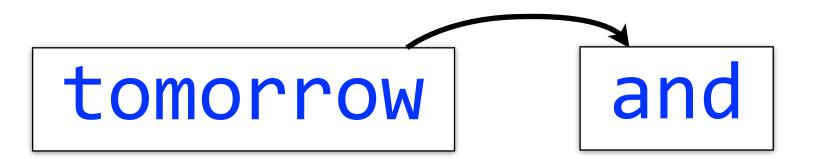
The de Bruijn Graph and its efficient representation

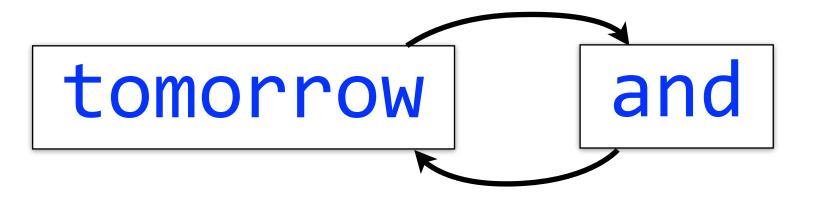


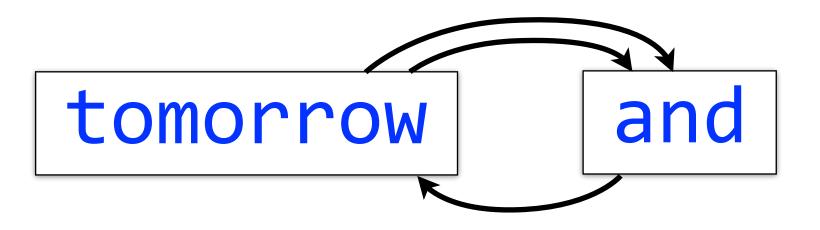
"tomorrow and tomorrow and tomorrow"

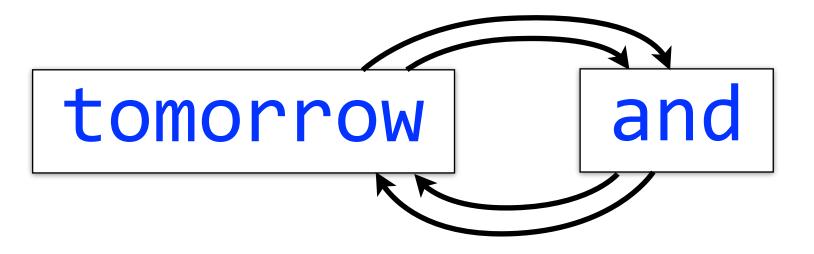
tomorrow

and

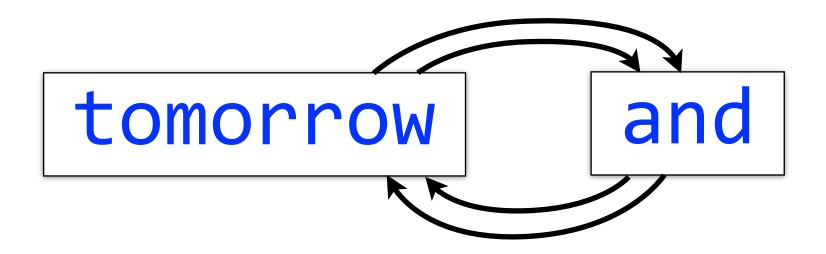






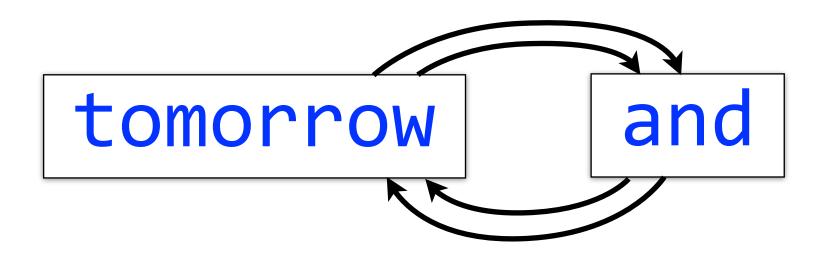


"tomorrow and tomorrow and tomorrow"



An edge represents an ordered pair of adjacent words in the input

"tomorrow and tomorrow and tomorrow"



An edge represents an ordered pair of adjacent words in the input

Multigraph: there can be more than one edge from node A to node B

genome: AAABBBBA

*

genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

L/R 2-mers: AA, AA

genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

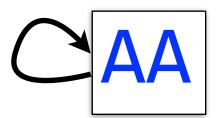
L/R 2-mers: AA, AA

AA

genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

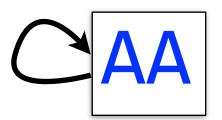
L/R 2-mers: AA, AA



genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

L/R 2-mers: AA, AA AA, AB

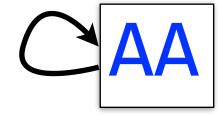


genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

L/R 2-mers: AA, AA AA, AB

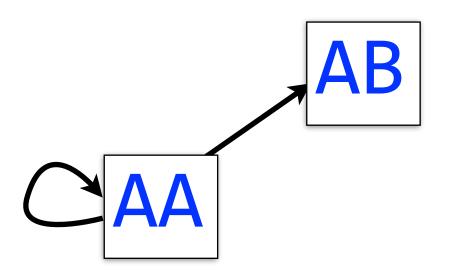
AB



genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

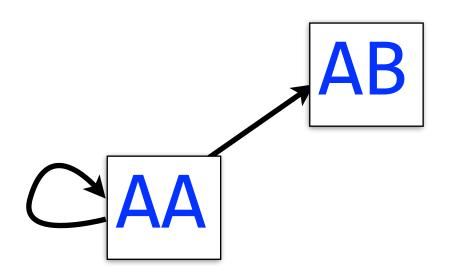
L/R 2-mers: AA, AA AA, AB



genome: AAABBBBA

3-mers: AAA, AAB, ABB, BBB, BBB, BBA

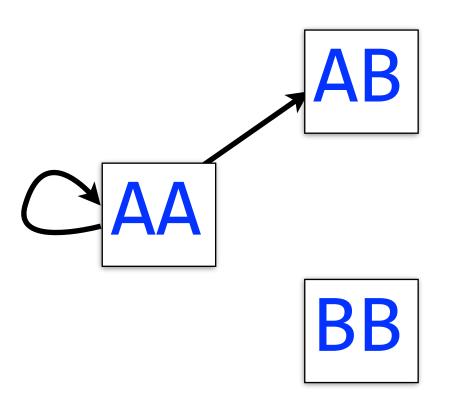
L/R 2-mers: AA, AA AA, AB AB, BB



genome: AAABBBBA

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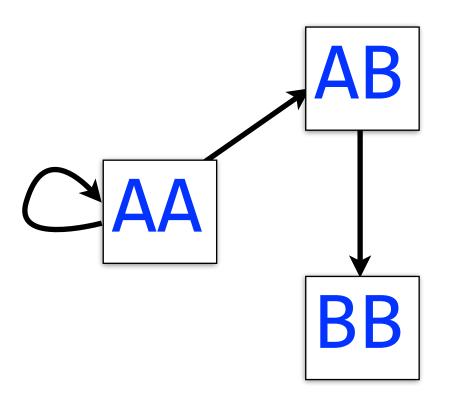
L/R 2-mers: AA, AA AA, AB AB, BB



genome: AAABBBBA

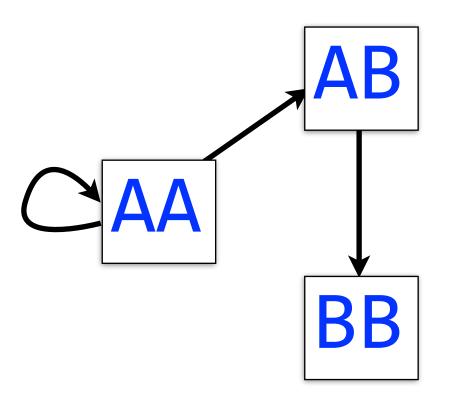
3-mers: AAA, AAB, ABB, BBB, BBB, BBA

L/R 2-mers: AA, AA AA, AB AB, BB



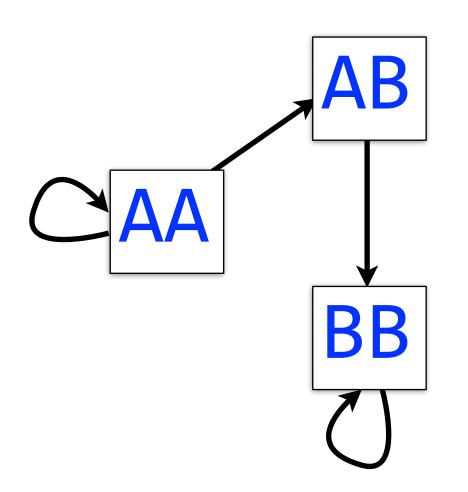
genome: AAABBBBA

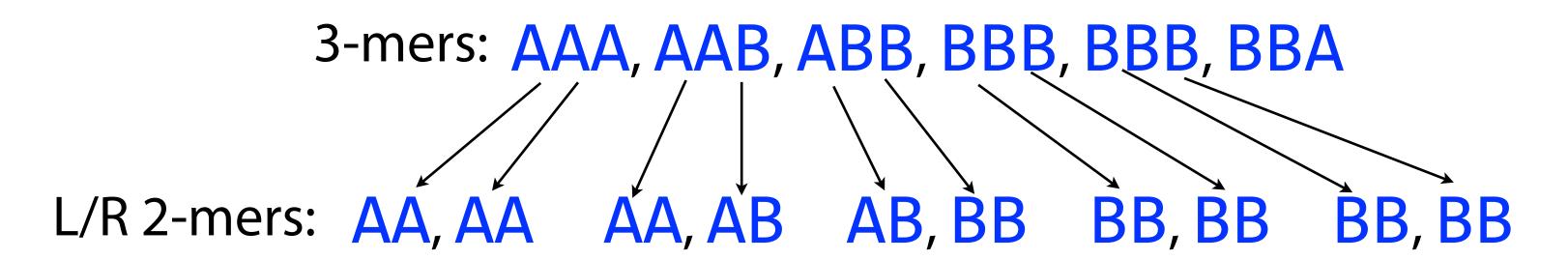
3-mers: AAA, AAB, ABB, BBB, BBB, BBA L/R 2-mers: AA, AA AA, AB AB, BB BB, BB

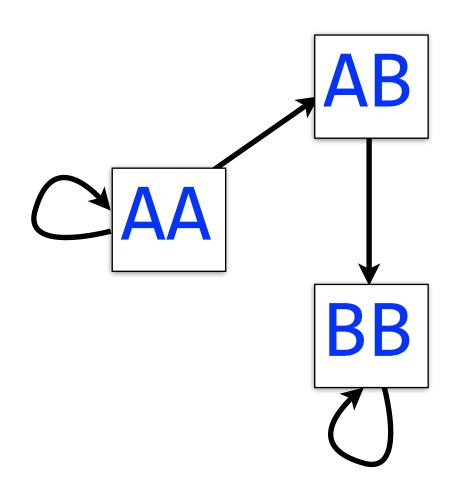


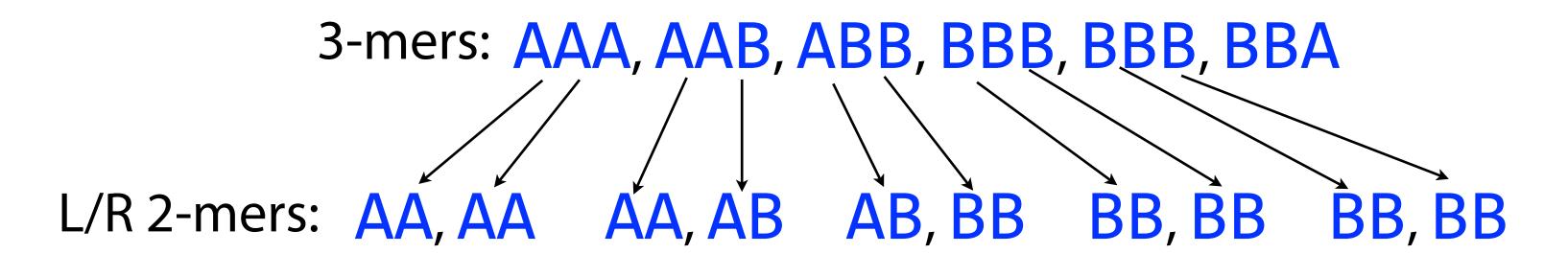
genome: AAABBBBA

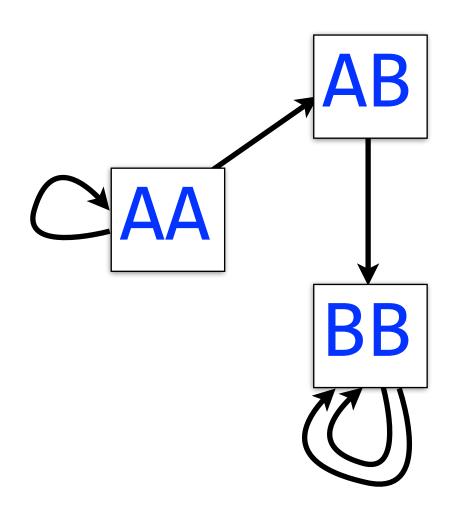
3-mers: AAA, AAB, ABB, BBB, BBB, BBA L/R 2-mers: AA, AA AA, AB AB, BB BB, BB

