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**GUIDELINES FOR THE SELECTION OF
WEIGHTING FUNCTIONS FOR
H-INFINITY CONTROL**

**BY JOHN E. BIBEL AND D. STEPHEN MALYEVAC
WEAPONS SYSTEMS DEPARTMENT**

JANUARY 1992

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**NAVAL SURFACE WARFARE CENTER
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Dahlgren, Virginia 22448-5000**

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FOREWORD

This report discusses general engineering guidelines for the design of weighting functions in H-Infinity (H_∞) Control. The H_∞ Control method is one approach to robust control that has been receiving much attention recently in the controls community. The Aeromechanics Branch (G23) has been examining H_∞ control and other robust control methods and their application to naval weapon systems (e.g., missile flight control and high-performance/high-precision pointing and tracking systems). The material presented herein is introductory in nature and, hopefully, provides useful information to the control system design engineer.

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Approved by:



D. L. BRUNSON, Head
Missile Systems Division



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ABSTRACT

This report provides insight into the selection of H-Infinity (H_∞) Control weighting functions that help shape the performance and robustness characteristics of systems designed using the H_∞ and μ -Synthesis Control methods. Background material regarding sensitivity functions, loopshaping, and H_∞ Control is followed by a discussion of general engineering guidelines for the design of H_∞ Control weighting functions. In addition, unresolved design issues and alternatives are presented. Thus, this report presents practical rules-of-thumb and identifies issues and alternatives in the design of weighting functions for H_∞ Control.

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INTRODUCTION

During the past decade, there have been many theoretical advances in the field of Robust Control^{1,2} and, in particular, H-Infinity (H_∞) Control. A breakthrough in the solution of the H_∞ Control problem in 1988³ led to the development of computationally feasible algorithms for finding H_∞ optimal controllers. Since then, analytical applications of H_∞ Control and μ -Synthesis (an iterative design technique that combines Structured Singular Value, μ , analysis with H_∞ Control synthesis) to real-world problems^{4,7} have demonstrated the potential of these techniques.

One of the key steps in the H_∞ Control design approach is the formulation of input and output weighting functions. These weighting functions are utilized to normalize the inputs and outputs and reflect the spatial and frequency dependency of the input disturbances and the performance specifications of the output (error) variables. Unfortunately, little work has been performed on finding reliable methods of selecting these weighting functions⁸.

This document provides insight into useful guidelines for selecting the input and output weighting functions utilized in H_∞ Control designs. This report is organized in the following manner. First, a brief description of feedback control systems, loopshaping, and background on H_∞ Control weighting functions is presented. Second, general guidelines on formulating these weighting functions are given. Third, some of the remaining problem-dependent issues regarding the control weights are discussed. A summary concludes this report.

BACKGROUND

We begin with an introductory level discussion of some of the technical background to the H_∞ Control weighting selection problem. This section includes a description of feedback control systems, loopshaping design concepts, and the definition of the H_∞ Control weight selection problem; it concludes with a missile autopilot example to further illustrate these concepts.

FEEDBACK CONTROL SYSTEMS AND LOOPSHAPING CONTROL DESIGN

We begin this section with a definition of control system terminology. The concepts of loopshaping to meet classical types of design criteria are then presented.

A general control system is pictured in Figure 1. This system could represent either a scalar (Single-Input Single-Output, SISO) system or a multivariable (Multiple-Input Multiple-Output, MIMO) system with scalar or vector input and output signals (as appropriate) and either scalar or matrix transfer function system blocks (as appropriate). We assume that the system can be modeled as a linear time-invariant (LTI) system of differential equations. In Figure 1, $G(s)$ represents the model of the physical plant to be controlled and $K(s)$ denotes the controller. The inputs to the system are $r(s)$, reference command signals; $d(s)$, disturbance inputs; and $n(s)$, measurement sensor noise. The output variables are given by $y(s)$. In addition, we may also be interested in monitoring the error signals, $e(s)$, and the control signals, $u(s)$.

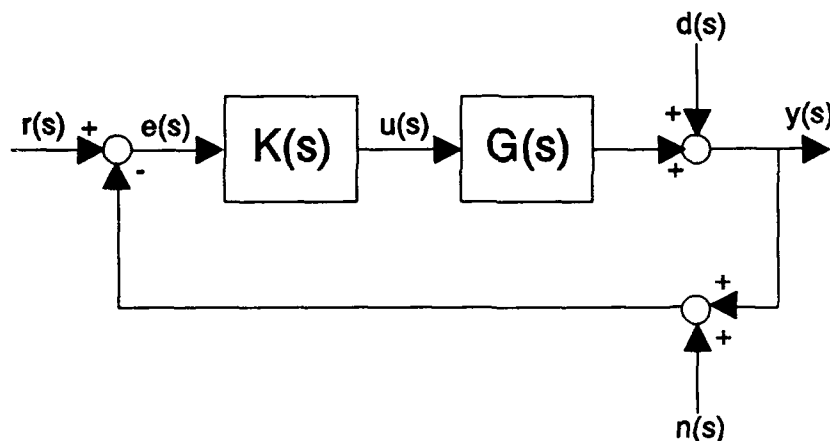


FIGURE 1. CONTROL SYSTEM DIAGRAM

Algebraically, the output $y(s)$ can be related to the three inputs as

$$y(s) = [I + G(s)K(s)]^{-1} \{G(s)K(s)r(s) + d(s) - G(s)K(s)n(s)\} \quad (1)$$

where I is the identity matrix. Likewise, the error and the control signals can be expressed, respectively, by

$$e(s) = [I + G(s)K(s)]^{-1} \{r(s) - d(s) - n(s)\} \quad (2)$$

$$u(s) = [I + K(s)G(s)]^{-1} K(s) \{r(s) - d(s) - n(s)\} \quad (3)$$

Definition of some common terminology follows:

$L(s) = G(s) K(s)$ = loop transfer matrix

$F_o = [I + G(s) K(s)]$ = (output) return difference transfer matrix

$S(s) = [I + G(s) K(s)]^{-1}$ = (output) sensitivity function matrix

$T(s) = [I + G(s) K(s)]^{-1} G(s)K(s)$ = (output) closed-loop transfer function matrix

Note that the closed-loop transfer function matrix, $T(s)$, relates the output of the system, $y(s)$, to the input reference signal, $r(s)$. It also specifies how the output $y(s)$ is affected by the noise, $n(s)$. The sensitivity function, $S(s)$, describes the output, $y(s)$, as a function of the disturbance input, $d(s)$. It also defines the response of the tracking error $e(s)$ to the reference input $r(s)$. Thus to summarize, $S(s) = y(s)/d(s) = e(s)/r(s)$ and $T(s) = y(s)/r(s) = -y(s)/n(s)$. From the definitions of $S(s)$ and $T(s)$, we find that

$$S(s) + T(s) = I \quad (4)$$

Thus, $T(s)$ is also referred to as the complementary sensitivity function matrix. Equation (4) is an important relationship that prescribes a limit on achievable performance.

In addition to the requirement for stability, other properties that the feedback control system should exhibit include

- Command Following. For good command following, we require the output to track the input reference signals. To accomplish this, we desire that $y(s) \approx r(s)$ and that $e(s) \approx 0$. For these to be true, we require that $T(j\omega) \approx I$ and $S(j\omega) \approx 0$, respectively. To achieve this, the loop transfer, $L(j\omega)$, should be large; i.e., $G(j\omega)K(j\omega) \gg 1$.
- Disturbance Rejection. For good disturbance rejection, the disturbance inputs should have a negligible effect upon the output $y(s)$. Thus, we require that $S(j\omega) \approx 0$. As before, this translates into the need for a large loop gain, $G(j\omega)K(j\omega) \gg 1$, in the frequency range of the disturbances.
- Sensor Noise Attenuation. In this case, we desire that the sensor noise inputs have a small effect upon the system outputs. For this to happen, we would like $T(j\omega)$ to be small, especially in the frequency range that the noise is concentrated. As a result, the loop gain should also be small in the appropriate frequency range, $G(j\omega)K(j\omega) \ll 1$.
- Control Sensitivity Minimization. Here, we desire to keep the control inputs small, so as to not saturate the servomechanism or amplify noise and disturbance signals mixed in the control signal. For this to happen, we would like $[I + K(s)G(s)]^{-1}K(s)$ to be near zero. In a SISO sense, then we desire

$$\frac{K(j\omega)}{1 + K(j\omega)G(j\omega)} = \frac{T(j\omega)}{G(j\omega)} \approx 0 \quad (5)$$

So for the SISO case, to minimize the control sensitivity, we would like to keep the complementary sensitivity small.

• Robustness to Modeling Errors. We recall that $G(s)$ represents a linear model of the true physical system. However, this model cannot represent the real system perfectly and, thus, contains modeling errors (or model uncertainties); e.g., high-frequency unmodeled dynamics. If we represent the plant model uncertainty as multiplicative uncertainty at the plant output, the true model of the system is

$$G_{\text{true}}(s) = [I + \Delta(s)]G(s) \quad (6)$$

where $\Delta(s)$ denotes a stable transfer function matrix representation of the uncertainties. To examine the sole effect of the uncertainties, assume that $r(s) = d(s) = n(s) = 0$. The system can then be configured as shown in Figure 2(a). Pulling out the $\Delta(s)$ into a separate block, the system is then represented by Figure 2(b). If the loop is broken on either side of the uncertainty $\Delta(s)$, the loop properties from the input $v(s)$ to output $w(s)$ are given by

$$w(s) = -G(s)K(s)[I + G(s)K(s)]^{-1}v(s) \quad (7)$$

So, to reduce the sensitivity of the system to modeling errors at the plant output, we desire to keep the quantity $G(j\omega)K(j\omega)[I + G(j\omega)K(j\omega)]^{-1}$ small in the frequency range of the expected model uncertainties. To accomplish this, the loop gain should be small, (i.e., $G(j\omega)K(j\omega) \ll 1$). In the SISO case, this simplifies to keeping $T(j\omega)$ and, therefore, $L(j\omega)$ small in the appropriate frequency range.

To summarize, for good command following and disturbance rejection, we would like to keep S small; to remain insensitive to sensor noise and modeling errors (at the plant output) and to reduce control sensitivity, we desire to keep T small. However, we cannot keep both S and T small over the whole frequency range because of the $S + T = I$ constraint. Thus, we must determine some tradeoff between the sensitivity and complementary sensitivity functions.

Usually, reference signals and disturbances occur at low frequencies, while sensor noise and modeling errors (e.g., high frequency unmodeled dynamics) are concentrated at high frequencies. The tradeoff, in a SISO sense, is to make $|S(j\omega)|$ small at low frequencies and $|T(j\omega)|$ small at high frequencies. For example, a Bode plot of a typical sensitivity function is illustrated in Figure 3.

