# Supplementary Materials for “Legislature Size and Welfare: Evidence from Brazil”

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## Contents



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## B.1 A Model of Legislature Size and Service Provision

### B.1.1 Primitives of the Model

Consider an strategic interaction between a mayor ($M$) and $N \geq 3$ city councilors, indexed in $i \in \{1, 2, \cdots, N\}$. The mayor is the head of the local executive branch, and has the prerogative of proposing a vector of policies, that is voted by the city councilors. Policy proposals are a combination of public goods provision $g$, rents for the mayor $r$, and a vector of transfer for city councilors $\curvearrowleft$.

The city council votes the mayor's proposal and if they accept it, it is implemented. Otherwise, a reversal policy takes place, where the decision leaves the mayor's hands and either immediately stops or transfers to the council. The types of reversal policies are crucial for our argument, as it changes the relative strength of the councilors in their bargaining with the mayor. We investigate three types of reversal policies: a *non-partisan* reversal mechanism, where city councilors have no partisan concerns; a *partisan* reversal mechanism, where the council is divided in support and opposition for the mayor; and finally a *hybrid* reversal mechanism, that combines partisan concerns with non-partisan rent extraction. In the main paper, we report only the hybrid model, as it is the more realistic scenario. Despite that, comparing the partisan and non-partisan reversal mechanisms illustrate clearly our argument that partisan a mechanism with only politics (city councilors have parties, that generates political benefits and costs), and an intermediate mechanism (a more realist mechanism, including both extremes).

To govern, the mayor has to convince at least half of the city councilors to support her policy proposals, and she has to use the policy instruments at her disposal. These instruments will vary depending on the mechanism that we will investigate, but they are two in nature: targeted goods for the city councilor's constituency and private goods for the city councilor. The targeted goods improve the councilor's reelection chances while the private goods can be understood as corruption, aimed at "buying off' ' the support of a recipient councilor. The use of the policy instruments, will generate a cost of govern, $C_G$, that will vary with the policy choices and the legislature size.

Finally, the mayor has to propose a level of public goods provision ($g$) and rents ($r$). Public goods provision help the mayor to get reelected. Rents are for direct mayor's consumption, and does not contribute to the mayor's electoral success. This makes the mayor's expected utility a sum of the utility from rents and benefits from reelection. Both the utilities from rents ($u(r)$) and the probability of reelection $\pi(g)$ are concave functions, meaning that more rents or public goods increase the utility at a decreasing rate. The probability of reelection is multiplied by the benefit from holding office $B_M > 0$. This benefit captures the tangible and intangible gains that the mayor perceives from holding the public office. The utility is multiplied by an indicator $\mathbb{I}$ that captures whether the mayor's proposal was accepted by the city council.[1] The expected utility for the mayor is:

$$\mathbb{E}U(r, g) \;=\; \mathbb{I}(\text{Approval})(u(r) + B_M \pi(g))$$

The policy choices of the mayor are subject to the municipal budget constraint. The municipality has $R$ resources, and we assume the municipality cannot run debts. Let $C_G$ the costs of govern. The budget balance constraint requires the offers that the mayor makes to the city councilors to satisfy the following rule:

$$r + g + C_G \;\leq\; R$$

The expected utility for the city councilors depend on the mechanism that will determine the reversal policy. We explore three different mechanisms. First, in the *non-partisan legislature* mechanism, we assume that when the council rejects the mayor's office, a reversal stage starts with the selection of one councilor randomly. Resources diminish by a factor of $\delta \in (0, 1)$, and the selected councilor will have to make a proposal. If her proposal is accepted, it is implemented. If it is rejected, the budget shrinks again by a factor $\delta$, and another city councilor is recognized to make the proposal. The process than repeats, until one proposal is finally accepted.

In the second reversal mode, the *partisan legislature* mechanism, we assume that each councilor has a party affiliation. Party affiliations are mutually exclusive (a councilor cannot belong to two parties at once). If a city councilor belongs to a party aligned with the mayor, we say that she belongs to the government coalition ($G \subset \{1, 2, \cdots, N\}$). Otherwise, the city councilor belongs to the opposition $O \not\subset G$. The party affiliation play an essential role in the *partisan legislature* mechanism: if a city councilor that belongs to the mayor's coalition rejects the mayor's proposal, she loses the election. Whereas, if the city councilor belongs to the opposition, she loses the election by supporting mayor's proposal. Although these assumptions are strong, they capture the main underlying political dynamics. Partisanship means in most political interactions that if a city councilor aligns with the mayor, her electorate has preferences toward policies that the mayor would implement. Voting against the mayor's proposals would represent to defy the partisan brand.