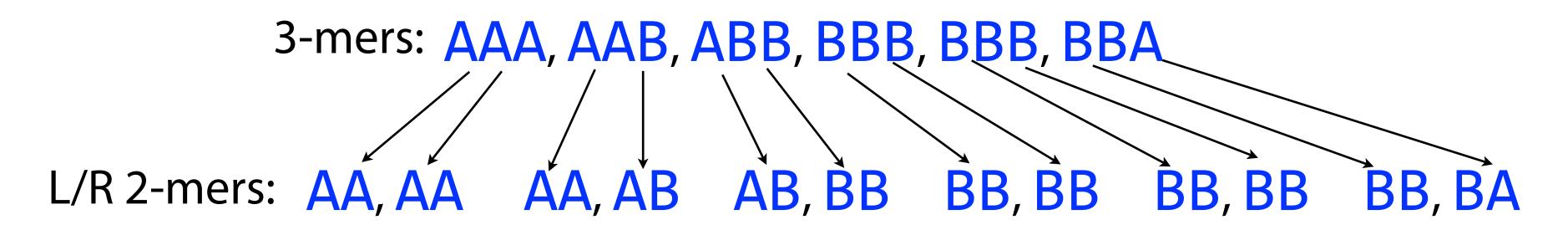


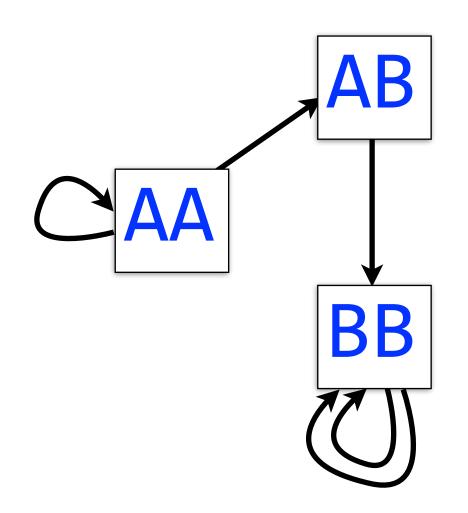


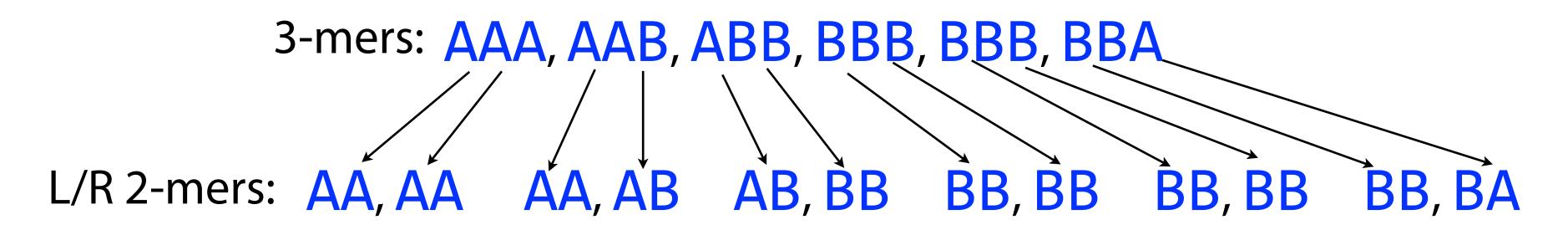


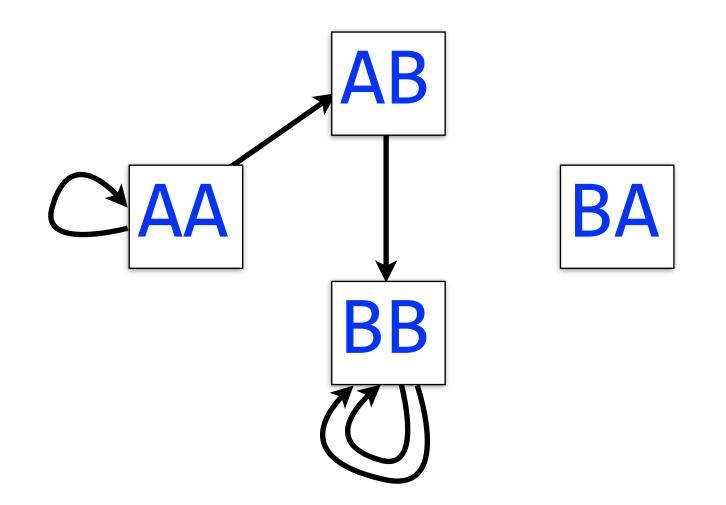


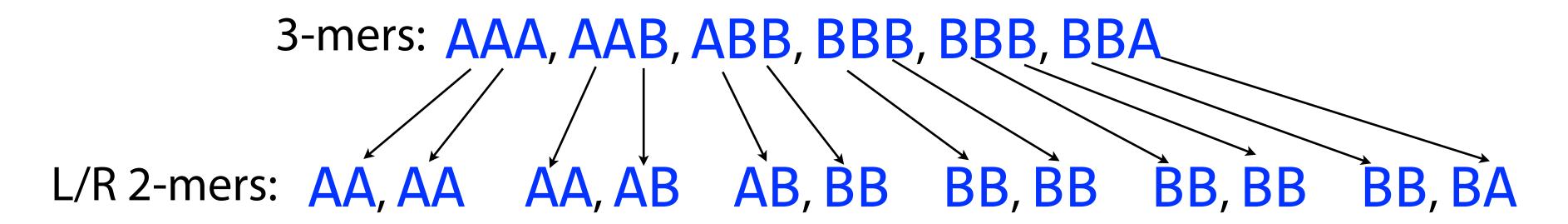


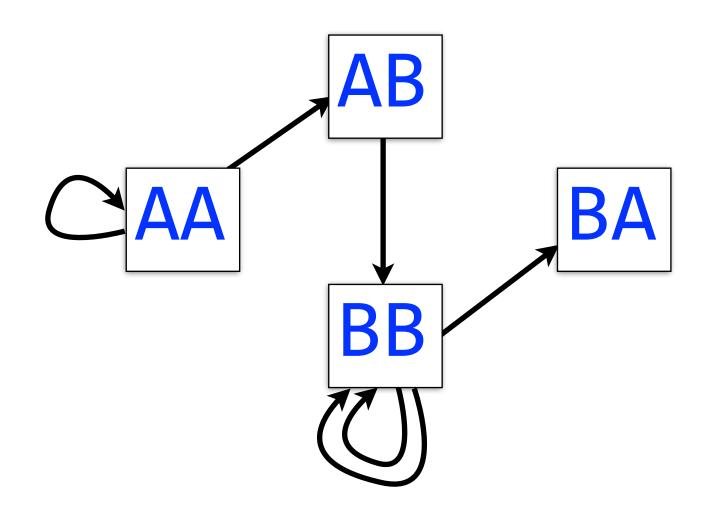


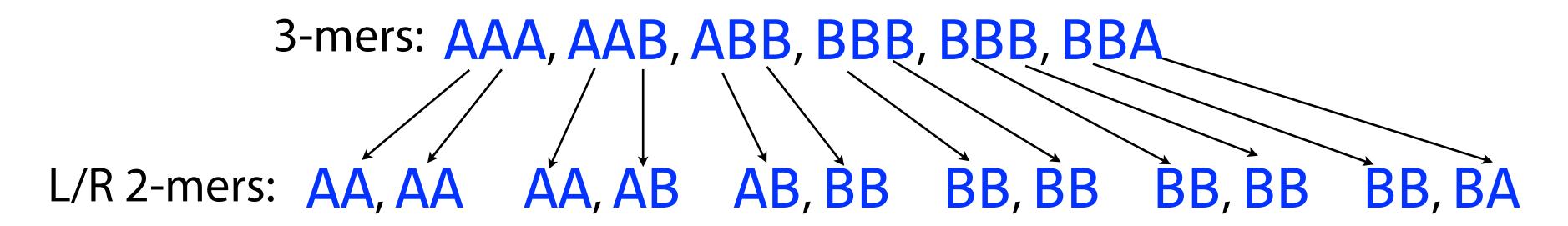


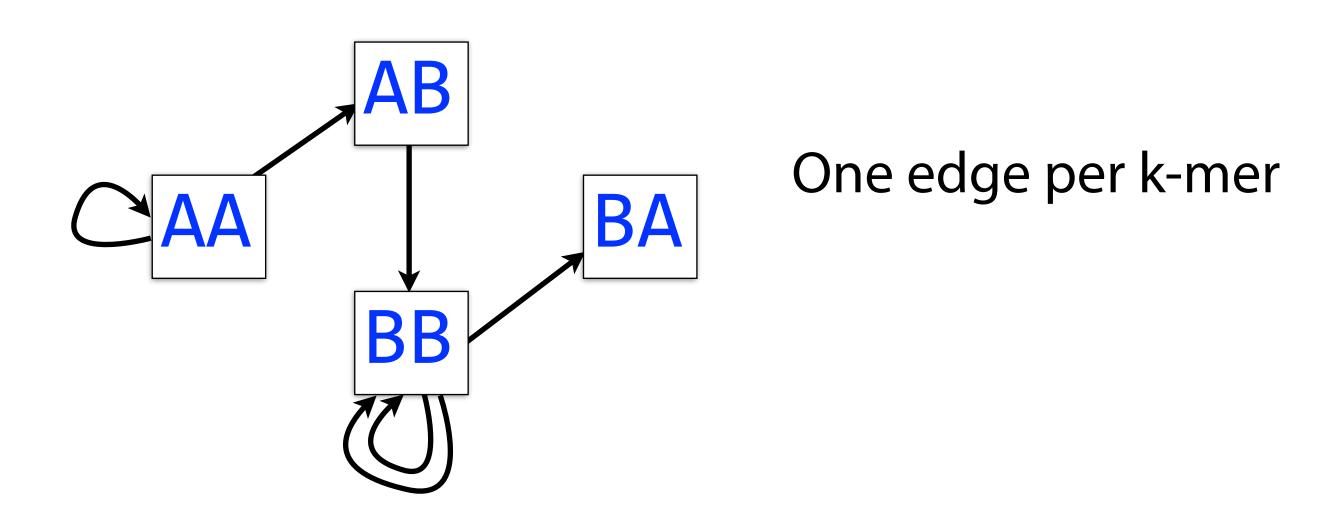


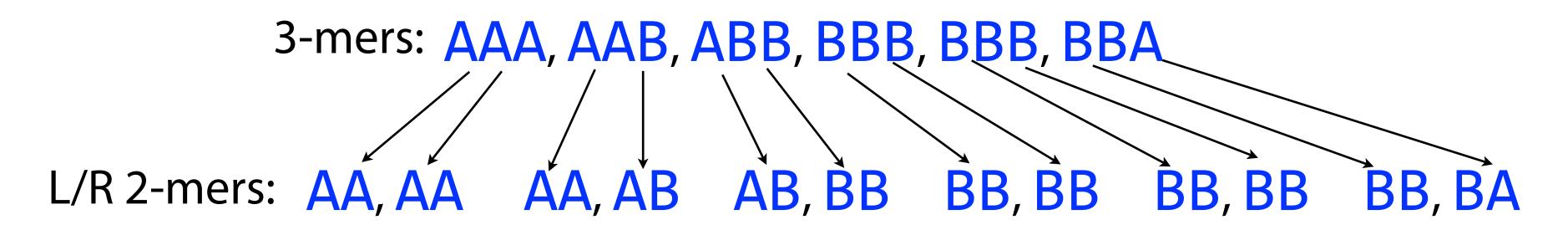


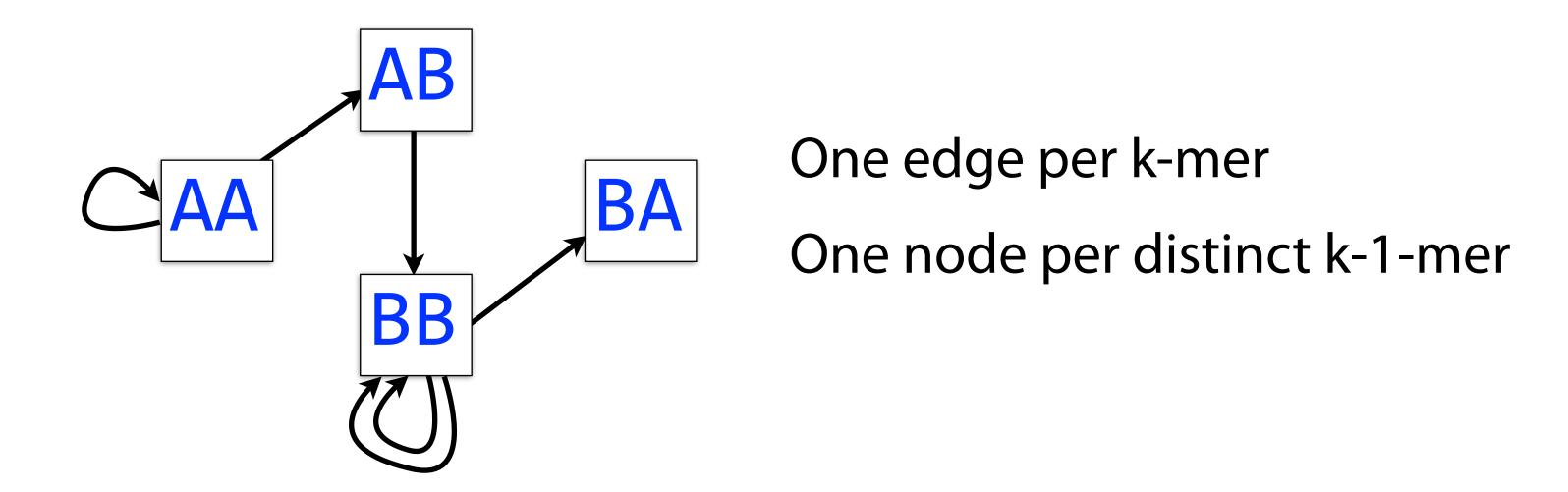


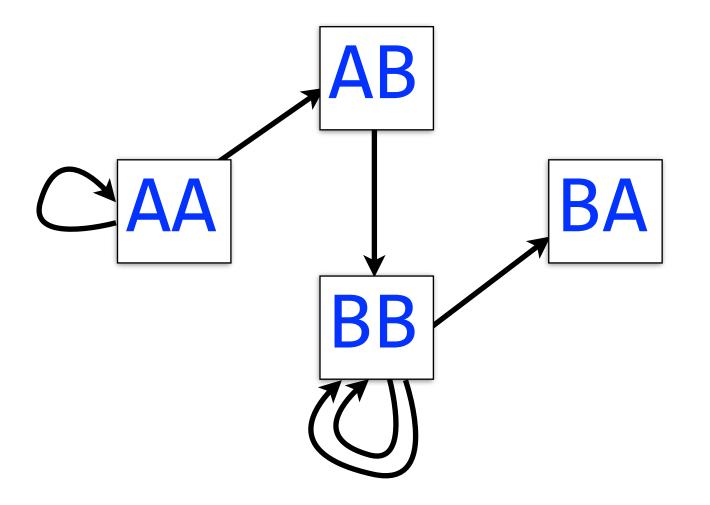


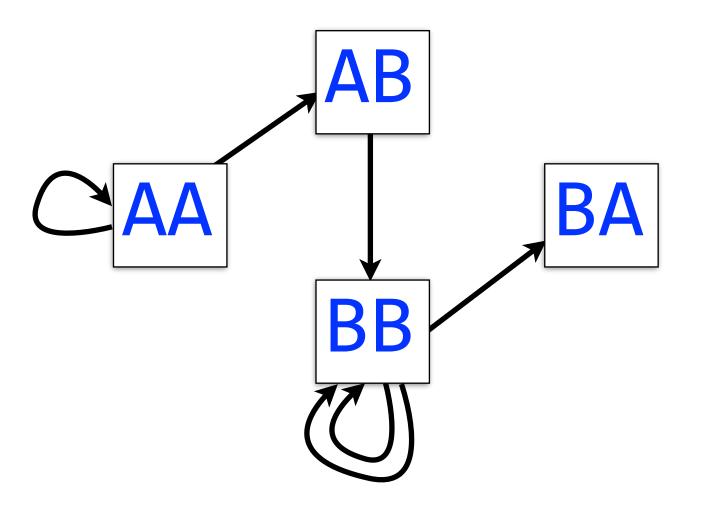


