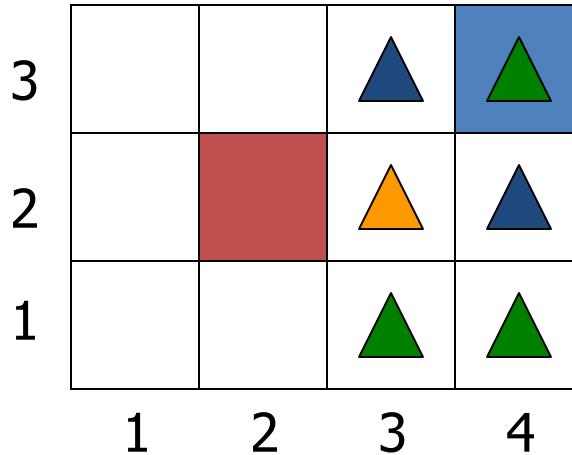
- $P([4,3] | (U,R).[3,2]) =$
 $P([4,3] | R.[3,3]) \times P([3,3] | U.[3,2])$
+ $P([4,3] | R.[4,2]) \times P([4,2] | U.[3,2])$
 - $P([3,3] | U.[3,2]) = 0.8$
 - $P([4,2] | U.[3,2]) = 0.1$

Probability of Reaching the Goal



- $\mathbf{P}([4,3] \mid (U,R).[3,2]) =$
 $\mathbf{P}([4,3] \mid R.[3,3]) \times \mathbf{P}([3,3] \mid U.[3,2])$
+ $\mathbf{P}([4,3] \mid R.[4,2]) \times \mathbf{P}([4,2] \mid U.[3,2])$
- $\mathbf{P}([4,3] \mid R.[3,3]) = 0.8$ • $\mathbf{P}([3,3] \mid U.[3,2]) = 0.8$
- $\mathbf{P}([4,3] \mid R.[4,2]) = 0.1$ • $\mathbf{P}([4,2] \mid U.[3,2]) = 0.1$

Probability of Reaching the Goal

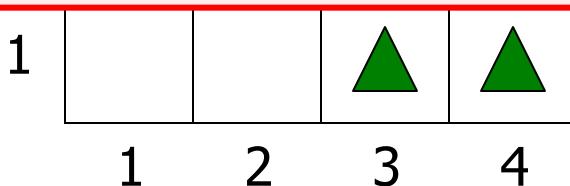


- $P([4,3] | (U,R).[3,2]) = P([4,3] | R.[3,3]) \times P([3,3] | U.[3,2]) + P([4,3] | R.[4,2]) \times P([4,2] | U.[3,2])$
- $P([4,3] | R.[3,3]) = 0.8$ • $P([3,3] | U.[3,2]) = 0.8$
- $P([4,3] | R.[4,2]) = 0.1$ • $P([4,2] | U.[3,2]) = 0.1$
- $P([4,3] | (U,R).[3,2]) = 0.65$

Probability of Reaching the Goal

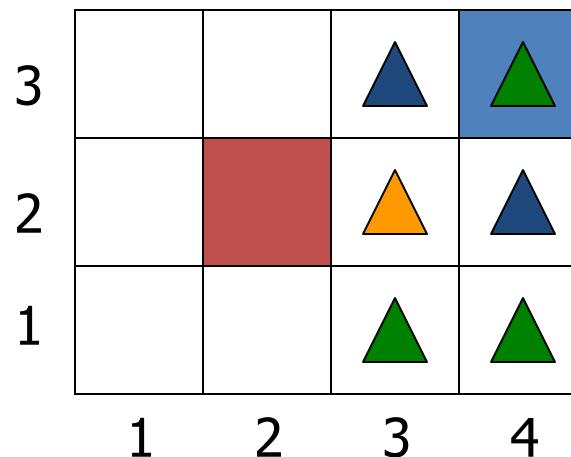


Note importance of Markov property
in this derivation



- $\mathbf{P}([4,3] \mid (\mathbf{U}, \mathbf{R}).[3,2]) =$
 $\mathbf{P}([4,3] \mid \mathbf{R}.[3,3]) \times \mathbf{P}([3,3] \mid \mathbf{U}.[3,2])$
 $+ \mathbf{P}([4,3] \mid \mathbf{R}.[4,2]) \times \mathbf{P}([4,2] \mid \mathbf{U}.[3,2])$
- $\mathbf{P}([4,3] \mid \mathbf{R}.[3,3]) = 0.8$ • $\mathbf{P}([3,3] \mid \mathbf{U}.[3,2]) = 0.8$
- $\mathbf{P}([4,3] \mid \mathbf{R}.[4,2]) = 0.1$ • $\mathbf{P}([4,2] \mid \mathbf{U}.[3,2]) = 0.1$
- $\mathbf{P}([4,3] \mid (\mathbf{U}, \mathbf{R}).[3,2]) = 0.65$

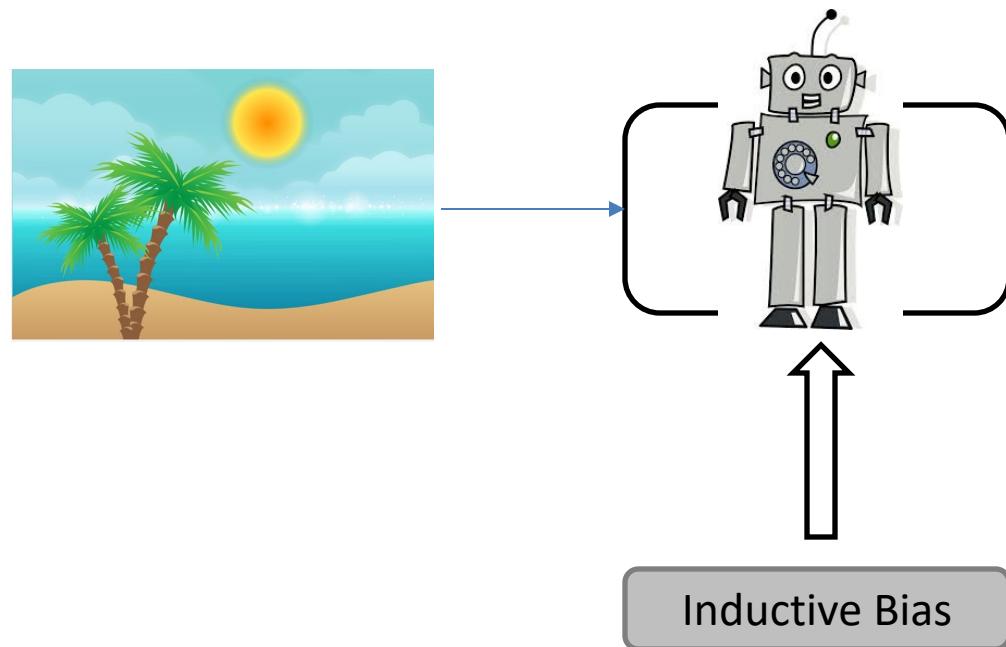
Probability of Reaching the Goal



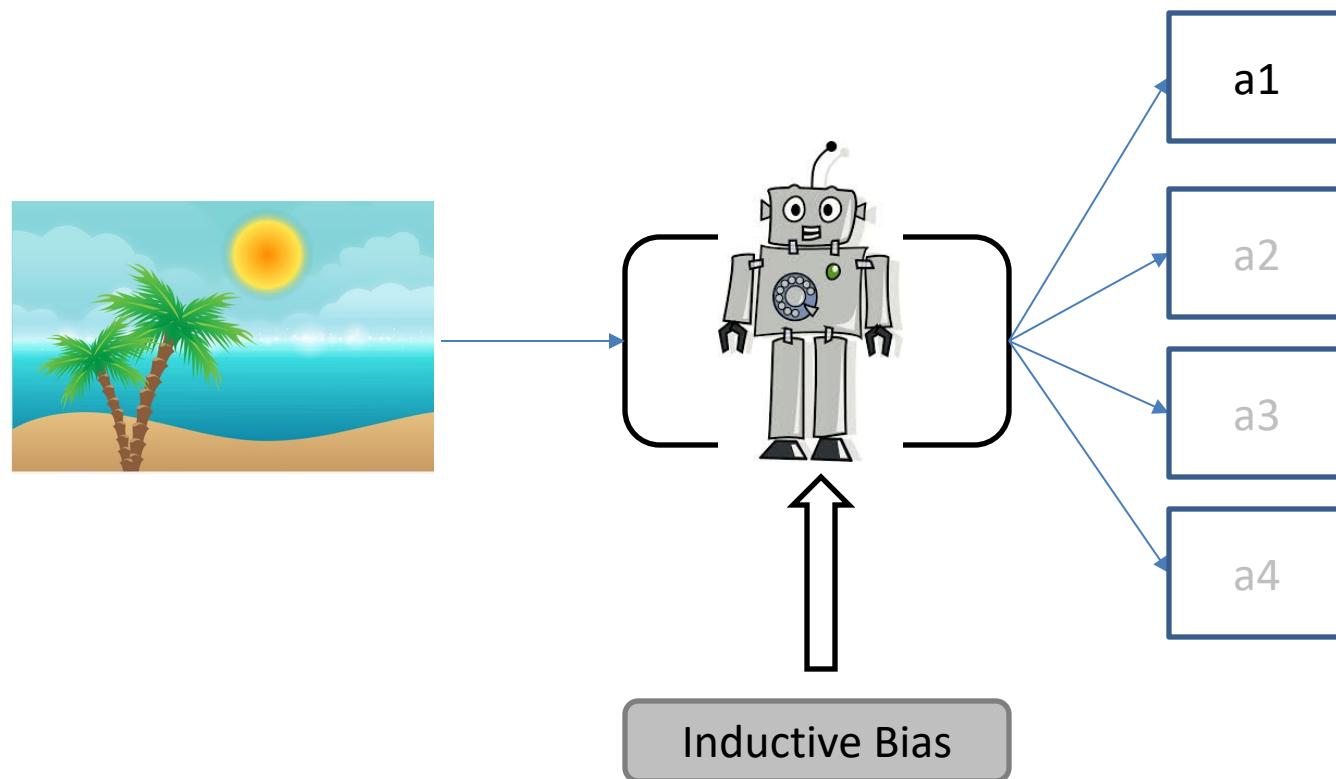
- Main idea: **multiply backward** probabilities of each step taken from end state reached (*because of our Markov/independence assumptions*)
- But we still need to consider different ways of reaching a state
 - Going all the way around the obstacle would be “worse”

But what about the
learning part of
reinforcement learning?

