

Introduction to Special Relativity and LHCb

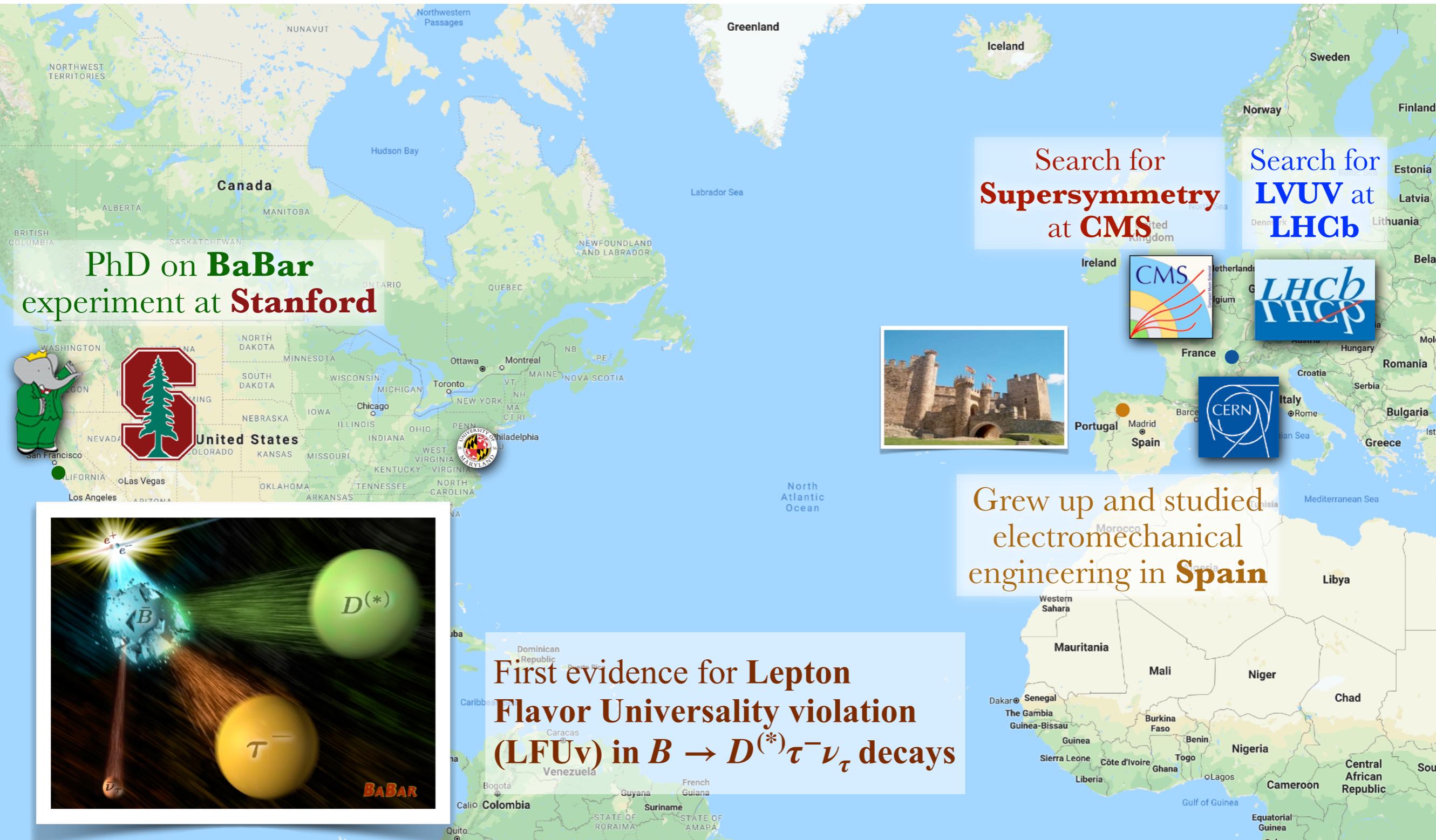


7th July 2022

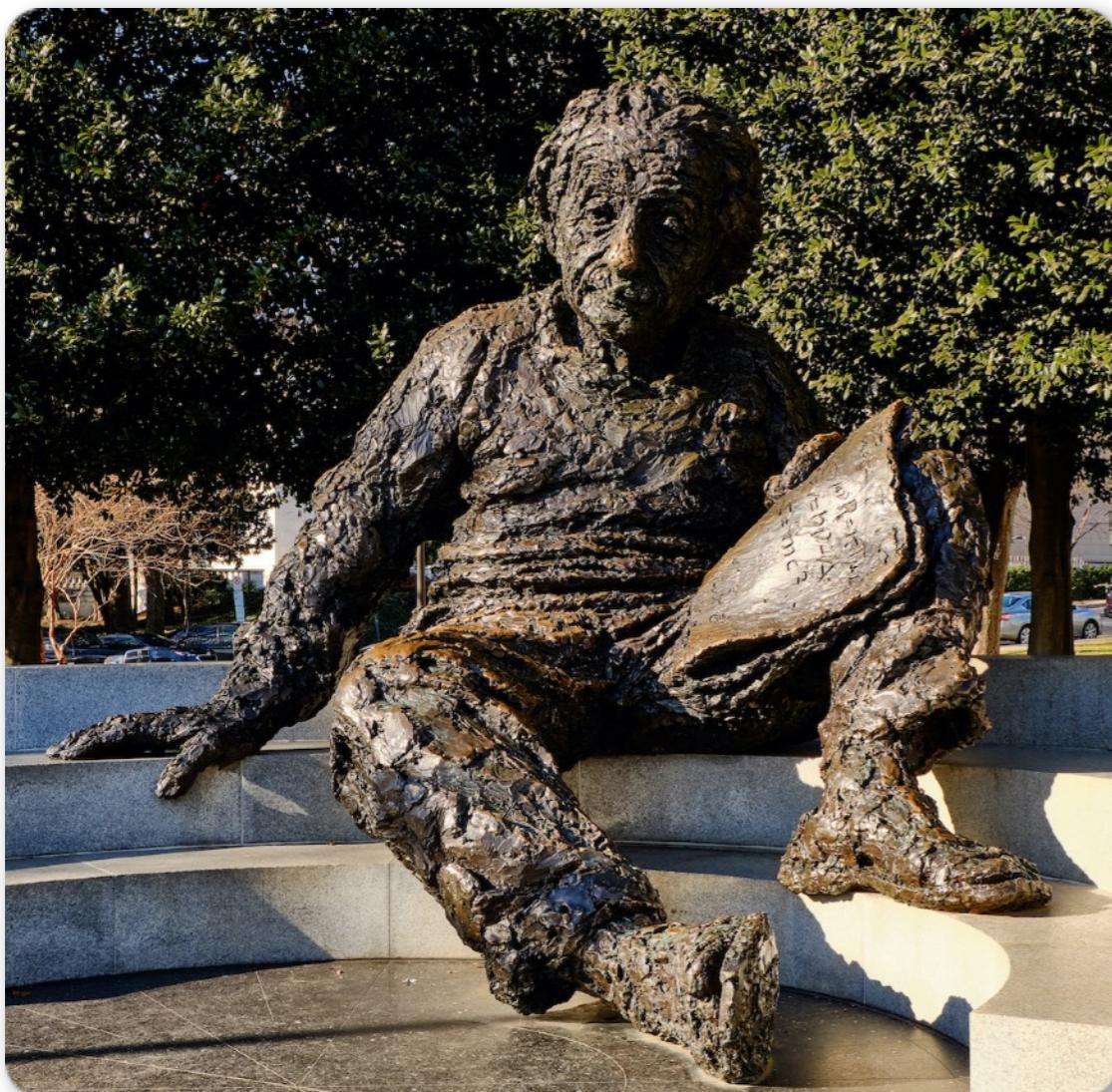
Manuel Franco Sevilla



Quick bio



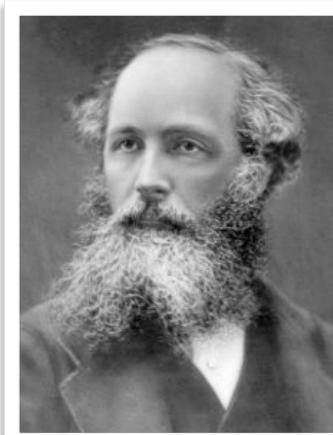
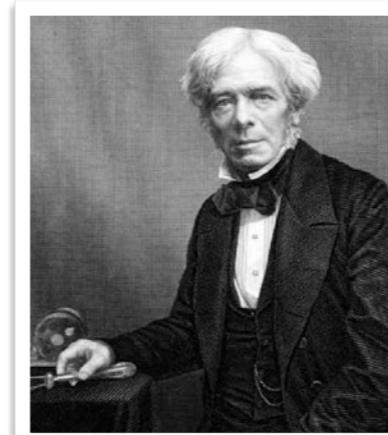
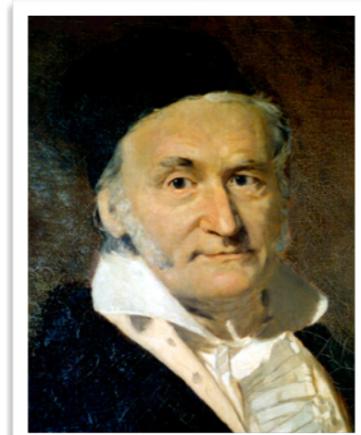
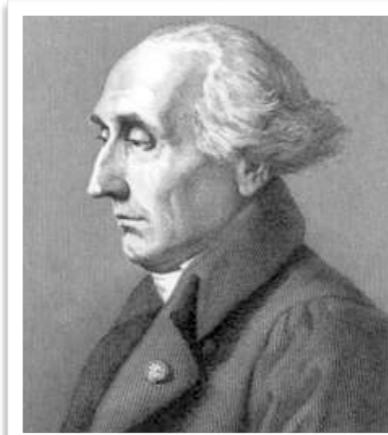
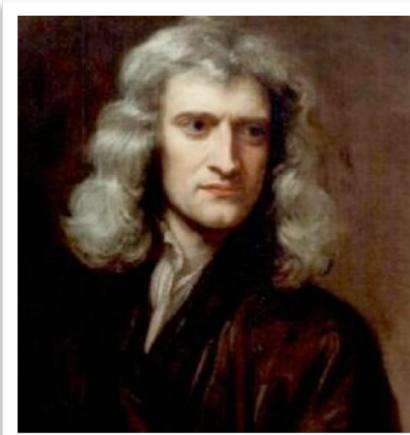
Outline



- ~ Overview of Modern physics
- ~ Introduction to Special Relativity
- ~ Introduction to LHCb

Physics complete in the XIX century?