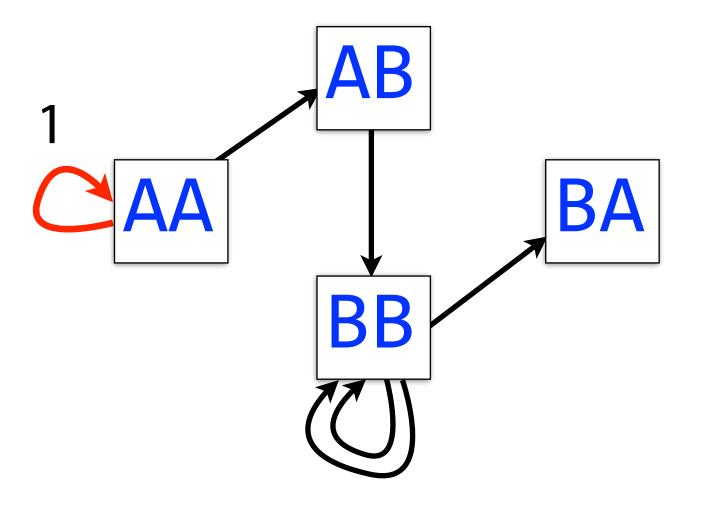




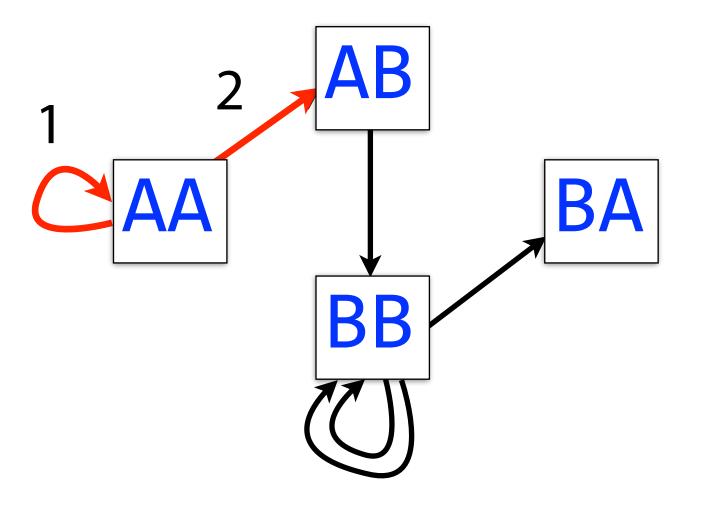




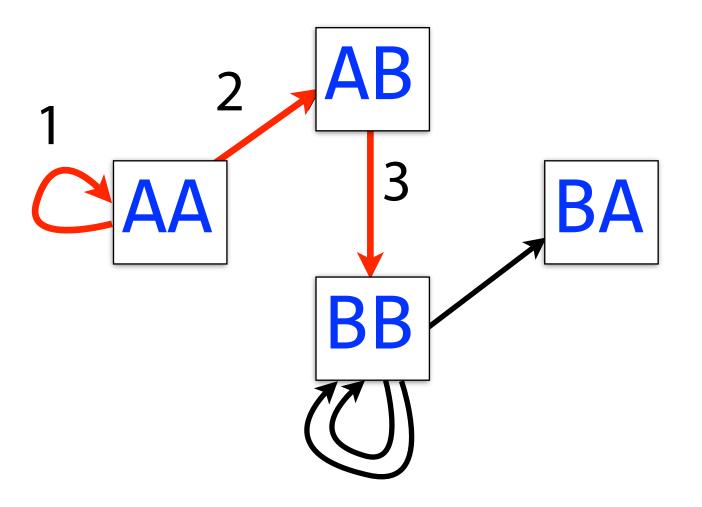




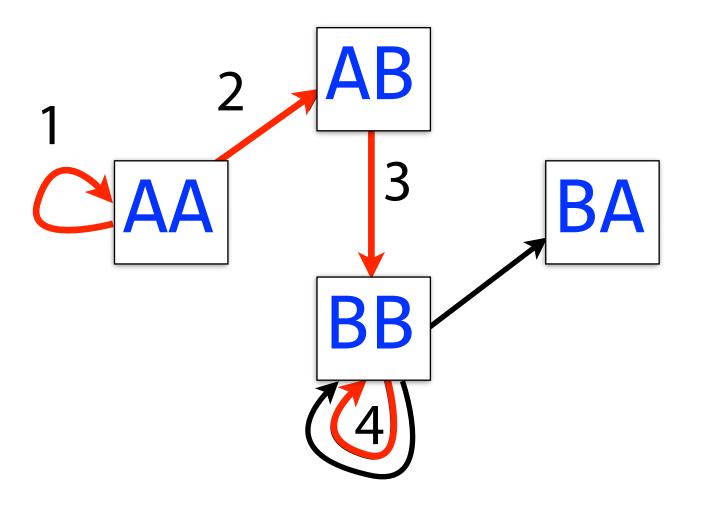
AAA



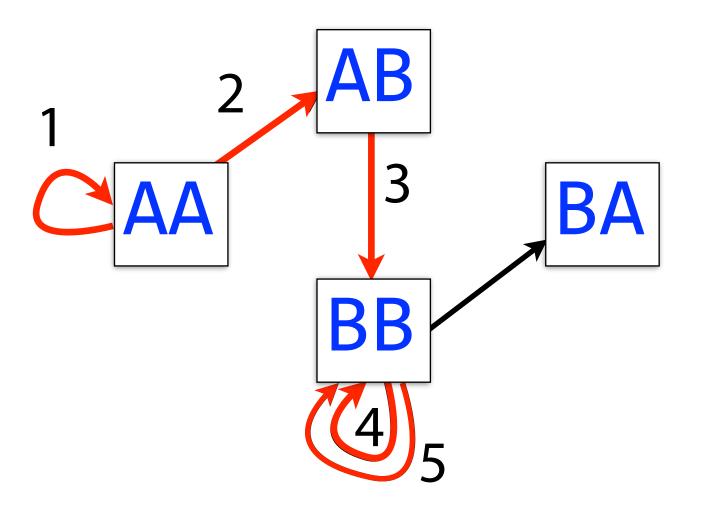
AAA B



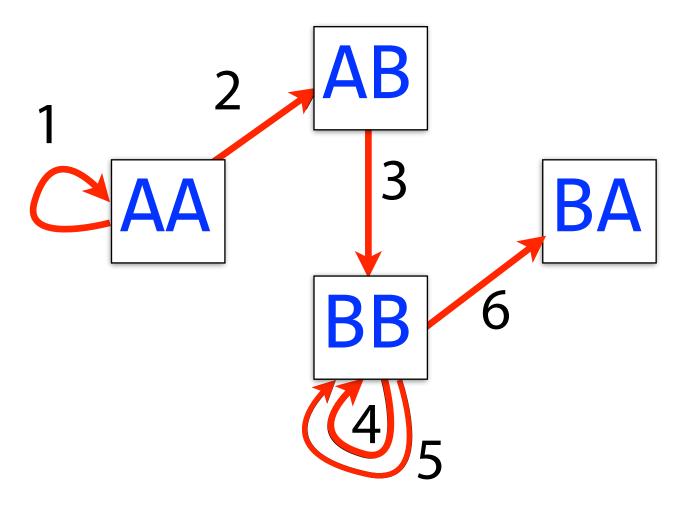
AAA BB



AAA BBB

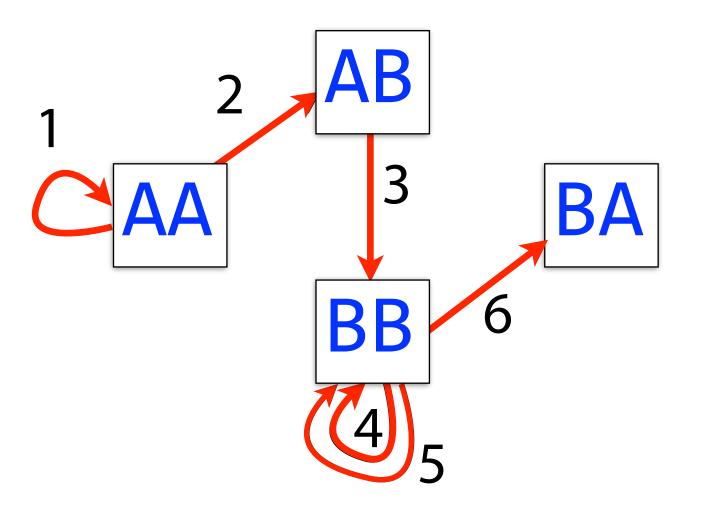


AAA BBBB



AAA BBBBA

Walk crossing each edge exactly once gives a reconstruction of the genome



AAA BBBBA

Walk crossing each edge exactly once gives a reconstruction of the genome . This is an Eulerian walk.

