DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning



Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z.F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao

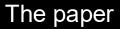
> A paperclub presentation for SDx by Keith Chester





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These slides



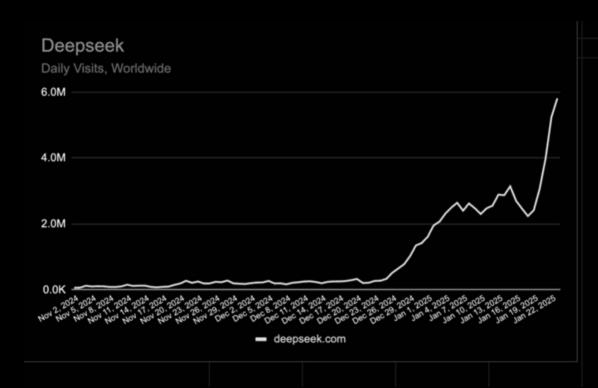
Me!



- Claims of the paper
 - O What was built?



- Claims of the paper
 - What was built?
- What's the big deal?



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- How do we train it?
 - Pipeline
 - Distillation!
 - o GRPO in depth



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 - Pipeline
 - Distillation!
 - GRPO in depth
- Cool experiments!





Reinforcement Learning is Key for Reasoning - even without Supervised Fine
 Tuning

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 - This experiment proved the importance of RL for reasoning
- DeepSeek-R1: A state-of-the-art reasoning model
 - Novel 4-Stage Training Pipeline for Reasoning LLMs
- Distilled DeepSeek-R1 Models: A suite of smaller, dense models that inherit the reasoning capabilities through knowledge distillation



Model	Rumored Cost to Train
DeepSeek R1	\$5 million
OpenAl GPT-4o	\$60 million +
OpenAl o1	\$100 million +
OpenAl o3-mini	\$??



Model	Input Cost per 1M tokens	Output Cost per 1M tokens
DeepSeek R1	\$0.14	\$0.28
OpenAl GPT-4o	\$2.50	\$10.00
OpenAl o1	\$15.00	\$60.00
OpenAl o3-mini	\$1.10	\$4.40



DeepSeek surpasses ChatGPT App Store as Chinese Al startup sends shockwaves through tech stocks



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DeepSeek hit by cyberattack as users flock to Chinese AI startup

By Reuters

January 27, 2025 1:44 PM PST · Updated 19 days ago





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OpenAl Says DeepSeek May Have Improperly Harvested Its Data

used data generated by OpenAI technologies to build new

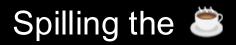
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OpenAl Says DeepSeek May Have Improperly Harvested Its Data

The San Francisco start-up claims that its Chinese rival may have used data generated by OpenAI technologies to build new systems.

- Bulk Export of Responses: someone bulk-exported model responses from OpenAI in late 2024. Unclear if this is linked to DeepSeek.
- Microsoft reportedly observed someone in China extracting large volumes of data from the OpenAl API (against ToS)
- Similar Responses: There are instances where DeepSeek provides responses that seem very similar to what ChatGPT would give, suggesting they learned from OpenAl's model.

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] Aa

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DeepSeek V3 and the actual cost of training frontier AI models

The \$5M figure for the last training run should models cost.

JAN 09, 2025



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Jan 24

Jiayi Pan 💮 @jiayi pirate

dems.

We reproduced DeepSeek R1-Zero in the CountDown game, and it just works

Through RL, the 3B base LM develops self-verification and search abilities all on its own

You can experience the Ahah moment yourself for < \$30 Code: github.com/Jiayi-Pan/TinyZer...







How do reasoning models work?

 Explicitly outputs its reasoning steps as it attempts to answer/solve their query

Key Elements of a **CoT** Prompt:

- Explicitly ask for step-by-step reasoning
- Use phrases like "Let's think step by step" or "Break down the problem"
- Include placeholder steps (e.g., "First... Second...")
- Request verification of the answer to encourage critical thinking

Yes / No

...and why?

CoT

Encourages the model through prompting to generate a sequence of thoughts *but* might not be refined, and might produce a bad chain of reasoning.

Reasoning

CoT

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Reasoning

Trained to produce multiple paths of reasoning, evaluating them, and selecting the most promised one.

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Trained to produce multiple paths of reasoning, evaluating them, and selecting the most promised one.

Why is this different?