- ~ From the **XVII to the XIX centuries, extraordinary progress** in our fundamental understanding of the universe
 - Newton's Gravitation, Classical and Statistical Mechanics, Thermodynamics, Electromagnetism



$$F = G \frac{Mm}{R^2}$$

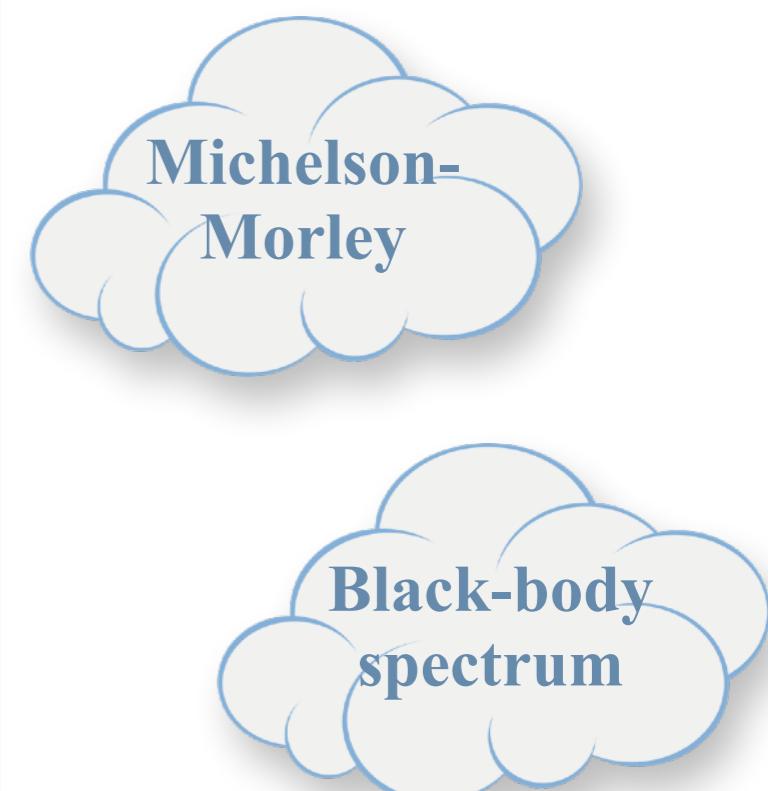
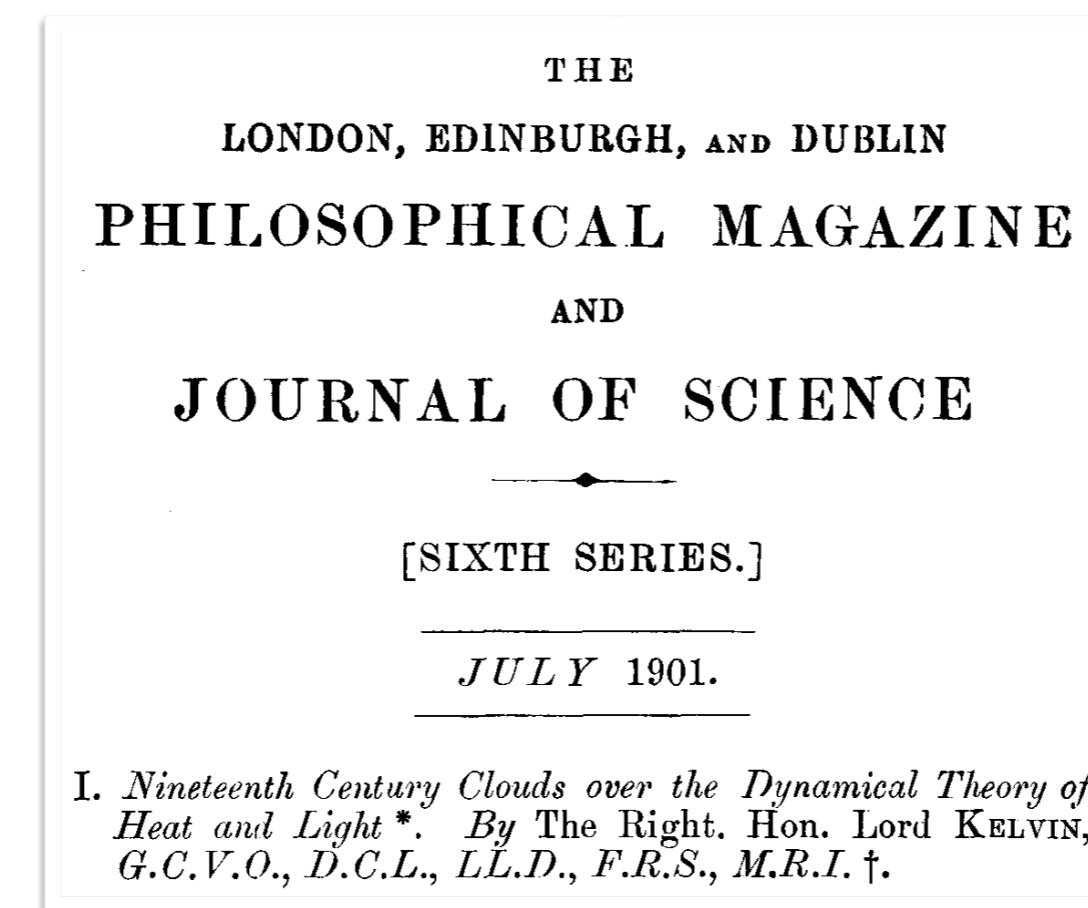
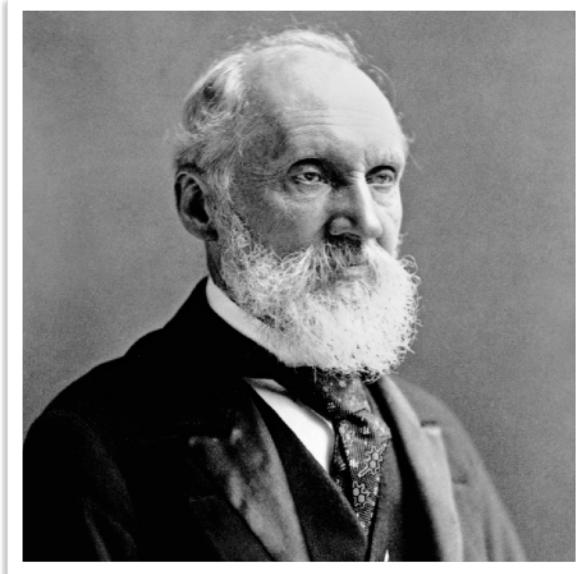
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right)$$

Feeling at the end of the XIX century

“There is nothing new to be discovered in physics now. All that remains is more and more precise measurements.”

Lord Kelvin's clouds

*“The beauty and clearness of the dynamical theory,
which asserts heat and light to be modes of motion, is
at present obscured by two clouds”*



Quantum mechanics in a nutshell

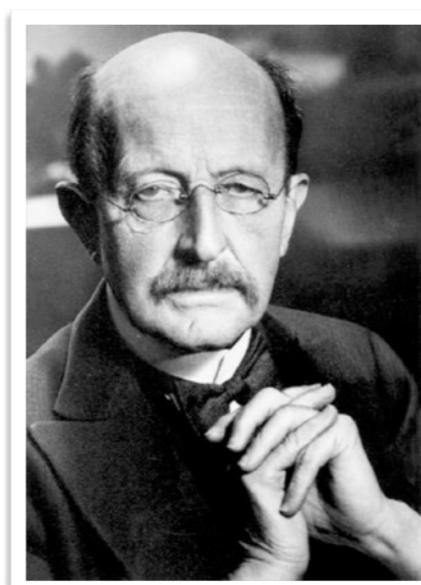
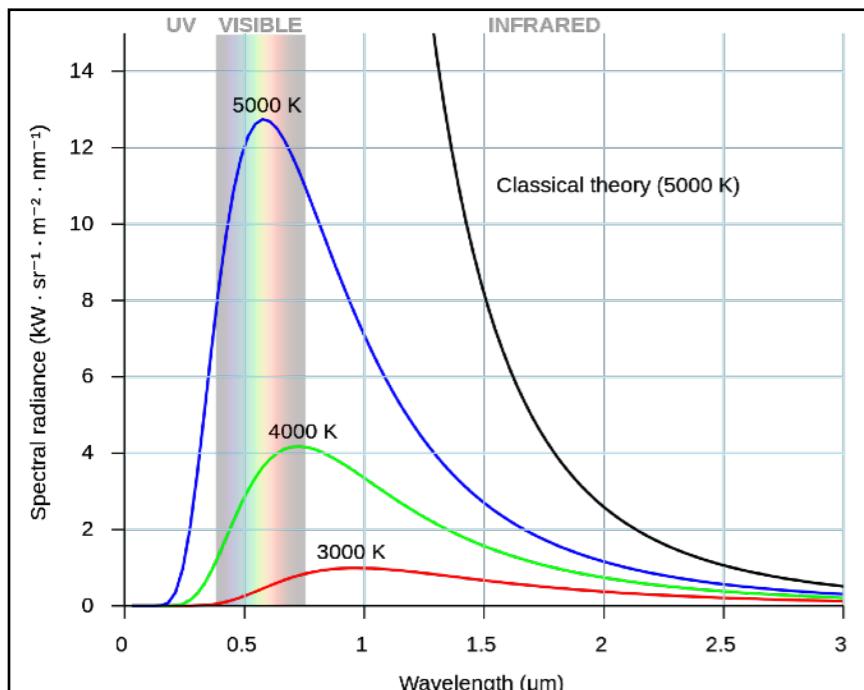
Black body radiation
Ultraviolet catastrophe



Energy quantization



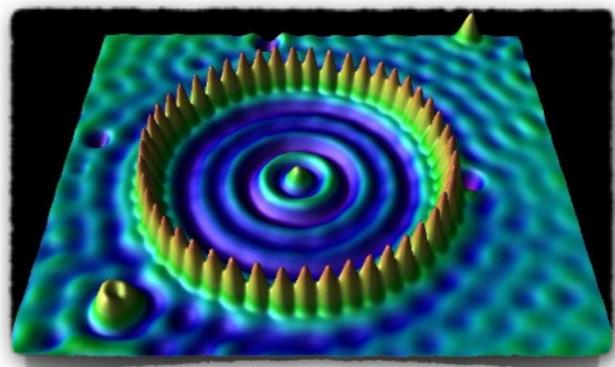
**Deeper understanding
of the Universe's
fundamental
constituents**



$$E = h\nu$$

+ many other experiments
(photoelectric effect...)

+ work from many others (Einstein,
Schrödinger, Heisenberg...)