Directed multigraph

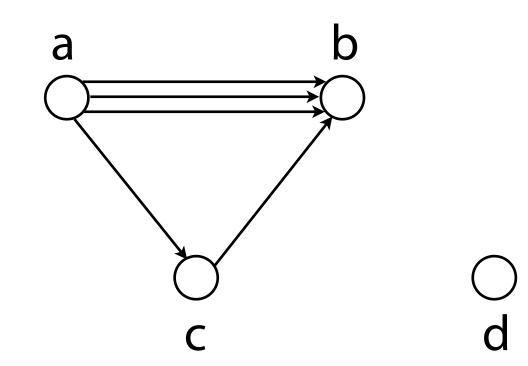
Directed multigraph G(V, E) consists of set of vertices, V and multiset of directed edges, E

Otherwise, like a directed graph

Node's indegree = # incoming edges

Node's outdegree = # outgoing edges

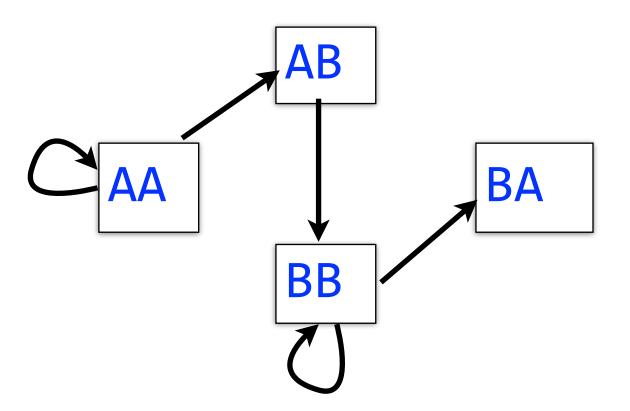
De Bruijn graph is a directed multigraph



$$V = \{ a, b, c, d \}$$

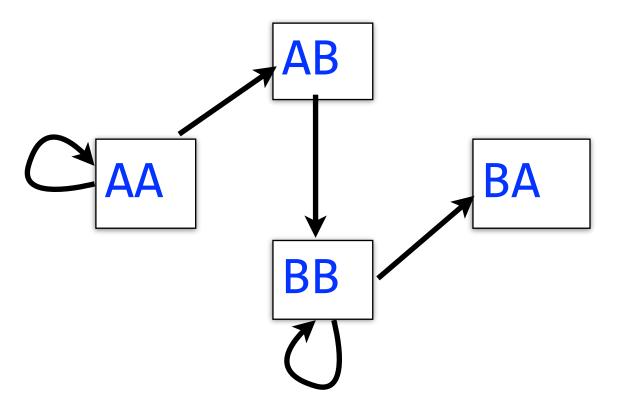
 $E = \{ (a, b), (a, b), (a, b), (a, c), (c, b) \}$
——Repeated ——

Node is balanced if indegree equals outdegree



Node is balanced if indegree equals outdegree

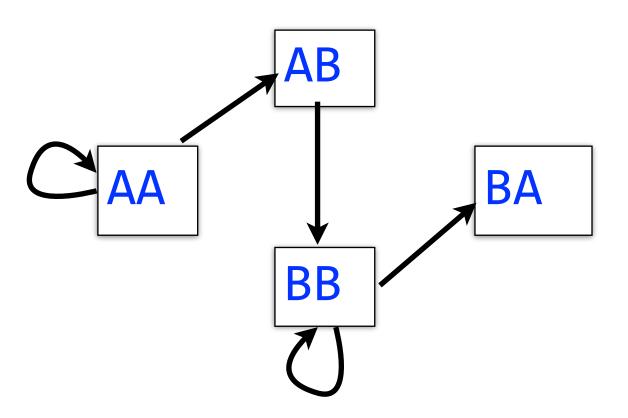
Node is semi-balanced if indegree differs from outdegree by 1



Node is balanced if indegree equals outdegree

Node is semi-balanced if indegree differs from outdegree by 1

Graph is connected if each node can be reached by some other node

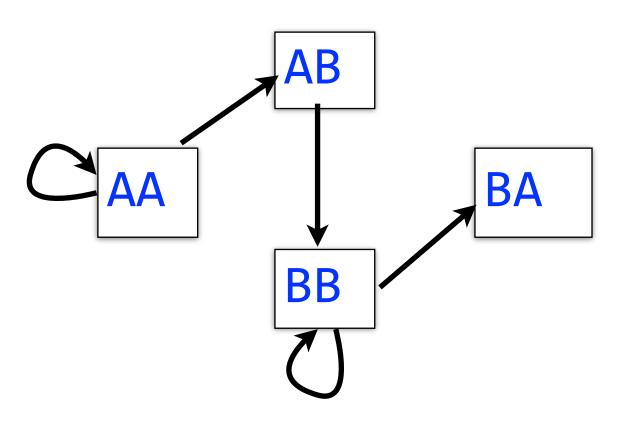


Node is balanced if indegree equals outdegree

Node is semi-balanced if indegree differs from outdegree by 1

Graph is connected if each node can be reached by some other node

Eulerian walk visits each edge exactly once



*

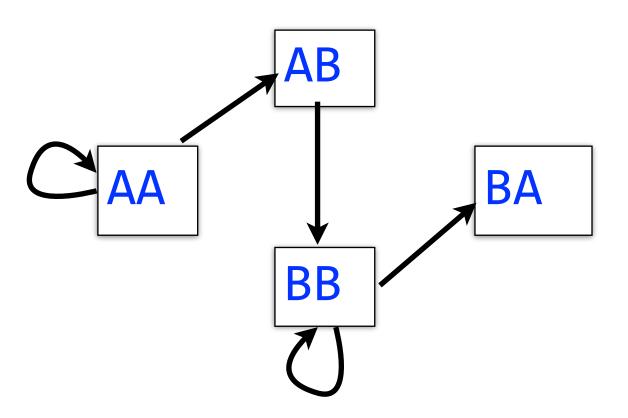
Node is balanced if indegree equals outdegree

Node is semi-balanced if indegree differs from outdegree by 1

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Eulerian walk visits each edge exactly once

Not all graphs have Eulerian walks. Graphs that do are Eulerian. (For simplicity, we won't distinguish Eulerian from semi-Eulerian.)



Node is balanced if indegree equals outdegree

Node is semi-balanced if indegree differs from outdegree by 1

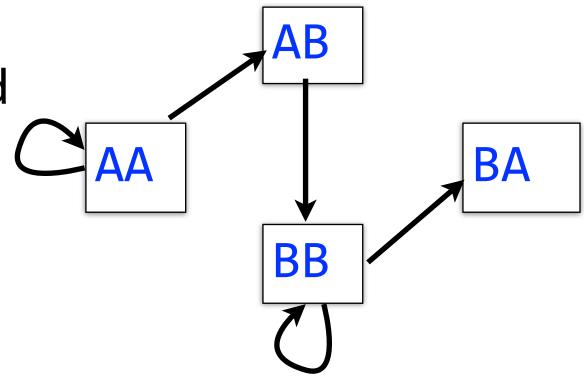
Graph is connected if each node can be reached by some other node

Eulerian walk visits each edge exactly once

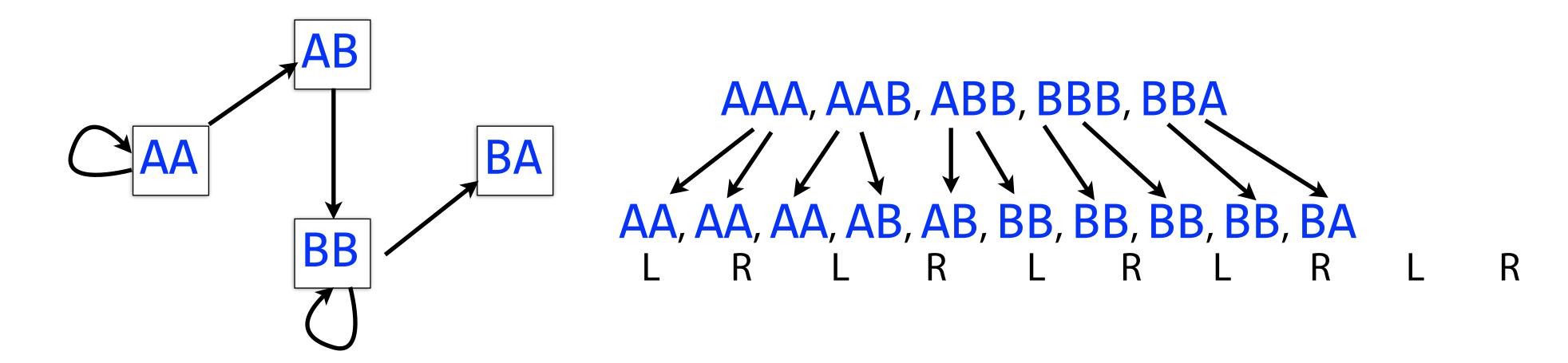
Not all graphs have Eulerian walks. Graphs that do are Eulerian. (For simplicity, we won't distinguish Eulerian from semi-Eulerian.)

A directed, connected graph is Eulerian if and only if it has at most 2 semi-balanced nodes and all other nodes are balanced

Jones and Pevzner section 8.8

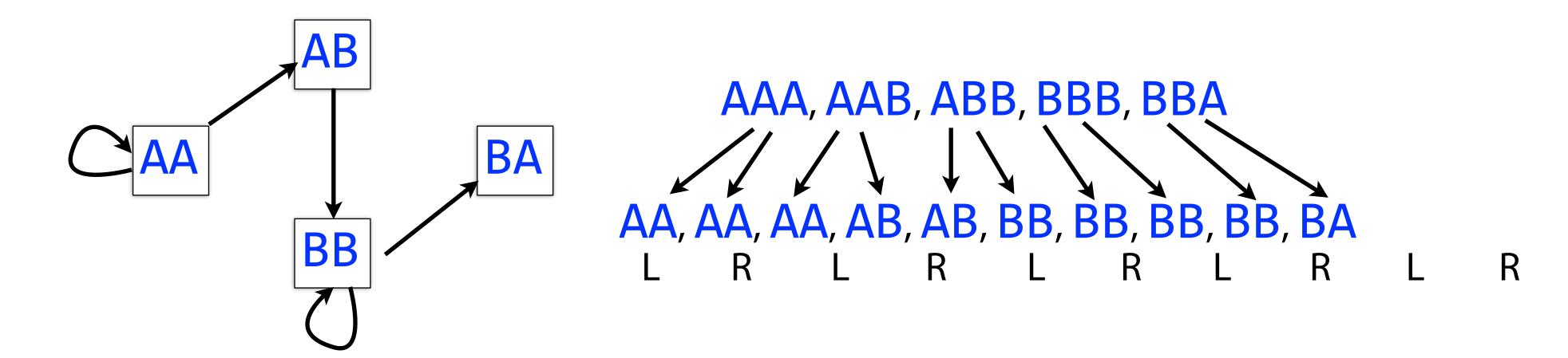


Back to de Bruijn graph



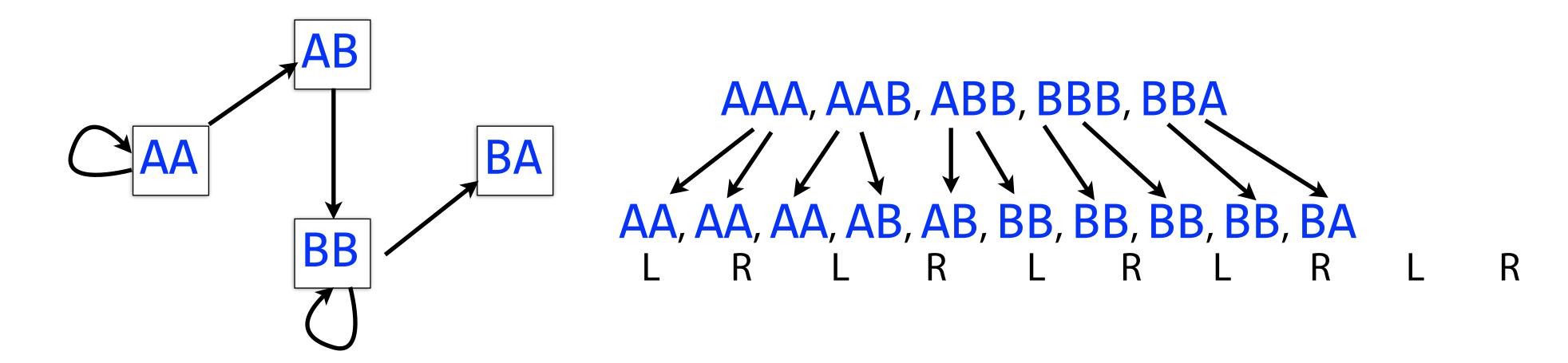
Is it Eulerian?