In the last reversal mode, which is the reversal mechanism we use in the paper, we combine both partisanship with legislature bargaining rents. Councilors in this mechanism receive a partisan utility from the mayor's policy, but if they reject the policy, a reversal policy similar to the *non-partisan legislature* mechanism takes place. In the mayor's proposal stage, city councilors that belong to the mayor's coalition receive a gain of $p > 0$ when the mayor's policy is implemented. Opposition members, on the other hand, receive $-p$ when the

[1]The indicator function is defined as

$$I(condition) \;=\; \begin{cases} 1 & \text{if Condition is satisfied.} \\ 0 & \text{otherwise} \end{cases} \quad .$$

mayor's proposal is implemented. If the mayor's proposal is rejected, as city councilors do not control the Executive, no public goods are provided ($p = 0$). Moreover, in equilibrium, we assume that city councilors would prefer to receive rents instead of providing public goods.

The timeline of the game is as follows:

1. The mayor learns how many government $|G|$ and opposition $|O|$ legislators were elected.
2. The mayor proposes a policy vector $(r, g, \mathbf{x})_{\mathbf{M}}$.
3. The city council votes the proposal.
   - If the council accepts, the policy is implemented and the game ends.
   - Otherwise, the reversal policy is implemented.
4. (Reversal Policy) Depends on which mechanism, we will have three different reversal policies. We will detail them below.

This is a bargaining game, along the lines of Rubinstein (1982). For a political science model similar to this model, see Baron and Ferejohn (1989).

The solution for this game consists in finding a Sub-game Perfect Nash Equilibrium using backward induction. In the model with infinitely repeated proposals, we extend the equilibrium concept to require that the equilibrium is Stationary. A Stationary Sub-game Perfect Equilibrium is based on the assumption that if a politician accepts an offer at time $t + 1$, she should accept the same offer at time $t$. This gives us an objective of finding the offer that will be accepted at any point in the game. Applying this reasoning, we can find the optimal offer at $t = 0$, which represents no delay in policy implementation.

**Definition 1.** *The solution for the game consists in a pair of strategic policy vectors that for the mayor, and the city councilors, indexed by the round of the implementation. The mayor's policy vector is comprised of a public goods provision $g^*$, rents $r^*$, $\approx$ XXXX*

### B.1.2 The Mayor's Stage

Assume that we have worked out the solution using backward induction, and when there are $N$ councilors, we find that the cost of govern for the mayor is $C_G(N)$. This assumption will be dropped when we work each of the cases below.

We now derive the optimal rents ($r$) and public goods provision ($g$) proposed by the mayor. The mayor benefits from public goods provision, as it increases the chances of her reelection. However, she prefers to contribute as minimum as possible for the public goods, and extract the remaining resources as political rents. Her objective is to maximize the following expected utility, subject to the municipal budget constraint.

$$
\begin{aligned}
\max_{r,g} \quad & u(r) + B_M \pi(g) \\
\text{s.t.} \quad & r + g + C_G(N) \leq R
\end{aligned}
$$

In equilibrium the budget binds, and the optimal public goods provision maximizes:

$$\max_g \{u(R - g - C_G(N)) + B_M \pi(g)\}$$

The first order condition for an optimal public goods provision makes the marginal benefits to increase rents, in this case, marginal costs of providing the public goods, equals to the marginal benefits of reelection:

$$u'(R - g - C_G(N)) = B_M \pi'(g)$$

This is sufficient for the equilibrium, as the second order condition reassures the concavity of the mayor's expected utility: $u''(R - g^* - C_G(N)) + B_M \pi''(g^*) < 0$.

**Proposition 1.** *The provision of public goods increase with legislature size if the costs of govern decreases in larger legislatures.*

*Proof.* In order to prove this result, we need to show that the mayor's expected utility satisfy the increasing differences in $g$ and $N$. This would mean that when increasing the size of the council, the optimal solution $g^*$ would also increase. However, as $N$ increases discretely, we cannot take a derivative on $N$ and use the implicit function theorem. Instead, we use monotone comparative statics [@milgrom1994monotone] to derive these results.