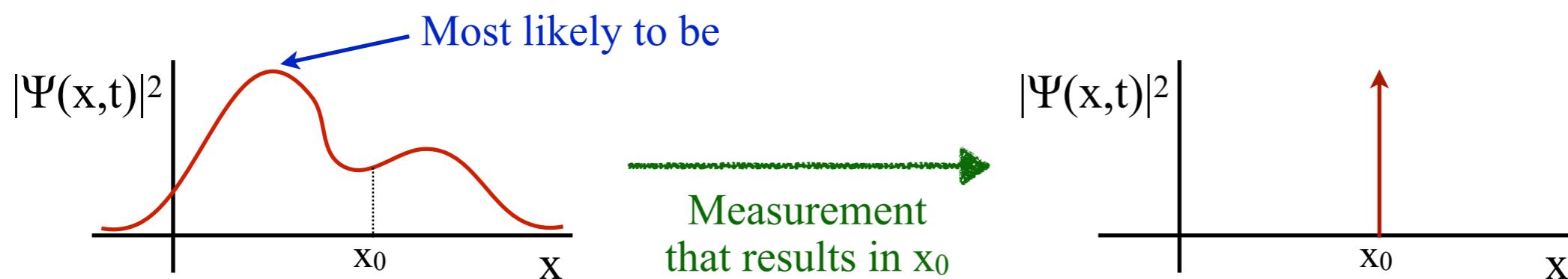


Quantum mechanics

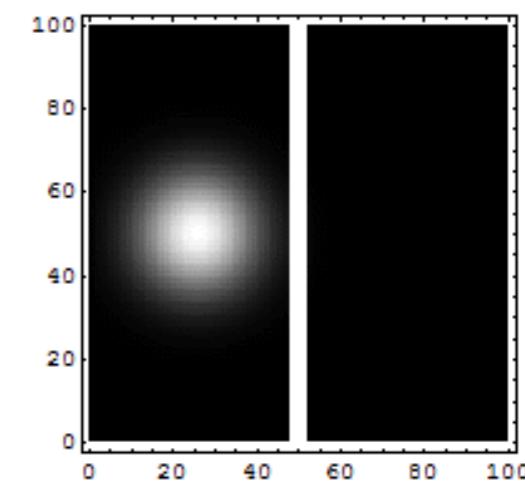
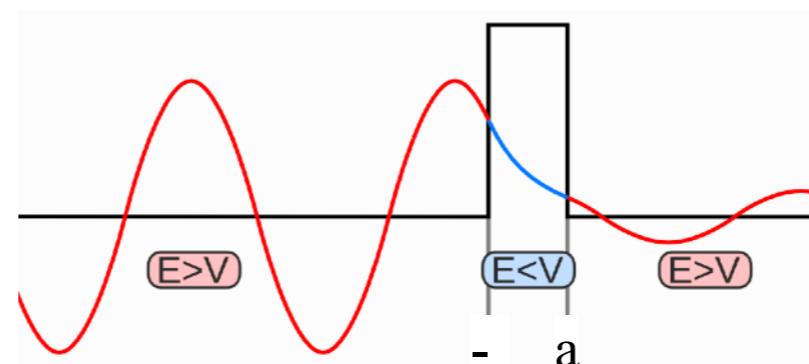
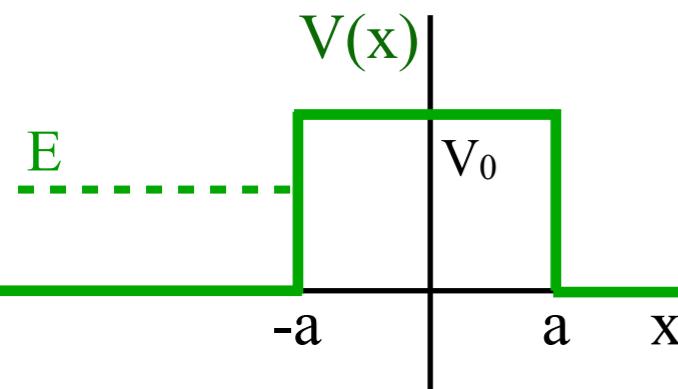
Particles have a wavefunction Ψ found by solving Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, y, z, t) = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(x, y, z, t) \right) \Psi(x, y, z, t)$$

$|\Psi(x, t)|^2 = \Psi^* \Psi$ is the probability of finding the particle between x and $x+dx$ at time t

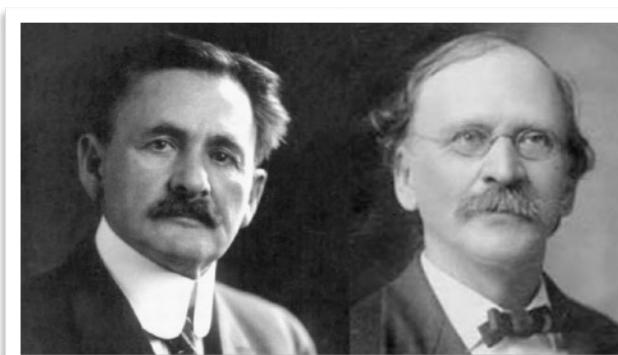


Tunneling of particle coming from left

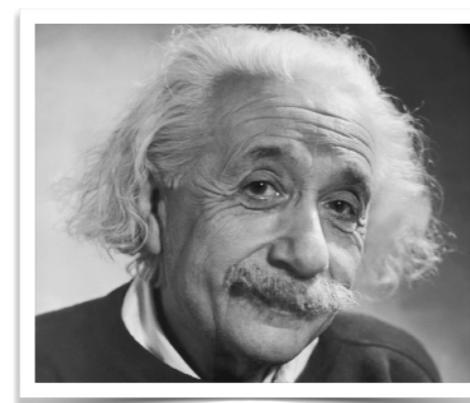


Relativity in a nutshell

Michelson-Morley experiment

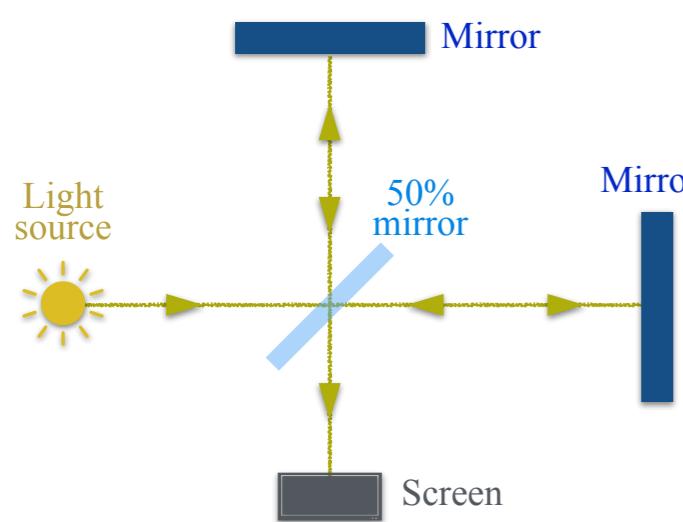


Einstein's postulates



Deeper understanding of the Universe's spacetime, mass, and energy

$$E = mc^2$$



+ experiments from many others (ether dragging...)

The laws of Physics have the same form in all inertial frames of reference

The speed of light in vacuum has the same value c in every direction in all inertial frames of reference

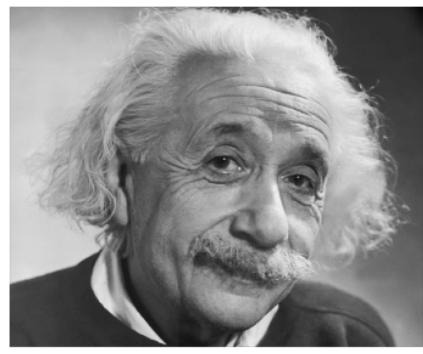
+ work from many others (Poincaré, Lorentz...)



$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$



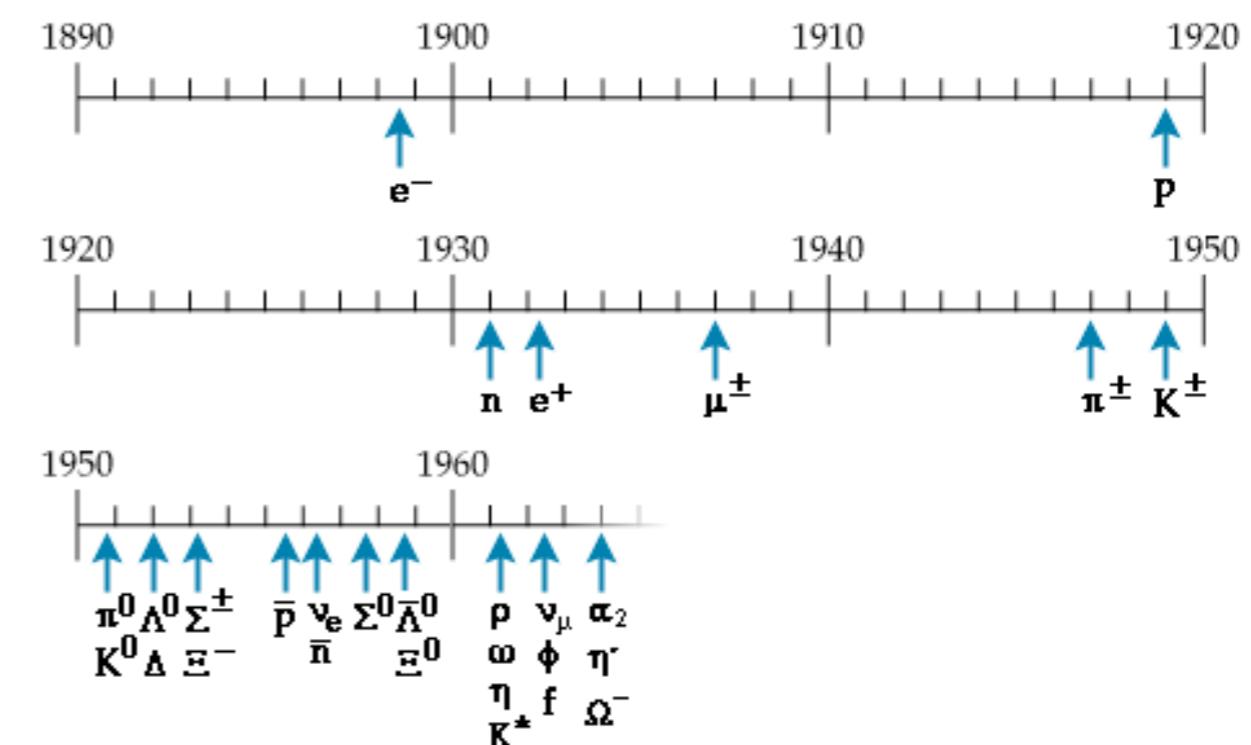
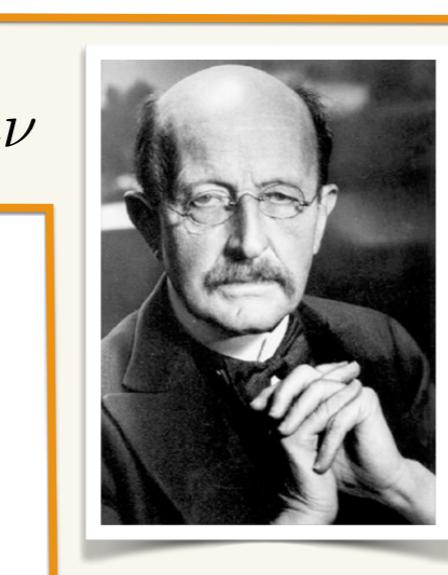
~ Break-neck pace of discovery in the XX century



$$E = h\nu$$

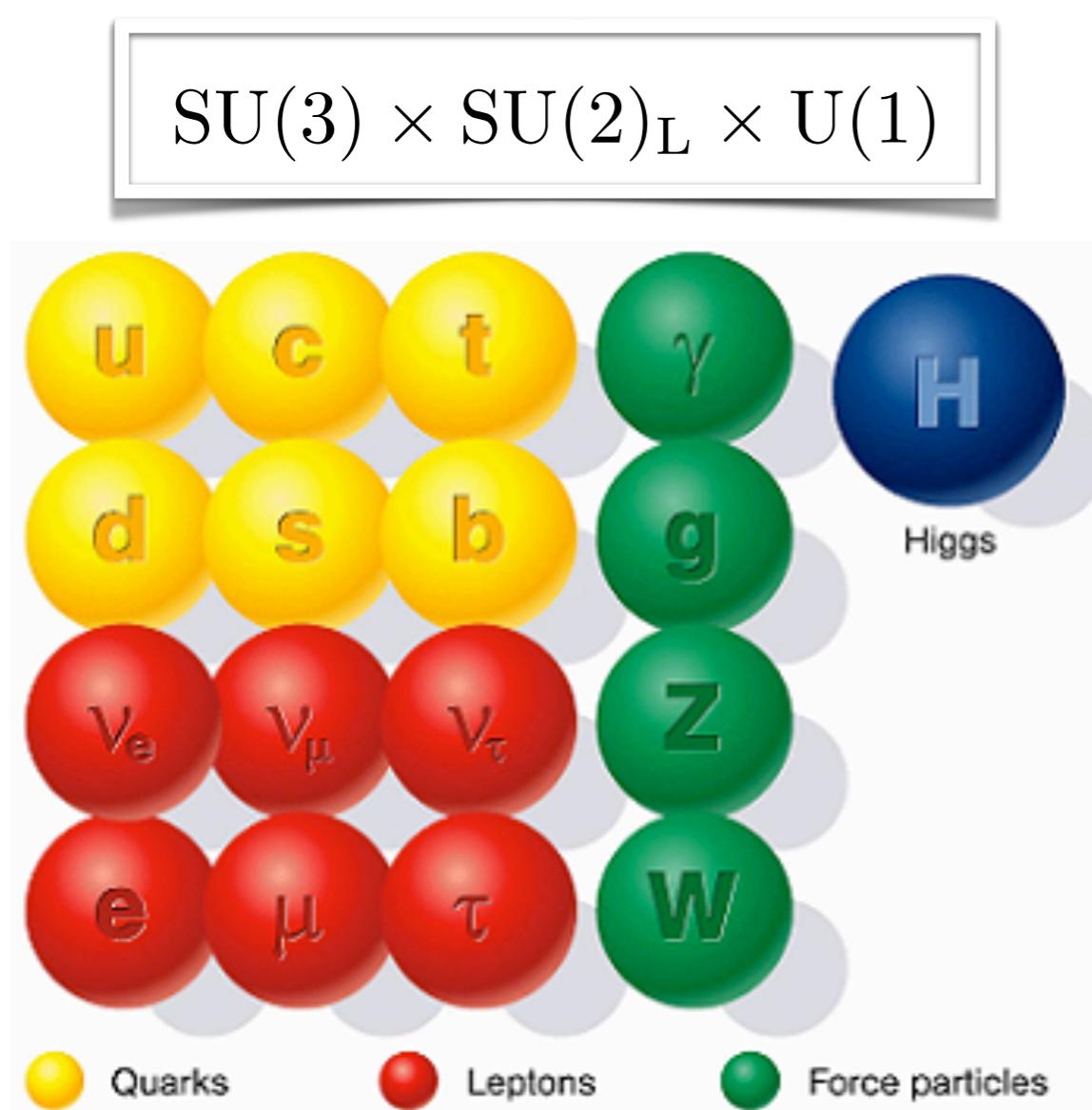
$$E = mc^2$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Physics complete in the XX century?

