Electoral Competition and Incentives for Political Corruption*

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Abstract

What are the effects of electoral competition on political corruption? In this paper, I propose a formal model of political corruption where an incumbent decides how much kickbacks to add to public goods expenditure, and challengers may run anti-corruption campaigns against the incumbent. To understand how the number of competitors affect the results, I study two benchmarks: in the first, called the non-excludable anti-corruption campaign, successful exposure of an incumbent opens her seats to all other competitors. In the second benchmark, the excludable anti-corruption campaign, the politician successfully exposing the corruption signals competence, and voters tend to prefer her vis-a-vis the other challengers. When anti-corruption campaigns are non-excludable, an increase in the number of challengers raises corruption. However, when anti-corruption campaigns are excludable, the number of challengers is endogenously limited. The excludability makes the free-riding in corruption exposure disappear. These predictions hold empirically using quasi-experimental data from Brazilian and Italian municipalities. This study has implications for the design of electoral institutions.

Keywords: Political Corruption; Competition; Negative Campaign; Welfare; Brazil

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1 Introduction

Political accountability requires that voters, upon XXXX, sanction XXXX. As the electoral process creates a XXXX, voters use elections to reward beneficial XXXX and to sanction XXXX. Different from markets, the electoral sanctioning is XXXX and XXXX (?).

However, when sanctioning political corruption, voters seem to have a more XXXX approach to XXXX. For instance, two cases in Latin America well illustrate this XXXX. In Argentina, XXXX. On the other hand, Jair Bolsonaro in Brazil XXXX. Albeit remarkable differences between the cases, in Bolsonaro successfully exploited the major corruption scandal uncovered in Brazilian history, and made its way to XXXX against the XXXX. In Argentina, Macri lost his reelection bid without going to runoff, because he could not credibly XXXX the uncover of XXXX that happened a few months earlier by the Argentinian justice. Therefore, what is the XXXX?

In this paper, I propose a simple theoretical model to understand the electoral sanctioning of political corruption. In the model, I investigate one main feature celebrated as a signal of a healthy democracy: the existence and abundance of electoral challengers, running against a XXXX incumbent.

My model has XXXX distinctive characteristics. First, I model political corruption as the XXXX of kickbacks in a public goods provision XXXX. The provision of the public goods usually hides the corruption involved and distract voters. However, a public good that costs considerably more than its market price XXXX. Second, challengers are the main XXXX in exposing corrupt politicians. XXXX. Finally, exposing corruption has an XXXX. Challengers have the incentive to always claim that XXXX.

I solve the model in three stages. First, I solve the model for one incumbent versus one challenger. In this case, exposing a corrupt politician has a direct effect on XXXX, as XXXX. Second, I solve the case of one incumbent versus many challengers assuming the *non-excludable benchmark*, where XXXX.

I show that, compared with the one challenger case, the *non-excludable* benchmark generates more corruption while the *excludable* benchmark generates less corruption. When there is one challenger, all the negative campaign benefits directly the challenger. In this sense, the costs of running a negative campaign are mostly paid off when taking down a corrupt politician. However, when there are more challengers, exposing corruption aggregates a free-riding incentive. Whenever the person denouncing the corruption gains directly from it, then the XXXX, On the other hand, when XXXX. The intensity of the free-riding is crucial to determine whether XXXX is XXXX.

XL.

This model contributes to four strands in the literature. First, it contributes with the literature on the effect of electoral systems on political corruption. XXXX. This paper shows that political competition also plays an important role to understand political corruption. But different from **?**, competition have a non-linear effect, depending on XXXX: competition in countries with widespread corruption generates free-riding incentives, as challengers know that a successful exposure benefit all of them. In countries with low corruption, XXXX. XXXX can have a beneficial or harmful effect, depending on how widespread the corruption in the country is. XXXX.

Second, this paper contributes to the literature on retrospective voting. XXXX.

Third, this paper contributes to a growing literature on the effects of political competitiveness on elections. XXXX. This paper aligns with XXXX, showing that competition can be harmful, especially in cases when XXXX.

Finally, this paper contributes to a growing literature on the effects of negative campaign. XXXX. Despite the nefarious effects, negative campaign by challengers, aimed at exposing a corruption, is crucial to hold politicians accountable. When the judicial prosecution fails, the best alternative to sanction corrupt politicians is to use the electoral system, and XXXX.