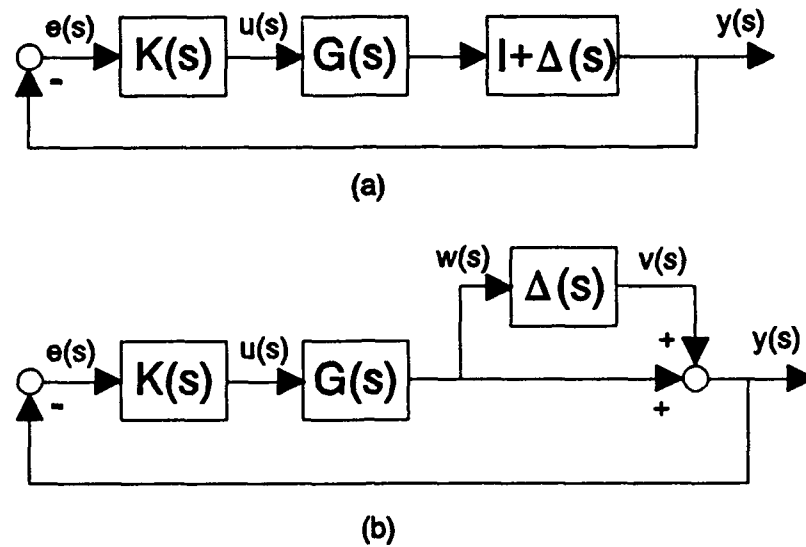


FIGURE 2. REPRESENTATIONS OF MODEL UNCERTAINTY

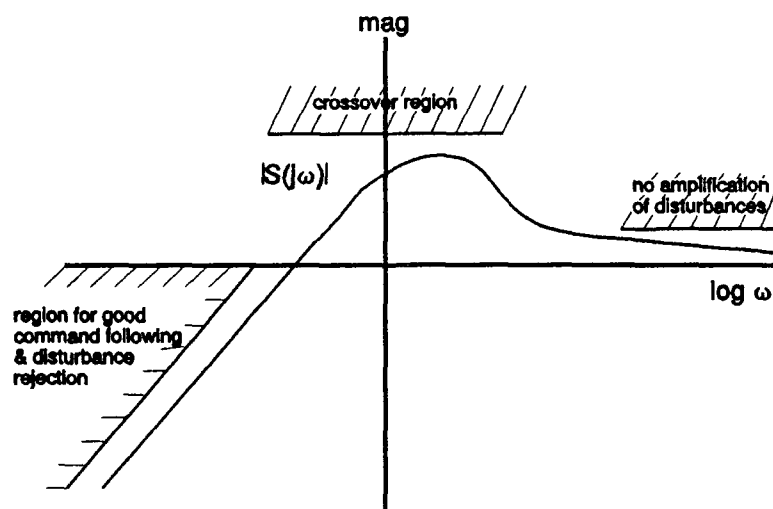


FIGURE 3. TYPICAL SENSITIVITY FUNCTION

Design guidelines for the loop transfer function can also be prescribed. A graphical representation of the loop gain $|g(j\omega)k(j\omega)|$ that satisfies the tradeoffs described in the previous pages is shown in Figure 4 for the SISO case.

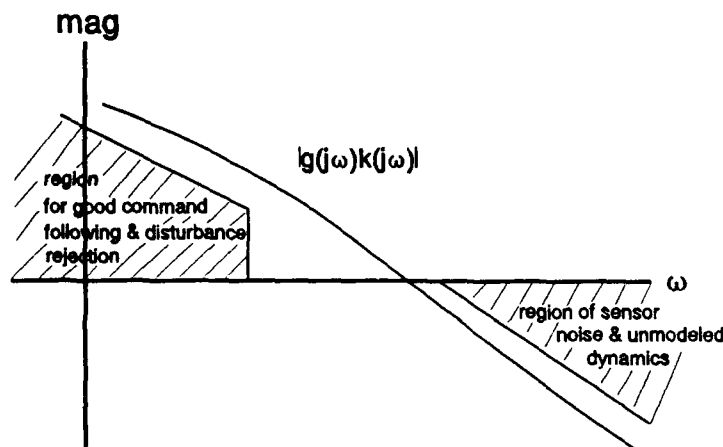
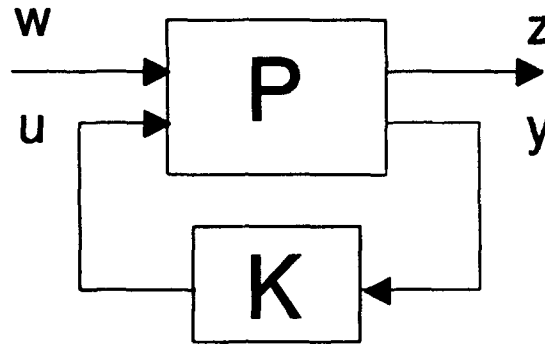


FIGURE 4. SHAPE OF THE LOOP GAIN

These concepts for loopshaping can also be extended to the design of multivariable control systems through the application of singular values. For more details regarding multivariable loopshaping and design constraints, see References 8 - 15. Next, we turn to the use of weights in H_∞ Control.

H_∞ CONTROL AND WEIGHTING FUNCTIONS

We now need to design a controller that meets the above loopshaping conditions. In the present work, we consider H_∞ Control as the design approach. H_∞ Control is a design technique with a state-space computational solution that utilizes frequency-dependent weighting functions to tune the controller's performance and robustness characteristics. A new design framework is given in Figure 5 for H_∞ Control, where $P(s)$ represents the generalized plant transfer function matrix and $K(s)$ is a linear transfer function matrix description of the controller. In this framework, w represents the exogenous inputs (e.g., reference commands, disturbances, and noise), z denotes the regulated performance output variables (e.g., tracking errors, performance variables, and actuator signals), u signifies control inputs, and y represents the measured output variables. The plant P has two inputs (w and u) and two outputs (z and y), and can be decomposed into four sub-transfer function matrices,

FIGURE 5. H_∞ CONTROL FRAMEWORK

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (8)$$

in which P_{ji} is the particular transfer function matrix from the i th input to the j th output. Closing the feedback control loop, the transformation (or mapping) from the input w to the output z , T_{zw} , is called the lower linear fractional transformation, $F_l(P, K)$,

$$z = T_{zw} w = F_l(P, K) w \quad (9)$$

where

$$T_{zw} = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (10)$$

The H_∞ Control problem is to find a controller that minimizes the infinity norm of the input-output map T_{zw} , or $\min \|T_{zw}\|_\infty$, over the space of all realizable controllers $K(s)$ that stabilize the closed-loop system. For a SISO system, the infinity norm of the transfer function $G(s)$, $\|G\|_\infty$, is equivalent to the maximum value of the Bode gain $|G(j\omega)|$. The infinity-norm thus gives the maximum amplification that a sinusoidal signal of frequency ω receives as it passes through G . If the input signal (energies) are normalized to unity, then the H_∞ controller minimizes, in one sense, the maximum energy gain between the exogenous inputs and the performance outputs. Or, in other words, the worst-case output is minimized.