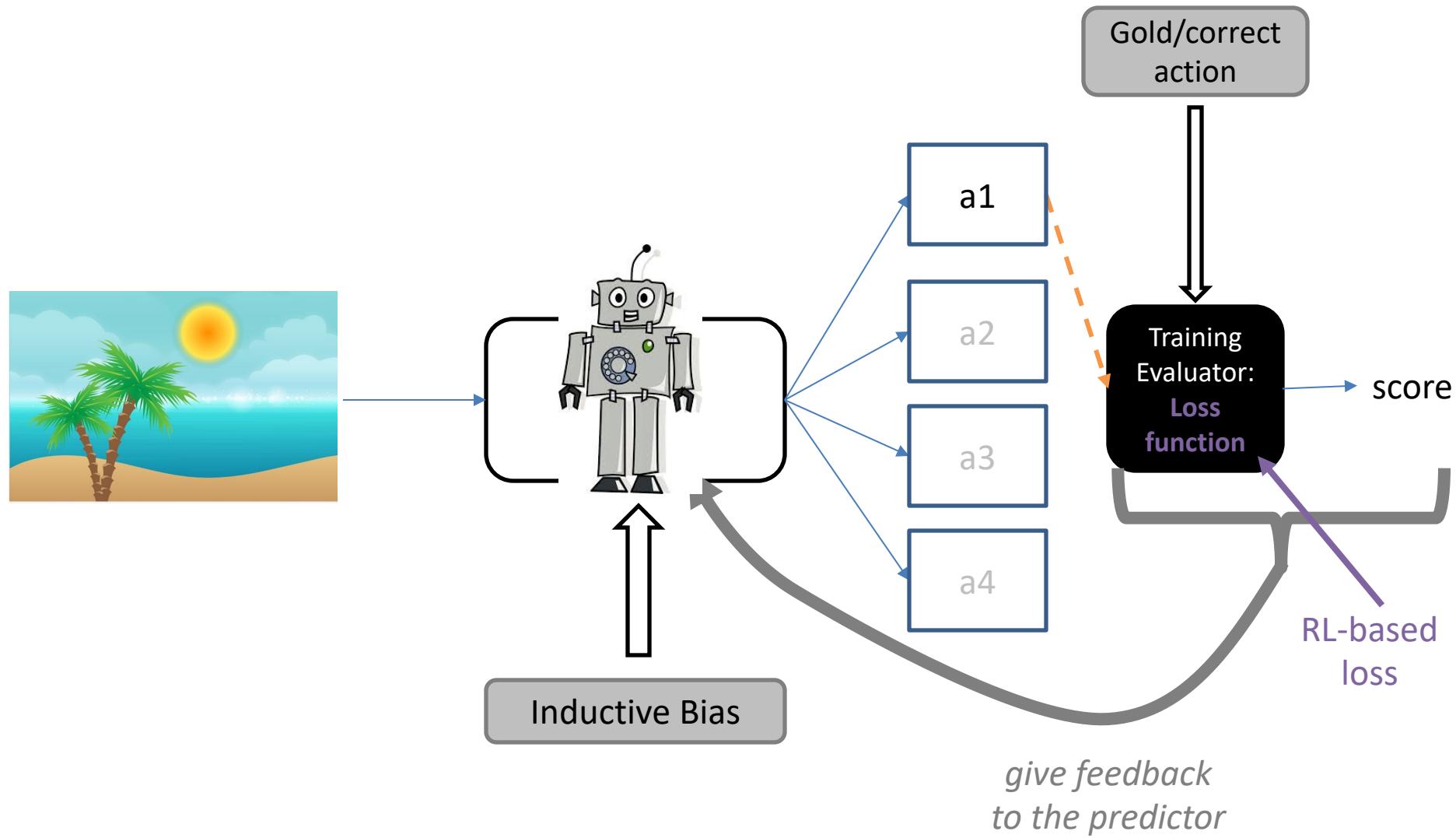
RL, in our ML framework



RL, in our ML framework



RL, in our ML framework



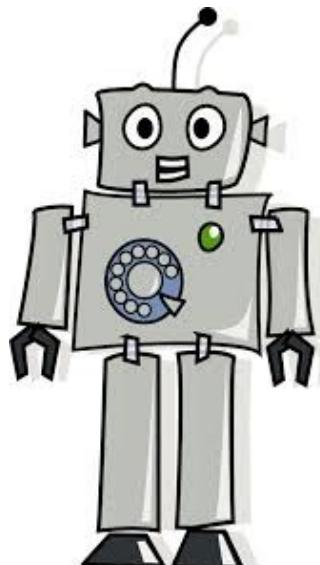
Markov Decision Process: Formalizing Reinforcement Learning



Markov Decision
Process:

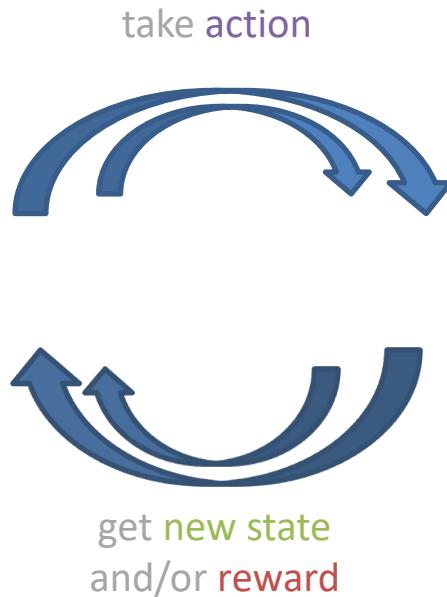
$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

Markov Decision Process: Formalizing Reinforcement Learning



agent

Markov Decision
Process:



$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

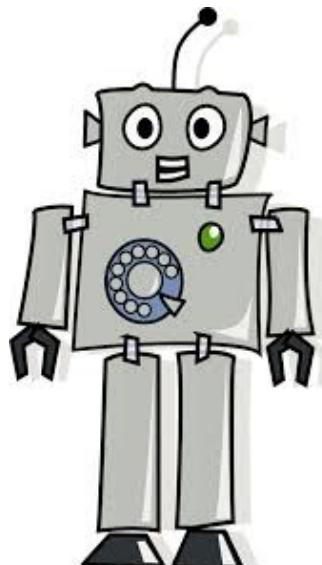
set of possible actions

set of possible states



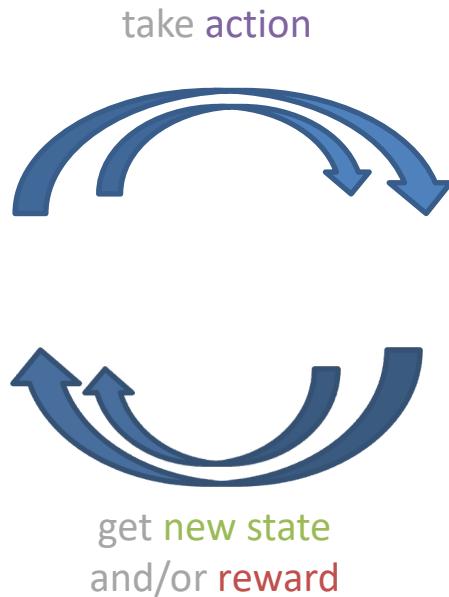
environment

Markov Decision Process: Formalizing Reinforcement Learning



agent

Markov Decision
Process:

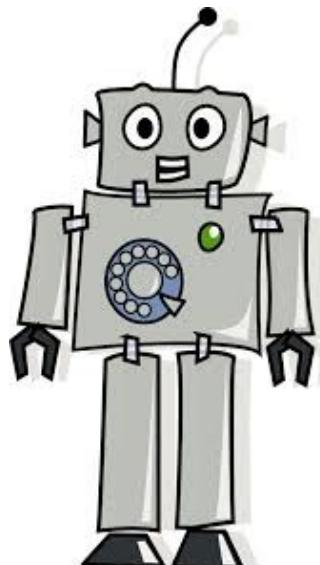


environment

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

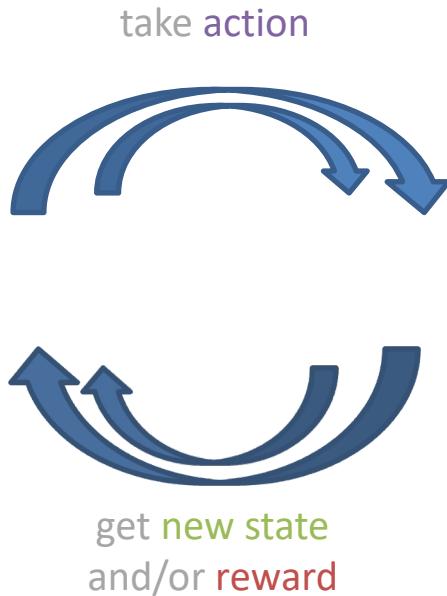
set of possible states set of possible actions reward of (state, action) pairs

Markov Decision Process: Formalizing Reinforcement Learning



agent

Markov Decision
Process:



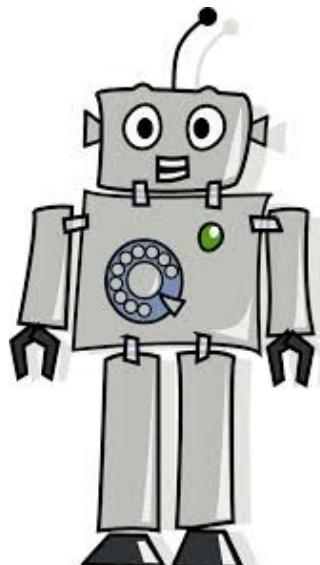
$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible states	reward of (state, action) pairs
set of possible actions	state-action transition distribution



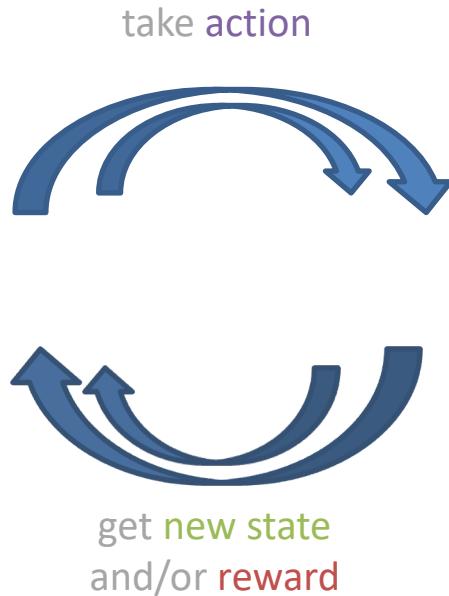
environment

Markov Decision Process: Formalizing Reinforcement Learning



agent

Markov Decision
Process:



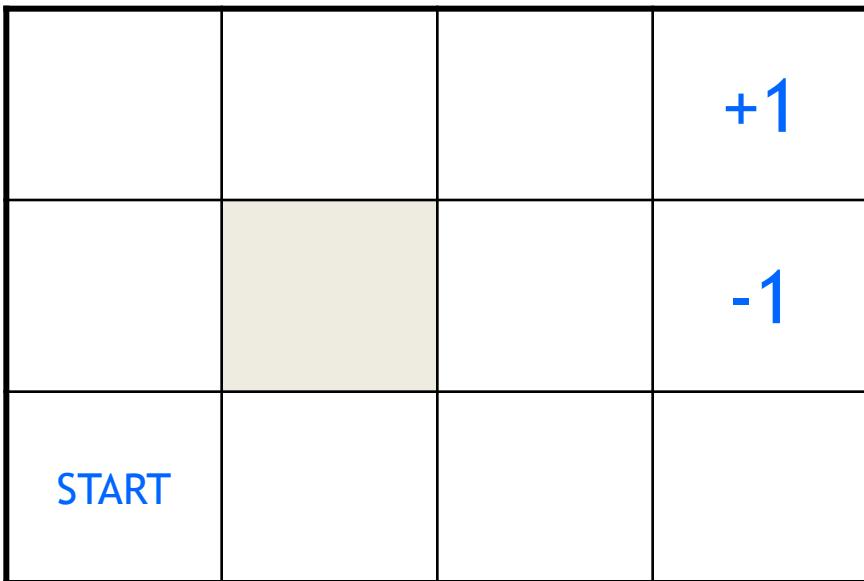
environment

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible states set of possible actions state-action transition distribution

reward of (state, action) pairs discount factor

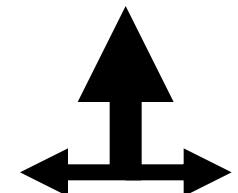
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

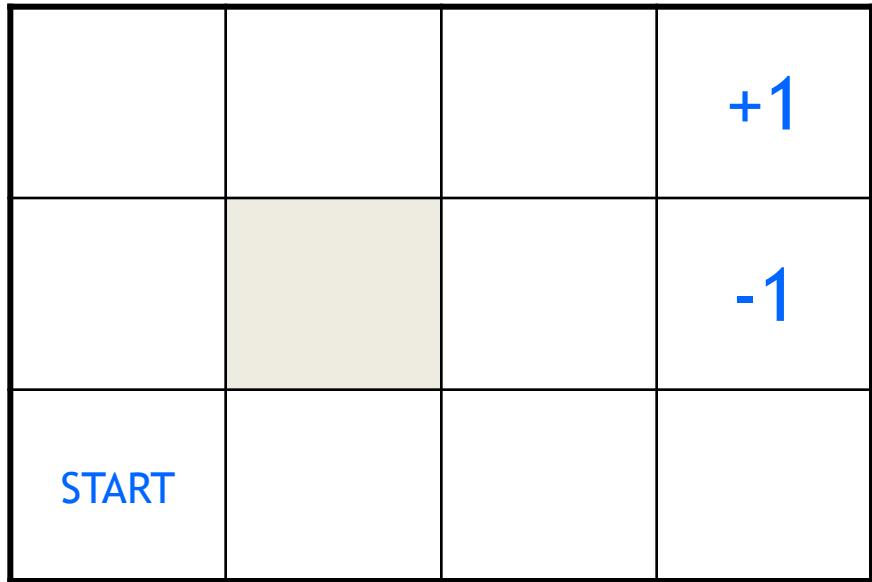
- 80% move UP
- 10% move LEFT
- 10% move RIGHT



reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

Goal: what's the strategy to achieve the maximum reward?

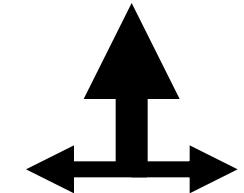
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

- 80% move UP
- 10% move LEFT
- 10% move RIGHT



reward +1 at [4,3], -1 at [4,2]

reward -0.04 for each step

states: current location

actions: where to go next

rewards

what is the solution? Learn a mapping from (state, action) pairs to new states

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution

set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution
set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:
choose action a_t

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution
set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

Policy
 $\pi: S \rightarrow A$

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

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Start in initial state s_0
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choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution

set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
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choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: choose
action over time
to maximize time-
discounted reward

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution
set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: choose action over time to maximize discounted reward

Consider all possible future times t

Reward at time t

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution
set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

objective: maximize
discounted reward

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

Consider all
possible future
times t

Discount at
time t

Reward at
time t

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution

set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: maximize discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

Consider all possible future times t

Discount at time t

Reward at time t

Example of Discounted Reward

objective: maximize
discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

Consider all possible future times t Discount at time t Reward at time t

- If the discount factor $\gamma = 0.8$ then reward
$$0.8^0 r_0 + 0.8^1 r_1 + 0.8^2 r_2 + 0.8^3 r_3 + \dots + 0.8^n r_n + \dots$$
- Allows you to consider all possible rewards in the future but preferring current vs. future self

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution

set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: maximize
discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

“solution”: the policy π^* that maximizes the expected (average) time-discounted reward

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision
Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible actions state-action transition distribution
set of possible states reward of (state, action) pairs discount factor

Start in initial state s_0
for $t = 1$ to ...:

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: maximize
discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

“solution” $\pi^* = \operatorname{argmax}_{\pi} \mathbb{E} \left[\sum_{t>0} \gamma^t r_t ; \pi \right]$

Expected Value of a Random Variable

random variable

$$X \sim p(\cdot)$$

Expected Value of a Random Variable

random variable

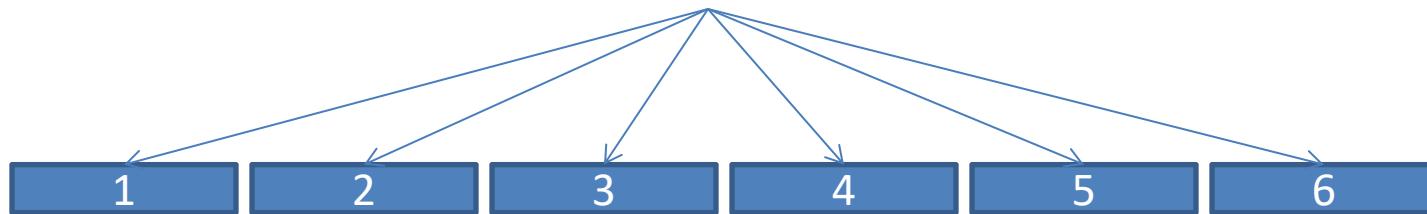
$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_x x p(x)$$

*expected value
(distribution p is
implicit)*

Expected Value: Example

uniform distribution of number of cats I have



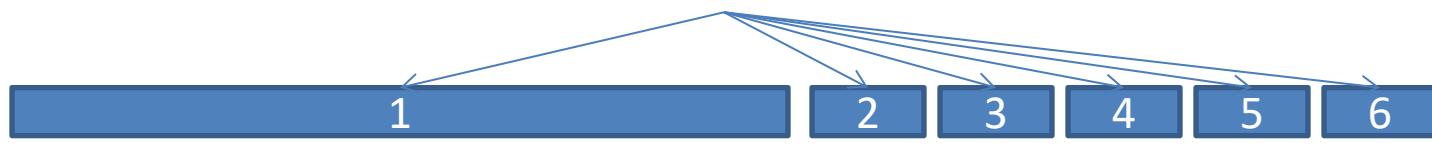
$$\mathbb{E}[X] = \sum_x x p(x)$$

\downarrow

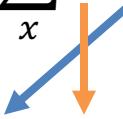
$$1/6 * 1 +
1/6 * 2 +
1/6 * 3 + \quad = 3.5
1/6 * 4 +
1/6 * 5 +
1/6 * 6$$

Expected Value: Example 2

*non-uniform distribution of number of cats a normal
cat person has*



$$\mathbb{E}[X] = \sum_x x p(x)$$



$$1/2 * 1 +
1/10 * 2 +
1/10 * 3 +
1/10 * 4 +
1/10 * 5 +
1/10 * 6 = 2.5$$

Expected Value of a Function of a Random Variable

$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_x x p(x)$$

$$\mathbb{E}[f(X)] = ???$$

Expected Value of a Function of a Random Variable

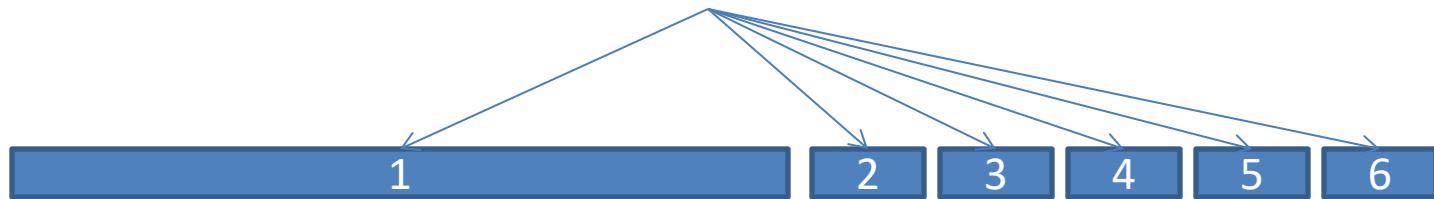
$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_x x p(x)$$

$$\mathbb{E}[f(X)] = \sum_x f(x) p(x)$$

Expected Value of Function: Example

non-uniform distribution of number of cats I start with



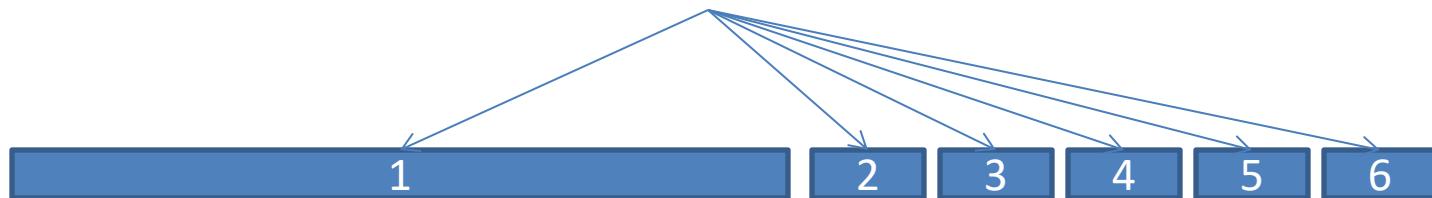
What if each cat magically becomes two?

$$f(k) = 2^k$$

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$$\begin{aligned} & 1/2 * 2^1 + \\ & 1/10 * 2^2 + \\ & 1/10 * 2^3 + \quad = 13.4 \\ & 1/10 * 2^4 + \\ & 1/10 * 2^5 + \\ & 1/10 * 2^6 \end{aligned}$$

Markov Decision Process: Formalizing Reinforcement Learning

Markov

Here, r_t is a function of random variable s_t .