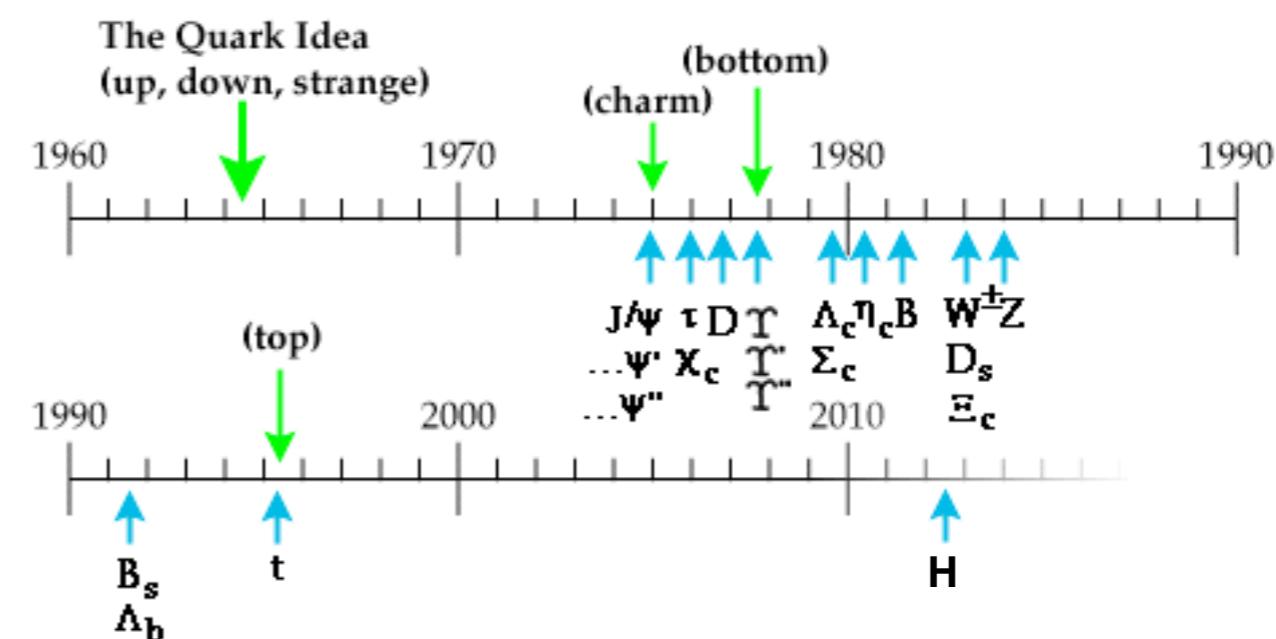
~ XX century **theoretical advances culminate** in the 70's with **SM of particle physics**



Anomalous magnetic dipole moment

$$\frac{g_e - 2}{2} \Big|_{\text{SM}} = 0.001\ 159\ 652\ 181\ 643(764)$$

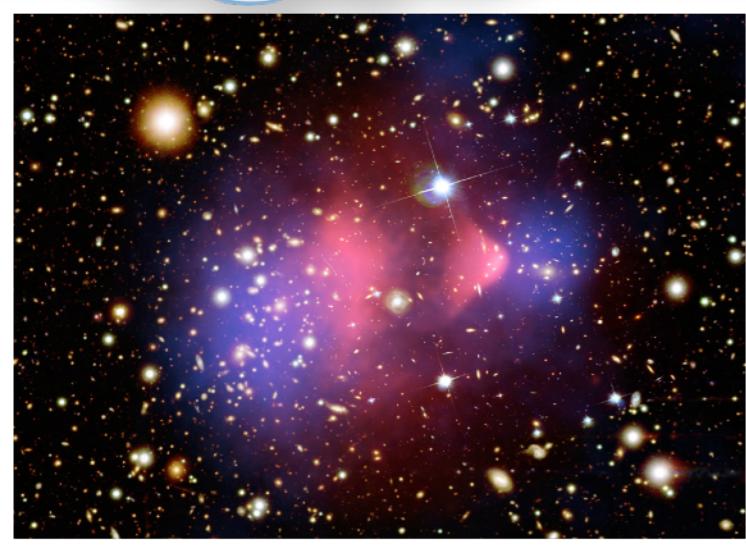
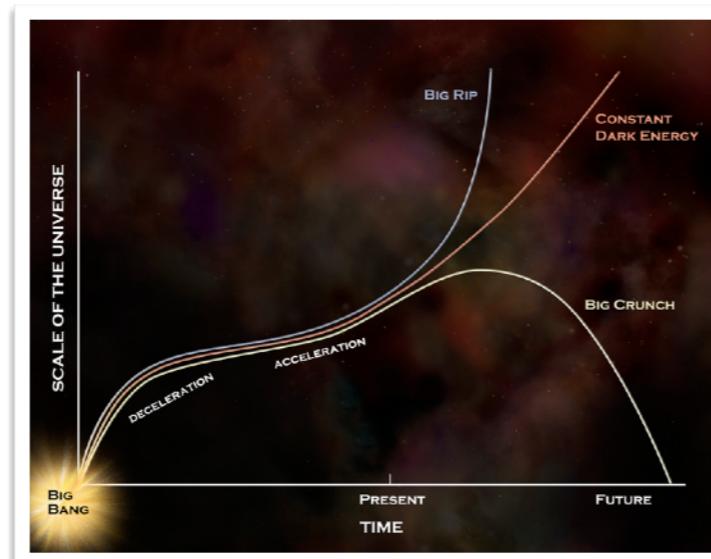
$$\frac{g_e - 2}{2} \Big|_{\text{exp}} = 0.001\ 159\ 652\ 180\ 73(28)$$



Anything left to discover?

Clouds today

NASA/CXC/M. Weiss

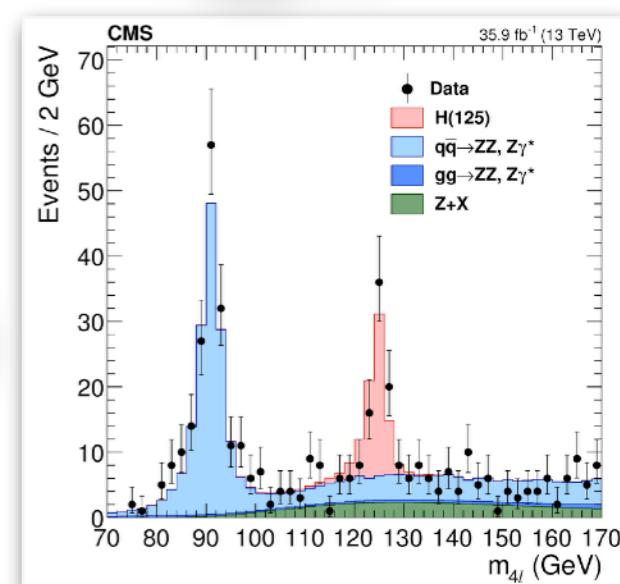


NASA/STScI; ESO WFI;
Magellan/U.Arizona/ D.Clowe et al.