2 Elections and Political Corruption

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3 A Model of Elections under XXXX

Consider a strategic interaction between three players: an incumbent politician (I), a challenger politician (C), and a representative voter (V).

The game starts in the *governing stage*, with a recently elected incumbent politician deciding the expenditure in public goods provision. Upon taking office, the incumbent learns the required expenditure to provide the public goods and services that voters need. For instance, the incumbent may learn that a school or hospital needs investments in infrastructure, and she decides how much to allocate for this. The required expenditure is only learned when the incumbent takes office, and it is her private information. I denote the required expenditure level $\theta \in [0, R]$, and the incumbent has a budget of $R > 0$ resources to invest in public goods.

But the *enacted* expenditure in public goods provision is under the incumbent's discretion. This means that, despite knowing the required investment levels, incumbents are under no obligation to spend optimally, and may invest more or less than this required investment level. For simplicity, I assume two consequences for the differences between required and enacted investments. If the incumbent decides to invest less than the required amount, I assume the public goods are not provided. This simplification matches the reality in most cases where restricting investments can cripple the entirety of the service provision. On the other hand, when they invest more than the optimal amount, they can pocket the difference as rents. These assumptions together imply that incumbents will always provide the public goods. In fact, if they care about rents, they will always over-provide the goods, to pocket the differences between the required and the proposed expenditure.

This provides an interesting micro-foundation for corruption: Political corruption here are kickbacks, and public goods are always provided by corrupt politicians. This matches most cases of corruption in procurement contracts, where the government provides the goods, at a higher-than-market prices. For instance, during the 2016 Brazilian World Cup, the stadiums constructed with public money were delivered for the world cup, despite the fact that they costed 1.5 to 2.5 times more than comparable stadiums in less corrupt settings (?).

After the governing stage, it starts the *campaign stage*. During the campaign, the challenger learns the expenditure enacted by the incumbent e, and she knows that e most likely higher than the required expenditure θ. However, she knows that the required expenditure is a random variable, distributed uniformly between zero and R, and with that information, she form a belief about the optimal expenditure using Bayes' rule.

The challenger then decides whether or not to run a negative campaign against the incumbent. As I assume that voters have an incumbent-bias,[1] so that negative campaign is the only instrument that makes voters to revise their bias toward an incumbent that successfully provided the public goods. Additionally, when the challenger decides to run negative campaign against an incumbent, she pays a value $c > 0$. This value captures the direct and indirect costs of collecting information about corruption. Moreover, it also represents the potential opportunity costs of trying to convince the voters that the incumbent is corrupt.

Finally, the *voting stage* of the game features the representative voter observing the information presented by the challenger, and deciding whether to sanction or not the incumbent. Challengers have an incentive to run negative campaign against incumbents, regardless of whether the incumbent engaged in corruption. The representative voter, knowing the challenger's incentives to falsely accuse incumbents, evaluates the presented evidence of corruption. The strength of the evidence correlates with the difference between the required expenditure and the proposed expenditure. Voters and challengers do not know the require expenditure, but the greater the distance between the required and the enacted expenditures, the easier for challengers to convince voters that the incumbents are corrupt. This assumption has empirical support is most accounting literature, as the higher the kickbacks, the harder it is to hide it from authorities (?).

The timeline for this strategic interaction is as follows:

1. Nature draws the required expenditure level to provide the public services.
2. The incumbent observes the required expenditure level and chooses the government's expenditure in public goods.

[1] Meaning that they prefer to reelect incumbents when the public goods were successfully provided. This correlates with incumbency advantage findings (?) as well as formal models of public service provision (?).

3. After the public goods provision, the election starts with the challenger deciding her campaign strategy.

4. The representative voter observes the campaign and the expenditure, and chooses whether to retain or dismiss the incumbent. If she dismisses the incumbent, she automatically selects the challenger.