However, the question still remains about how to weight the H_∞ problem so as to shape the loop transfer as discussed previously. Recall that for good command following and disturbance rejection, we need $S(s)$ to be small; and for noise attenuation and insensitivity to unmodeled dynamics, we would like $T(s)$ to be small. So then, one could minimize the infinity norm of S and/or T ; e.g., $\min \|S(j\omega)\|_\infty$. This is equivalent, in the SISO case, to $\min \max |S(j\omega)|$ over all frequencies. To put the feedback system in the proper

framework, consider for example the sensitivity function. Recall that the sensitivity function relates the output to the disturbance input as shown in Figure 6(a). This diagram is rearranged into the H_∞ form as depicted in Figure 6(b), where now $T_{zw} = [I + GK]^{-1} = S_o$, the (output) sensitivity function. Likewise, T describes how the output relates to the commanded reference signals, and is shown in Figure 7(a). The transformation of the system to the H_∞ framework is illustrated in Figure 7(b). The input/output transformation, T_{zw} , in this case is $[I + GK]^{-1}GK$, which is T_o , the (output) complementary sensitivity function. Figures 6 and 7 show how the sensitivity functions can be obtained in the H_∞ framework for minimization.

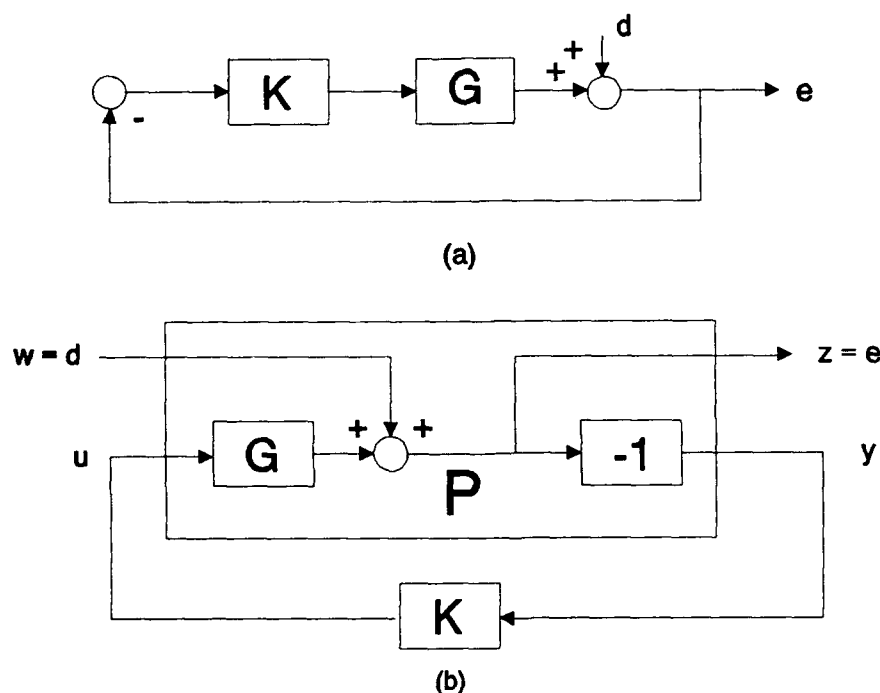


FIGURE 6. SENSITIVITY FUNCTION DIAGRAMS

However, since S and T cannot be minimized together over all frequencies because of inherent design constraints and limitations, weights are introduced to shape the solution. Not only can S and T be weighted, but other regulated performance variables and inputs can be weighted as well. This is depicted in Figure 8. Multiple objectives (e.g., minimization of weighted sensitivity, weighted complementary sensitivity, and weighted control input) can be considered within this framework. The weights on the input and output variables are selected to reflect the spatial and frequency dependence of the respective signals and performance specifications. Thus, these weights help shape or tune the open- and closed-loop response characteristics. These input and output weighting functions are defined as rational, stable, minimum-phase transfer functions (i.e., no poles or zeros in the right half plane).

