`dennm`: (DEN)sity of (N)on-(M)arkovian phenomena

A generalised density solver for non-Markovian phenomena written in `Rust` and using the `ArrayFire` wrapper (see here) to make it nice and fast.

The computational framework for `dennm` is derived using the formalism from the publicly-available book: Diffusing Ideas. Let's begin by writing the generalised non-Markovian master equation for stochastic phenomena which was given in this chapter of the book

$$P_{\mathsf{t}+1}(x|z) = \int_{\Omega_{\mathsf{t}}} \mathrm{d}X' P_{\mathsf{t}+1}(X|z) = \int_{\Omega_{\mathsf{t}}} \mathrm{d}X' P_{\mathsf{t}}(X'|z) P_{(\mathsf{t}+1)\mathsf{t}}(x|X', z).$$

For now, let's imagine that x is just a scalar (as opposed to a row vector) for simplicity in the expressions. We can then write down a kind of Kramers-Moyal expansion of the equation above

$$\begin{aligned} P_{\mathsf{t}+1}(x|z) = \int_{\Omega_{\mathsf{t}}} \mathrm{d}X' P_{\mathsf{t}}(X'|z) \bigg[&\delta(x-x') + \sum_{\mathsf{t}'=0}^{\mathsf{t}} \partial_x \delta(x-x') [\mu_1]_{\mathsf{t}'}(X'', z) \\ &+ \sum_{\mathsf{t}'=0}^{\mathsf{t}} \partial_x \delta(x-x') \sum_{\mathsf{t}''=0}^{\mathsf{t}'} \partial_{x'} \delta(x'-x'') [\mu_2]_{\mathsf{t}'\mathsf{t}''}(X'', z) \dots \bigg], \end{aligned}$$

which, when truncating the expansion terms up to second order, results in the following difference equation

$$\begin{aligned} P_{\mathsf{t}+1}(x|z) - P_{\mathsf{t}}(x|z) \simeq &- \sum_{\mathsf{t}'=0}^{\mathsf{t}} \frac{\partial}{\partial x} \Big\{ P_{\mathsf{t}'}(X|z) [\mu_1]_{\mathsf{t}'}(X, z) \Big\} \Big|_{X_{\mathsf{t}'}=x} \\ &+ \sum_{\mathsf{t}'=0}^{\mathsf{t}} \frac{\partial}{\partial x} \sum_{\mathsf{t}''=0}^{\mathsf{t}'} \frac{\partial}{\partial x'} \Big\{ P_{\mathsf{t}'}(X|z) [\mu_2]_{\mathsf{t}'\mathsf{t}''}(X, z) \Big\} \Big|_{X_{\mathsf{t}'}=x \wedge X_{\mathsf{t}''}=x'} . \end{aligned}$$