To differentiate between strategies and actions, I assume that the incumbent's decision for the expenditure is a function $\varepsilon : [0, R] \to [0, R]$, i. e., the decision-making process receives the information about the required expenditure level, and returns the expenditure decision. For the challenger there is a similar reasoning, and the decision to investigate the incumbent is a function $\varphi : [0, R] \to [0, 1]$, that receives the expenditure decision and returns a probability of running the negative campaign. Condensing the discussion above, the incumbent, upon observing the optimal public goods provision level $\theta \in [0, R]$, decides the expenditure $\varepsilon \in [0, R]$. The benefits from pocketing resources are captured by a function $u(.)$. In the main paper, I assume it is equal to the identity function but that in the appendix I solve the model for a generic concave function. The expected utility of the incumbent also adds the benefit from winning the election $B > 0$ times the probability of this event, given the optimal and enacted expenditures and the decision to investigate φ.

$$U_I(\varepsilon; \theta, \varphi) \;=\; u(\varepsilon - \theta) + B\mathbb{P}(\text{re-election}; \varepsilon, \theta, \varphi)$$

The expected utility for the challenger involves a decision in terms of the probability to win the election and the costs of running a negative campaign. This benefit from holding office $B > 0$ is multiplied by the probability that the challenger replaces the incumbent.[2] The probability that the challenger wins the election depend on the decision to run a negative campaign, the enacted expenditure, and on her beliefs about the required expenditure, when she observes the enacted expenditure. The beliefs are captured by a probability rule $\mu(\theta|\varepsilon)$. The challenger also decides the anti-corruption campaign strategy $\varphi \in [0, 1]$ and, if she pursues the investigation, she pays a cost of $c > 0$.

$$U_C(\varphi; \varepsilon) \;=\; B \int_0^{\varepsilon} \mathbb{P}(\text{win election}|\varphi, \varepsilon, \theta)\mu(\theta|\varepsilon)d\mu - c\varphi$$

Finally, any voter's decision is sequentially rational, as it happens after the incumbent implements her expenditure strategy and the challenger runs the anti-corruption campaign. They cannot change the incumbent's or the challenger's decisions ex-post, and this makes it easier for me to set the decision rule of the voter. The voter's decision will be to remove the incumbent from office whenever the challenger shows credible evidence that the incumbent is corrupt.[3] The chance that the challenger convinces the voter is a

[2] I assume the material and non-material benefits from office the same for both the challenger and the incumbent. This simplifies the exposition, and makes it easier to think about these benefits as salaries or other types of pecuniary benefits.

[3] In the appendix XXXX, I discuss why this decision rule may be optimal from the perspective of the voter. Intuitively, when compared with reelection decisions such as @ferejohnXXXX, that has a single threshold for dismissal, this decision rule partitions the

function $\lambda(\varepsilon - \theta)$, with $\lambda' > 0$ to capture the idea that more kickbacks represent a higher chance of detect corruption.[4]

I solve this game for a *Perfect Bayesian Equilibrium* (**??**), and as this is a signaling game, I also use *universal divinity* as the equilibrium selection criteria (**?**). The restriction imposed by the universal divinity requires that the challenger, facing a deviation from the equilibrium path, assign a probability one on for the type that is more likely to deviate. Moreover, as I have a continuum of possible expenditure strategies, I use the approach by **?**, which consists in finding the monotonicity in the payoffs and single crossing conditions, characterizing the behavior of the types inside the intervals of the type space, analyzing the incentives to pool or separate among the types.

Definition 1. *A Perfect Bayesian Equilibrium for the strategic interaction described consists in strategic choices of the expenditure ε and the probability of running negative campaign φ such that:*

1. *For a given required expenditure level, $\theta \in [0, R]$, the proposed expenditure ε maximizes the incumbent's expected utility $U_I(\varepsilon; \varphi^*(\varepsilon), \theta)$.*

2. *The challenger observes the incumbent's equilibrium expenditure level ε^*, and decides her negative campaign strategy φ to maximize the expected utility $U_C(\varphi; \varepsilon^*, \mu(\theta|\varepsilon^*))$, given her beliefs $\mu(\theta|\varepsilon^*)$ about the required expenditure level θ.*

3. *The voter observes ε^* and the realization of φ^* (which is either zero or one), plus the result of the anti-corruption campaign $\lambda(.) \in \{0, 1\}$. She dismisses the incumbent if the anti-corruption campaign reveals corruption, and retain otherwise.*

4. *The beliefs are derived using Bayes' Rule, whenever possible. The *universal divinity* also requires that for all expenditure levels that are off-the-equilibrium path, the challenger beliefs $\mu(\theta|\varepsilon) > 0$ (in fact, 1) only if the deviant type $\widehat{\varepsilon}$ is in $\widehat{\varepsilon} \in \arg\min_\theta p(\theta, \widehat{\varepsilon})$, where $p(\theta, \widehat{\varepsilon})$ is the profit from deviating from the equilibrium expenditure ε to a deviation expenditure $\widehat{\varepsilon}$, when the required expenditure level is θ.*

Note that this model has no ideological proximity, or affinity between the voter and any of the politicians. This because the paper's objective is to explore the incentives for corruption, not the trade-offs between ideology and corruption.[5] In the appendix, we model a situation where there is a positive valence shock towards one of the candidates, that is known before the election. My findings, qualitatively, remain the same, with the caveat that depending on the direction of the valence shock, the incumbent extracts even more rents in equilibrium.