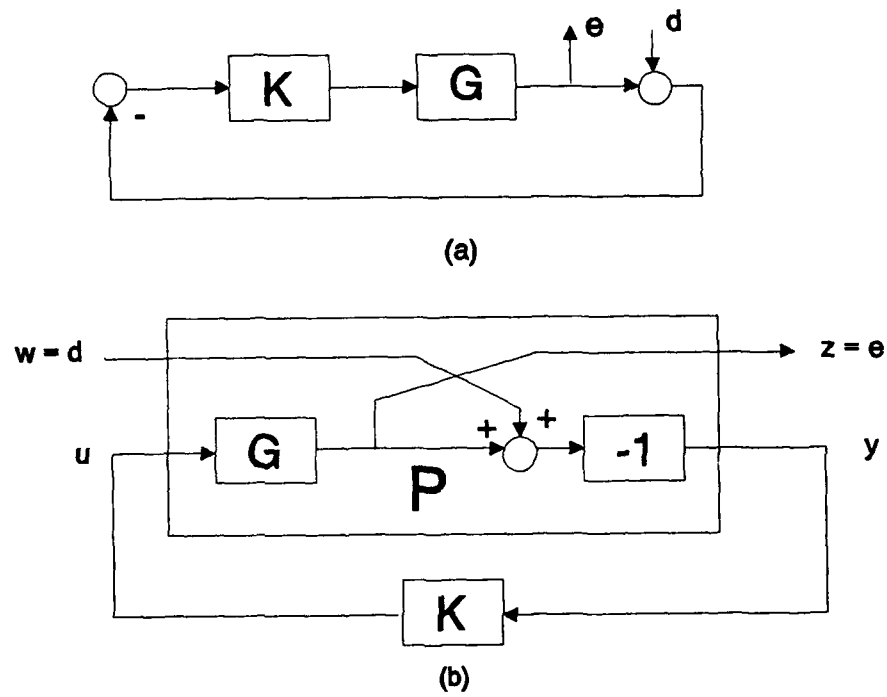


FIGURE 7. COMPLEMENTARY SENSITIVITY FUNCTION DIAGRAMS

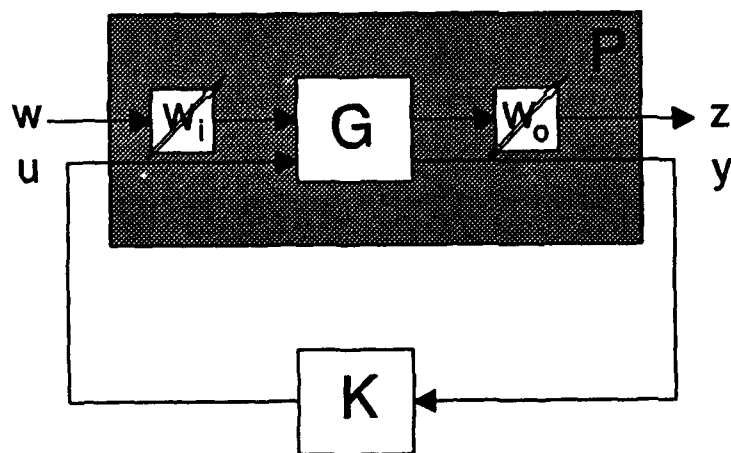


FIGURE 8. INPUT AND OUTPUT WEIGHTS IN H_∞ FRAMEWORK

In addition, these weights are chosen to normalize the input and output energies to one (here, energy refers to the 2-norm of the input or output variable). This is illustrated by the sequence of Figure 9 for a SISO example, which shows how a weight is selected for some desired sensitivity function shape. After specifying the sensitivity function, it is bounded in magnitude [Figure 9(a)]. This upper bound magnitude is inverted to obtain the sensitivity function weight, $W_s(s)$, as shown in Figure 9(b). So, when $S(s)$ is multiplied by its weight $W_s(s)$, the magnitude of $W_s S(s)$ is less than or equal to one over all frequencies [Figure 9(c)]. This example is termed the "weighted sensitivity problem", since the H_∞ controller minimizes the maximum magnitude of $W_s S$ found over all frequencies (i.e., $\min \|W_s S\|_\infty$).

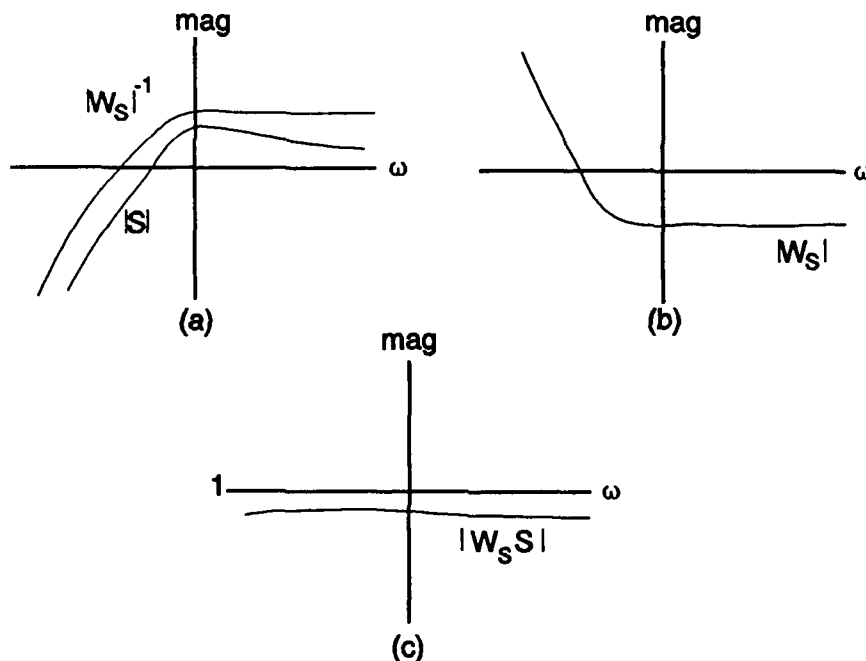


FIGURE 9. WEIGHT SELECTION FOR SENSITIVITY FUNCTION

Similarly, we can define a weighting for the complementary sensitivity function, $W_T(s)$. As alluded to previously, in H_∞ Control, the designer chooses which exogenous inputs to include and what regulated outputs to minimize, then formulates appropriate weighting functions. The input and output weights are then incorporated with the plant model $G(s)$ to form the generalized plant, $P(s)$, in the H_∞ Control framework. Thus, the input/output weights are the "control knobs" of the H_∞ design method and have a strong impact on the resulting controller design.

MISSILE AUTOPILOT DESIGN EXAMPLE

To further illustrate these concepts, we present an example of setting up a missile autopilot design in the H_∞ design framework. This example will be drawn upon throughout this report.