CoT

A clever prompting technique to elicit reasoning steps from instruct capable language models that aren't trained to reason

Reasoning

Is *explicitly* trained through reinforcement learning to produce **correct** reasoning steps

CoT

A clever prompting technique to elicit reasoning steps from instruct capable language models that aren't trained to reason

Prompted to produce things that sound logical

Reasoning

Is *explicitly* trained through reinforcement learning to produce **correct** reasoning steps

Trained to produce *logical* steps

DeepSeek-R1-Zero

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DeepSeek-V3-Base - a previously created MoE model made with SFT

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 - Shows that the typical SFT-then-RL approach wasn't needed

DeepSeek-R1-Zero

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- Initial test (hence Zero) to see if a reasoning LLM can be taught purely through reinforcement learning (GRPO)
 - Shows that the typical SFT-then-RL approach wasn't needed
 - Emergent reasoning behaviors:
 - Self-Verification
 - Reflection
 - Longer Chain of Thoughts

DeepSpak_R1_7aro

Question: If a > 1, then the sum of the real solutions of $\sqrt{a} - \sqrt{a + x} = x$ is equal to

- Deep Response: <think>
- To solve the equation $\sqrt{a-\sqrt{a+x}}=x$, let's start by squaring both \cdots Initial

through
$$\left(\sqrt{a-\sqrt{a+x}}\right)^2 = x^2 \implies a-\sqrt{a+x} = x^2$$
.

Rearrange to isolate the inner square root term:

$$(a-x^2)^2 = a+x \implies a^2 - 2ax^2 + (x^2)^2 = a+x \implies x^4 - 2ax^2 - x + (a^2-a) = 0$$

Wait, wait. Wait. That's an aha moment I can flag here.

Let's reevaluate this step-by-step to identify if the correct sum can be · · ·

We started with the equation:

$$\sqrt{a-\sqrt{a+x}}=x$$

First, let's square both sides:

$$a - \sqrt{a + x} = x^2 \implies \sqrt{a + x} = a - x^2$$

Next, I could square both sides again, treating the equation: ...

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$$(\sqrt{a-\sqrt{a+x}})^2$$
 of $x=x^2$.

Rearrange to is of thought lare Self-identifies
$$(a-x^2)^2 = a+x \implies a^2 - 2ax^2 + (x^2)^2 \text{ turning point in its}$$

$$ax^2 - x + (a^2 - a) = 0$$

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• Researchers designed a reward for encouraging reasoning

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Focus	What it measured
Accuracy	Correctly providing answers in reasoning tasks
Format	Using <think> and <answer> tags correctly</answer></think>

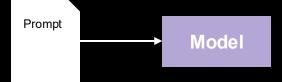
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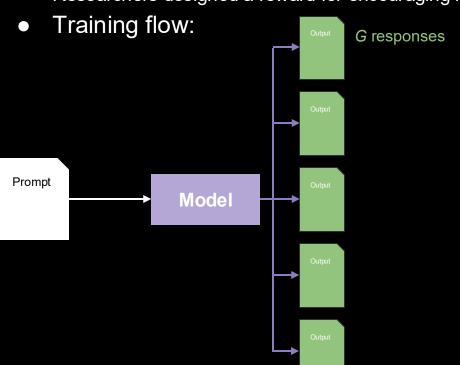
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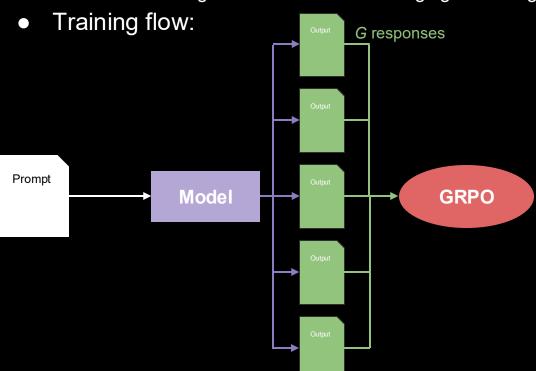


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Training flow:

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Prompt

Model

What the heck is **GRPO**?