Start in initial state
for $t = 1$ to ...

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$
get reward $r_t = \mathcal{R}(s_t, a_t)$

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

“solution” $\pi^* = \operatorname{argmax}_{\pi} \mathbb{E} \left[\sum_{t>0} \gamma^t r_t ; \pi \right]$

Markov Decision Process: Formalizing Reinforcement Learning

Here, r_t is a function of random variable s_t . →

The expectation is over the different states s_t the agent could be in at time t (equiv. actions the agent could take).

Start in initial state s_0

for $t = 1$ to ...

choose action a_t

“move” to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

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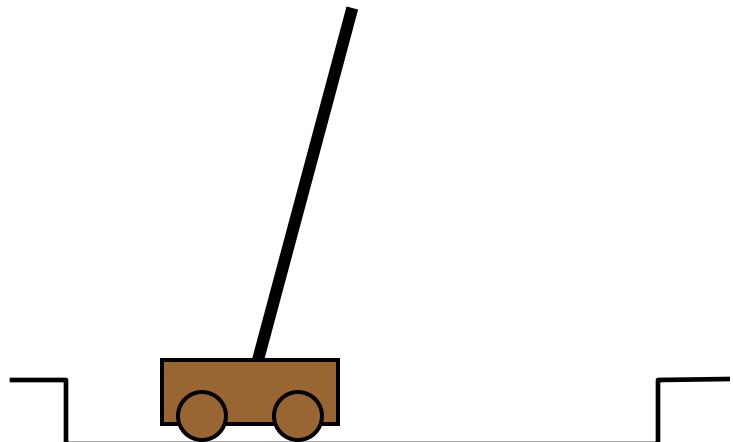
Some Challenges

1. Representing states (and actions)
2. Defining our reward
3. Learning our policy

State Representation

Task: pole-balancing

state representation?



move car left/right to
keep the pole balanced

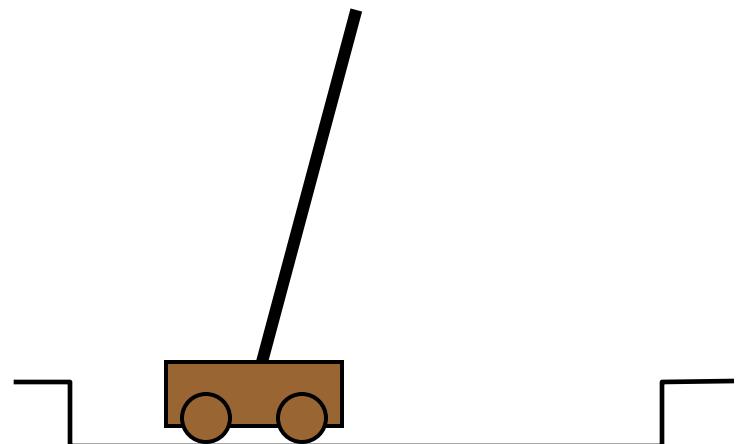
State Representation

Task: pole-balancing

state representation

position and velocity of car

angle and angular velocity of pole



move car left/right to
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what about *Markov property*?

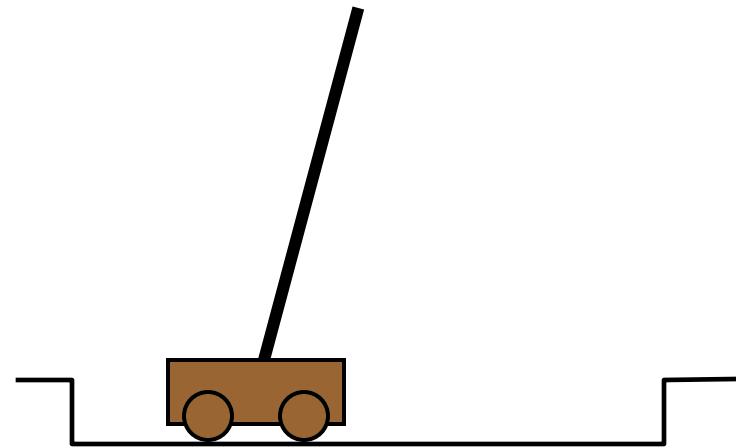
State Representation

Task: pole-balancing

state representation

position and velocity of car

angle and angular velocity of pole



move car left/right to
keep the pole balanced

what about *Markov property*?

would need more info

noise in sensors, temperature,
bending of pole

Some Challenges

1. Representing states (and actions)
2. Defining our reward
3. Learning our policy

Designing Rewards

robot in a maze

episodic task, not discounted, +1 when out, 0 for each step

chess

GOOD: +1 for winning, -1 losing

BAD: +0.25 for taking opponent's pieces

high reward even when lose

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rewards indicate **what** we want to accomplish

NOT **how** we want to accomplish it

shaping

positive reward often very “far away”

rewards for achieving subgoals (domain knowledge)

also: adjust initial policy or initial value function



Simple Reinforcement Learning

- Feedback is at the end, on a **series** of actions.
- Very early concept in Artificial Intelligence!
- Arthur Samuels' checker program was a simple reinforcement based learner, initially developed in 1956.
- In 1962 it beat a human checkers master.



www-
03.ibm.com/ibm/history/ibm100/us/en/icons/ibm700series/impacts/

Example: Animals Game

- Human: I'm thinking of an animal.

Example: Animals Game

- Human: I'm thinking of an animal.
- Computer: Is it a bird?

Example: Animals Game

- Human: I'm thinking of an animal.
- Computer: Is it a bird?
- Human: No.

Example: Animals Game

- Human: I'm thinking of an animal.
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- Computer: What is it?
- Human: A mouse.

Example: Animals Guessing Game Architecture

- All of the parts of ML Architecture:
 - The Representation is a sequence of questions and pairs of yes/no answers (decision tree).
 - The Actor “walks” the tree, interacting with a human; at each question it chooses whether to follow the “yes” branch or the “no” branch.
 - The Evaluator/loss function is the human player telling the game whether it has guessed correctly.
 - The Learner elicits new questions and adds questions, guesses and branches to the tree.

Reinforcement Learning (cont.)

- Goal: agent acts in the world to maximize its rewards
- Agent has to figure out what it did that made it get that reward/punishment
 - This is known as the **credit assignment problem**

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Example: Animals Game

- Human: I'm thinking of an animal.
- Computer: Is it a bird?
- Human: No.
- Computer: Is it a frog?
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- Computer: What is it?
- Human: A mouse.
- Computer: **Tell me a question that distinguishes a frog from a mouse.**

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- Human: A mouse.
- Computer: **Tell me a question that distinguishes a frog from a mouse.**
- Human: Does it have fur?
- Computer: What is the answer for a frog?
- Human: No.

Example: Animals Behind the Scene

Computer: Is it a bird?

Human: No

Computer: Is it a frog?

Human: No

Computer: What is it?

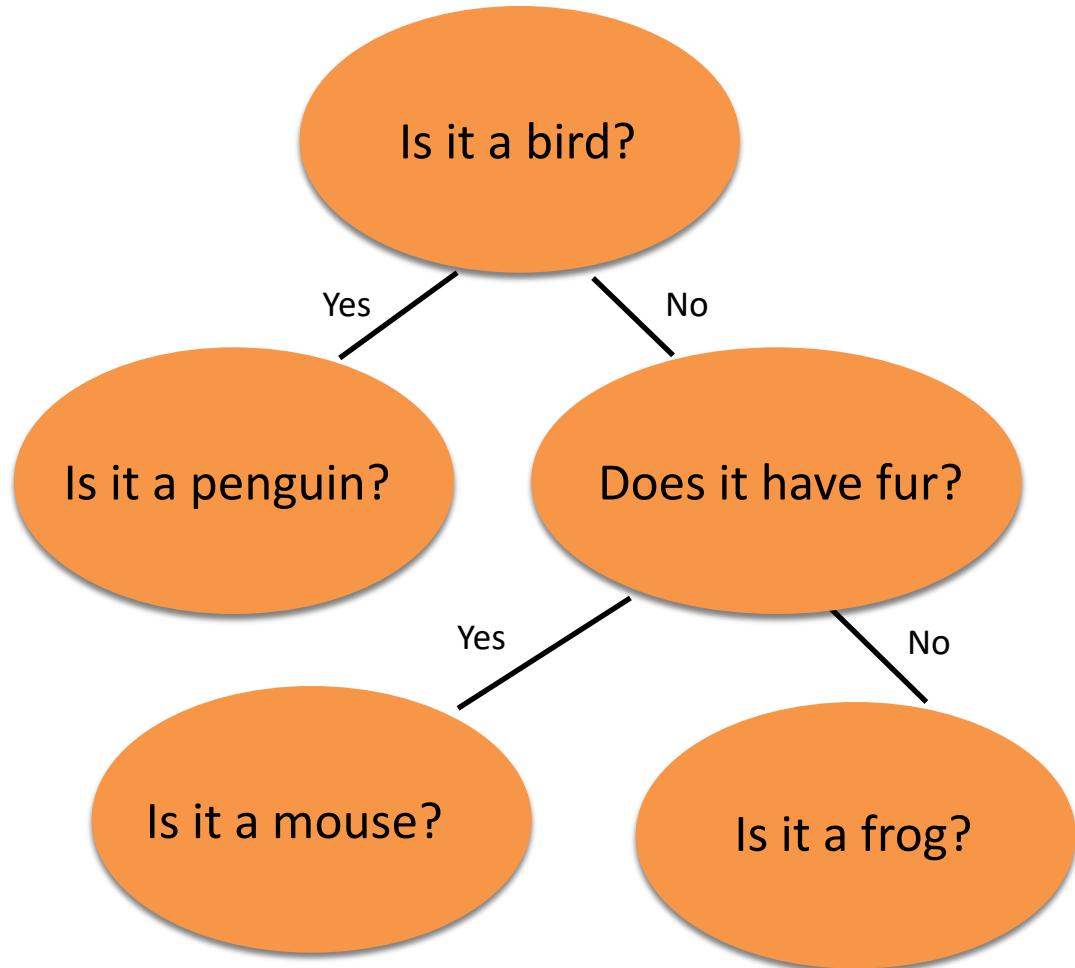
Human: A mouse

Computer: Tell me a question
that distinguishes a frog from
a mouse.