The problem is to design a missile pitch (or yaw) plane autopilot to track acceleration commands from the guidance system. The autopilot determines fin deflection commands that are sent to the tail surface servos. By deflecting the tail fins, aerodynamic forces and moments are generated to maneuver the missile. Rate gyro and accelerometer measurements are processed by the flight control system to close the feedback control loop. This missile flight control loop is depicted in Figure 10.

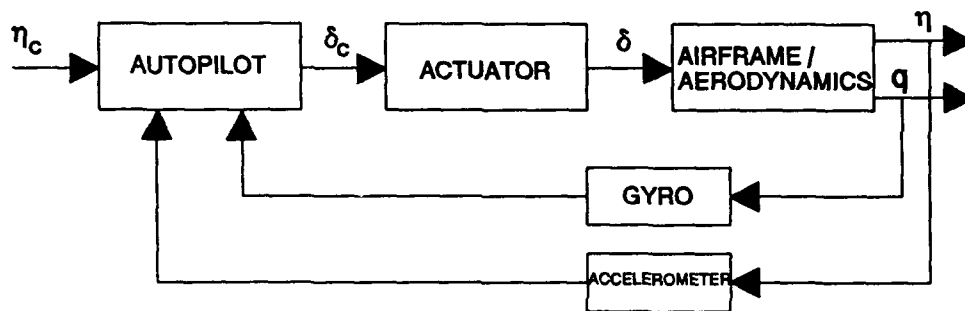
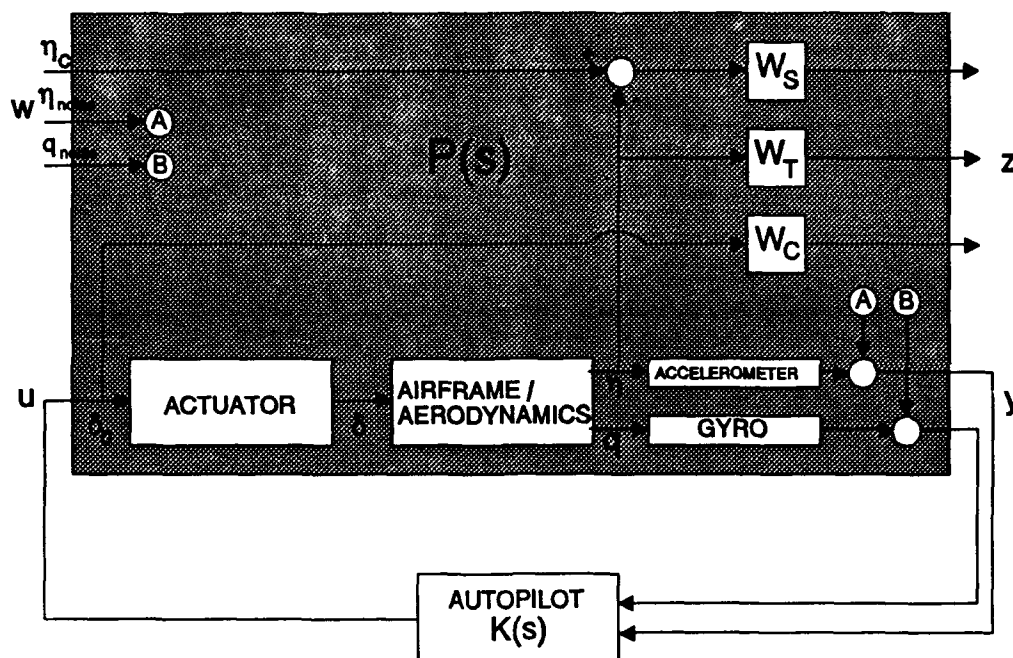


FIGURE 10. MISSILE FLIGHT CONTROL SYSTEM

To formulate the H_∞ Control framework for this example, we first select the inputs and outputs. Inputs for this example include the acceleration command and measurement sensor noises. As outputs, we might select the sensitivity function, the complementary sensitivity function, and the control input to the actuator, δ_c . The next step is to select appropriate weighting functions for the design to achieve the design specifications. The framework for this example is shown in Figure 11. Note that the sensitivity function is taken at the acceleration error (between the commanded acceleration and the achieved acceleration) signal and that the complementary sensitivity function is taken at the acceleration output signal. Here, the acceleration command can be considered as a disturbance at the input to the plant. However, the problem is how to select the best weighting functions to optimize the controller design.

As evident in this example, the H_∞ design method is dependent on weighting functions to tune the controller design. However, little information currently exists on how this should be done. As a result, weight selection is currently an art that relies heavily on ad hoc procedures and control system design experience. To offset this deficiency, the next section presents some practical guidelines for choosing these H_∞ Control weighting functions to

FIGURE 11. H₂ FRAMEWORK FOR MISSILE AUTOPILOT EXAMPLE

optimize the control system design and shows how they relate to quantities of engineering significance.

GENERAL GUIDELINES FOR WEIGHT SELECTION

In this section, some general guidelines for selecting the frequency-dependent weighting functions used to specify the controller design goals will be discussed. These are empirical guidelines based on past experience with the H_∞ design method. In particular, the discussion will address the selection of weights for shaping the time and frequency response. This includes the selection of the sensitivity weighting function, the complementary sensitivity weighting function, and control input (or actuator) weights.

The weighting function on the sensitivity transfer function from reference input to output tracking error, $W_s(s)$, is selected to reflect the desired performance (i.e., time response) characteristics. As discussed in the previous section, the sensitivity function should have low gain at low frequencies for good tracking performance and high gain at high frequencies to limit overshoot. This is accomplished by selecting a weighting function W_s such that W_s^{-1} reflects the desired shape of the sensitivity function.