Group Relative Policy Optimization - an evolution from PPO

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Group Relative Policy Optimization - an evolution from PPO

$$\begin{split} \mathcal{J}_{GRPO}(\theta) &= \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)] \\ &\frac{1}{G} \sum_{i=1}^G \left(\min\left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{old}}(o_i|q)} A_i, \operatorname{clip}\left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{old}}(o_i|q)}, 1 - \varepsilon, 1 + \varepsilon\right) A_i \right) - \beta \mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right) \right), \\ &\mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right) = \frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - \log\frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - 1, \end{split}$$

Proximal Policy Optimization (PPO), briefly

- Proximal Policy Optimization is a technique published by OpenAl in 2017
 - Probability distribution output (Often Gaussian for continuous, Categorical for discrete, etc)
 - Mean (μ) and standard deviation/variance (σ or σ^2 , respectively) are the layer's outputs
- Utilizes Advantage (A) to train the Actor

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Critic Evaluation

Discounted Rewards

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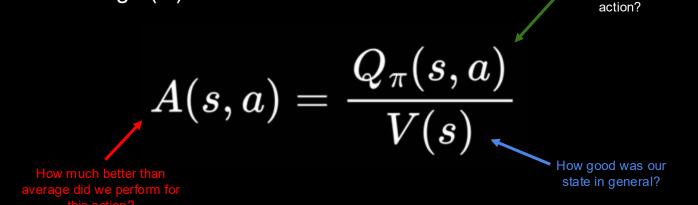
$$A(s,a) = rac{Q_{\pi}(s,a)}{V(s)}$$
How good was our state in general?

How good was this action?

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How good was this

- Utilizes Advantage (A) to train the Actor



Clipped objective function to avoid large policy changes in the actor

$$A(s,a) = rac{Q_{\pi}(s,a)}{V(s)}$$

How much better than average did we perform for this action?

Clipped objective function to avoid large policy changes in the actor

$$A(s,a) = rac{Q_\pi(s,a)}{V(s)} \hspace{1cm} r = rac{\pi_ heta(a_t|s_t)}{\pi_{ heta_k}(a_t|s_t)}$$

How much better than average did we perform for this action?

Clipped objective function to avoid large policy changes in the actor

Probability the current policy would choose this action.

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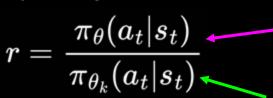
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Probability the current policy would choose this action.

Probability the **previous** policy would choose this action.

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How much better than average did we perform for this action?

$$r = rac{\pi_{ heta}(a_t|s_t)}{\pi_{ heta_k}(a_t|s_t)}$$

Probability the **previous** policy would choose this action.

$$loss = min(r, clamp(1-\epsilon, 1+\epsilon, r)) * A$$

Clipped objective function to avoid large policy changes in the actor

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Clipped objective function to avoid large policy changes in the actor

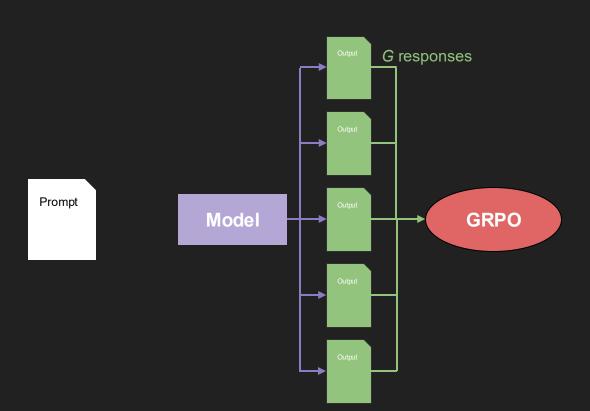
Probability the **current** policy would choose this action.

$$A(s,a)=rac{Q_{\pi}(s,a)}{V(s)}$$
 How much better than average did we perform for

this action? $loss = min(r, clamp(1-\epsilon, 1+\epsilon, r)) *A$

Probability the **previous** policy would choose this action.

This prevents large changes to the policy for any one action, creating a pessimistic lower bound



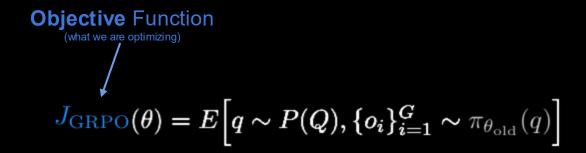
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Group Relative Policy Optimization (GRPO), briefly

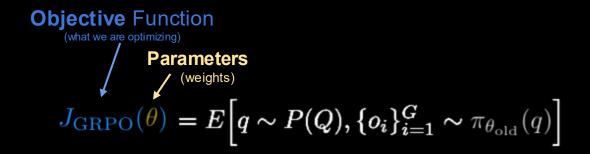
What the heck is **GRPO**?