Back to de Bruijn graph



Is it Eulerian? Yes

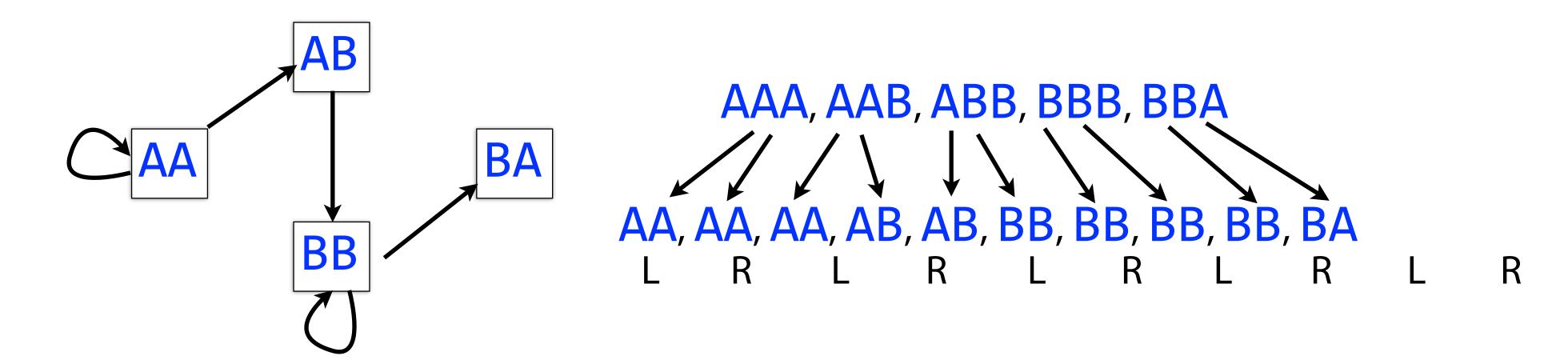
Back to de Bruijn graph



Is it Eulerian? Yes

Argument 1: $AA \rightarrow AA \rightarrow AB \rightarrow BB \rightarrow BB \rightarrow BA$

Back to de Bruijn graph



Is it Eulerian? Yes

Argument 1: $AA \rightarrow AA \rightarrow AB \rightarrow BB \rightarrow BB \rightarrow BA$

Argument 2: AA and BA are semi-balanced, AB and BB are balanced

Bloom Filters

Originally designed to answer *probabilistic* membership queries:

Is element e in my set S?

If yes, **always** say yes

If no, say no with large probability

False positives can happen; false negatives cannot.

Bloom Filters

For a set of size N, store an array of M bits Use k different hash functions, $\{h_0, ..., h_{k-1}\}$ To insert e, set $A[h_i(e)] = 1$ for 0 < i < k To query for e, check if $A[h_i(e)] = 1$ for 0 < i < k

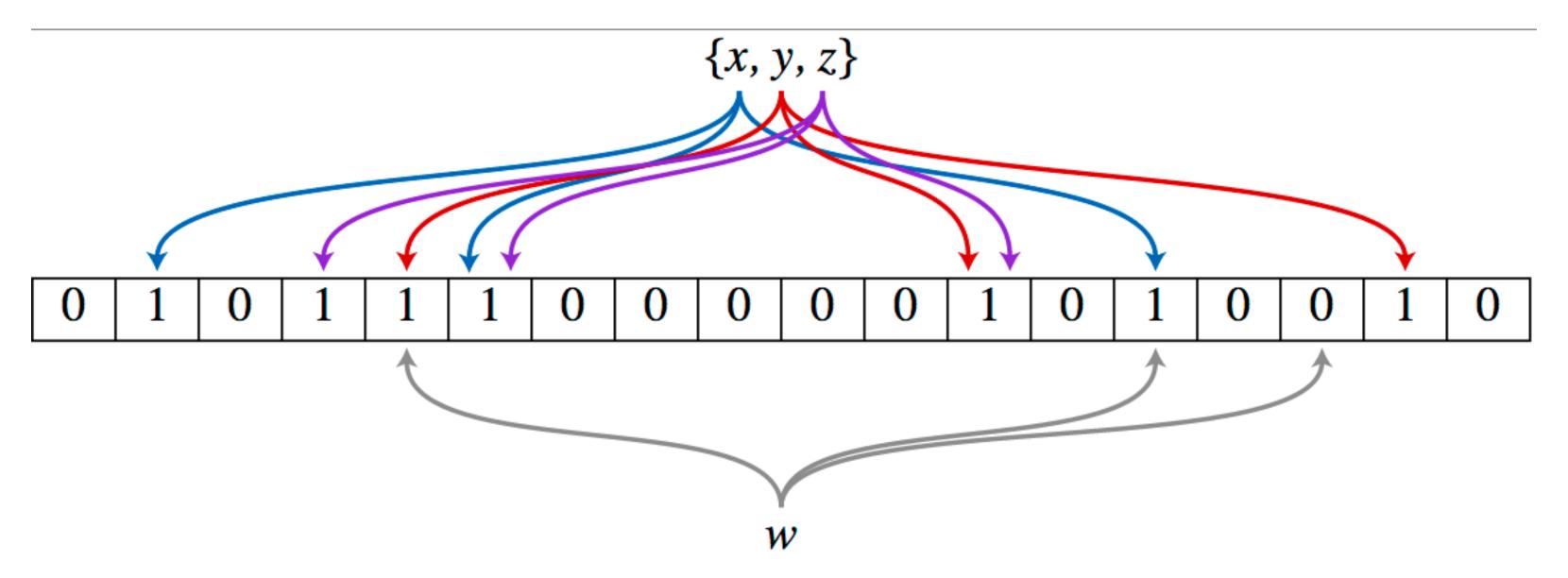


Image by David Eppstein - self-made, originally for a talk at WADS 2007

Bloom Filters

If hash functions are good and sufficiently independent, then the probability of false positives is low and controllable.

How low?

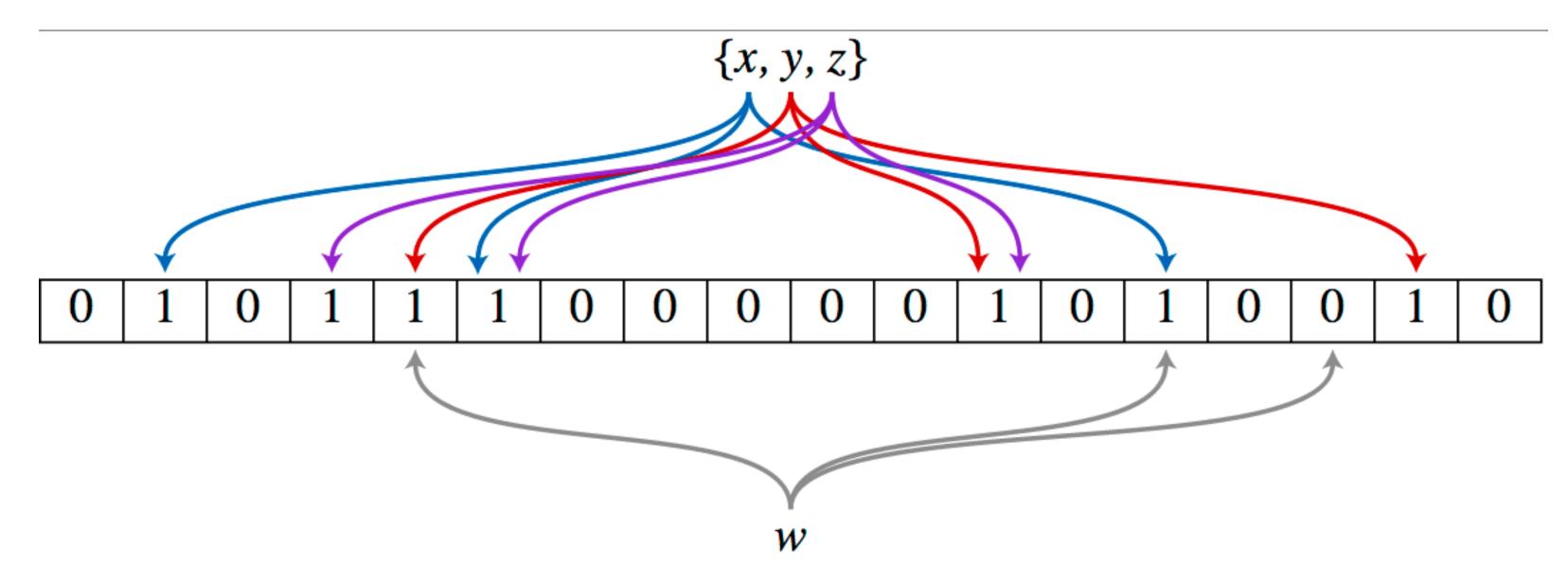


Image by David Eppstein - self-made, originally for a talk at WADS 2007

Let q be the fraction of the m-bits which remain as 0 after n insertions.

The probability that a randomly chosen bit is 1 is 1-q.

But we need a 1 in the position returned by k different hash functions; the of this is $(1-q)^k$

We can derive a formula for the expected value of q, for a filter of m bits, after n insertions with k different hash functions:

$$E[q] = (1 - 1/m)^{kn}$$

Mitzenmacher & Unfal used the Azuma-Hoeffding inequaltiy to prove (without assuming the probability of setting each bit is independent) that

$$\Pr(|q - E[q]| \ge \frac{\lambda}{m}) \le 2\exp(-2\frac{\lambda^2}{m})$$

That is, the random realizations of q are highly concentrated around E[q], which yields a false positive prob of:

$$\sum_{t} \Pr(q = t)(1 - t)^{k} \approx (1 - E[q])^{k} = \left(1 - \left[1 - \frac{1}{m}\right]^{kn}\right)^{k} \approx (1 - e^{-\frac{kn}{m}})^{k}$$

^{*}analysis of Mitzenmacher and Upfal

$$\sum_{t} \Pr(q = t)(1 - t)^{k} \approx (1 - E[q])^{k} = \left(1 - \left[1 - \frac{1}{m}\right]^{kn}\right)^{k} \approx (1 - e^{-\frac{kn}{m}})^{k}$$

This lets us choose optimal values to achieve a target false positive rate. For example, assume m & n are given. Then we can derive the optimal k

$$k = (m/n) \ln 2 \implies 2^{-k} \approx 0.6185 \, m/n$$

We can then compute the false positive prob

$$p = (1 - e^{-(\frac{m}{n} \ln 2) \frac{n}{m}})^{(\frac{m}{n} \ln 2)} \Longrightarrow$$

$$\ln p = -\frac{m}{n} (\ln 2)^2 \Longrightarrow$$

$$m = -\frac{n \ln p}{(\ln 2)^2}$$

$$\sum_{t} \Pr(q = t)(1 - t)^{k} \approx (1 - E[q])^{k} = \left(1 - \left[1 - \frac{1}{m}\right]^{kn}\right)^{k} \approx (1 - e^{-\frac{kn}{m}})^{k}$$

This lets us choose optimal values to achieve a target false positive rate. For example, assume m & n are given. Then we can derive the optimal k

$$k = (m/n) \ln 2 \Rightarrow 2^{-k} \approx 0.6185 \, \text{m/n}$$

We can then compute the false positive prob

$$p = (1 - e^{-(\frac{m}{n} \ln 2) \frac{n}{m}})^{(\frac{m}{n} \ln 2)} \Longrightarrow \text{given an expected} \\ \ln p = -\frac{m}{n} (\ln 2)^2 \Longrightarrow \text{and a desired} \\ \text{false positive rate} \\ \text{we can compute} \\ \text{the optimal size and} \\ \text{# of has functions}$$

Bloom Filters & De Bruijn Graphs

How could the Bloom filter be useful for representing a De Bruijn graph?

Say we have a bloom filter B, for all of the k-mers in our data set, and say I give you one k-mer that is truly present.

Remember this term, it will come up in a couple of slides

We now have a "navigational" representation of the De Bruijn graph (can return the set of neighbors of a node, but not select/iterate over nodes); why?

Detour: Bloom Filters & De Bruijn Graphs

How could this data structure be useful for representing a De Bruijn graph?



A given (k-1)-mer can only have $2^*|\Sigma|$ neighbors; $|\Sigma|$ incoming and $|\Sigma|$ outgoing neighbors — for genomes $|\Sigma| = 4$

To navigate in the De Bruijn graph, we can simply query all possible successors, and see which are actually present.

Bloom Filters & De Bruijn Graphs

But, a Bloom filter still has false-positives, right?

May return some neighbors that are not actually present.

Pell et al., PNAS 2012, use a lossy Bloom filter directly

Chikhi & Rizk, WABI 2012, present a *lossless* data structure based on Bloom filters

Salikhov et al., WABI 2013 extend this work and introduce the concept of "cascading" Bloom filters

Pellow, Filippova & Kingsford, RECOMB 2016. Take advantage of "independence" of false positives to lower FP rate for Bloom Filter representations

First, some bounds

JOURNAL OF COMPUTATIONAL BIOLOGY Volume 22, Number 5, 2015 © Mary Ann Liebert, Inc. Pp. 336–352 DOI: 10.1089/cmb.2014.0160

MEMB queries.