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voter's information set more finely. The partitioning reduces the amounts of kickbacks at a range for the random variable larger than when only the voter sanctions.

[4]For the simulations, I use $\lambda(\varepsilon - \theta) = \dfrac{\varepsilon - \theta}{R}$. In the online appendix, I model the decision for a generic functional form for $\lambda(.)$.

[5]For papers exploring these trade-offs, see XXXX.

The benchmark for the model is the one incumbent x one challenger setting. By comparing this situation with the one incumbent x multiple challengers, I show the impact of the competitiveness on the equilibrium rents. Moreover, using Monotone Comparative Statics on the equilibrium, I can show the conditions under which the rent extraction decreases as the number of challenger increases. This threshold is characterized by an XXXX of excludability η that determines XXXX.

3.1 One Incumbent x One Challenger

Applying backward induction, the voters retain the incumbent if the incumbent provided the public goods and no corruption was found. From the challenger's perspective, as she faces no entry costs, she decides whether to run an anti-corruption campaign. She does so when the expected utility of run a negative campaign is higher than or equal the expected utility of not running a negative campaign, given her beliefs about the required level of public goods investment θ. The expected utility of running a negative campaign is equal to the benefit from office B times the probability of the anti-corruption campaign be successful. This probability depends on the beliefs about the true value of the required public goods investment, derived by Bayes' rule ε, $\mu(\theta|\varepsilon) = \dfrac{f(\theta)}{\int_0^{\varepsilon} f(\theta)d\theta}$, and the chance that the anti-corruption campaign works $\lambda(\varepsilon - \theta)$. All this minus the cost $c > 0$ of running an anti-corruption campaign. When the anti-corruption campaign strategy is $\varphi(\varepsilon)$, the expected benefit is equal to:

$$\varphi(\varepsilon)\left[B\int_0^{\varepsilon} \lambda(\varepsilon-\theta)\frac{f(\theta)}{\int_0^{\varepsilon} f(\theta)d\theta}d\theta - c\right] \geq 0$$

Running a negative campaign is a risky bet: it costs c for sure, but the benefits are uncertain. The incumbent calibrates her rent extraction to make the challenger's cost-benefit equation $B\int_0^{\varepsilon}\left(\lambda(\varepsilon-\theta)\dfrac{f(\theta)}{\int_0^{\varepsilon} f(\theta)d\theta}\right)d\theta - c$ never always positive or always negative. If the challenger's cost-benefit equation is always positive, the incumbent is extracting too much rents. In this case, it is beneficial for challengers to run a negative campaign for sure ($\varphi(\varepsilon) = 1$). If the challenger's cost-benefit equation is always negative, on the other hand, the incumbents are extracting to little rents. In this case, the challenger should never run a negative campaign ($\varphi(\varepsilon) = 0$). In the always positive case, the incumbent can do better by extracting *fewer* rents. In the always negative case, the incumbent can do better by extracting *more* rents. If we consider that these two forces are at play simultaneously, they converge to a point where the challenger's cost-benefit to run an anti-corruption campaign is equal to zero. Zeroing the cost-benefit decision makes the challenger indifferent between running or not running an anti-corruption campaign. However, it could be the case that either the office benefits are too small, or the costs are too high, that even when incumbents extract all the rents possible, no anti-corruption campaign happens in equilibrium. To rule out this possibility, I assume the following:

Assumption 1. *To avoid a trivial equilibrium where there is no anti-corruption campaign in equilibrium, I* *assume that the benefits from office B and the costs to run a negative campaign c are such that:*

$$\frac{c}{B} < \int_0^R \lambda(R-\theta)f(\theta)d\theta = Q^*$$

I.e., the costs are low enough and the benefits from office are high enough to avoid a trivial equilibrium with no anti-corruption campaign.