Group Relative Policy Optimization - an evolution from PPO

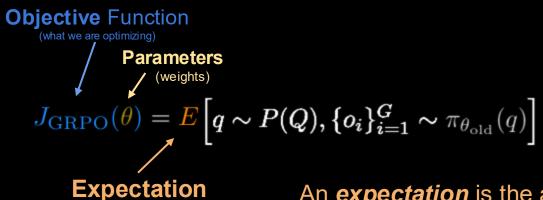
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Objective functions represents a quantity to be optimized in a problem

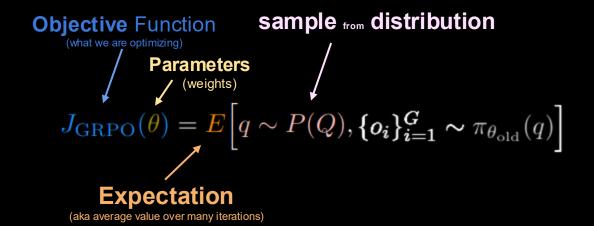


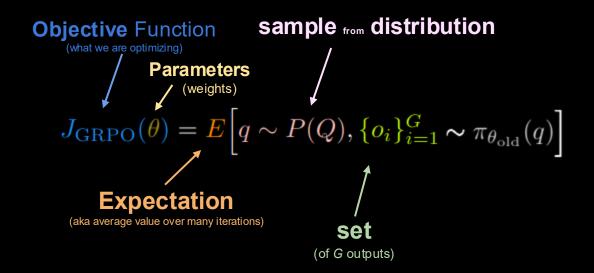
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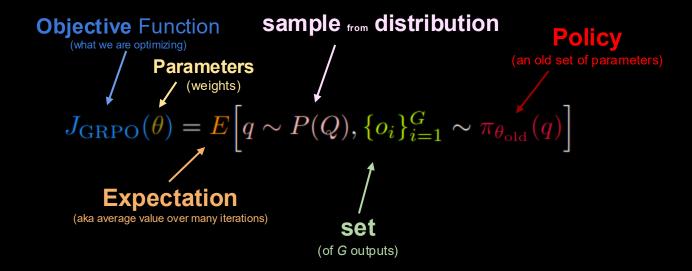


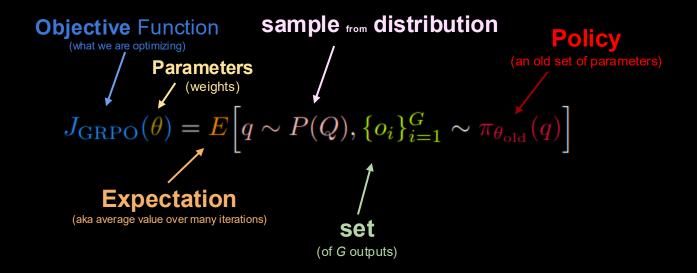
(aka average value over many iterations)

An **expectation** is the average over many samples or repetitions.









This says:

Sample a query from a collection of prompts, creating a set of *G* outputs using the old policy, then take the average of its scores over many iterations to iteratively improve our weights.

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Take the average

$$\frac{1}{G} \sum_{i=1}^{G} \min \left(\frac{\pi_{\theta}(o_{i}|q)}{\pi_{\theta_{\text{old}}}(o_{i}|q)} A_{i}, \operatorname{clip} \left(\frac{\pi_{\theta}(o_{i}|q)}{\pi_{\theta_{\text{old}}}(o_{i}|q)} (o_{i}|q), 1 - \epsilon, 1 + \epsilon \right) A_{i} \right) - \beta D_{\text{KL}}(\pi_{\theta}||\pi_{\text{ref}})$$

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Probability ratio

Take the average
$$A(s,a) = \frac{Q_{\pi}(s,a)}{V(s)}$$
 Advantage
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 Probability **ratio**

Take the average
$$A_i = \frac{r_i - \operatorname{mean}(\{r_1, r_2, \dots, r_G\})}{\operatorname{std}(\{r_1, r_2, \dots, r_G\})}$$