A low pass weight is used on the sensitivity function with the low frequency magnitude set approximately equal to the inverse of the desired steady state tracking error and high frequency gain set to limit overshoot. In general, the larger the magnitude of the high frequency gain, the more the overshoot is limited. However, as one would expect, limiting the overshoot is accomplished by adding more damping at the expense of response speed, which sets up a design tradeoff between overshoot and response speed. Experience suggests that a good high frequency gain for the sensitivity weighting is in the range of $[0.1, 0.5]$ to effectively limit overshoot while allowing for a fast response time. The crossover frequency of the sensitivity weighting function should be chosen to limit the maximum desired closed-loop time constant. For example, if a time constant less than or equal to 0.25 sec is desired, the sensitivity weighting function should have a crossover frequency of approximately 4 rad/sec.

The response of the system to reference inputs and sensor noise inputs is given by the complementary sensitivity function, T . Recall that we desire T near unity for good tracking of the reference input and near zero for noise suppression. Again, trade-offs over frequency ranges must be made. The complementary sensitivity function is weighted to achieve stability robustness characteristics (e.g., insensitivity to noise and unmodeled dynamics). Because noise usually has most of its energy concentrated at high frequencies while reference input commands occur at low frequencies, a high pass weight is used on the complementary sensitivity function. This amounts to keeping the weighted complementary sensitivity function near unity at low frequencies and low at high frequencies. The magnitude of this weighting function at low frequencies can be set to limit the achievable system response. For instance, in a missile autopilot design, the low frequency gain of the complementary sensitivity weighting function $W_T(s)$ can be set to the inverse of the maximum allowable missile acceleration (i.e., structural g-limit of the missile airframe). For this example, $W_T(s)$ is applied to the achieved missile acceleration as shown in Figure 11. The crossover frequency of the complementary sensitivity weighting function is chosen to limit the closed-loop bandwidth and the high frequency gain is set high to provide sensor noise rejection and high frequency gain attenuation. For example, the high frequency magnitude of $W_T(s)$ can be set as the inverse of a high frequency attenuation design requirement.

When using both sensitivity and complementary sensitivity weighting functions it is important to make sure that the magnitude of these weights at the frequency where they cross is less than one. This is necessary to prevent violation of the conservation law given by Equation (4). Typical weighting functions for $W_s(s)$ and $W_T(s)$ are shown in Figure 12.

Another method of limiting the controller bandwidth and providing high frequency gain attenuation is to use a high pass weight on an unmodeled dynamics uncertainty block that may be added from the plant input to the plant output. This type of weighting is depicted in Figure 13. The characteristic of this weighting function is very similar to the complementary sensitivity weighting function. When using an unmodeled dynamics weighting function, it is usually chosen to represent the expected frequency content of the higher order dynamics. In other words, the weight is chosen to cover the expected worst case magnitude of the unmodeled dynamics. In addition, this weight may also be selected so as to satisfy

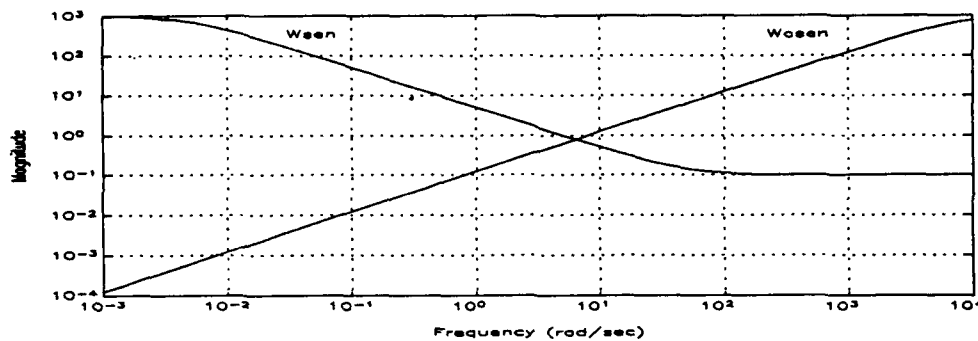


FIGURE 12. TYPICAL S AND T FUNCTION WEIGHTS

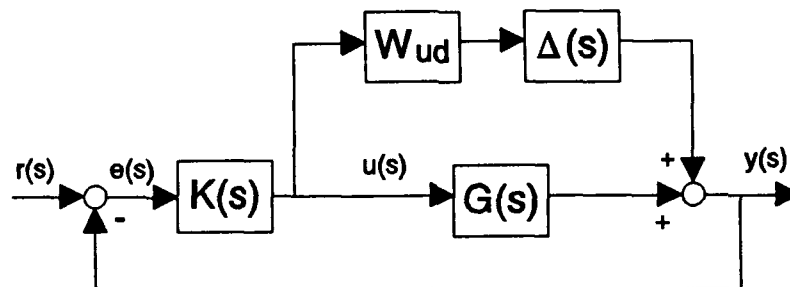


FIGURE 13. UNMODELED DYNAMICS MODEL

some high frequency attenuation specifications. A typical unmodeled dynamics weighting function is shown in Figure 14.

Other performance weights that might be included in the H_∞ control design model are those placed on the control inputs and actuator variables. For example in a missile autopilot design, we might weight the control input to the actuator and the actuator rate. The purpose of these weights is to prevent actuator saturation (both position and rate) and limit amplification of sensor noise signals on the control input signal. Typically actuator input weights are constant over frequency and are set at the inverse of the saturation limit. However, if the frequency content of the saturation is known, the weight could be frequency dependent.