Human: Does it have fur?

Computer: What is the
answer for a frog?

Human: no



After several rounds...

Reinforcement Learning (cont.)

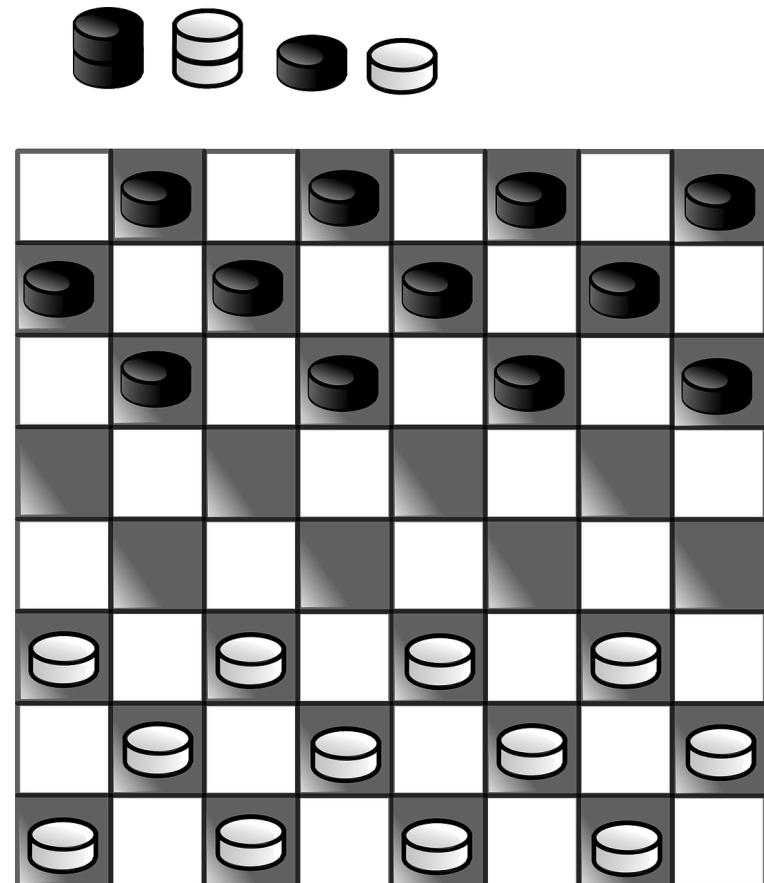
- Goal: agent acts in the world to maximize its rewards
- Agent has to figure out what it did that made it get that reward/punishment
 - This is known as the **credit assignment problem**
- RL can be used to train computers to do many tasks
 - Backgammon and chess playing
 - Job shop scheduling
 - Controlling robot limbs

Reactive Agent

- This kind of agent is a reactive agent
- The general algorithm for a reactive agent is:
 - Observe some state
 - If it is a terminal state, stop
 - Otherwise choose an action from the actions possible in that state
 - Perform the action
 - Recur.

Simple Example

- Learn to play checkers
 - Two-person game
 - 8x8 boards, 12 checkers/side
 - relatively simple set of rules:
<http://www.darkfish.com/checkers/rules.html>
 - Goal is to eliminate all your opponent's pieces



Representing Checkers

- First we need to represent the game
- To completely describe one step in the game you need
 - A representation of the game board.
 - A representation of the current pieces
 - A variable which indicates whose turn it is
 - A variable which tells you which side is “black”
- There is no history needed
- A look at the current board setup gives you a complete picture of the state of the game

Representing Checkers

- Second, we need to represent the rules
- Represented as a **set of allowable moves** given board state
 - If a checker is at row x , column y , and row $x+1$ column $y \pm 1$ is empty, it can move there.
 - If a checker is at (x,y) , a checker of the opposite color is at $(x+1, y+1)$, and $(x+2,y+2)$ is empty, the checker must move there, and remove the “jumped” checker from play.
- There are additional rules, but all can be expressed in terms of the state of the board and the checkers.
- Each rule includes the outcome of the relevant action in terms of the state.
- What's a good reward?

A More Complex Example

- Consider an agent which must learn to drive a car
 - State?
 - Possible actions?
 - Rewards?

Some Challenges

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2. Defining our reward
3. Learning our policy

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Dynamic programming

use value functions to structure the search for good policies

Dynamic programming

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- policy evaluation: compute V^π from π
- policy improvement: improve π based on V^π

Dynamic programming

use value functions to structure the search for good policies

policy evaluation: compute V^π from π ↗
policy improvement: improve π based on V^π ↘

start with an arbitrary policy

repeat evaluation/improvement until convergence

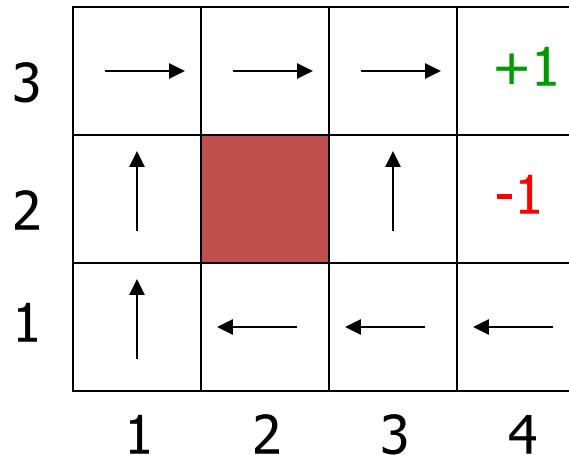
Reactive Agent Algorithm

Repeat:

- ◆ $s \leftarrow$ sensed state
- ◆ If s is a terminal state then exit
- ◆ $a \leftarrow$ choose action (given s)
- ◆ Perform a

Accessible or
observable state

Policy (Reactive/Closed-Loop Strategy)



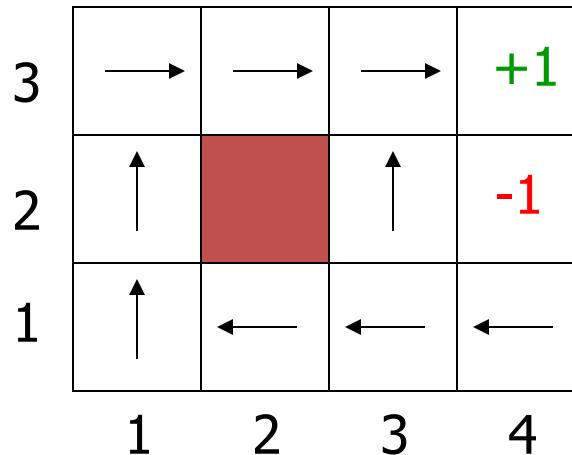
- In every state, we need to know what to do
- The **goal** doesn't change
- A **policy** (Π) is a complete mapping from *states* to *actions*
 - "If in [3,2], go up; if in [3,1], go left; if in..."

Reactive Agent Algorithm

Repeat:

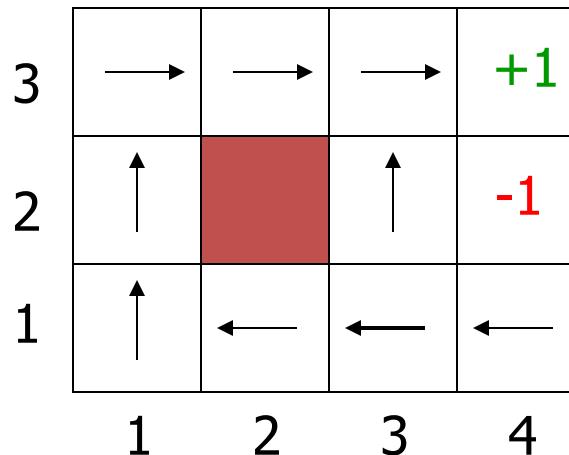
- ◆ $s \leftarrow$ sensed state
- ◆ If s is terminal then exit
- ◆ $a \leftarrow \Pi(s)$
- ◆ Perform a

Optimal Policy



- A **policy Π** is a complete mapping from states to actions
- The **optimal policy Π^*** is the one that always yields a history (sequence of steps ending at a terminal state) with maximal ***expected*** utility

Optimal Policy



- A **policy Π** is a complete rule for choosing actions
- The **optimal policy Π^*** is the policy that chooses the action with maximal expected utility

This problem is called a
Markov Decision Problem (MDP)

How to compute Π^* ?

Defining Value Function

- Problem:
 - When making a decision, we only know the reward so far, and the possible actions

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 - What is the value function of a particular **state** in the middle of decision making?
 - Need to compute ***expected value function*** of possible future histories/states

Defining Value Function

$$V^\pi(s) = \mathbb{E} [R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \dots \mid s_0 = s, \pi].$$

$V^\pi(s)$ is simply the expected sum of discounted rewards upon starting in state s , and taking actions according to π .¹

Given a fixed policy π , its value function V^π satisfies the **Bellman equations**:

$$V^\pi(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^\pi(s').$$

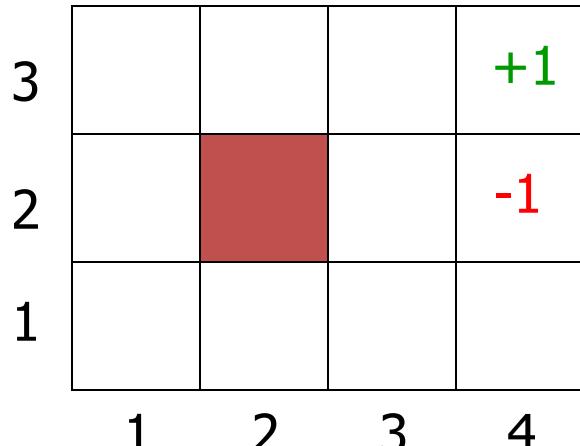
- What is the value function of a particular **state** in the middle of decision making?
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Value Iteration

Algorithm 4 Value Iteration

- 1: For each state s , initialize $V(s) := 0$.
- 2: **for** until convergence **do**
- 3: For every state, update

$$V^*(s) = R(s) + \max_{a \in A} \gamma \sum_{s' \in S} P_{sa}(s') V^*(s'). \quad (15.4)$$



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3	0.812 →	0.868 →	??? →	+1
2	0.762 ↑	0.660 ↑	-1	
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EXERCISE: What is $V^*([3,3])$ (assuming that the other V^* are as shown)?