$$\frac{1}{G} \sum_{i=1}^{G} \min \left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{\mathrm{old}}}(o_i|q)} A_i, \operatorname{clip} \left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{\mathrm{old}}}(o_i|q)} (o_i|q), 1 - \epsilon, 1 + \epsilon \right) A_i \right) - \beta D_{\mathrm{KL}}(\pi_{\theta}||\pi_{\mathrm{ref}})$$
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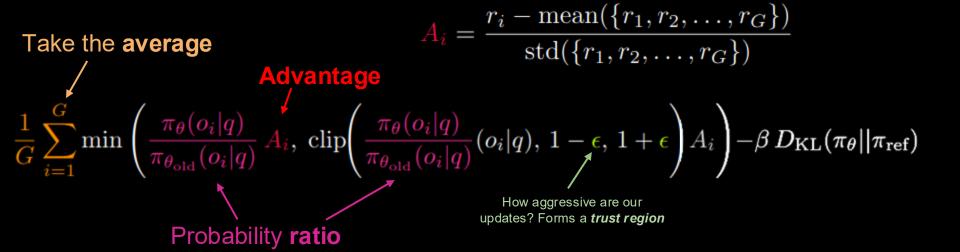
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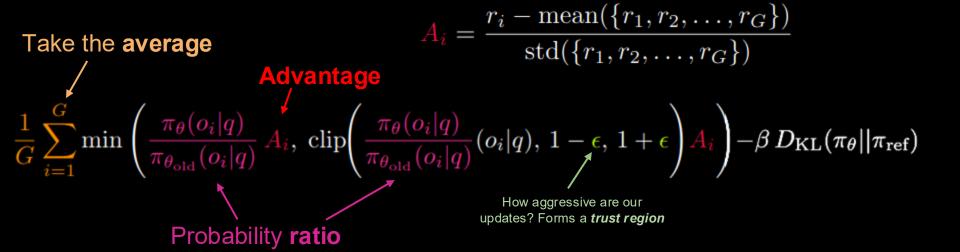
Here, advantage is looking at relative improvement by comparing each response to the group's average, driving model updates towards better relative responses

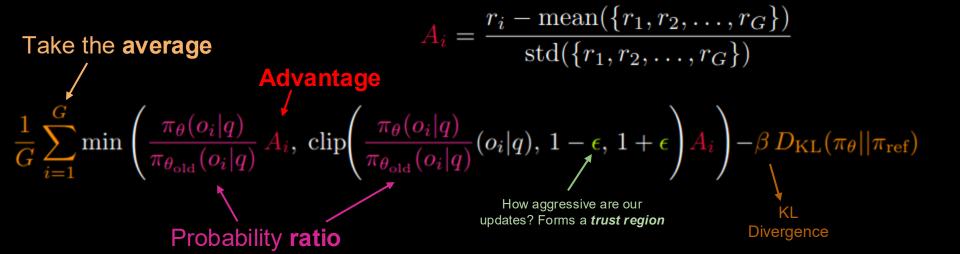
Also, by not having to train a critic model, we are more stable and more efficient!

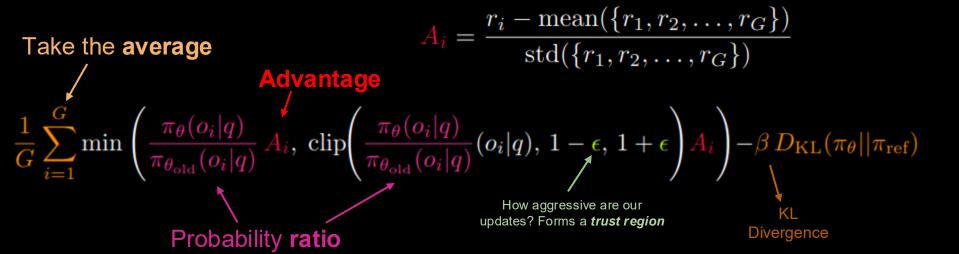
Take the average
$$A_i = \frac{r_i - \operatorname{mean}(\{r_1, r_2, \dots, r_G\})}{\operatorname{std}(\{r_1, r_2, \dots, r_G\})}$$

$$\frac{1}{G} \sum_{i=1}^{G} \min \left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{\mathrm{old}}}(o_i|q)} A_i, \operatorname{clip}\left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{\mathrm{old}}}(o_i|q)}(o_i|q), 1 - \epsilon, 1 + \epsilon\right) A_i \right) - \beta D_{\mathrm{KL}}(\pi_{\theta}||\pi_{\mathrm{ref}})$$
 Probability **ratio**







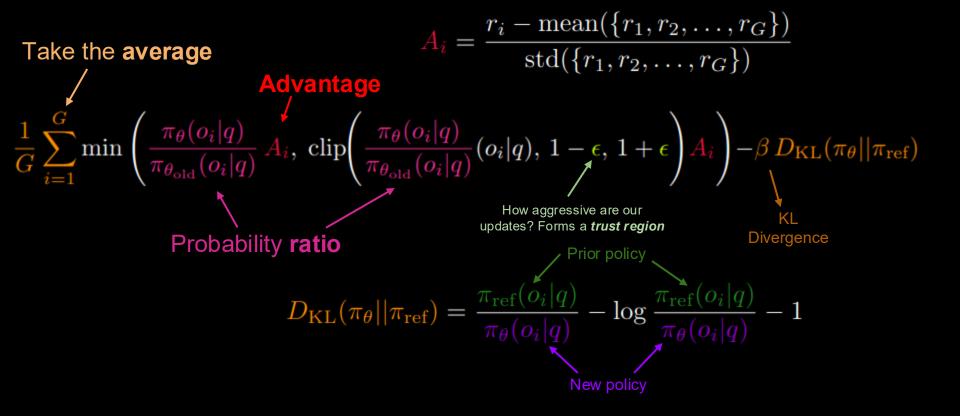


KL Divergence (Kullback–Leibler), also known as relative entropy, measures how different two probability distributions are.