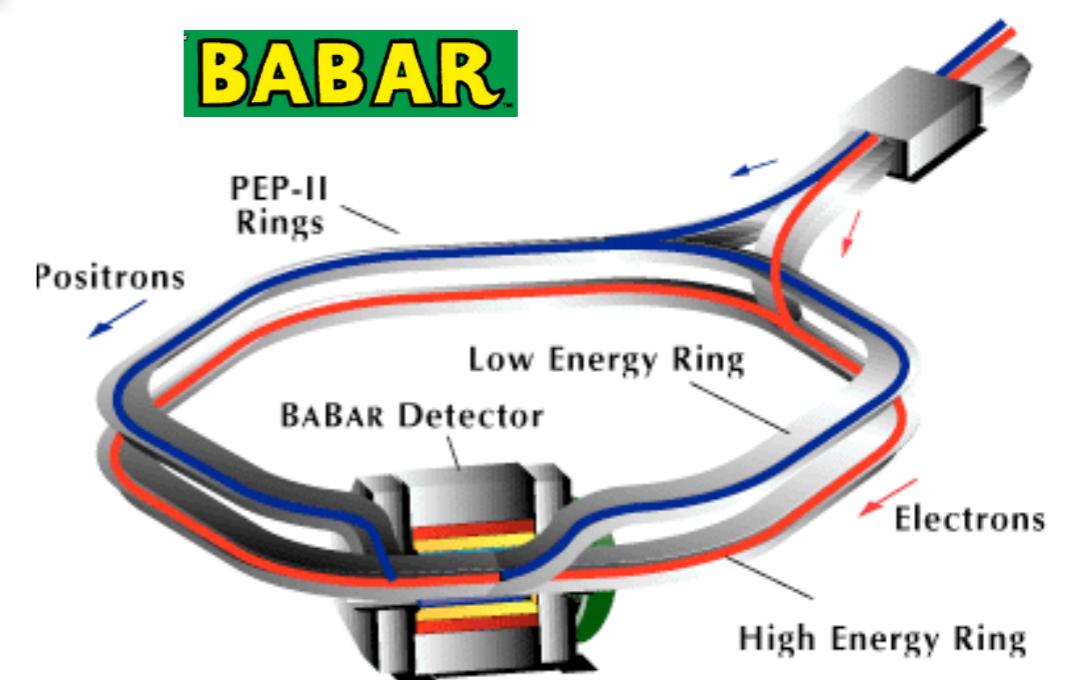
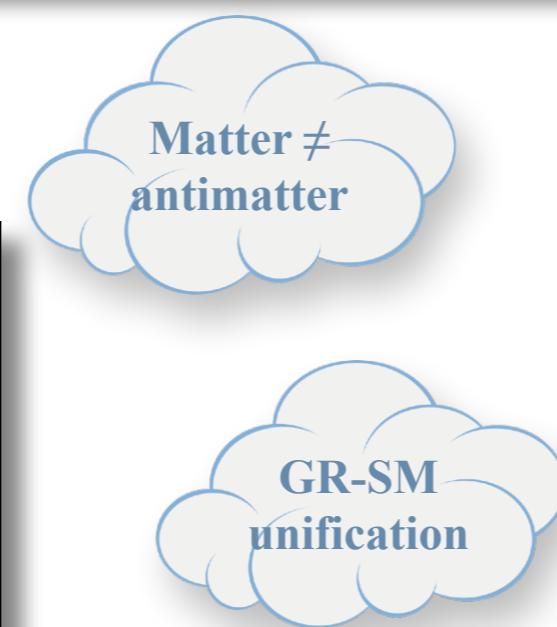
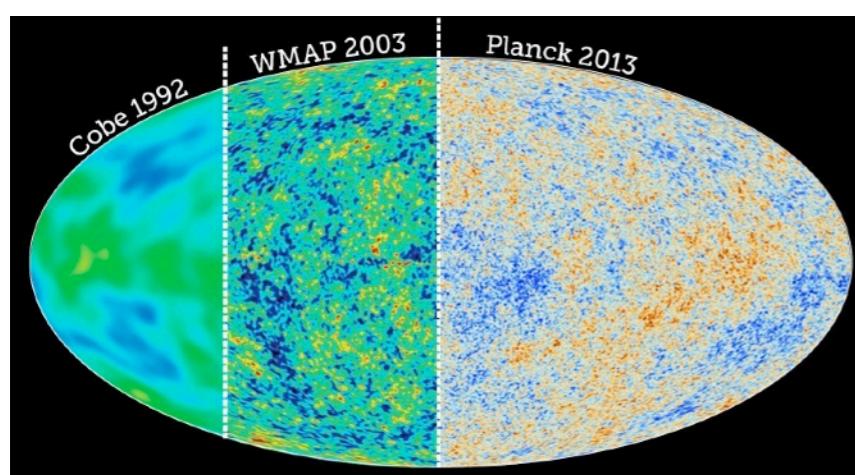
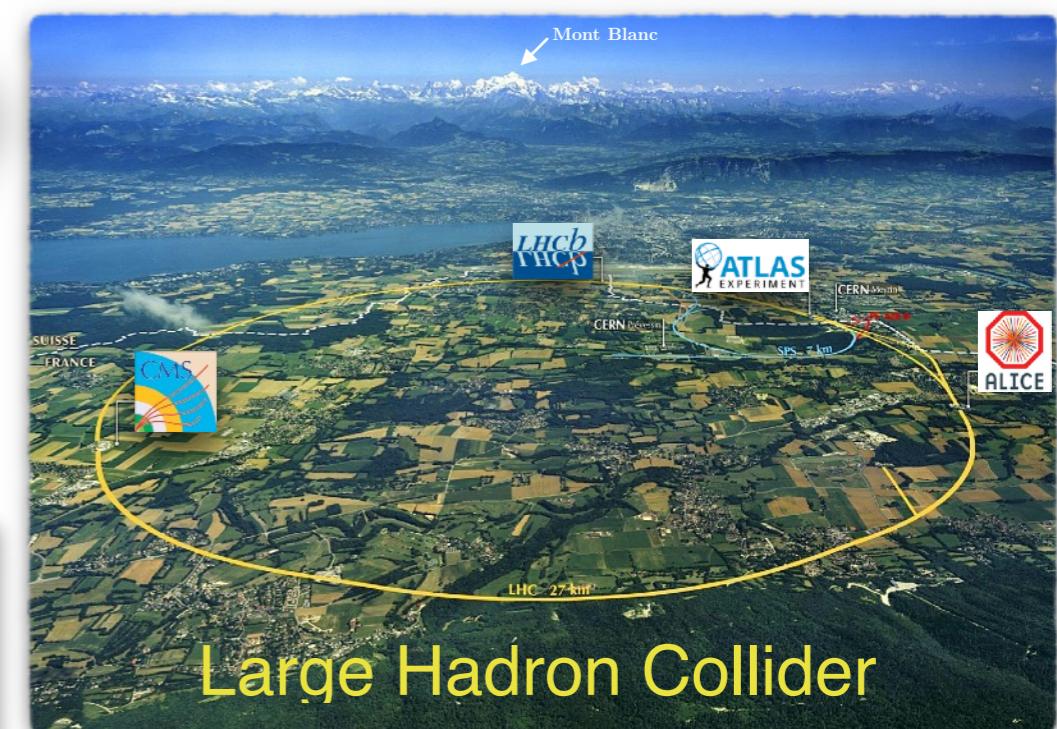
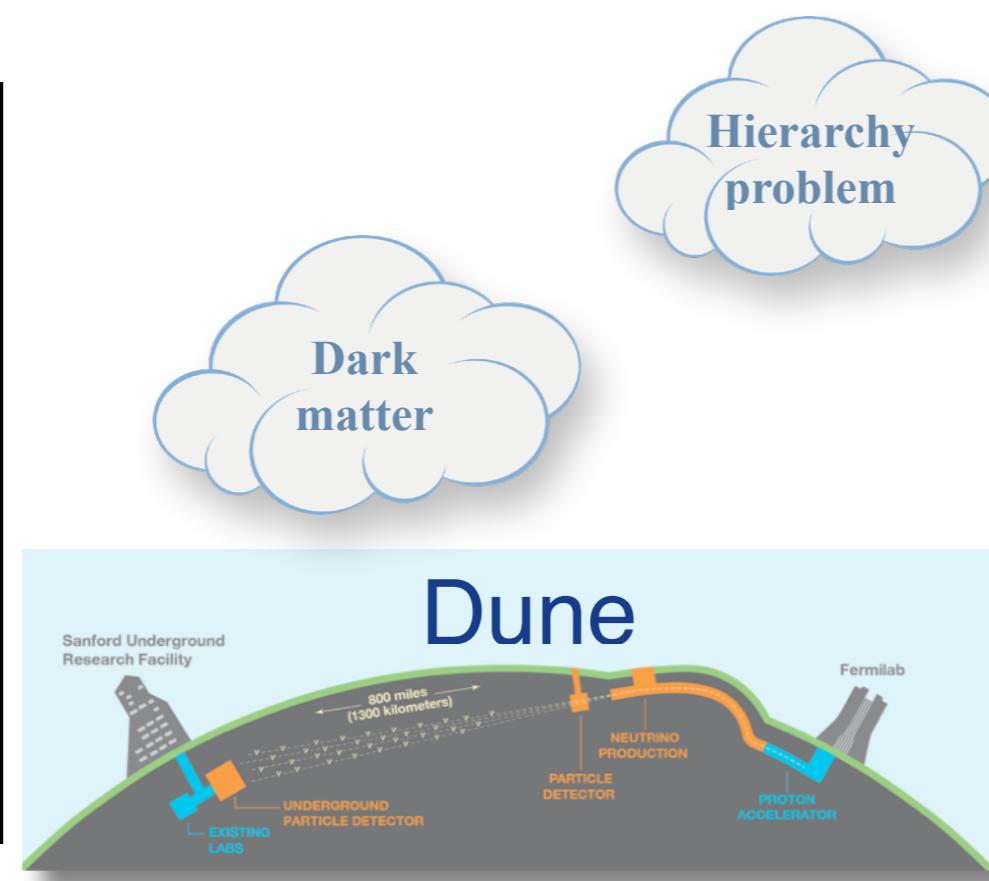
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\text{SU}(3) \times \text{SU}(2)_L \times \text{U}(1)$$



JHEP 11 (2017) 047

Dispelling the new clouds: experiments



Modern physics timeline

Electron discovery

Ultraviolet catastrophe

Energy quantization

General Relativity

Positron discovery

Particle zoo

Michelson-

Maxwell equations

Morley experiment

Special Relativity

Schrödinger equation

QED Higgs QCD mechanism

Neutrino masses

Higgs discovery

XIX century

XX century

XXI

Classical

Modern Physics

	Low speeds	High speeds
Large sizes	<i>Classical physics</i>	<i>Relativity</i>
Small sizes	<i>Quantum mechanics</i>	<i>Relativistic quantum</i>

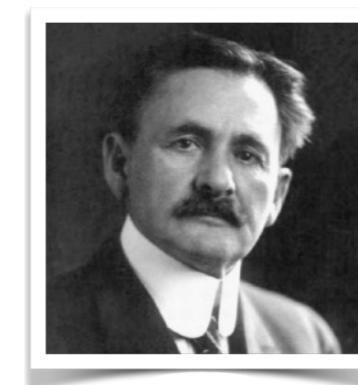
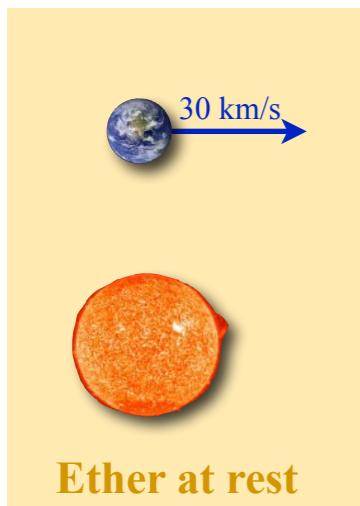
In **particle physics** we deal with small sizes and high energies (speeds), so we use a **relativistic quantum** framework called **Quantum Field Theory (QFT)**



Special Relativity

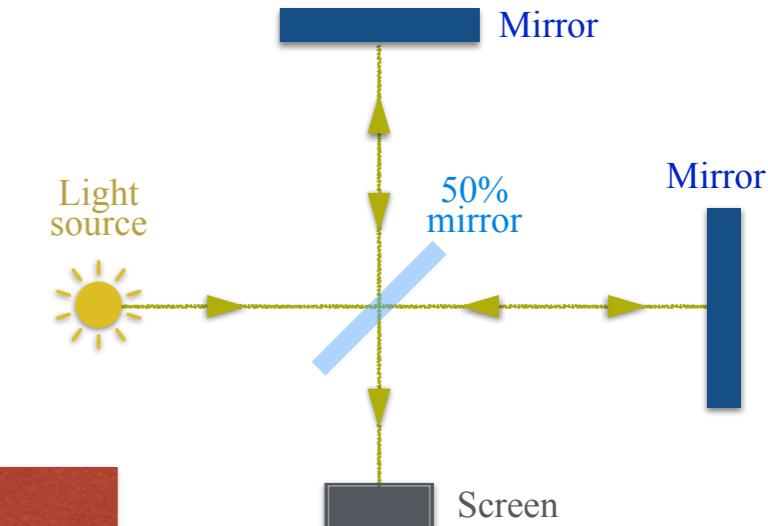
Einstein's postulates

Hypothesis was that **Earth** was moving at least at 30 km/s with respect to Ether

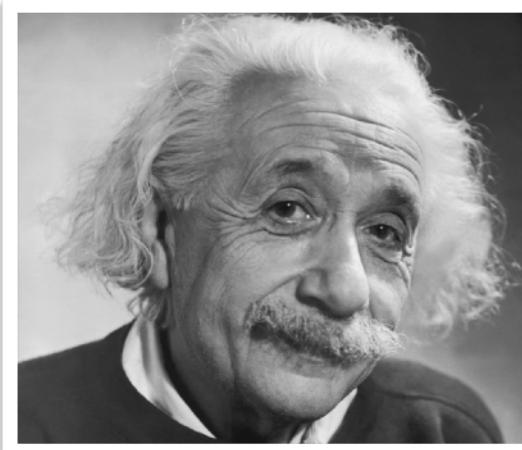


If light moved at c with respect to Ether, beam would take **different times** when aligned with Earth's speed versus transverse to it

With an **interferometer**, invented by **Michelson** in **1881** (Naval academy, Annapolis).