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1 2 3 4

$$V^*_{3,3} = R_{3,3} + [P_{3,2} V^*_{3,2} + P_{3,3} V^*_{3,3} + P_{4,3} V^*_{4,3}]$$

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	1	2	3	4

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 V^*_{3,3} &= R_{3,3} + \\
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 &= -0.04 + \\
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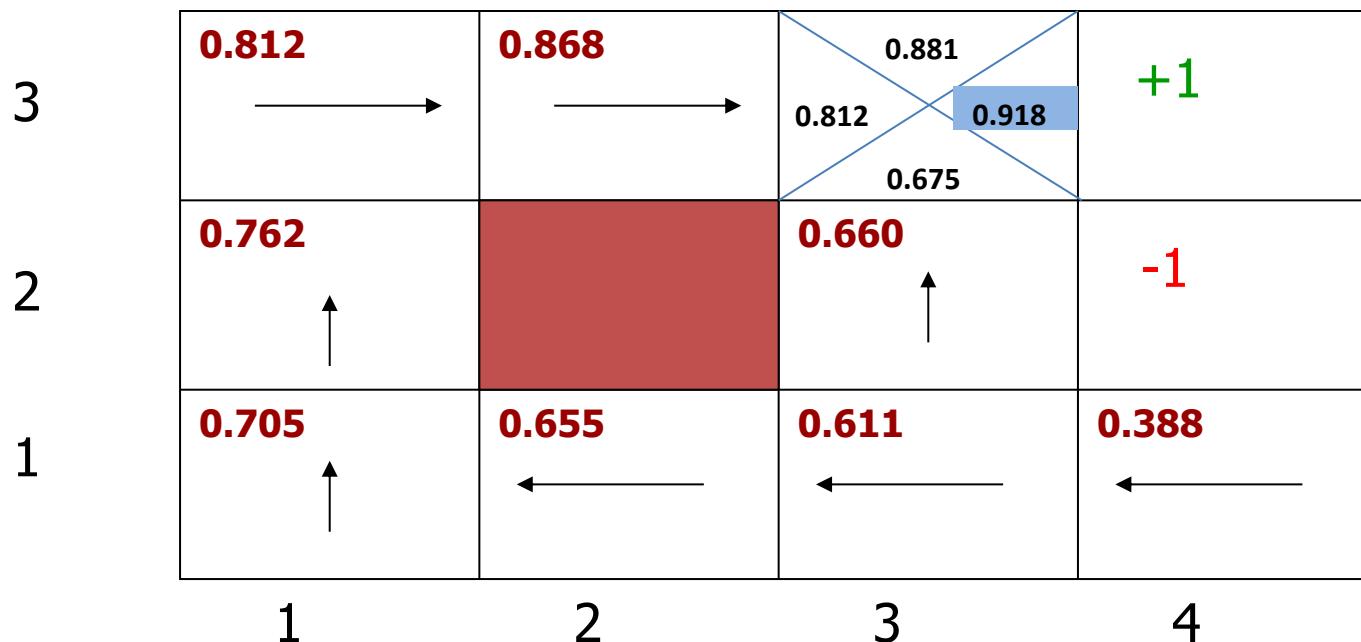
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More Breakdown

Value Iteration

In (3, 3), since → action gave us the **maximum expected future reward**, we choose to keep → in our policy. Same thing was done for all states.



Optimal Policy

$$\pi^*(s) = \arg \max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s').$$

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Whichever is higher becomes next action for (3, 1)

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Whichever is higher becomes next action for (3, 1)

Policy Iteration

- Pick a policy Π at random
- Repeat:
 - Compute Value function of each state for Π

$$V(s) := V^\pi(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s')V^\pi(s').$$

- Compute the policy Π' given these value functions

$$\pi'(s) := \arg \max_{a \in A} \sum_{s'} P_{sa}(s')V(s').$$

- If $\Pi' = \Pi$ then return Π

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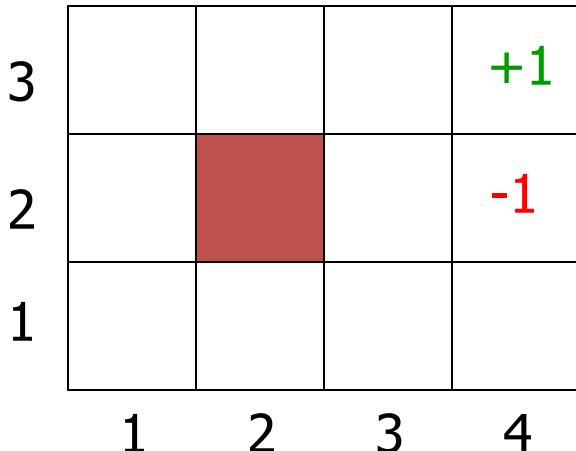
Or solve the set of linear equations:
(often a sparse system)

$$\pi'(s) := \arg \max_{a \in A} \sum_{s'} P_{sa}(s')V(s').$$

- If $\Pi' = \Pi$ then return Π

Infinite Horizon

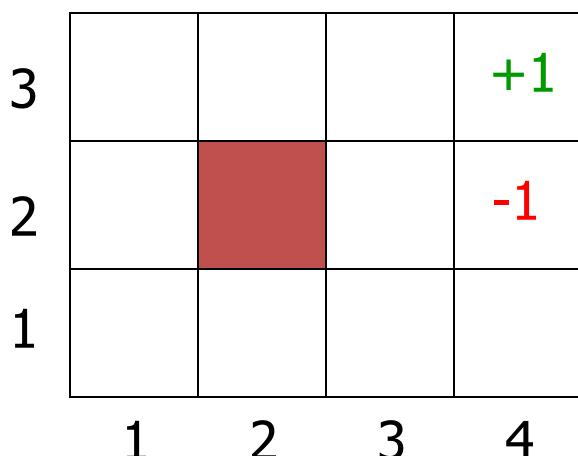
In many problems, e.g., the robot navigation example, histories are potentially unbounded and the same state can be reached many times



Advanced
topic

Infinite Horizon

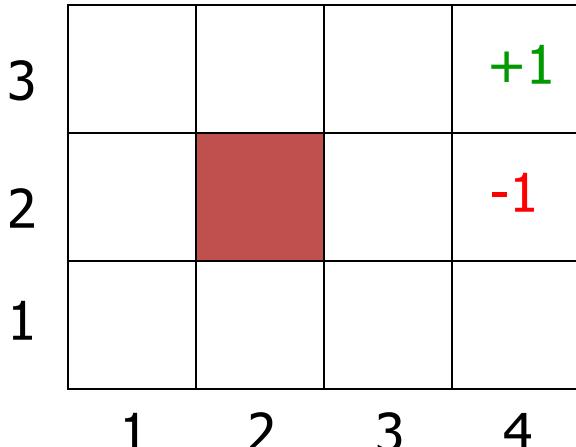
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What if the robot lives forever?

Infinite Horizon

In many problems, e.g., the robot navigation example, histories are potentially unbounded and the same state can be reached many times



What if the robot lives forever?

One trick:
Use discounting to make an infinite horizon problem mathematically tractable

Value Iteration: Summary

- Initialize state values (expected utilities) randomly
- Repeatedly update state values using best action, according to current approximation of state values
- Terminate when state values stabilize
- Resulting policy will be the best policy because it's based on accurate state value estimation

Policy Iteration: Summary

- Initialize policy randomly
 - Repeatedly update state values using best action, according to current approximation of state values
 - Then update policy based on new state values
 - Terminate when policy stabilizes
 - Resulting policy is the best policy, but state values may not be accurate (may not have converged yet)
 - Policy iteration is often faster (because we don't have to get the state values right)
-
- **Both methods have a major weakness: They require us to know the transition function exactly in advance!**

Exploration vs. Exploitation

- Problem with naïve reinforcement learning:
 - What action to take?
 - **Best apparent action, based on learning to date**
 - Greedy strategy
 - Often prematurely converges to a suboptimal policy!
 - **Random (or unknown) action**
 - Will cover entire state space
 - Very expensive and slow to learn!
 - When to stop being random?
 - Balance exploration (try random actions) with exploitation (use best action so far)

More on Exploration

- Agent may sometimes choose to explore suboptimal moves in hopes of finding better outcomes
 - Only by visiting all states frequently enough can we guarantee learning the true values of all the states
- When the agent is **learning**, ideal would be to get accurate values for all states
 - Even though that may mean getting a negative outcome
- When agent is **performing**, ideal would be to get optimal outcome
- A learning agent should have an **exploration policy**

Exploration Policy

- Wacky approach (exploration): act randomly in hopes of eventually exploring entire environment
 - Choose any legal checkers move
- Greedy approach (exploitation): act to maximize utility using current estimate
 - Choose moves that have in the past led to wins
- Reasonable balance: act more wacky (exploratory) when agent has little idea of environment; more greedy when the model is close to correct
 - Suppose you know no checkers strategy?
 - What's the best way to get better?

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Q-learning

$$Q: (\textcolor{orange}{s}, \textcolor{teal}{a}) \rightarrow \mathbb{R}$$

Goal: learn a function that
computes a “goodness” score
for taking a particular action $\textcolor{teal}{a}$
in state $\textcolor{orange}{s}$

Q-learning

previous algorithms: on-policy algorithms

start with a random policy, iteratively improve
converge to optimal

Q-learning: off-policy

use any policy to estimate Q

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Q directly approximates Q^* (Bellman optimality equation)

independent of the policy being followed

only requirement: keep updating each (s,a) pair

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$R(s_t)$

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Deep/Neural Q-learning

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network

desired optimal solution

Deep/Neural Q-learning

$$Q(s, a; \theta) \approx Q^*(s, a)$$

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Approach: Form (and learn)
a neural network to model
our optimal Q function

Deep/Neural Q-learning

Learn weights
(parameters) θ of our
neural network



$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network

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Approach: Form (and learn)
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Overview: Learning Strategies

Dynamic Programming

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Monte Carlo policy evaluation