Take the average
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 How aggressive are our updates? Forms a *trust region* Divergence

$$D_{\mathrm{KL}}(\pi_{\theta}||\pi_{\mathrm{ref}}) = \frac{\pi_{\mathrm{ref}}(o_i|q)}{\pi_{\theta}(o_i|q)} - \log \frac{\pi_{\mathrm{ref}}(o_i|q)}{\pi_{\theta}(o_i|q)} - 1$$

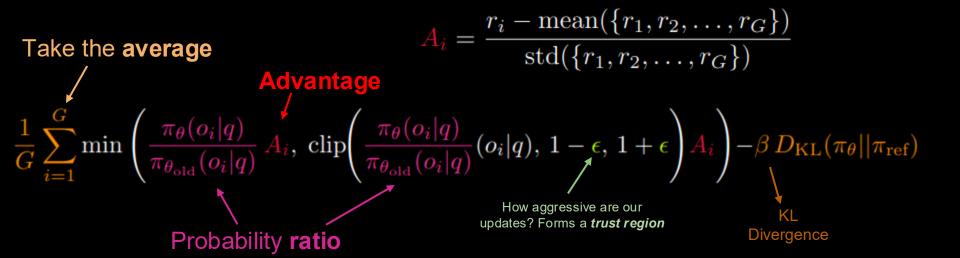


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 KL divergence is seeing how different the new policy is from the old policy - we are

penalizing large changes!



This says:

Update its **strategy** based on how much better an action performed compared to the average (**advantage**). However, it only makes small, careful adjustments (**trust region**) by clipping the size of the update and discouraging big changes (**KL** divergence), then averages these adjustments over multiple experiences to find the best overall improvement.

$$\begin{split} \mathcal{J}_{GRPO}(\theta) &= \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)] \\ &\frac{1}{G} \sum_{i=1}^G \left(\min\left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{old}}(o_i|q)} A_i, \operatorname{clip}\left(\frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{old}}(o_i|q)}, 1 - \varepsilon, 1 + \varepsilon\right) A_i \right) - \beta \mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right) \right), \\ &\mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right) = \frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - \log\frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - 1, \end{split}$$

$$J_{\text{GRPO}}(\theta) = E\left[q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(q)\right]$$

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 - Lack of markdown formatting or structure in thinking tokens
 - Few steps noted
 - Jumps in logic
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 - Not human friendly/aligned
- Mixed language responses/thinking
 - No RL penalty, so it mixed languages as long as accuracy improved
- Reward hacking
 - RL resulted in focusing on the reward, not a useful model

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	1820
OpenAI-o1-0912	74.4	83.3	94.8	77.3	63.4	1843
DeepSeek-R1-Zero	71.0	86.7	95.9	73.3	50.0	1444

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AIME 2024

American Invitational Mathematics
Examination by the Mathematical
Association of America (MAA)

- 15 question 3-hour high school mathematics exam
- Problem-solving skills and mathematical knowledge in:
 - Algebra
 - Geometry
 - Number theory
 - Combinatorics

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MATH-500

MATH-500 is a comprehensive mathematics benchmark that consists of 500 problems spanning various mathematical topics, including:

- Algebra
- Calculus
- Probability
- Geometry

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GPQA Diamond

- Graduate-Level Google-Proof Q&A Benchmark
- 448 multiple-choice questions covering:

- Biology
- Physics
- Chemistry
- Diamond includes an additional 198 PhD level science questions

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LiveCode Bench

- Diverse coding benchmark of 600+ high quality coding problems
- "Contamination Free" problems
 LLM's haven't been trained on nor
 contains LLM generated solutions

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	Codeforces	Skill Level	ELO Range
•	A competitive programming	Newbie	<i>Up to</i> 999
	benchmark to give an ELO rating to a	Pupil	1000-1199
	programmer's problem solving and	Specialist	1400-1599
	algorithmic	Expert	1600-1799

• DeepSeek-Zero gave us a good blueprint, but can we fix its limitations?