By solving for the amount of rents that makes the challenger indifferent between run or not an anti-corruption campaign, I find the incumbents rent extraction strategy.

$$\varepsilon^*(\theta) \ = \ \theta + \lambda^{-1}\left(\frac{c}{B}\right)$$

The derivation, with all the details, is in the appendix. Note that strategic expenditure is equal to the required expenditure plus kickbacks. A simple inspection shows that the proposed expenditure is increasing in the cost to run the anti-corruption campaign c, decreasing in the office benefits B. Moreover, the proposed expenditure is linear on the amount of kickbacks $\lambda^{-1}\left(\frac{c}{B}\right)$: the proposed expenditure is equal to the required expenditure θ plus the amount of kickbacks $\lambda^{-1}\left(\frac{c}{B}\right)$. Note also that it could be the case that the required expenditure is high, making the strategic expenditure to be higher than the budget for some levels of θ. In this case, define $\overline{\theta} = R - \lambda^{-1}\left(\frac{c}{B}\right)$ such that when the required expenditure surpasses this threshold, then the expenditure proposed by the incumbent equals the maximal expenditure, $\varepsilon^*(\theta) = R$.

By solving for the indifference in the challenger's expected utility, I find the expenditure strategy for the incumbent. This is how a perfect Bayesian equilibrium works: by solving the game backwards, the decision of one player is usually derived by maximizing the utility of another player. To find the challenger's anti-corruption investigation strategy, I need to show that the incumbent's expenditure strategy maximizes her expected utility. Proceeding backwards, the incumbent's expected utility is equal to the utility accrued from rents $u(\varepsilon-\theta)$ plus the benefits from holding office times the chance of win the election. The incumbent wins the election when there is no anti-corruption campaign $(1-\varphi(\varepsilon))$ or when the anti-corruption campaign fails $(\varphi(\varepsilon)(1-\lambda(\varepsilon-\theta)))$.

$$U_I(\varepsilon;\theta,\varphi(\varepsilon)) = u(\varepsilon-\theta) + B\left(1-\varphi(\varepsilon)+\varphi(\varepsilon)(1-\lambda(\varepsilon-\theta))\right)$$

To maximize this equation, we take the derivative in ε and equate the result to zero. This gives us the following differential equation:

$$\frac{d\varphi}{d\varepsilon} + \varphi(\varepsilon)\frac{B\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}{c} = \frac{u'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}{c}$$

The solution of the above equation characterizes the anti-corruption campaign decision by the challenger. Note that the equation has an unique solution when the boundary condition is well characterized. The boundary condition here states that when the challenger proposes $\varepsilon^* = \lambda^{-1}\left(\frac{c}{B}\right)$, this is the minimum amount of kickback possible, and it means that $\theta = 0$. In this situation, $\varphi\left(\lambda^{-1}\left(\frac{c}{B}\right)\right) = 0$. I solve this equation for $\varphi(\varepsilon)$ in details in the online appendix. The result is:

$$\varphi(\varepsilon) = \frac{u'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}{B\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}\left(1 - e^{\frac{B}{c}\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)\left(\lambda^{-1}\left(\frac{c}{B}\right)-\varepsilon\right)}\right)$$

A simple derivative check shows that when ε increases, the chance of running the anti-corruption campaign increases; when the costs c increase, the chance of running an anti-corruption campaign decreases; and when the benefits from office increase, the chances for running an anti-corruption campaign increases.

This anti-corruption campaign strategy works well for all incumbent's expenditures between $\lambda^{-1}\left(\frac{c}{B}\right)$ and $\overline{\theta}$. However, what if the incumbent expenditure is between 0 and $\lambda^{-1}\left(\frac{c}{B}\right)$ or between $\overline{\theta}$ and R? These are *out-of-path-of-play* expenditures, and under the *universal divinity* equilibrium, we should best-respond based on the types that are more likely to deviate. Any expenditure between 0 and $\lambda^{-1}\left(\frac{c}{B}\right)$ should be met with $\varphi(\varepsilon) = 0$, as the challenger's cost-benefit to run the anti-corruption campaign is negative. The interesting case is when the incumbent proposes an expenditure between $\overline{\theta}$ and R. In this case, the required expenditure θ is surely between 0 and $\overline{\theta}$. Otherwise, the incumbent would have selected R. This signals that the incumbent is trying to get more rents than the equilibrium prescribes, and this makes the cost-benefit for the challenger positive. The challenger's best response in this case is to set $\varphi(\varepsilon) = 1$.