No evidence for light traveling at different speeds in different directions



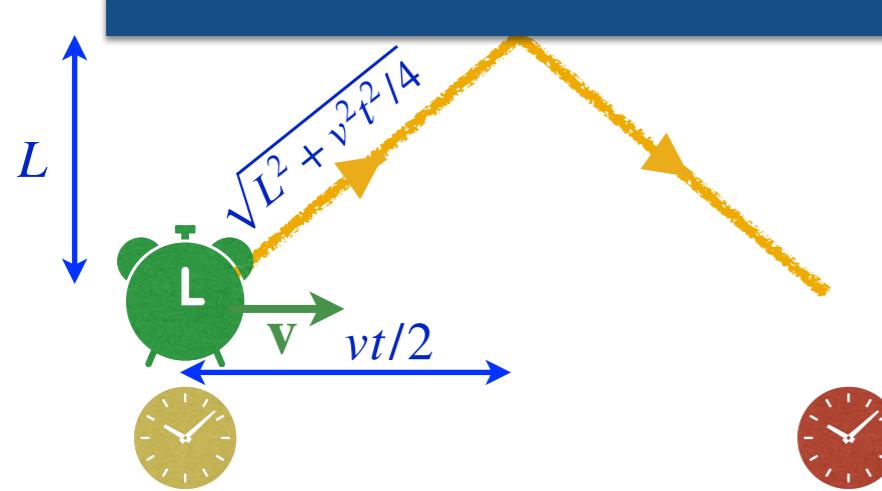
Relativity principle: all laws of Physics have the same form in all inertial (no acceleration) frames of reference

The speed of light in vacuum has the same value c in every direction in all inertial frames of reference

Time dilation

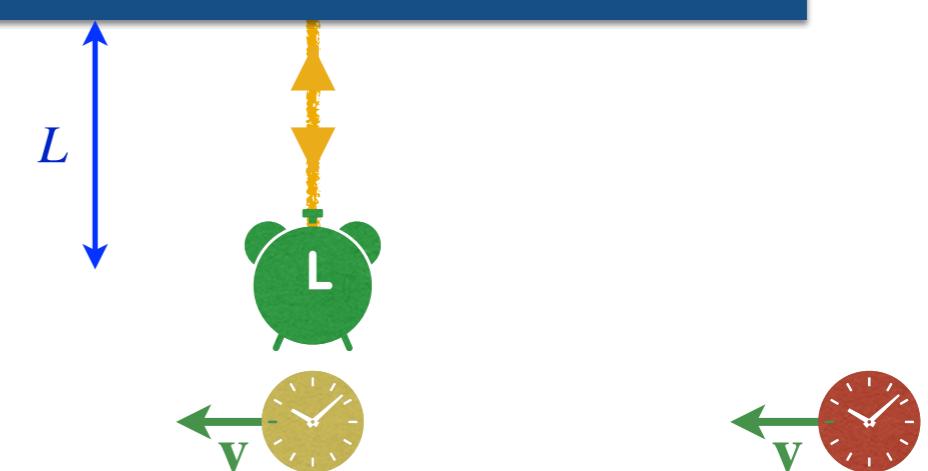
~ What's the **time** passed **between light is emitted and absorbed** by the **green clock**?

Rest frame



$$t = 2 \frac{\sqrt{L^2 + v^2 t^2 / 4}}{c} \Rightarrow t = \frac{2L}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Green clock frame



Total time between events is simply total distance covered by light divided by speed

$$t' = \frac{2L}{c}$$

We define a new quantity γ

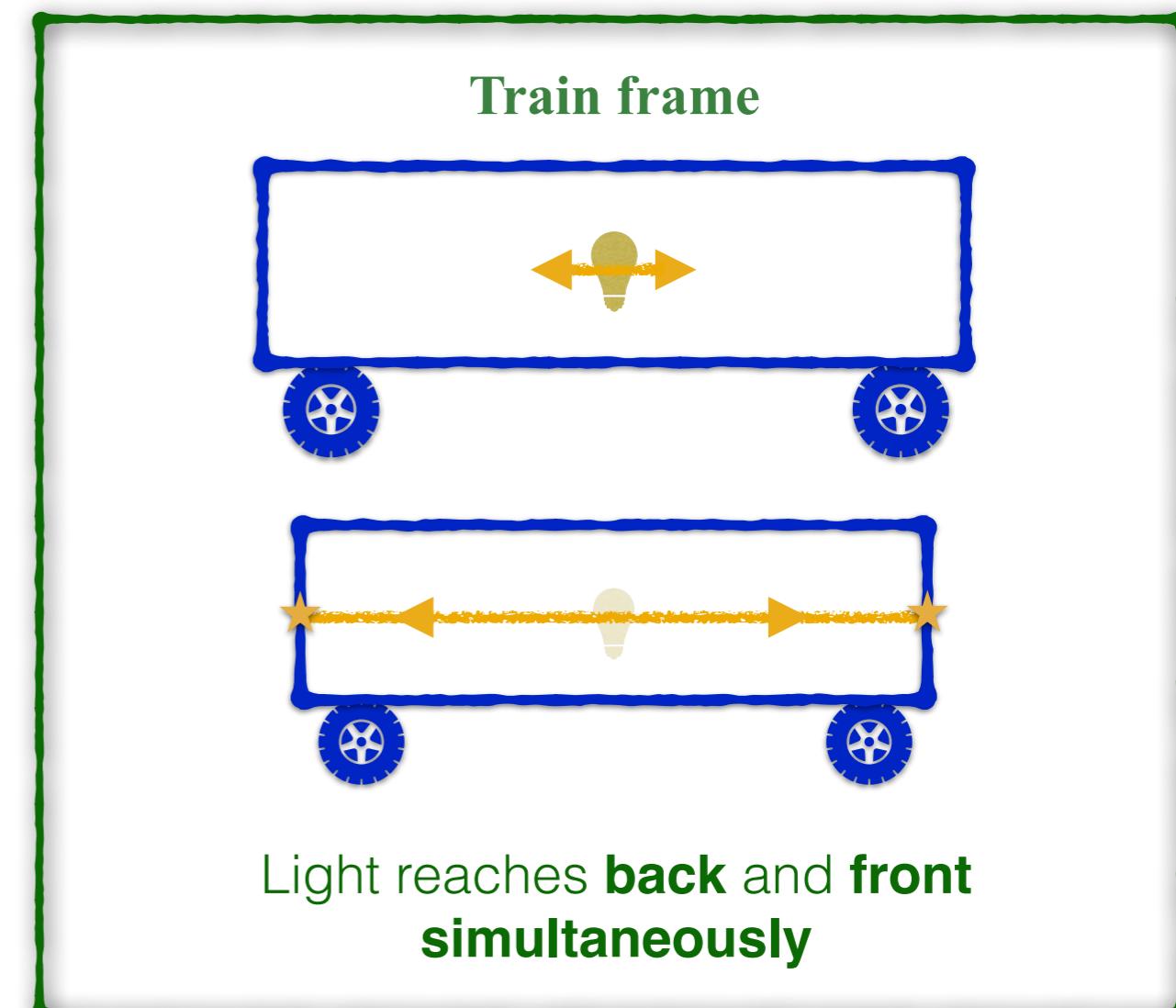
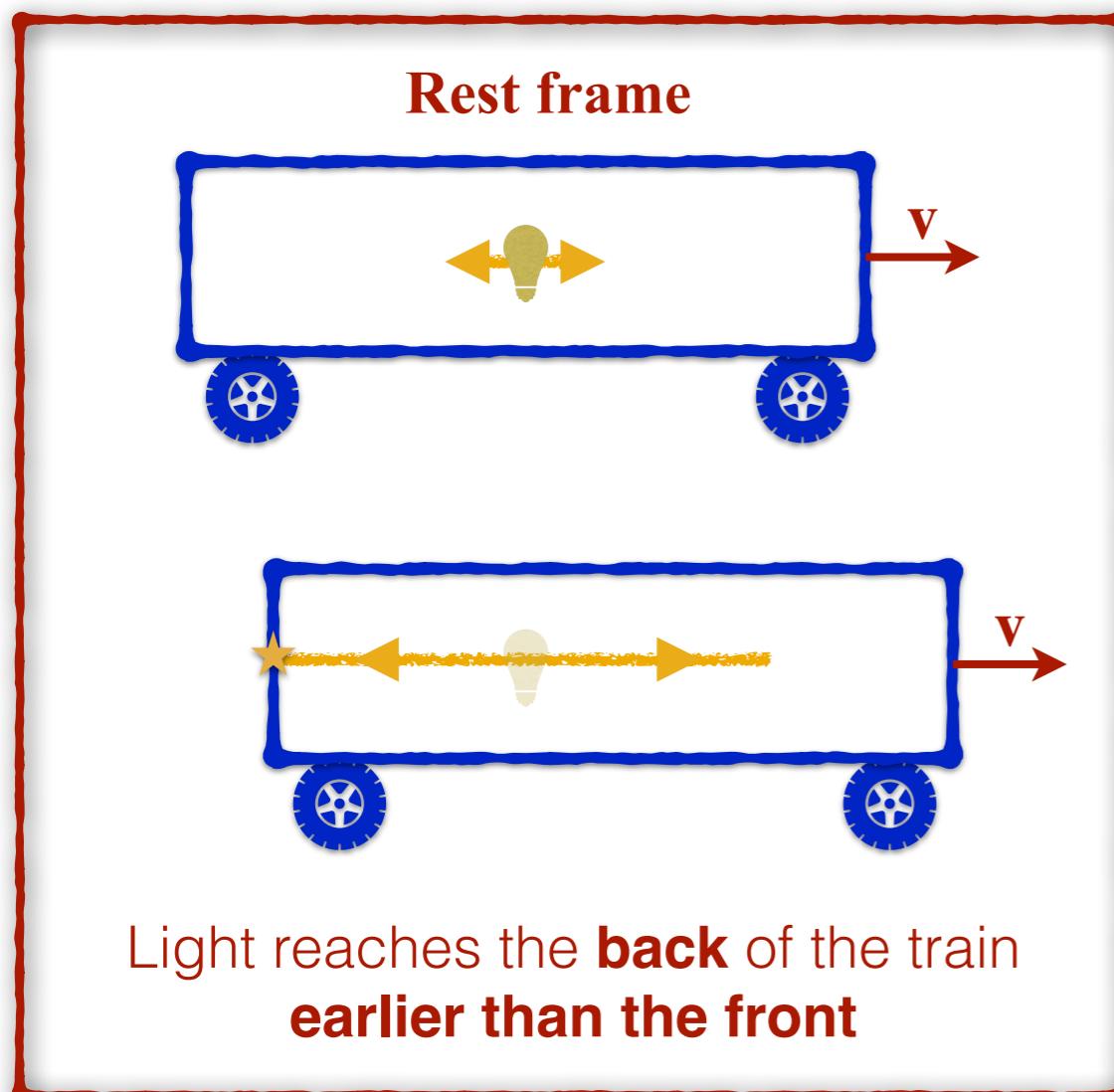
$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = t' \gamma$$

Based on Einstein postulates alone, **time passes more slowly when moving**

Simultaneity is frame dependent

- ~ A light goes off in the middle of a moving train
 - Do the light rays **reach the front/back of the train at the same time?**



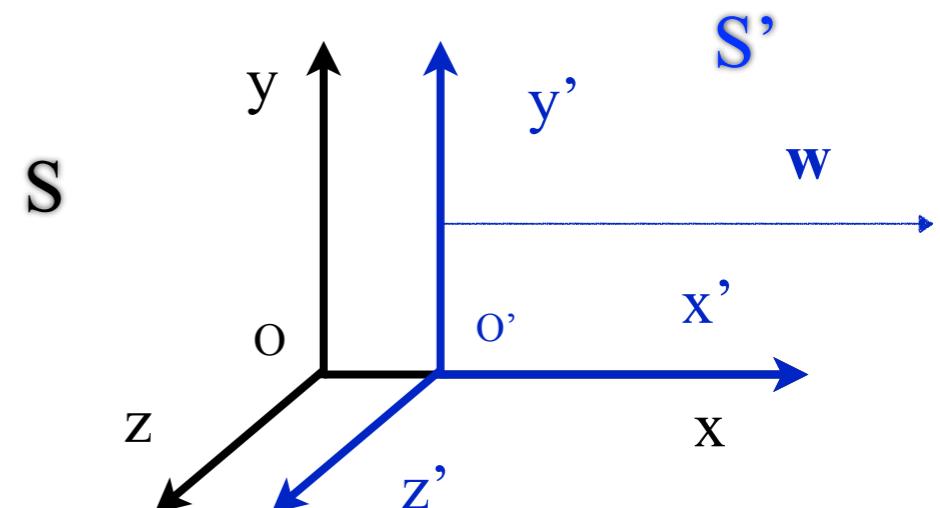
Whether two events that occurred far apart are simultaneous or not depends on the frame of reference