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 - Fine tune DeepSeek-V3-Base high quality human-readable CoT examples

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- 1. 🗱 Cold Start 🚇
 - Fine tune DeepSeek-V3-Base high quality human-readable CoT examples
 - Improve Readability demonstrate format
 - Get rid of language mixing

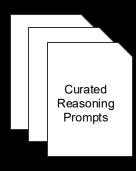
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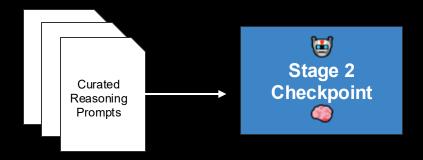
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 - RL w/ a focus on reasoning-intensive tasks (coding, math, science, logic)

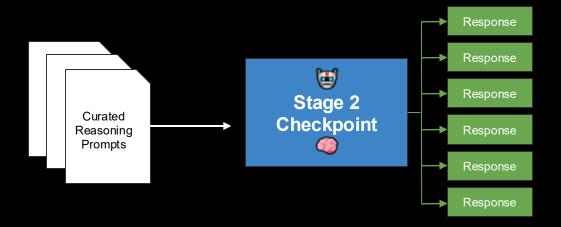
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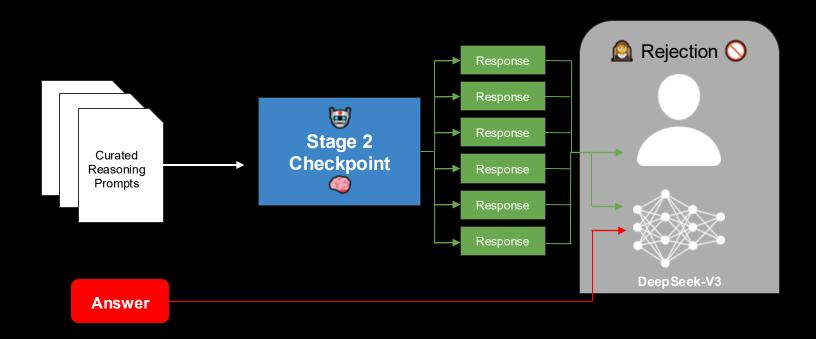
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- 1. 🙉 Refined Reasoning Rejection Sampling SFT 🚫

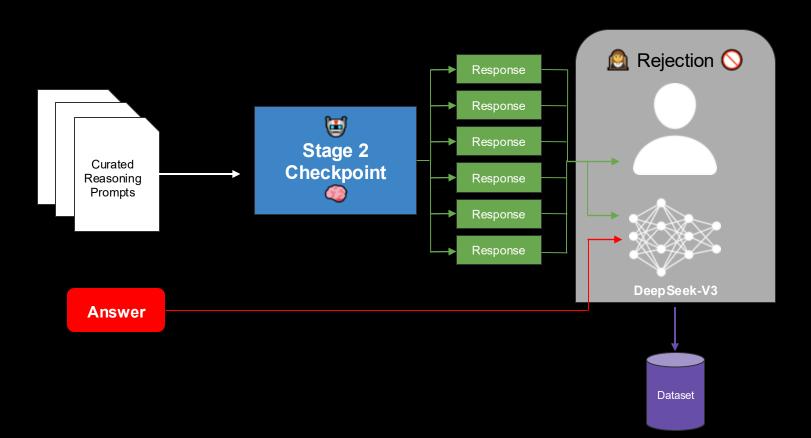
DeepSeek-R1 Stage 3 - Refined Reasoning Rejection Sampling SFT