This section concludes with a few remarks on the order of weighting functions. Because the order of the optimal controller is equal to the order of the nominal plant model plus the order of the weights, the complexity of the controller is increased as the order of the weights increases. While higher-order weighting functions allow more flexibility in shaping the response of the system and ease the controllers ability to satisfy the design specifications, they also yield higher-order controllers. We have also observed convergence problems in the DK iteration used in the μ -Synthesis controller design process as the order of the weighting functions is increased. This is evidence of the known property that the μ -

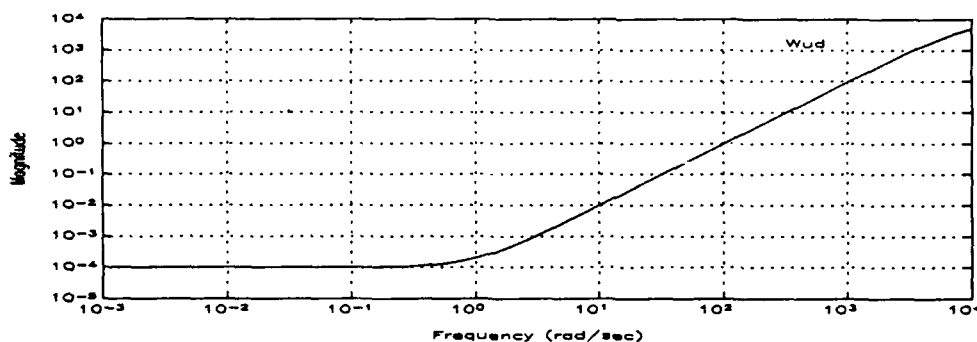


FIGURE 14. EXAMPLE OF UNMODELED DYNAMICS WEIGHT

Synthesis DK iterations are not globally convex, which means that μ -Synthesis is not always guaranteed to converge to an improved solution. Our experience suggests that the order of the weights should be kept reasonably low to reduce the order of the resulting optimal compensator and avoid potential convergence problems in the DK iterations.

PROBLEM-DEPENDENT ISSUES

While the guidelines discussed above in general apply for all controller designs, several issues regarding the proper selection of performance weights that we have found to be problem-dependent follow:

- Performance vs Stability/Robustness Tradeoffs and Weight Interaction. The primary tradeoff the designer must make in selecting the weighting functions is between performance and stability robustness characteristics. For example, increasing the high frequency magnitude of output weights to add extra robustness to high frequency unmodeled dynamics usually detracts from the desired output time response characteristics (e.g., slower time response). This is due in part to the limitation expressed by Equation (4) as well as other limitations. These type of tradeoffs are common to all control design methods. For μ -synthesis, these tradeoffs are complicated by the interaction effects of the uncertainty weights with the input disturbance and output error weighting functions. The effects of the weight interactions are complex and nonlinear and vary from problem to problem.
- Fine-Tuning of the Time Constant. An example of the above tradeoffs is isolation of the effects of interactions between sensitivity and complementary sensitivity weights on the overall system time constant. With the addition of the complementary

sensitivity weight, the relationship between the sensitivity weighting function crossover point and closed-loop system time constant seems to be problem-dependent. The question of where to exactly cross W_s and W_T and at what magnitude requires thoughtful consideration.

- Convergence Problems with High-Order Weights. In some cases, nonconvex behavior in the DK iteration has been observed when using high order weighting functions. We have found this to be problem-dependent to some degree; but, in general, we find that low-order weights are better all around. In addition, the fit of the D-scales in the DK iteration process may also have an influence on the convergence behavior, especially if the D-scale fits are of high-order and have extreme discontinuities. Using D-scale fits with extreme discontinuities and peaks in effect adds lightly damped modes to the design model.

- Approach to High Frequency Weighting. Several issues concerning weighting functions that have most of their energy concentrated at high frequencies have been identified. In general, they relate to the best approach for handling the high frequency loopshaping. These can be summarized as follows: If an unmodeled dynamics weight is used, what, if any, benefit is there to also weighting the complementary sensitivity function? There may be cases where a complementary sensitivity weight will be required to meet specific design goals or vice versa for an unmodeled dynamics weight. For some problems, a multiplicative uncertainty weight for high frequency attenuation can be used at the plant input in place of the unmodeled dynamics weight and/or the complementary sensitivity weight.

- Issues Regarding the Performance Block. Consider robust performance analysis using the structured singular value. The performance block can be thought of as a fictitious unstructured uncertainty block that relates the regulated performance outputs to the exogenous (or disturbance) inputs. Certain trade-offs between performance and robustness must be considered when specifying the performance output weights. The point at which performance can be over specified will depend on the uncertainty description of the system. The size of the performance block will also be a factor in the ability to achieve lower structured singular values, μ . Because the performance block is treated as a full block, interactions between each component of the full block will effect the design. For instance, including the actuator and high frequency attenuation elements in the full block will give higher μ -values than if these were treated as individual uncertainties included in the diagonal uncertainty Δ_U block. However, if they are included in the full block, the number of D-scales that must be fit (and hence the order of the controller) will be reduced. Thus, if you can get robust performance with the larger full block, then this approach can be used to speed up the design and keep the order of the controller down by reducing the number of uncertainties in the design model. Again, whether or not this can be done depends on the particular problem.

The main purpose of the above discussion is to make the designer aware of the alternatives when selecting weighting functions to specify design goals and the potential pitfalls that should be avoided. It is important to realize that the optimum weighting strategies in H_∞ control and μ -Synthesis are problem-dependent and require engineering judgement. However, the guidelines suggested previously can help speed up the formulation of these weighting strategies.

SUMMARY AND CONCLUSIONS

In this report, we discussed the selection of weighting functions in H_∞ Control. Sensitivity functions were defined and loopshaping design concepts were presented. After describing how the design weights serve as important "tuning knobs" in H_∞ Control, guidelines were given on how these weights can be selected and how they relate to system characteristics. This was followed by a discussion of advanced problem dependent issues yet to be resolved regarding the general weight selection process.

This report provides the H_∞ Control system designer with practical rules-of-thumb for designing weighting functions to shape the response characteristics of the system and also points out other options available to the designer and their impact on the design. Through the use of the guidelines presented herein, the H_∞ Control design process should be more streamlined and, thus, result in savings in development time and cost of new control systems.

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