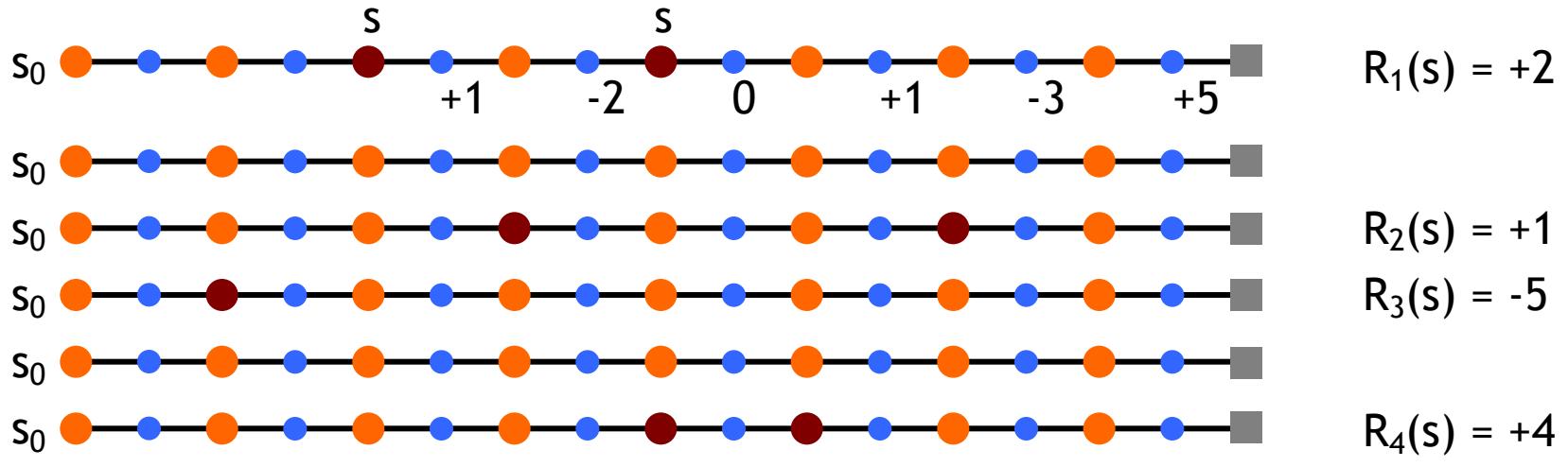
don't need full
knowledge of
environment (just
(simulated) experience)

want to estimate $V^\pi(s)$

Monte Carlo policy evaluation

don't need full knowledge of environment (just **(simulated)** experience)

want to estimate $V^\pi(s)$
expected return starting from s and following π
estimate as average of observed returns in state s



$$V^\pi(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$$

Maintaining exploration

key ingredient of RL

deterministic/greedy policy won't explore all actions

- don't know anything about the environment at the beginning
- need to try all actions to find the optimal one

maintain exploration

- use *soft* policies instead: $\pi(s,a) > 0$ (for all s,a)

ϵ -greedy policy

- with probability $1-\epsilon$ perform the optimal/greedy action

- with probability ϵ perform a random action

- will keep exploring the environment

- slowly move it towards greedy policy: $\epsilon \rightarrow 0$

RL Summary 1:

- **Reinforcement learning systems**
 - Learn **series** of actions or decisions, rather than a single decision
 - Based on feedback given at the end of the series
- A reinforcement learner has
 - A goal
 - Carries out trial-and-error search
 - Finds the best paths toward that goal

RL Summary 2:

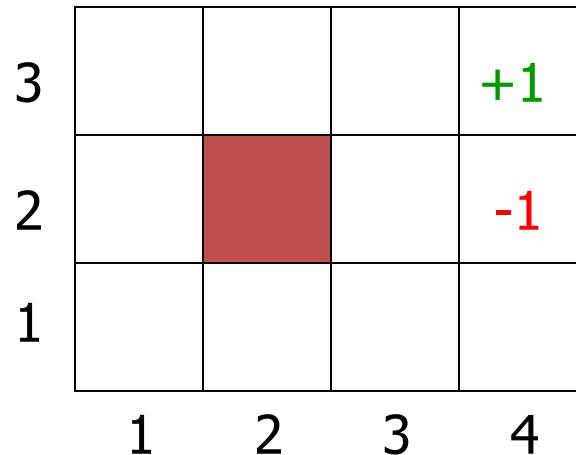
- A typical reinforcement learning system is an active agent, interacting with its environment.
- It must balance:
 - Exploration: trying different actions and sequences of actions to discover which ones work best
 - Exploitation (achievement): using sequences which have worked well so far
- Must learn **successful sequences of actions** in an uncertain environment

RL Summary 3

- Very hot area of research at the moment
- There are **many** more sophisticated RL algorithms
 - Most notably: probabilistic approaches
- Applicable to game-playing, search, finance, robot control, driving, scheduling, diagnosis, ...

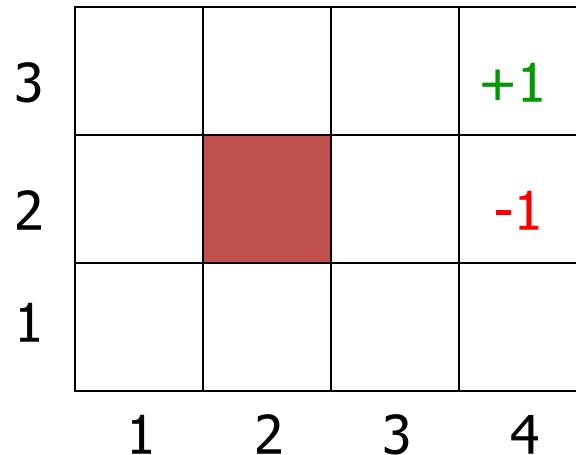
EXTRA SLIDES

Utility Function



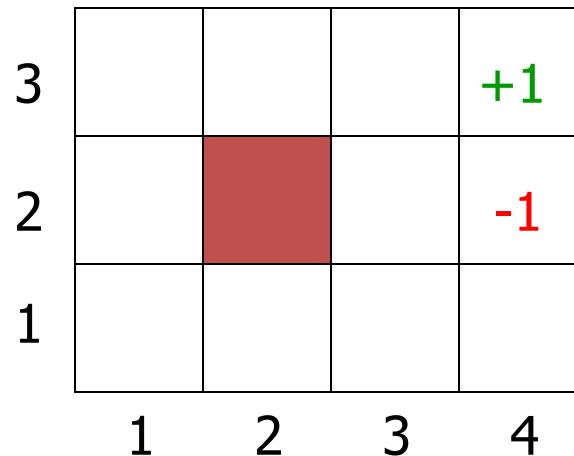
- [4,3] provides power supply
- [4,2] is a sand area from which the robot cannot escape

Utility Function



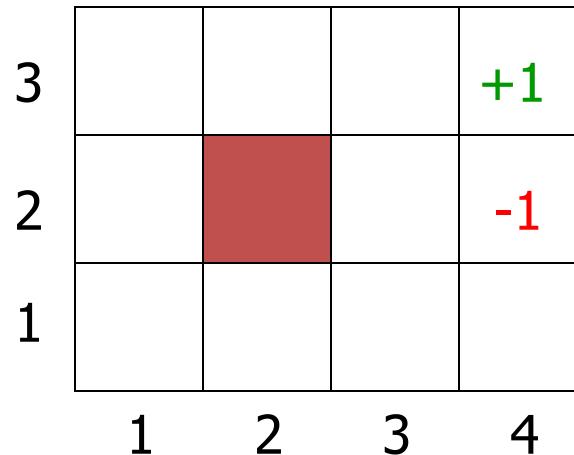
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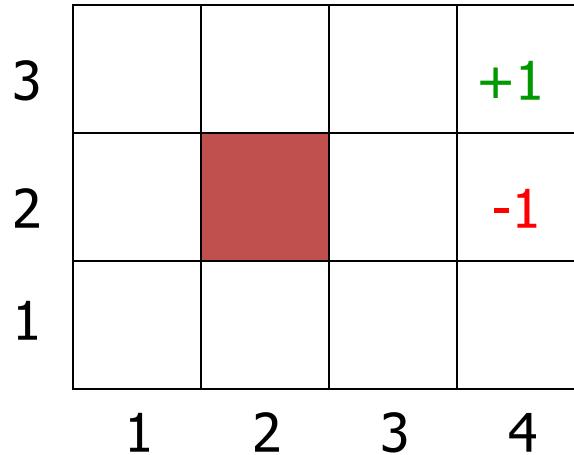
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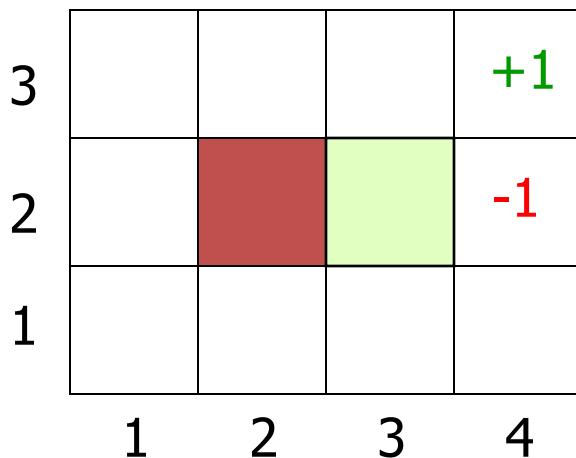
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- [4,3] and [4,2] are terminal states
- Histories have utility!

Utility of a History



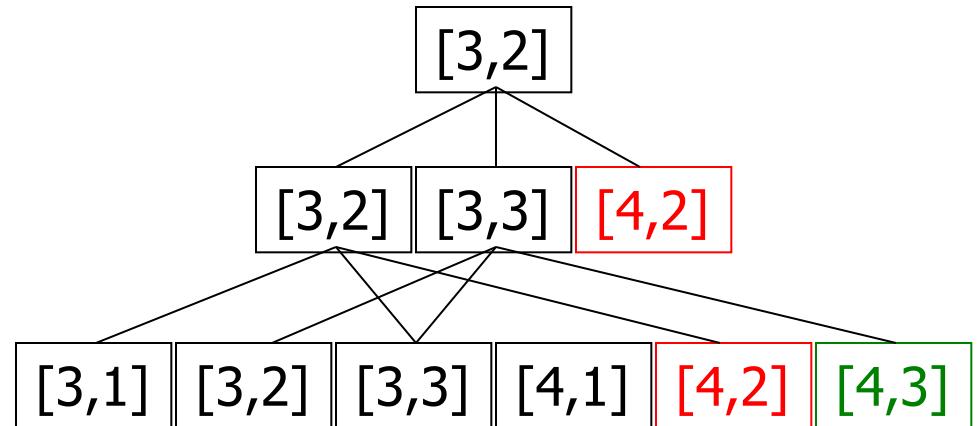
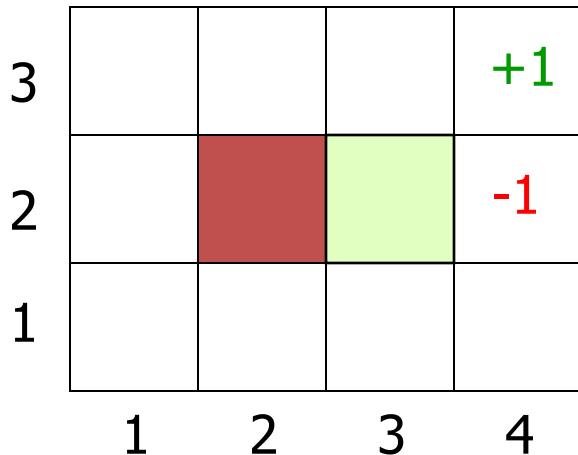
- [4,3] provides power supply
- [4,2] is a sand area from which the robot cannot escape
- The robot needs to recharge its batteries
- [4,3] or [4,2] are terminal states
- Histories have utility!
- The utility of a history is defined by the utility of the last state (+1 or -1) minus $n/25$, where n is the number of moves
 - Many utility functions possible, for many kinds of problems.