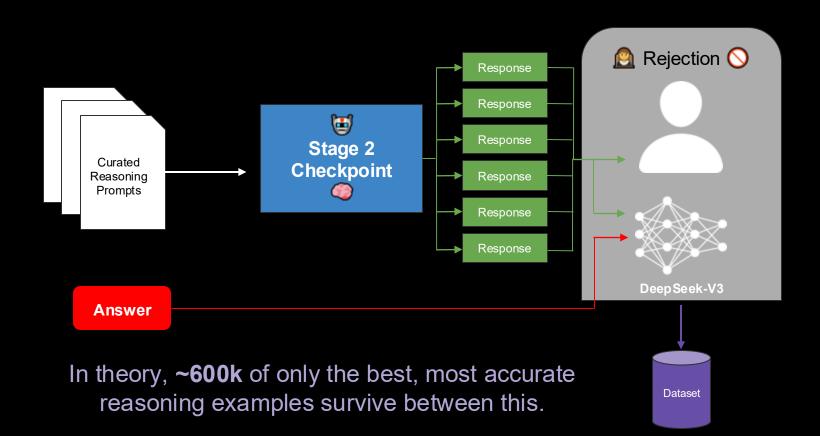




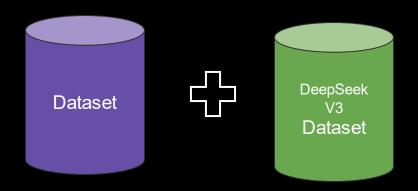


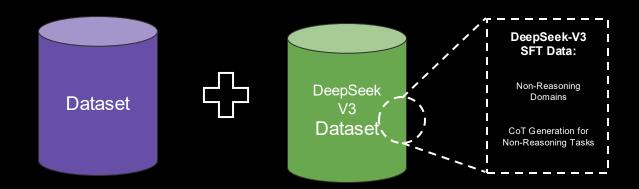












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 - Take <u>DeepSeek-V3-Base</u> (not Stage 2 Checkpoint) and combined dataset train for 2 epochs

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Why?

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Speculation:

Probably to avoid **overfitting** the rejection SFT dataset and to better incorporate **broader language** capabilities

DeepSeek-Zero gave us a good blueprint, but can we fix its limitations?

PIPELINE:

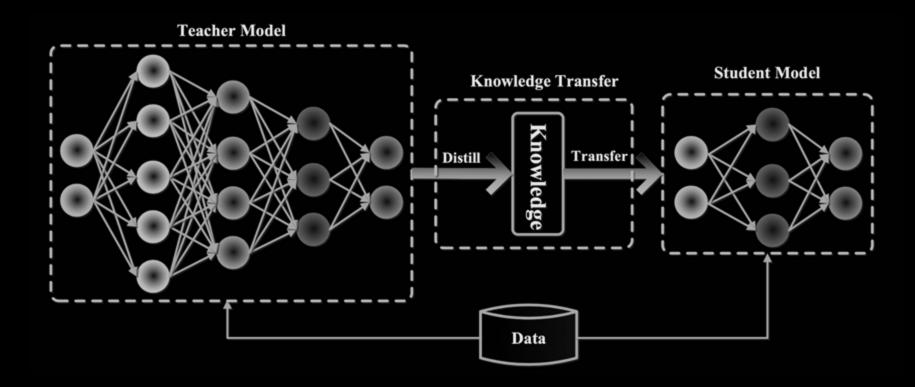
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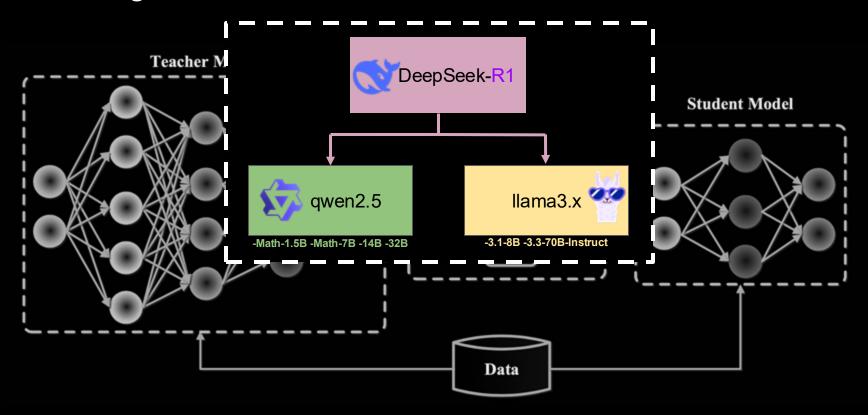
This is essentially knowledge distillation.

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- 1. 🖊 Alignment RL Helpfulness, Harmlessness, & Reasoning in All Scenarios 🤽





3.2. Distilled Model Evaluation

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-40-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	1820
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	72.6	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	86.7	94.5	65.2	57.5	1633

Table 5 | Comparison of DeepSeek-R1 distilled models and other comparable models on reasoning-related benchmarks.

PRM - Perplexity Reward Model

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- Challenging to define a fine-grain step in reasoning
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- Reward hacking

MCTS (Monte Carlo Tree Search)

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- Abandoned because:
 - Language models have a huge search space as it's an unstructured space setting search depth limits risked getting stuck in local optima
 - Couldn't effectively create a value model to guide token-level search in complex reasoning tasks

Questions?