Lorentz transformation

~ Lorentz boost for

- Two inertial frames
- Same origin at $t=0$ and S'
- Moving at speed v **along the x axis**

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$



Often we use this 2×2 matrix as nothing interesting may be happening in y and z

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\beta \equiv \frac{v}{c}$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

4-momentum

~ Similar to space-time, **energy and momentum are intrinsically connected**

- Represent with 4-component vector called **4-momentum**
 - ◆ Classical momentum is $\vec{p} = m\vec{v}$, while relativistic is $\vec{p} = m\gamma\vec{v}$

$$\begin{bmatrix} p'_0 \\ p'_1 \\ p'_2 \\ p'_3 \end{bmatrix} = \begin{bmatrix} E'/c \\ p'_x \\ p'_y \\ p'_z \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

Important

- Energy and momentum ($p^2 = p_x^2 + p_y^2 + p_z^2$) of a particle of mass m are related as

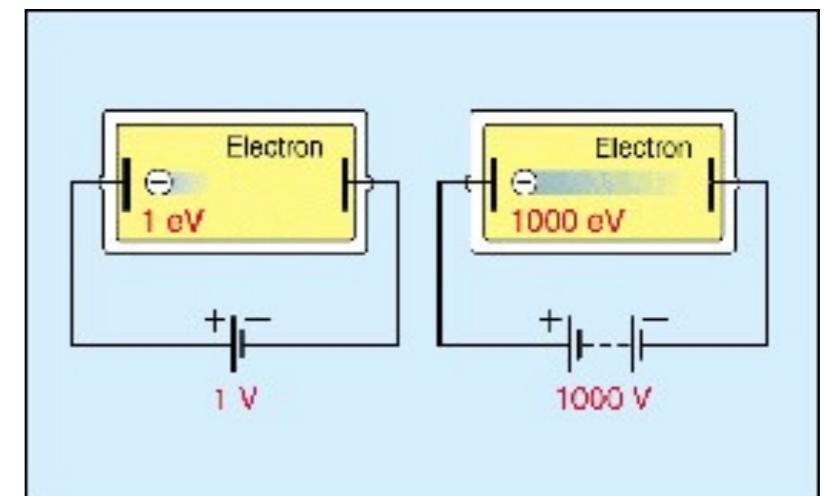
$$E^2 = p^2 c^2 + m^2 c^4 \Rightarrow p = \sqrt{E^2/c^2 - m^2 c^2} \Rightarrow m = \sqrt{E^2/c^4 - p^2/c^2}$$

$$p^\nu = \begin{bmatrix} myc \\ m\gamma\vec{v} \end{bmatrix}_\nu = \begin{bmatrix} \frac{E}{c} \\ \vec{p} \end{bmatrix}_\nu = \begin{bmatrix} \sqrt{p^2 + m^2 c^2} \\ \vec{p} \end{bmatrix}_\nu$$

Total 4-momentum is always conserved

Units in particle physics

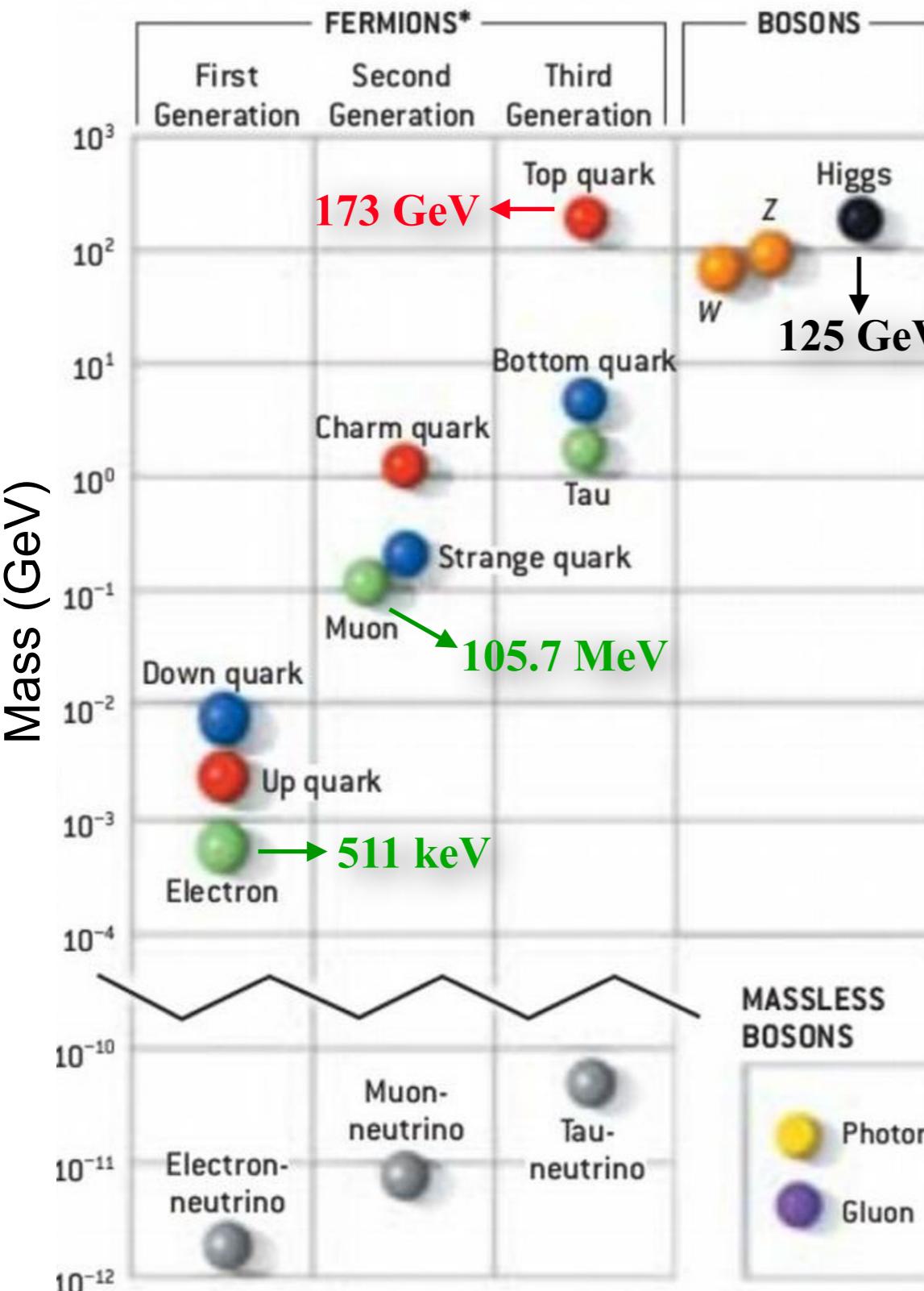
- ~ Typically use **electronvolts (eV)** to measure energy
 - Energy of an electron accelerated by a 1 V potential
 - Very useful in the original experiments
 - $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
- ~ Use **eV/c** for momentum
- ~ Use **eV/c²** for mass
- ~ What is the energy of an **electron ($m_e = 511 \text{ keV}/c^2$)** that has a **momentum of 1 MeV/c**?



$$E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4} = \sqrt{(1 \text{ MeV}/c)^2 c^2 + (0.511 \text{ MeV}/c^2)^2 c^4} = 1.12 \text{ MeV}$$

- ~ Very often, we just set **c = 1**, so **energy, mass, and momentum are measured in eV**

Particle masses

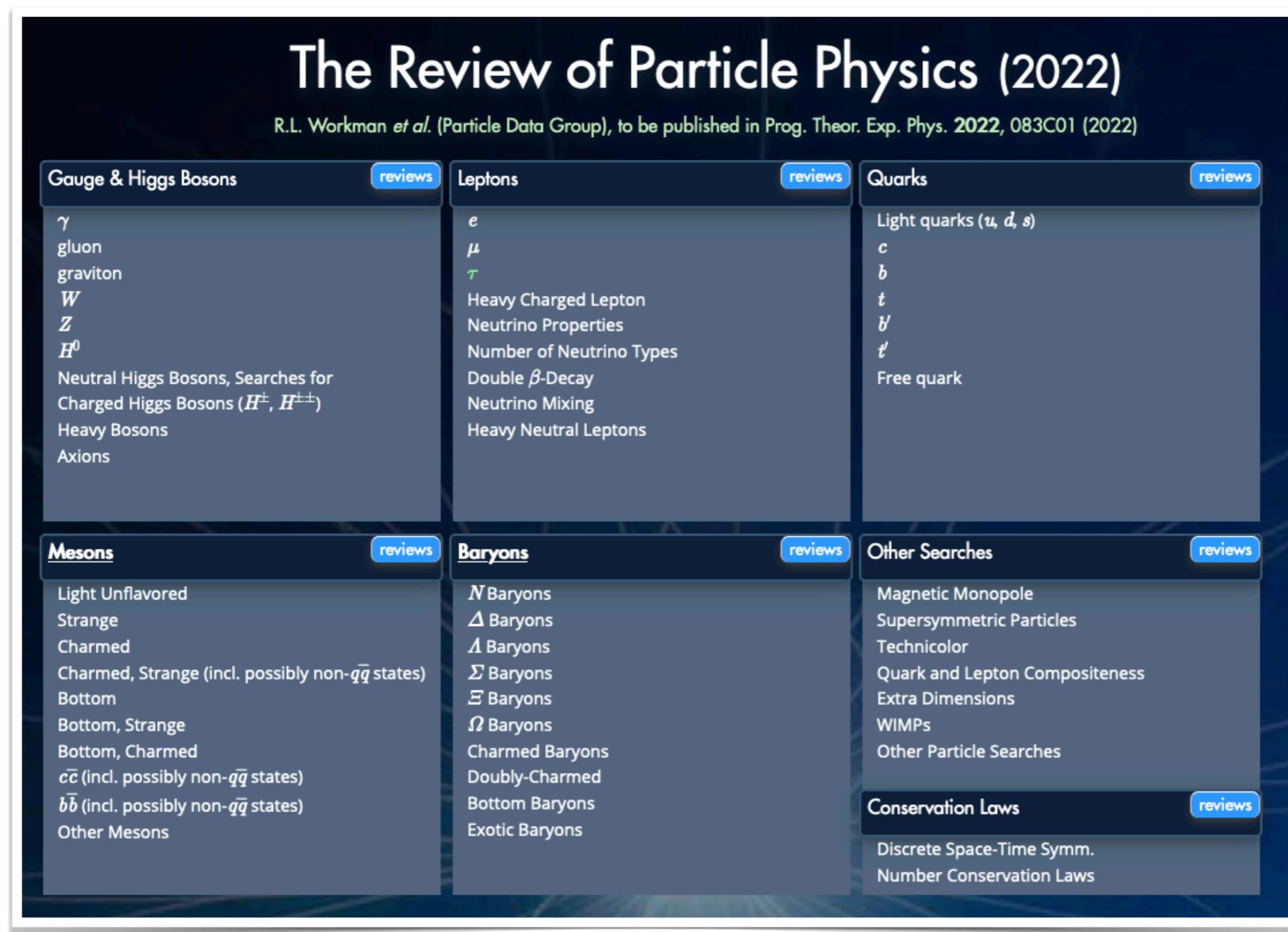


- ~ Elementary particles masses cover a large range
 - Photon/gluon are massless
 - Neutrinos ~ meV
 - 1st generation fermions ~ 1 MeV
 - 2nd generation fermions ~ 0.1 - 1 GeV
 - 3rd generation fermions ~ 1 - 173 GeV
 - W, Z bosons ~ 90 GeV
 - Higgs boson ~ 125 GeV