Combining all these results together gives us the following lemma, that characterizes the equilibrium for the one incumbent versus one challenger case.

Lemma 1. *A Perfect Bayesian Equilibrium for the game when there is one incumbent and one challenger is a group of strategies and beliefs such that:*

1. *For the incumbent, upon observing the required expenditure θ, she proposes the expenditure level:*

$$\varepsilon^*(\theta) = \begin{cases} \theta + \lambda^{-1}\left(\frac{c}{B}\right) & if\, 0 \leq \theta \leq \overline{\theta} \\ A & if\, \theta > \overline{\theta} \end{cases}$$

2. *For the challenger, upon observing the incumbent's investment strategy ε, she runs an anti-corruption*

campaign with the probability:

$$\varphi^*(\varepsilon) = \begin{cases} 0 & if\, \varepsilon < \lambda^{-1}\left(\frac{c}{B}\right) \\ \frac{u'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}{B\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}\left(1 - e^{\frac{B}{c}\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)\left(\lambda^{-1}\left(\frac{c}{B}\right)-\varepsilon\right)}\right) & if\, \lambda^{-1}\left(\frac{c}{B}\right) \leq \varepsilon \leq \overline{\theta} \\ 1 & if\, \overline{\theta} < \varepsilon < A \\ \frac{u'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}{B\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)}\left(1 - e^{\frac{B}{c}\lambda'\left(\lambda^{-1}\left(\frac{c}{B}\right)\right)\left(\lambda^{-1}\left(\frac{c}{B}\right)-A\right)}\right) & if\, \varepsilon = A \end{cases}$$

3. *For the voter, upon observing the anti-corruption investigation and the proposed expenditure by the incumbent, she replaces the incumbent if there is evidence of corruption.*

And the beliefs are updated using Bayes' Rule, whenever possible.

The remainder of the proof and the demonstration that this strategy is indeed an equilibrium is in the online appendix. Note three critical features on this equilibrium: First, an increase in the value of office (B) decreases the kickbacks extracted and the chance that the challenger runs an anti-corruption campaign. Decreases kickbacks because it becomes more profitable to be reelected vis-a-vis extract rents, and lowers the chance of an anti-corruption campaign because lower rent extraction it makes it harder to detect the corrupt behavior. Second, increasing the costs of running an anti-corruption campaign (c), increases the kickbacks and decreases the chance of running an anti-corruption campaign. The This because amount of exposure in equilibrium. Finally, increasing the gains from extracting rents (increasing u') does not change the amount of extraction, but increases the chance that the challenger runs an anti-corruption campaign, as rents became more valuable for the incumbents. These results are intuitive, but they reinforce that the model is grounded in reasonable logic.

3.2 One Incumbent x Many Challengers

Consider a change in the previous setup whereby we add multiple challengers running against the incumbent. Solving the model

an strategic interaction comprised of one incumbent, L challengers, and a representative voter. Starting with the voters, suppose that she learns something about the challenger that brought the corrupt incumbent down with probability $\eta \in (0, 1)$. This is the case that, for example, the challenger that successfully ran the anti-corruption campaign receives a large quantity of media exposure, which helps her election bid, making the voter to prefer her vis-a-vis the other challengers. On the other extreme, there is a chance of $1 - \eta$ that the voter selects one of the challengers randomly regardless of her running or not an anti-corruption campaign. In this case, each challenger has a chance of $1/L$ to get elected. . . fHowever. I label η the degree of excludability in the anti-corruption campaign, i.e., the extent through which the her optimal decision rule is XXXX.

their optimal strategy is to retain the incumbent if the incumbent provides the public good and no corruption has been uncovered. If the voters dismiss the incumbent, they randomly select a challenger with equal probability.