The mayor's expected utility satisfy the increasing differences in $g$ and $N$ when, for $g > g'$ and $N + 1 > N$, we have:

$$\mathbb{E}\text{U}(g, N+1) - \mathbb{E}\text{U}(g', N+1) \geq \mathbb{E}\text{U}(g, N) - \mathbb{E}\text{U}(g', N)$$

And after substituting, we have:

$$u(R{-}g{-}C_G(N{+}1)){+}B_M\pi(g){-}u(R{-}g'{-}C_G(N{+}1)){-}B_M\pi(g') \geq u(R{-}g{-}C_G(N)){+}B_M\pi(g){-}u(R{-}g'{-}C_G(N)){-}B_M\pi(g')$$

Which is equal to:

$$u(R - g - C_G(N+1)) - u(R - g' - C_G(N+1)) \geq u(R - g - C_G(N)) - u(R - g' - C_G(N))$$

Multiplying both sides by $\dfrac{1}{g-g'}$ and taking the limit when $g \longrightarrow g'$, we have:

$$u'(R-g-C_G(N+1)) \geq u'(R-g-C_G(N))$$

This is true when the cost of govern with a council size of $N+1$ is lower than the cost of govern when the council size has $N$ councilors:

$$C_G(N+1) \leq C_G(N)$$

Therefore, to satisfy increasing differences in the public goods provision and in the legislature size, the costs of govern has to decrease with legislature size. □

This provides us our first empirical hypothesis to test: *if the cost of govern decreases when the legislature increases, then the public goods provision increase when the legislature size increase.*

At this stage, we need to solve for the costs of govern to determine whether this hypothesis is true or not. Below we show that it is true for the *partisan* and *hybrid* mechanisms, but it is not true for the *non-partisan* mechanism.

### B.1.3 Non-Partisan Legislature

In the *non-partisan legislature* mechanism, we assume that when the council rejects the mayor's office, a reversal stage starts with the selection of one councilor randomly. Suppose that there were $k$ rejections, and we are at the $k+1$-th stage in the game. A given councilor will accept the proposer's offer if, and only if, accept the offer is better than wait until the next stage. If the offer is $x_i$, then:

$$x_i \geq \frac{1}{N}\left[\delta^{k+1}R - \frac{N}{2}x_i\right] + \left(1-\frac{1}{N}\right)\left(\frac{1}{2}\right)[x_i]$$

In the left-hand side, we place the offer. In the right-hand side, there are two components. The first is the amount the councilor $i$ gets when she is the proposer. It is equal to the budget in the next round minus the offers that she has to make to convince half of the councilors. The second part is the gains if she happens to reject but still receive an offer in the following round. Note that the offer she makes when she is the proposer is the same as the offer she wants to receive. This because the city councilors are exchangeable, and the solution is symmetric for all councilors receiving an offer (this means that we could have dropped the $i$ in the solution). After some algebra, the offer $x_i$ has to be greater than or equal to:

$$x_i \geq \frac{2\delta^{k+1}R}{2N+1} \equiv \underline{x}(k)$$

The proposer always offer the minimum possible to get the proposal approved. In the case, the offer at any given stage $k$ is going to be equal to $\underline{x}(k)$.

Proceeding backwards, at the mayor's proposal stage $k = 0$ and the mayor is going to offer $\underline{x}(0, N) = \frac{2\delta R}{2N+1}$ to half of the councilors. In this context, the cost of govern is equal to:

$$C_G(N) = \frac{N}{2N+1}(\delta R)$$

**Proposition 2.** *In the *non-partisan legislature* mechanism, the costs of govern increase as N increases.*

*Proof.* The differences in costs when there are $N + 1$ versus when there are $N$ legislators are equal to:

$$\begin{aligned} C_G(N+1) - C_G(N) &= \frac{N+1}{2(N+1)+1}(\delta R) - \frac{N}{2N+1}(\delta R) \\ &= \frac{\delta R}{(2N+3)(2N+1)} \\ &> 0 \end{aligned}$$

Therefore, the costs always increase in $N$.

□

**Corollary 1.** *In the *non-partisan legislature*, public goods provision decreases as the legislature size increases.*

This result is crucial to understand the role of the politics in our model. If the costs of govern increase with legislature size, then any increase in public goods provision associated with legislature size has come from a different mechanism. In our case, we show that there is a *political* reason for increasing the public goods provision.

### B.1.4 Partisan Legislature

We now look into the other extreme, which is a fully politicized legislature. In this case, when the mayor implements the policy, it generates a value of $p$. A legislator aligned with the mayor receives a benefit $p$ while an opposition legislator receives a value of $-p$. Moreover, as in the case of the non-partisan legislature, the mayor can also distribute $x_i$ private goods for the legislator $i$. In this case, when the policy is implemented, utility of an aligned legislator is equals to $x_i + p$. When the legislator is not aligned with the mayor, her utility is equals to $x_i - p$. If the policy is not implemented, then we assume a reversal policy of zero in all choice vectors $g = x_i = p = 0$.