Utility of an Action Sequence



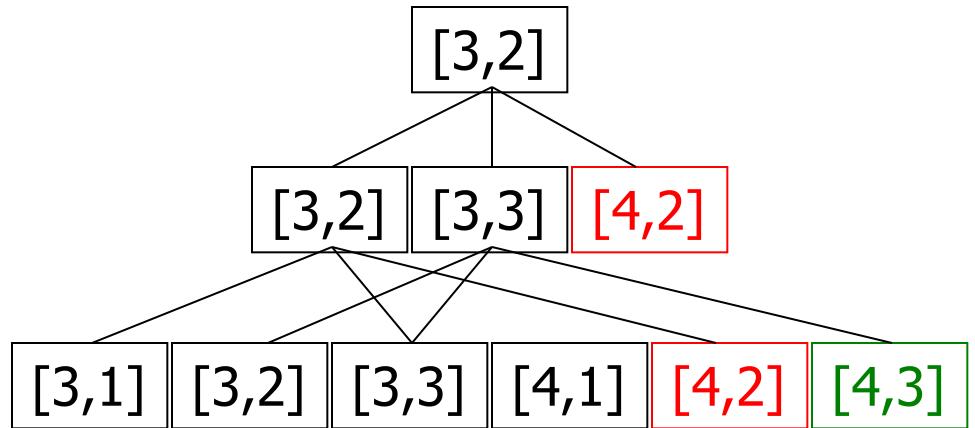
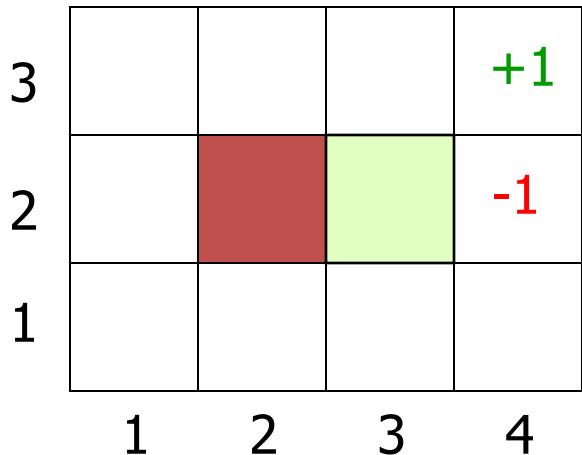
- Consider the action sequence (U,R) from [3,2]

Utility of an Action Sequence



- Consider the action sequence (U,R) from $[3,2]$
- A run produces one of 7 possible histories, each with some probability

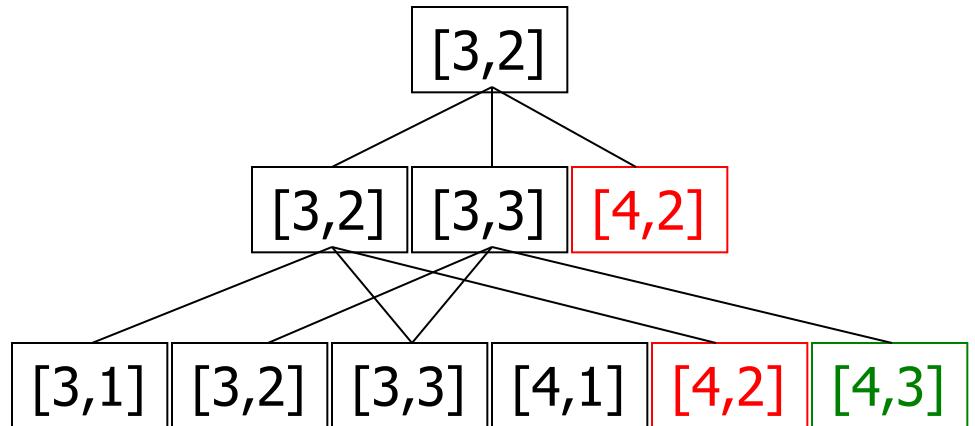
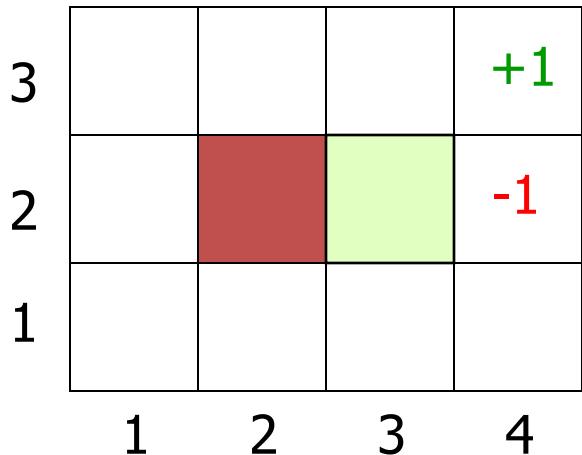
Utility of an Action Sequence



- Consider the action sequence (U, R) from $[3, 2]$
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- The **utility of the sequence** is the expected utility of the histories:

$$U = \sum_h U_h P(h)$$

Optimal Action Sequence

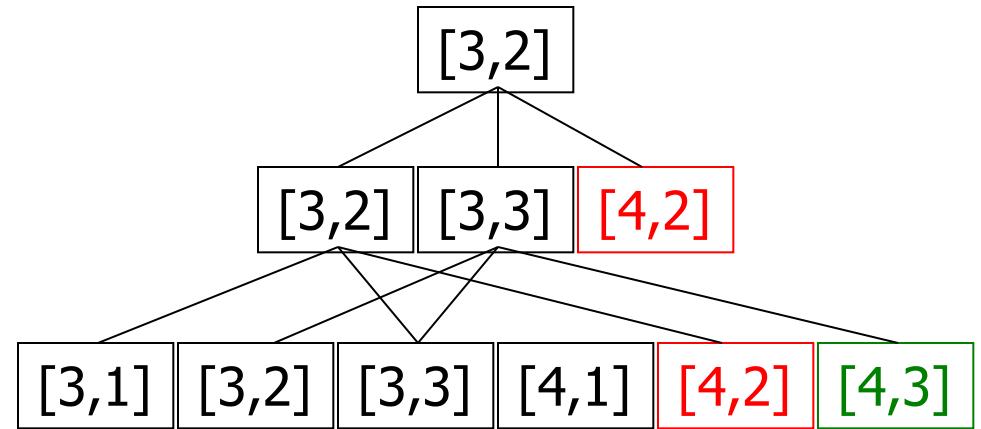
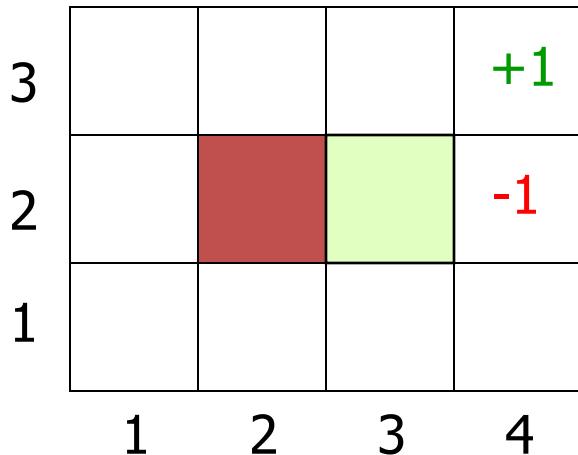


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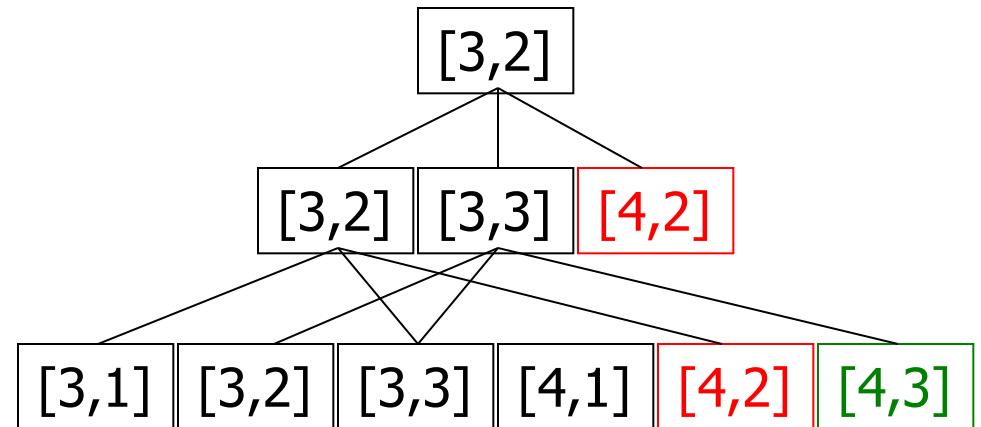
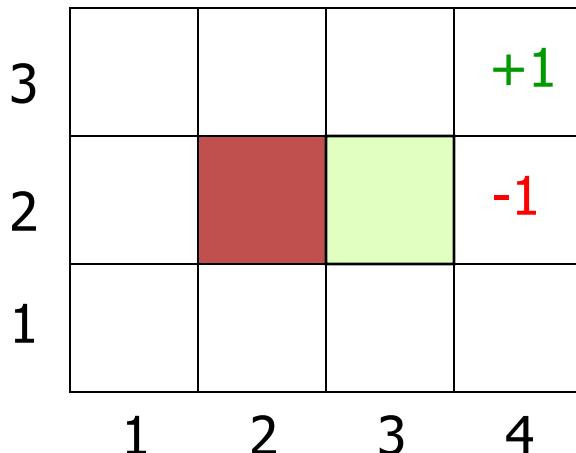
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Optimal Action Sequence



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- The **optimal sequence** is the one with maximal utility
- **But is the optimal action sequence what we want to compute?**

Optimal Action Sequence



- Consider the action sequence (U,R) from [3,2]
- A run produces utility **only if the sequence is executed blindly!**
- The utility of the sequence is the expected utility of the histories
- The **optimal sequence** is the one with maximal utility
- **But is the optimal action sequence what we want to compute?**