Research Articles

On the Representation of De Bruijn Graphs

RAYAN CHIKHI,^{1,6} ANTOINE LIMASSET,³ SHAUN JACKMAN,⁴ JARED T. SIMPSON,⁵ and PAUL MEDVEDEV^{1,2,6}

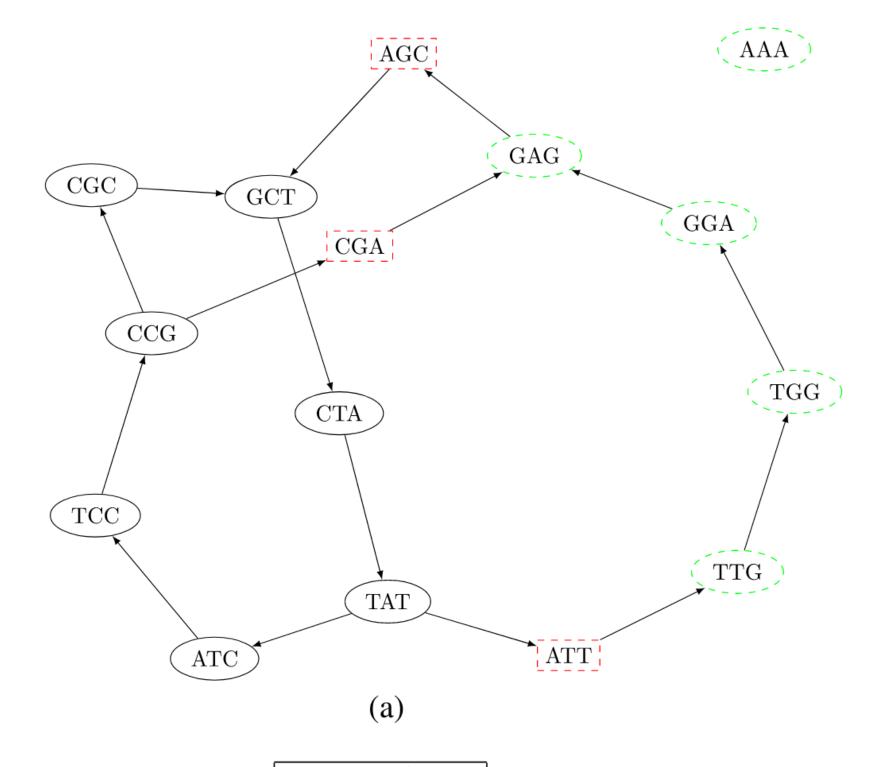
We use the term <u>membership data structure</u> to refer to a way of representing a dBG and answering k-mer membership queries. We can view this as a pair of algorithms: (CONST, MEMB). The CONST algorithm takes a set of k-mers S (i.e., a dBG) and outputs a bit string. We call const a constructor, since it constructs a representation of a dBG. The MEMB algorithm takes as input a bit string and a k-mer x and outputs true or false. Intuitively, MEMB takes a representation of a dBG created by CONST and outputs whether a given k-mer is present. Formally, we require that for all $x \in \Sigma^k$, MEMB(CONST(S), x) is true if and only if $x \in S$.

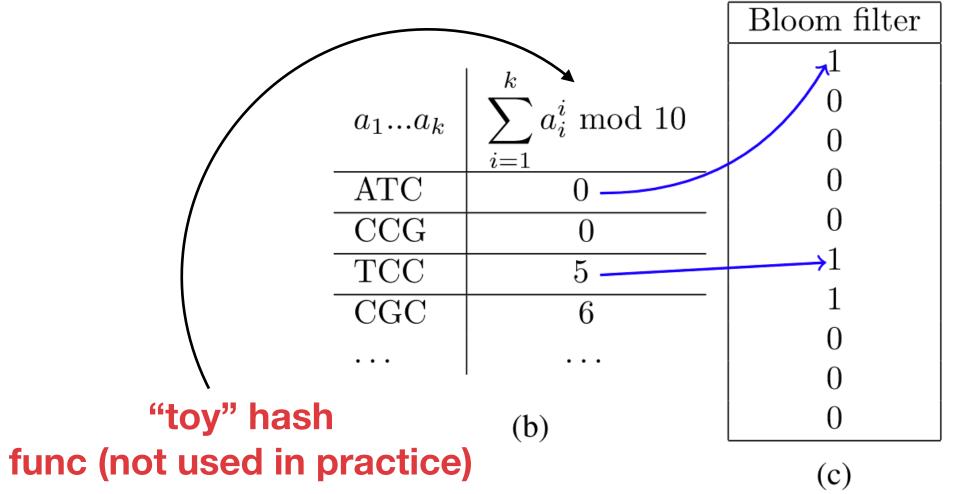
An NDS is a pair of algorithms, const and NBR. As before, const takes a set of k-mers and outputs a bit string. NBR takes a bit string and a k-mer and outputs a set of k-mers. The algorithms must satisfy that for every dBG S and a k-mer $x \in S$, NBR(const(S), x) = $ext(x) \cap S$. Note that if $x \notin S$, then the behavior of NBR(const(S), x) is undefined. We observe that a membership data structure immediately implies an NDS because an NBR query can be reduced to eight

In this section, we prove that a navigational data structure on de Bruijn graphs needs at least 3.24 bits per k-mer to represent the graph:

Theorem 1. Consider an arbitrary NDS and let CONST be its constructor. For any $0 < \epsilon < 1$, there exists a k and $x \subseteq \Sigma^k$ such that $|\text{CONST}(x)| \ge |x| \cdot (c - \epsilon)$, where $c = 8 - 3\lg 3 \approx 3.25$.

Critical False Positives





Nodes self-information:

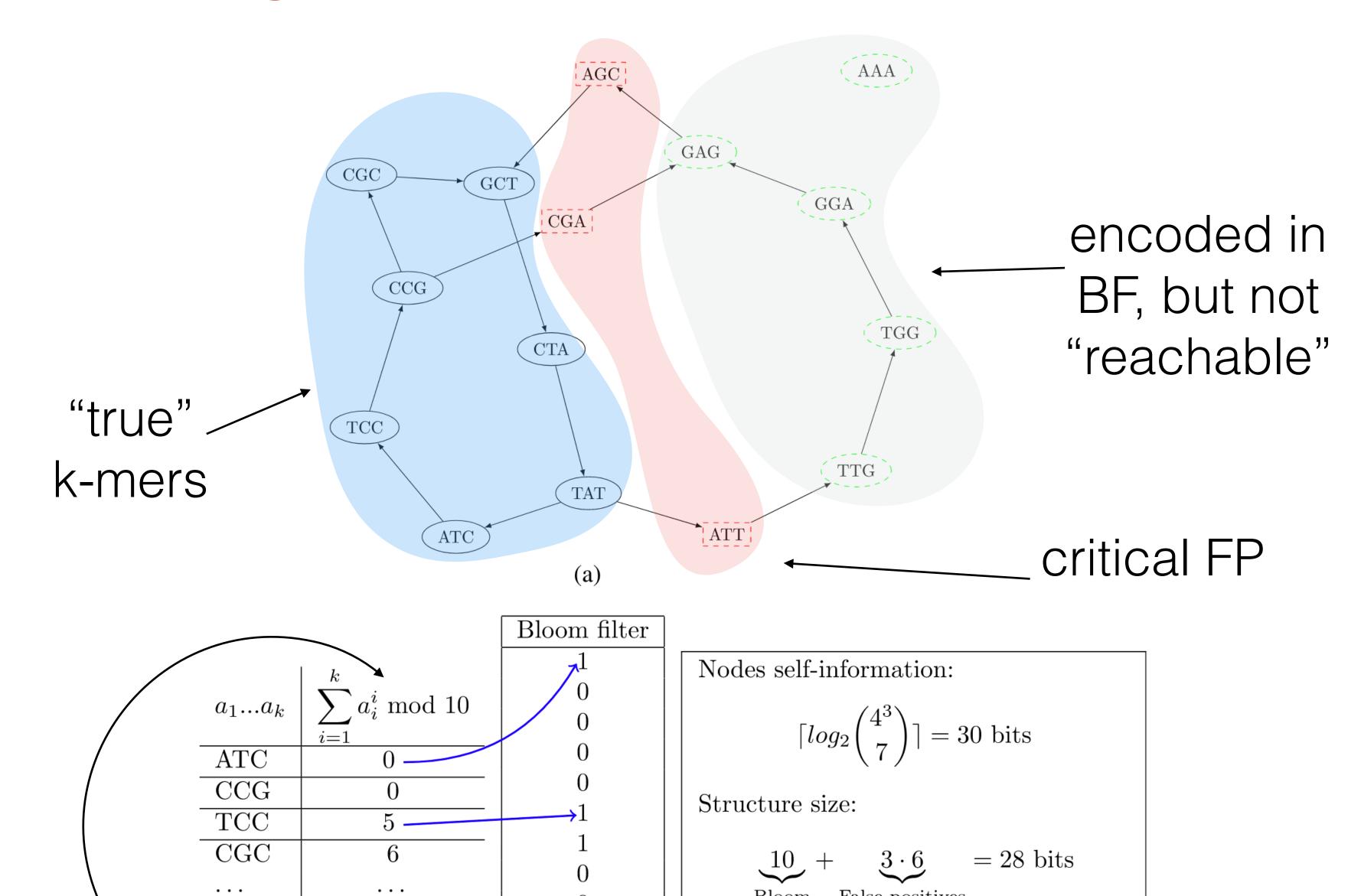
$$\lceil log_2 \binom{4^3}{7} \rceil = 30 \text{ bits}$$

Structure size:

$$\underbrace{10}_{\text{Bloom}} + \underbrace{3 \cdot 6}_{\text{False positives}} = 28 \text{ bits}$$

(d)

Critical False Positives



(c)

Bloom False positives

(d)

. . .

(b)

"toy" hash

func (not used in practice)

Chikhi & Rizk

Idea of Chkhi and Rizk

Assume we want to represent specific set T0 of k-mers with a Bloom filter B1

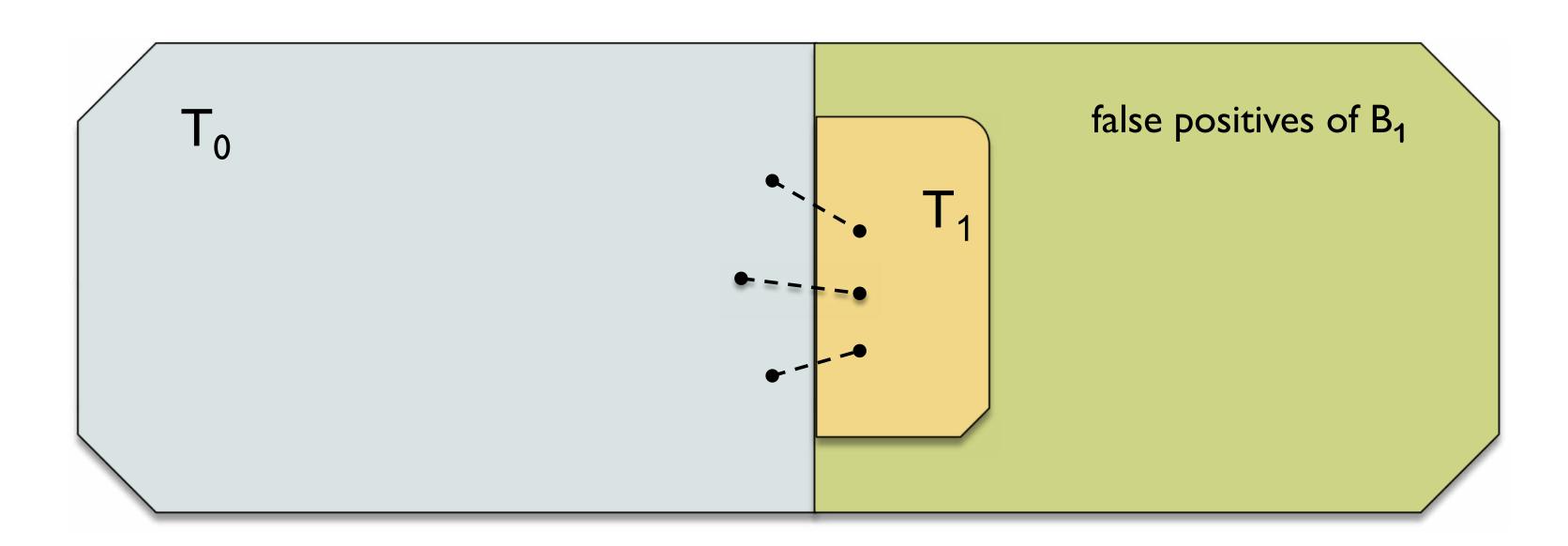
Key observation: in assembly, not all k-mers can be queried, only those having k-1 k-mers known to be in the graph.