Challengers observe ψ and ε and have to simultaneously compute the time to invest in negative campaign φ_i. As challengers have the same utility function, we solve for a symmetric equilibrium. Let $\lambda(\varepsilon - \eta) = \lambda_\varepsilon$. If we have k players investing in a negative campaign against the incumbent politician, the probability that the incumbent gets successfully exposed and lose the election is:

$$f(\varphi, x) = \sum_{k=0}^{L-1} \binom{L}{k} \varphi^k (1-\varphi)^{L-1-k} \cdot (1 - (1-\lambda_\varepsilon)^{k+x})$$

So when i decides to commit all her resources in a negative campaign, we let $x = 1$ in f. This strategy allows us to compute the expected utility for the challenger i as:

$$\frac{B}{L} \int_0^A f(\varphi, 0) \mu(\eta|\varepsilon) d\mu + \varphi_i \left[\frac{B}{L} \int_0^A (f(\varphi, 1) - f(\varphi, 0)) \, \mu(\eta|\varepsilon) d\mu - c \right]$$

Sequential rationality requires that $\varphi_i > 0$ when this quantity is greater than zero, and zero otherwise. After some algebra, we can rewrite the critical term in the decision process as an equation G:

$$G(\lambda, \varphi; L) = (1 - \varphi\lambda(\varepsilon - \theta))^{L-1} \lambda(\varepsilon - \theta) - \frac{c}{B} L$$

Where Q is defined as in the previous section: $Q(B, c) = \frac{c}{B}$. This equation yields to the optimal ε. The problem is that we will have, for each value of ε, a corresponding optimal value of φ, and vice-versa. As the incumbent plays first, she can choose the max-min value for λ, which is the maximum value for ε that still delivers the lowest φ possible. For a numerical example, if we assume $Q = 0.15$ and L ranging from 1 to 5, we can have a contour for the possible values for λ in Figure **??**.

Making $G = 0$ yields to the following optimal incumbent expenditure strategy:

$$\varepsilon^*(\eta) = \eta + \lambda^{-1} \left(Q(B, c) \cdot L \right) = \eta + \kappa(L)$$

Again, the actual value required to provide the service plus a kickback comprises the incumbent strategy. The kickback has the same characteristics expressed in the case with one challenger, except now it is augmented linearly by the number of challengers contesting the seat. In order to this expenditure profile be incentive compatible it has to satisfy the following differential equation in $\delta(\varepsilon)$:

$$\delta'(\varepsilon) - \delta(\varepsilon)\frac{\lambda'(\kappa(L))}{\lambda(\kappa(L))} = \frac{u'(\kappa(L))}{BQ(B,c)L^2}$$

With the boundary condition $\delta(\kappa(L)) = 0$, we have the following optimal exposure strategy:

$$\delta(\varepsilon) = \frac{u'(\kappa(L))}{BQ(B,c)L^2}\left(1 - e^{\frac{\lambda'(\kappa(L))}{\lambda(\kappa(L))}(\kappa(L)-\varepsilon)}\right)$$

Collecting all the results leads us to the following Lemma.

Lemma 2. *A Perfect Bayesian Equilibrium for the game when there is one incumbent and L challengers is a pair of strategies and beliefs such that*

1. *For the incumbent*

$$\varepsilon^*(\eta) = \begin{cases} \eta + \kappa(L) & if\, 0 \leq \eta \leq \overline{\eta} \\ A & if\, \eta > \overline{\eta} \end{cases}$$

2. *For each of the challengers*

$$\varphi^*(\varepsilon) = \begin{cases} 0 & if\, \varepsilon < \kappa(L) \\ \delta(\varepsilon) & if\, \varepsilon \in [\underline{\eta},\ \overline{\eta}] \\ \delta(A) & otherwise \end{cases}$$

3. *And the voter dismiss the incumbent if there is evidence of corruption, electing one of the L challengers with probability* $1/L$*.*

Where $\lambda(.)$ *is the challenger's campaign technology function. The optimal rent extraction is:*

$$\kappa(L) = \lambda^{-1}\left(L\frac{c}{B}\right)$$

And the $\delta(.)$ *defined as in the Appendix.*

The proof with the step-by-step derivations is in the Appendix. Note here that all the comparative statics with relation to the benefit from holding office, the costs of exposing corrupt politicians, and the utility of rent extractions remain unchanged. The fundamental differences here are the effect of the number of challengers: it increases the kickbacks while decreasing the incentives for investigating the corrupt incumbent. The reason is intuitive: the more challenger added, the more the collective action problem faced by the challengers. For every individual challenger, it is better to let someone else pay the cost of a negative campaign but if all challengers reason similarly no exposures are performed in equilibrium. Thus, adding challengers increase the incentives for corruption.

References