A legislator aligned with the mayor will always support the mayor's policy, regardless of any private goods. This because as $p > 0$, the councilor always receive positive benefits from the policy. However, a politician not aligned with the mayor will require a compensation if her vote is needed. In this case, if the mayor needs the support of an opposition politician to pass her policy proposal, then she will need to compensate her for the policy costs. In this case:

$$x_i \geq p$$

Optimality dictates that the mayor will offer $x_i = p$ to an opposition legislator. Let $\gamma$ be the ex-ante probability of electing a politician aligned with the mayor. Then, the costs of govern when the legislature size is $N$ is equal to the expected number of politicians in the opposition that the mayor will have to compensate:

$$C_G(N) = p\left(\frac{N}{2} - \gamma N\right)$$

**Proposition 3.** *In the *partisan legislature* mechanism, if the chance of electing a mayor-aligned politician is greater than one-half, then the costs of govern decrease as* $N$ *increases.*

*Proof.* The differences in costs when there are $N+1$ versus when there are $N$ legislators are equal to:

$$\begin{aligned} C_G(N+1) - C_G(N) &= p\left(\frac{N+1}{2} - \gamma(N+1)\right) - p\left(\frac{N}{2} - \gamma N\right) \\ &= p\left(\frac{1}{2} - \gamma\right) \end{aligned}$$

Therefore, when $\gamma < \frac{1}{2}$, the costs of govern increase. Otherwise, the costs decrease.

□

**Corollary 2.** *In the *partisan legislature*, if the chance of electing a mayor-aligned politician is greater than* $1/2$*, then public goods provision increase as the legislature size increases.*

This result shows that when the chances of electing an aligned politician is sufficiently high, then the costs of govern decrease. As a response, the public goods provision (and also rents), increase. The rate distribution between these two vectors will be determined by the marginal change in the utility from rents and the utility from reelection. In any case, the amount allocated for both increase.

### B.1.5 Hybrid Partisan and Non-partisan Legislatures

The hybrid mechanism combines both partisan and non-partisan motivations. In the hybrid mechanism, a city councilor aligned with the government favors the mayor's offer if:

$$x_i \geq \frac{2\delta R}{2N+1} - p$$

An opposition politician, on the other hand, favors the mayor's offer when:

$$x_i \geq \frac{2\delta R}{2N+1} + p$$

Inspecting these equations we can decompose these costs into costs in terms of rents and costs (or benefits) from political alignment. In this context, the cost of govern will be a weighted average between the expected payments for councilors that belong to the mayor's coalition, versus the expected payments for the councilors that belongs to the opposition. The chance of a opposition member be elected is equal to $\gamma$, and taking the weighted averages, the costs become:

$$C_G(N) = \frac{N}{2}\left(\frac{2\delta R}{2N+1} - p\right) + \left(\frac{N}{2} - \gamma N\right)p$$

**Proposition 4.** *In the *hybrid legislature*, if* $\gamma \leq \frac{1}{p}\left[\frac{1}{(2N+1)(2N+3)}\right]$, *then the costs of govern increase as the legislature size increases. Otherwise, the costs decrease when the legislature size increases.*

*Proof.* The differences in costs when there are $N+1$ versus when there are $N$ legislators are equal to:

$$\begin{aligned} C_G(N+1) - C_G(N) &= \frac{N+1}{2}\left(\frac{2\delta R}{2(N+1)+1} - p\right) + \left(\frac{N+1}{2} - \gamma(N+1)\right)p \\ &- \frac{N}{2}\left(\frac{2\delta R}{2N+1} - p\right) - \left(\frac{N}{2} - \gamma N\right)p \\ &= \frac{\delta R}{(2N+3)(2N+1)}\left[1 - \gamma(2N+1)(2N+3)p\right] \end{aligned}$$

The conditions for the differences in governing costs be decrease when the legislature size increases is:

$$\gamma \geq \frac{1}{p}\left[\frac{1}{(2N+3)(2N+1)}\right] \equiv \underline{\gamma}$$

□

**Corollary 3.** *In the *hybrid legislature* mechanism, public goods provision decreases as the legislature size increases.*

Also, note that the threshold $\gamma$ for the probability of electing a politician aligned with the government is decreasing in both the political outcome $p$ and the size of the legislature $N$. This means that more political benefits from policy and larger legislatures makes it easier to satisfy the electoral constraint.

### B.1.6 Main Hypotheses

Taking stocks, the models provide two empirically testable hypotheses. Consider a legislature that previously had $N$ legislators, but now needs to increase its legislature by $N+1$. Then:

H1. The provision of public goods increase when the costs to govern decrease with legislature size.

H2. The costs to govern decrease with legislature size when the chance of electing a government aligned legislator is sufficiently high.

## References

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