The set T1 of "critical false positives" (false neighbors of true k-mers) is *much* smaller than the set of all false positives and can be stored explicitly

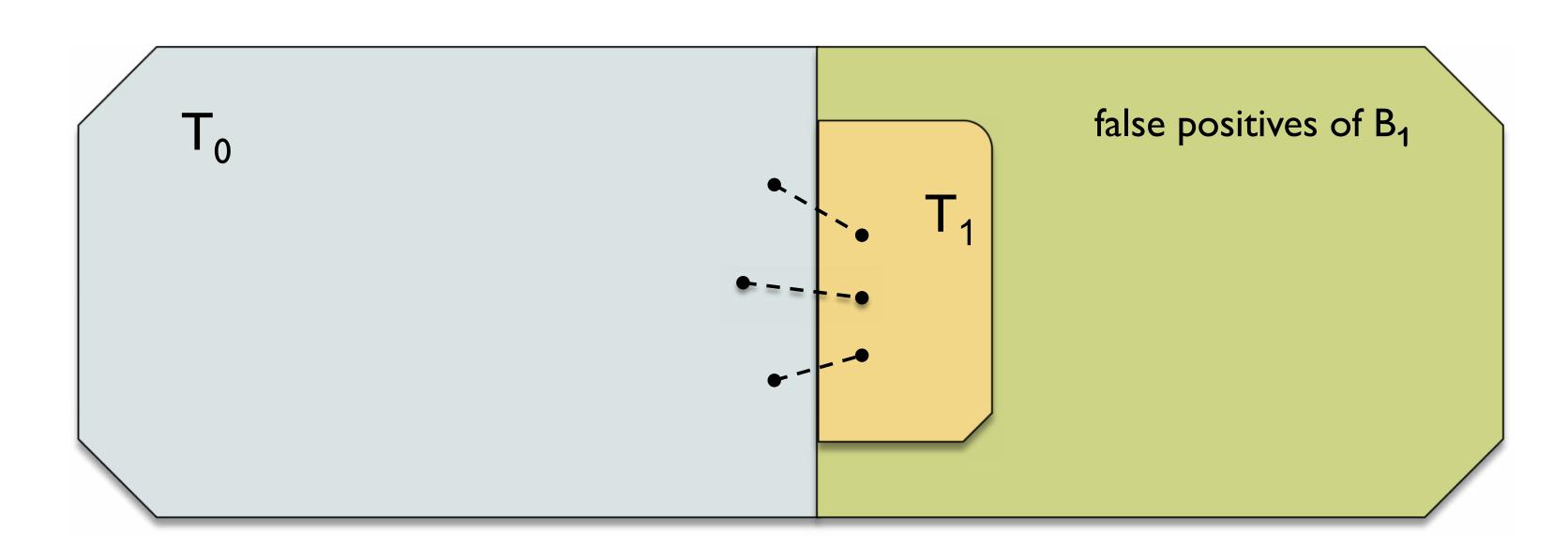
Storing B1 and T1 is much more space efficient that other exact methods for storing T0. Membership of w in T0 is tested by first querying B1, and if $w \in B1$, check that it is *not* in T1.



▶ Represent T₀ by Bloom filter B₁



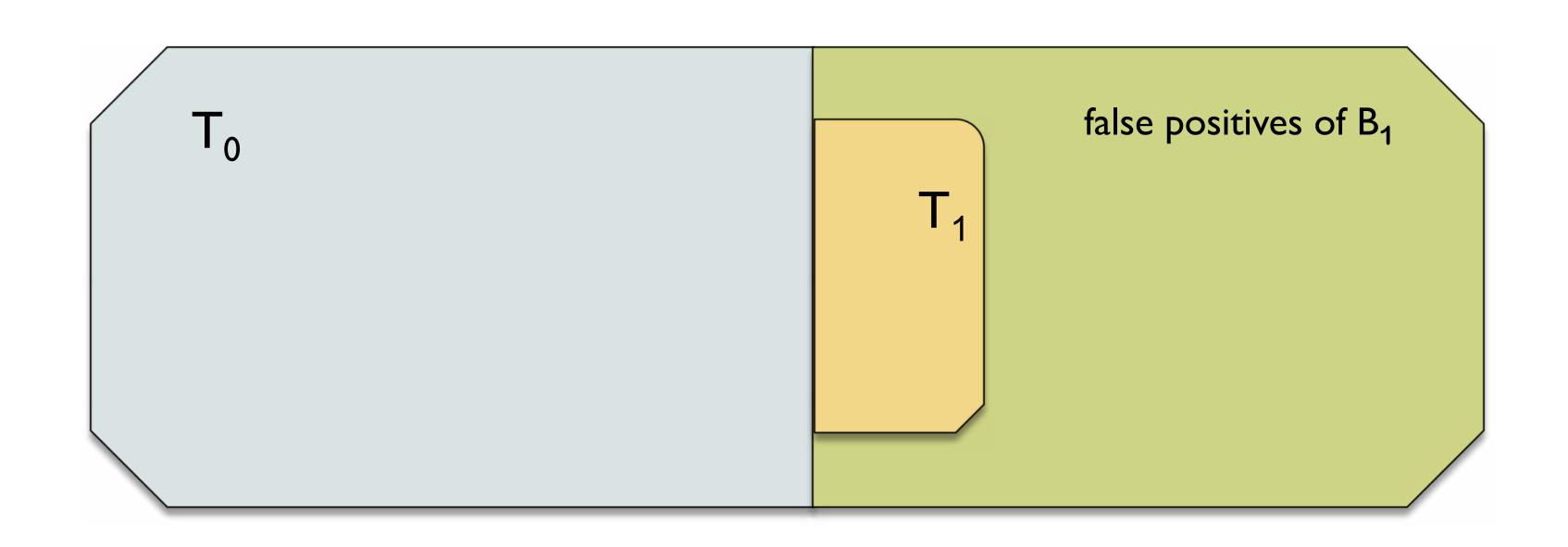
- ▶ Represent T₀ by Bloom filter B₁
- ▶ Compute T₁ ('critical false positives') and represent it e.g. by a hash table

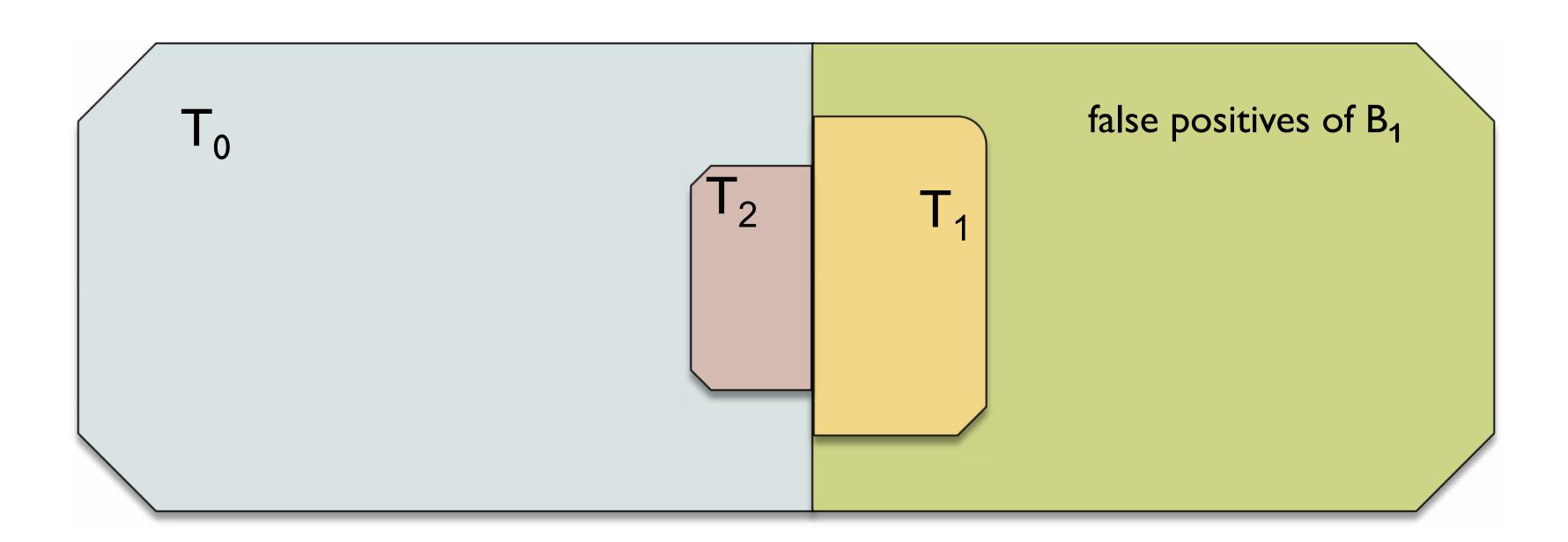


- ▶ Represent T₀ by Bloom filter B₁
- ▶ Compute T₁ ('critical false positives') and represent it e.g. by a hash table
- ▶ Result (example): 13.2 bits/node for k=27 (of which 11.1 bits for B_1 and 2.1 bits for T_1)

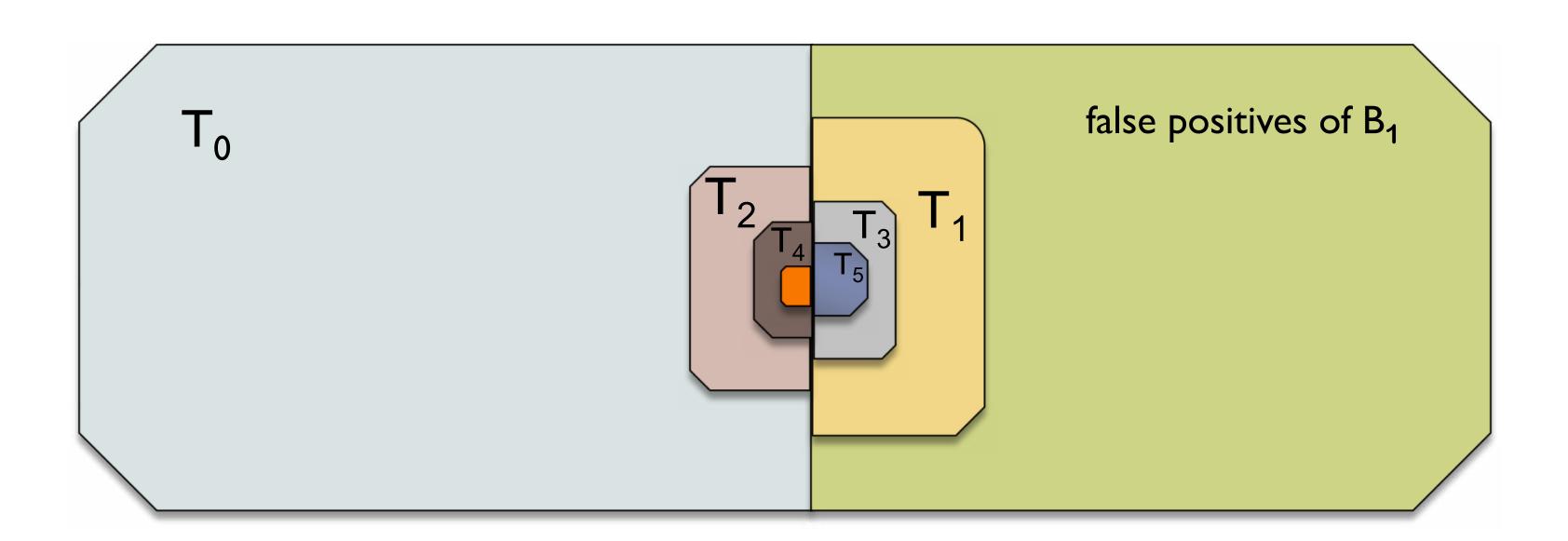
Improving on Chikhi and Rizk's method

- Main idea: iteratively apply the same construction to T_1 i.e. encode T_1 by a Bloom filter B_2 and set of 'false-false positives' T_2 , then apply this to T_2 etc.
- cascading Bloom filters

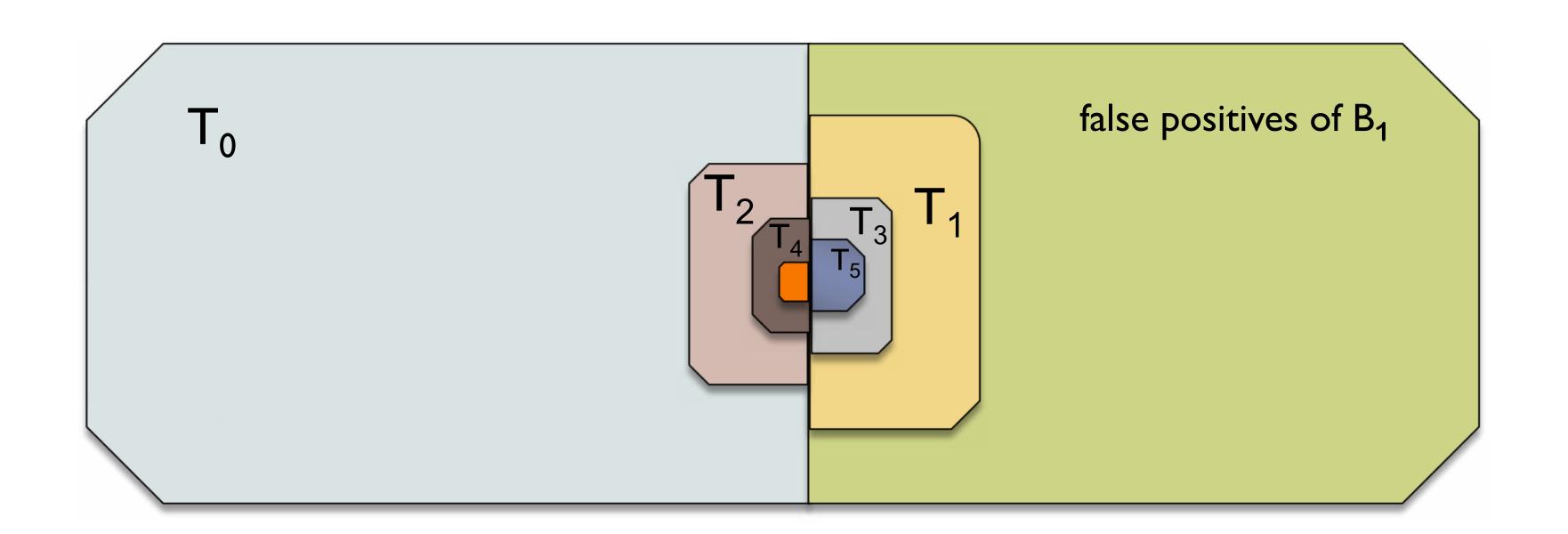




▶ further encode T_1 via a Bloom filter B_2 and set T_2 , where $T_2 \subseteq T_0$ is the set of k-mers stored in B_2 by mistake ('false² positives')



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- iterate the construction on T₂
- we obtain a sequence of sets $T_0, T_1, T_2, T_3, ...$ encode by Bloom filters $B_1, B_2, B_3, B_4, ...$ respectively
- $T_0 \supseteq T_2 \supseteq T_4 \supseteq ..., T_1 \supseteq T_3 \supseteq T_5 \supseteq T_5 \supseteq T_6 \supseteq$



Lemma [correctness]: For a k-mer w, consider the smallest i such that $w \notin B_{i+1}$. Then $w \in T_0$ if i is odd and $w \notin T_0$ if i is even.

- if $w \notin B_1$ then $w \notin T_0$
- ▶ if $w \in B_1$, but $w \notin B_2$ then $w \in T_0$
- if $w \in B_1$, $w \in B_2$, but $w \notin B_3$ then $w \notin T_0$
- etc.

Assuming infinite number of filters

Let $N=|T_0|$ and $r=m_i/n_i$ is the same for every B_i . Then the total size is

$$rN + 6rNc^{r} + rNc^{r} + 6rNc^{2r} + rNc^{2r} + ... = N(1+6c^{r})\frac{r}{1-c^{r}}$$
 $|B_{1}|$ $|B_{2}|$ $|B_{3}|$ $|B_{4}|$ $|B_{5}|$

The minimum is achieved for r=5.464, which yields the memory consumption of 8.45 bits/node

Infinity difficult to deal with;)

- In practice we will store only a small finite number of filters $B_1, B_2, ..., B_t$ together with the set T_t stored explicitely
- t=1 → Chkhi&Rizk's method
- The estimation should be adjusted, optimal value of r has to be updated, example for t=4

k	optimal r	bits per k -mer
16	5.776737	8.555654
32	6.048557	8.664086
64	6.398529	8.824496
128	6.819496	9.045435

Table: Estimations for t=4. Optimal *r* and corresponding memory consumption

Compared to Chikhi&Rizk's method

k	"Optimal" (infinite)	Cascading Bloom Filter	Data structure
	Cascading Bloom Filter	with $t = 4$	of Chikhi & Rizk
16	8.45	8.555654	12.0785
32	8.45	8.664086	13.5185
64	8.45	8.824496	14.9585
128	8.45	9.045435	16.3985

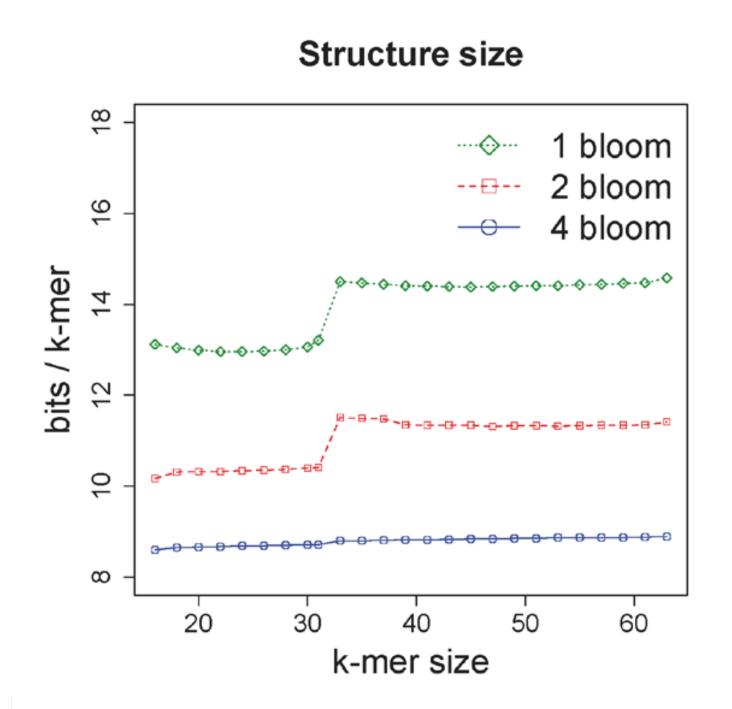
Table: Space (bits/node) compared to Chikhi&Rizk for t=4 and different values of k.

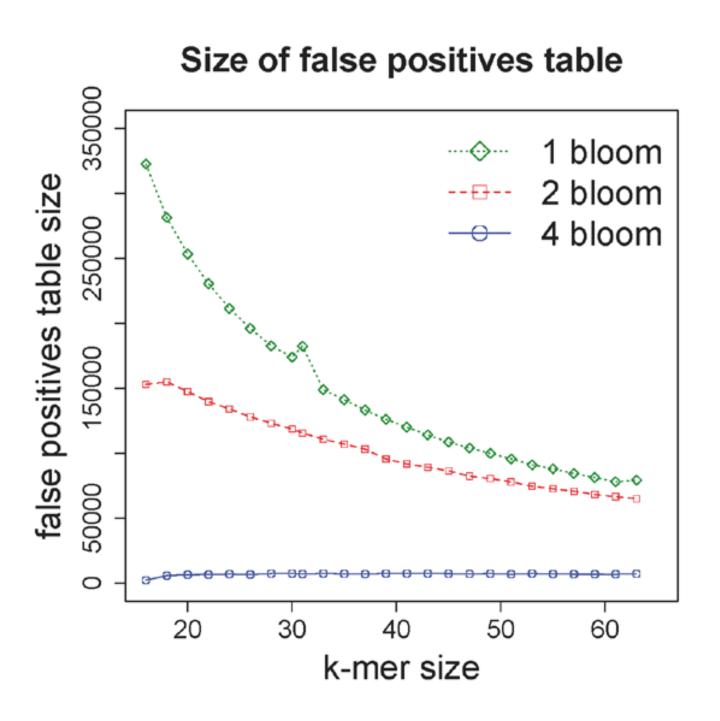
We can cut down a bit more ...

- Rather than using the same r for all filters B_1, B_2, \ldots , we can use different properly chosen coefficients r_1, r_2, \ldots
- This allows saving another 0.2 0.4 bits/k-mer

Experiments I: E.Coli, varying k

- 10M E.Coli reads of 100bp
- 3 versions compared: 1 Bloom (=Chikhi&Rizk), 2 Bloom (*t*=2) and 4 Bloom (*t*=4)





^{*} slide courtesy of Salikhov, Sacomoto & Kucherov

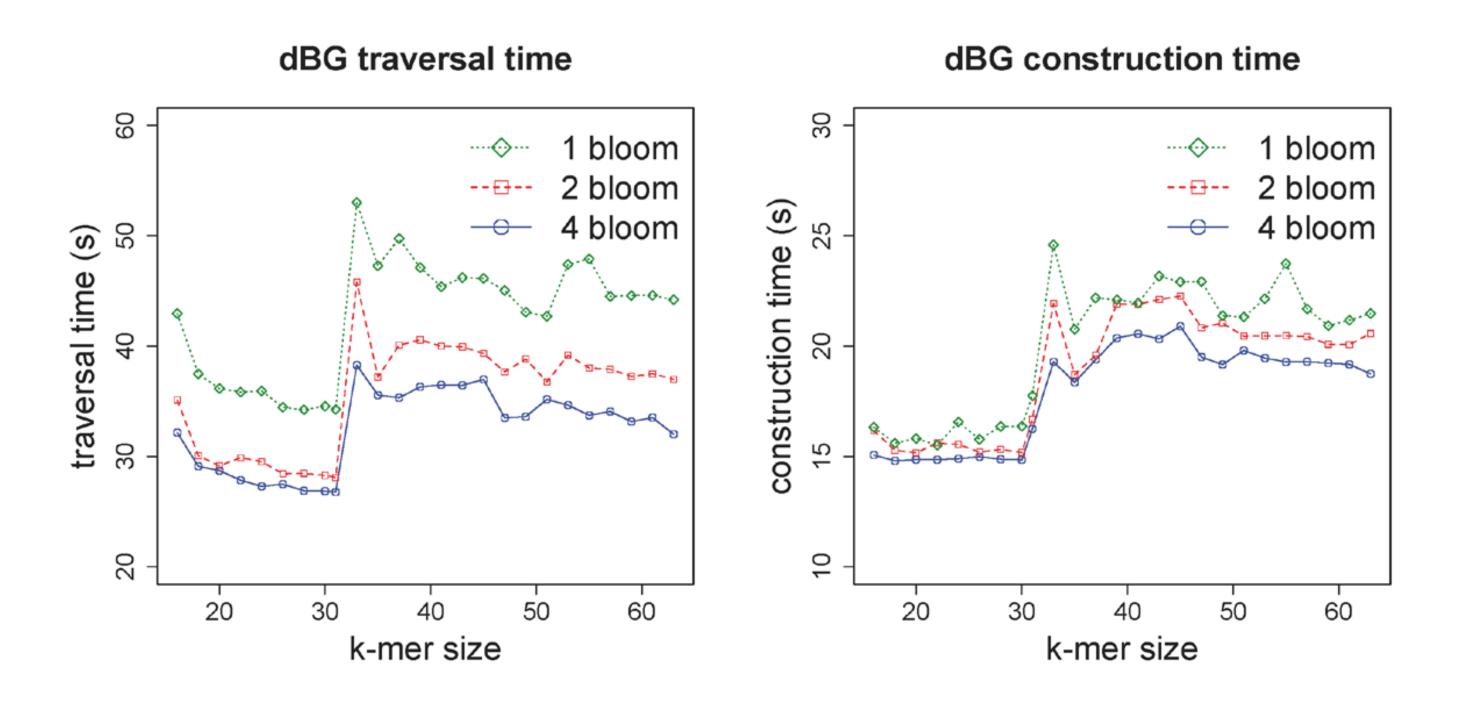
Experiments II: Human dataset

564M Human reads of 100bp (~17X coverage)

Method	1 Bloom	2 Bloom	4 Bloom
Construction time (s)	40160.7	43362.8	44300.7
Traversal time (s)	46596.5	35909.3	34177.2
$r ext{ (bits)}$	11.10	8.10	6.56
	$B_1 = 3250.95$	$B_1 = 2372.51$	$B_1 = 1921.20$
Bloom filters size (MB)		$B_2 = 292.65$	$B_2 = 496.92$
Diooni mucis size (wib)			$B_3 = 83.39$
			$B_4 = 21.57$
False positive table size (MB)	$T_1 = 545.94$	$T_2 = 370.96$	$T_4=24.07$
Total size (MB)	3796.89	2524.12	2547.15
Size (bits/ k -mer)	12.96	10.37	8.70

^{*} slide courtesy of Salikhov, Sacomoto & Kucherov

Experiments I (cont)



Efficiently enumerating cFP

Algorithm 1 Constant-memory enumeration of critical false positives

```
1: Input: The set S of all nodes in the graph, the Bloom
    filter constructed from S, the maximum number M
    of elements in each partition (determines memory
    usage)
 2: Output: The set cFP
 3: Store on disk the set \mathcal{P} of extensions of \mathcal{S} for which
    the Bloom filter answers yes
 4: Free the Bloom filter from memory
 5: D_0 \leftarrow \mathcal{P}
 6: i \leftarrow 0
 7: while end of S is not reached do
 8: P_i \leftarrow \emptyset
      while |P_i| < M do
         P_i \leftarrow P_i \cup \{\text{next } k\text{-mer in } \mathcal{S}\}
       for each k-mer m in D_i do
          if m \notin P_i then
12:
            D_{i+1} \leftarrow D_{i+1} \cup \{m\}
     Delete D_i, P_i
15: i \leftarrow i + 1
16: cFP \leftarrow D_i
```

Bloom filters & De Bruijn Graphs

So, we can make very small representation of the dBG. But it's navigational! We can also make them:

Bioinformatics, 2018, 1–7

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Original Paper

OXFORD

and even

weighted

Dynamic & membership

Bioinformatics, 33, 2017, i133–i141 doi: 10.1093/bioinformatics/btx261 ISMB/ECCB 2017



Sequence analysis

Practical dynamic de Bruijn graphs

Victoria G. Crawford^{1,†}, Alan Kuhnle^{1,†}, Christina Boucher¹, Rayan Chikhi² and Travis Gagie^{3,*}

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deBGR: an efficient and near-exact representation of the weighted de Bruijn graph

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Succinct k-mer Sets Using Subset Rank Queries on the Spectral Burrows-Wheeler Transform *

Jarno N. Alanko, Simon J. Puglisi, Jaakko Vuohtoniemi doi: https://doi.org/10.1101/2022.05.19.492613

This article is a preprint and has not been certified by peer review [what does this mean?].



Current static membership state-of-the-art for (non-compacted) de Bruijn Graphs