- ~ Masses of non-elementary particles heavily determined by binding energy
 - eg, $m(u) = 2.3 \text{ MeV}$, $m(d) = 4.8 \text{ MeV}$
 - ♦ Mass of proton (uud) is 938 MeV

PDG (Particle Data Group)

~ Find all particle properties at <https://pdglive.lbl.gov/Viewer.action>



The Review of Particle Physics (2022)

R.L. Workman *et al.* (Particle Data Group), to be published in Prog. Theor. Exp. Phys. **2022**, 083C01 (2022)

Gauge & Higgs Bosons reviews

- γ
- gluon
- graviton
- W
- Z
- H^0
- Neutral Higgs Bosons, Searches for Charged Higgs Bosons ($H^\pm, H^{\pm\pm}$)
- Heavy Bosons
- Axions

Leptons reviews

- e
- μ
- τ
- Heavy Charged Lepton
- Neutrino Properties
- Number of Neutrino Types
- Double β -Decay
- Neutrino Mixing
- Heavy Neutral Leptons

Quarks reviews

- Light quarks (u, d, s)
- c
- b
- t
- b'
- t'
- Free quark

Mesons reviews

- Light Unflavored
- Strange
- Charmed
- Charmed, Strange (incl. possibly non- $q\bar{q}$ states)
- Bottom
- Bottom, Strange
- Bottom, Charmed
- $c\bar{c}$ (incl. possibly non- $q\bar{q}$ states)
- $b\bar{b}$ (incl. possibly non- $q\bar{q}$ states)
- Other Mesons

Baryons reviews

- N Baryons
- Δ Baryons
- Λ Baryons
- Σ Baryons
- Ξ Baryons
- Ω Baryons
- Charmed Baryons
- Doubly-Charmed
- Bottom Baryons
- Exotic Baryons

Other Searches reviews

- Magnetic Monopole
- Supersymmetric Particles
- Technicolor
- Quark and Lepton Compositeness
- Extra Dimensions
- WIMPs
- Other Particle Searches

Conservation Laws reviews

- Discrete Space-Time Symm.
- Number Conservation Laws

Important

Particle decays

~ A particle α decays into two other particles λ and ω : $\alpha \rightarrow \lambda \omega$

General conservation of 4-momentum

$$\begin{bmatrix} \sqrt{p_\alpha^2 + m_\alpha^2 c^2} \\ p_{\alpha,x} \\ p_{\alpha,y} \\ p_{\alpha,z} \end{bmatrix} = \begin{bmatrix} \sqrt{p_\lambda^2 + m_\lambda^2 c^2} \\ p_{\lambda,x} \\ p_{\lambda,y} \\ p_{\lambda,z} \end{bmatrix} + \begin{bmatrix} \sqrt{p_\omega^2 + m_\omega^2 c^2} \\ p_{\omega,x} \\ p_{\omega,y} \\ p_{\omega,z} \end{bmatrix}$$

$$p_\alpha = \sqrt{p_{\alpha,x}^2 + p_{\alpha,y}^2 + p_{\alpha,z}^2}$$

$$p_\lambda = \sqrt{p_{\lambda,x}^2 + p_{\lambda,y}^2 + p_{\lambda,z}^2}$$

$$p_\omega = \sqrt{p_{\omega,x}^2 + p_{\omega,y}^2 + p_{\omega,z}^2}$$

Conservation of 4-momentum in two-body decay

$$\begin{bmatrix} m_\alpha c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_\lambda^2 + m_\lambda^2 c^2} \\ p_\lambda \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \sqrt{p_\omega^2 + m_\omega^2 c^2} \\ p_\omega \\ 0 \\ 0 \end{bmatrix}$$

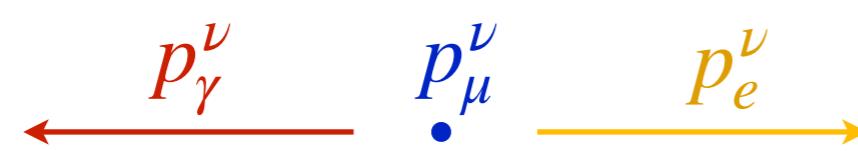
	Degrees of freedom			Total	Equations (constraints)
	α	λ	ω		
Original	4	4	4	12	4
Masses	3	3	3	9	4
α at rest	0	3	3	6	4
Symmetry	0	1	1	2	2



Cannot know λ and ω direction (isotropic decay), but we know that they will **both travel along the same line in opposite directions** by **conservation of momentum**
→ Choose x axis along the direction of the decay

Particle decay

- ~ A muon μ^- is a particle very similar to an electron e^- but with a higher mass $m_\mu > m_e$. What is the **energy** of the (**massless**) photon γ emitted in the decay $\mu^- \rightarrow e^- + \gamma$ assuming the **muon is initially at rest?**



Conservation of 4-momentum

$$\left[\frac{\sqrt{p_\mu^2 + m_\mu^2 c^2}}{p_\mu} \right] = \left[\frac{\sqrt{p_e^2 + m_e^2 c^2}}{p_e} \right] + \left[\frac{\sqrt{p_\gamma^2 + m_\gamma^2 c^2}}{p_\gamma} \right]$$

Cannot know γ and e^- direction (isotropic decay), but we know that they will **both travel along the same line in opposite directions by conservation of momentum**

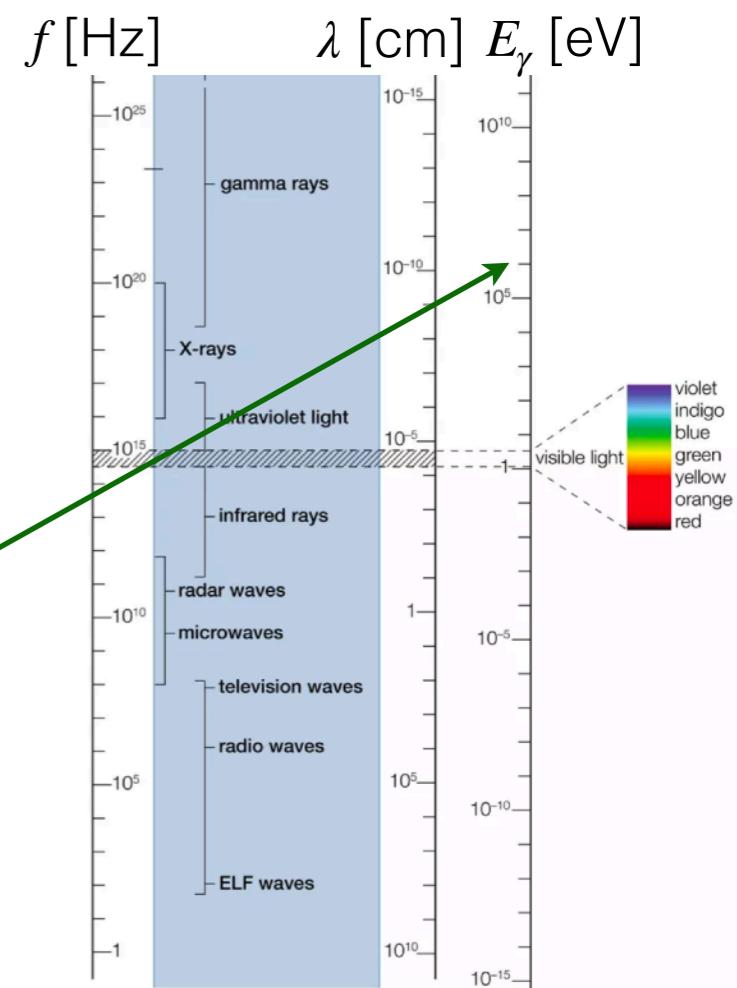
Since the muon is at rest, $\mathbf{p}_\mu = \mathbf{0}$, the photon is massless, $\mathbf{m}_\gamma = \mathbf{0}$

$$\begin{bmatrix} m_\mu c \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_e^2 + m_e^2 c^2} \\ p_e \end{bmatrix} + \begin{bmatrix} |p_\gamma| \\ p_\gamma \end{bmatrix} \Rightarrow \begin{aligned} \sqrt{p_e^2 + m_e^2 c^2} &= m_\mu c - |p_\gamma| \\ 0 &= p_e + p_\gamma \end{aligned}$$

Solve for positive p_γ

$$p_\gamma^2 + m_e^2 c^2 = m_\mu^2 c^2 - 2m_\mu c p_\gamma + p_\gamma^2 \Rightarrow p_\gamma = \frac{m_\mu^2 c - m_e^2 c}{2m_\mu}$$

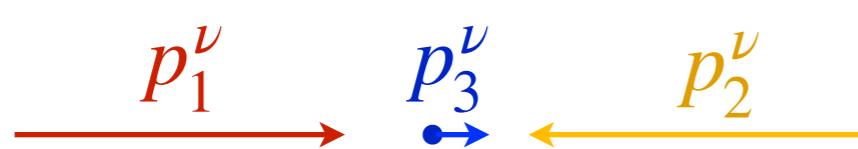
$$E_\gamma = p_\gamma c = \frac{m_\mu^2 c^2 - m_e^2 c^2}{2m_\mu} = 52.83 \text{ MeV}$$



Particle collisions

- Two particles of mass m_1 and m_2 collide each carrying momentum p_1 and p_2 what is the **mass and momentum of the particle they produce?**

Conservation of 4-momentum



$$\left[\frac{\sqrt{p_1^2 + m_1^2 c^2}}{p_1} \right] + \left[\frac{\sqrt{p_2^2 + m_2^2 c^2}}{p_2} \right] = \left[\frac{\sqrt{p_3^2 + m_3^2 c^2}}{p_3} \right]$$

$$p_3 = p_2 + p_1 \Rightarrow m_3 = \sqrt{\left(\sqrt{p_1^2/c^2 + m_1^2} + \sqrt{p_2^2/c^2 + m_2^2} \right)^2 - (p_1 + p_2)^2/c^2}$$

For instance, at the LHC we collide **two protons** (mass $\sim 1 \text{ GeV}/c^2$) of energy **7 TeV each**

$$|p_1| = |p_2| = \sqrt{E^2/c^2 - m_p^2 c^2} \approx E/c = 7 \text{ TeV}/c$$

$$p_3 = p_1 + p_2 = (7 \text{ TeV}/c) + (-7 \text{ TeV}/c) = 0$$

$$m_3 = \sqrt{(7 \text{ TeV}/c^2)^2 + (1 \text{ GeV}/c^2)^2} + \sqrt{(-7 \text{ TeV}/c^2)^2 + (1 \text{ GeV}/c^2)^2} \approx 14 \text{ TeV}/c^2$$

Because protons are not elementary, the highest mass that can be produced is smaller than this

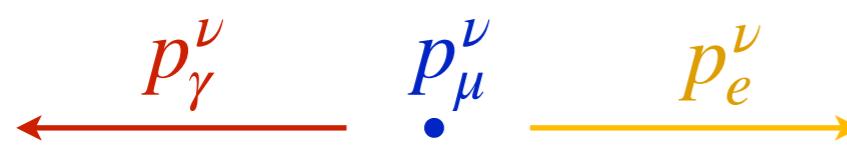
Dynamics summary

~ Relationship between $\gamma(\beta)$, m, E, and p in Relativity

$$p^\nu = \begin{bmatrix} m\gamma c \\ m\gamma \vec{v} \end{bmatrix} = \begin{bmatrix} \frac{E}{c} \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \sqrt{p^2 + m^2 c^2} \\ \vec{p} \end{bmatrix} \quad \text{with} \quad \vec{p} = m\gamma \vec{v}$$

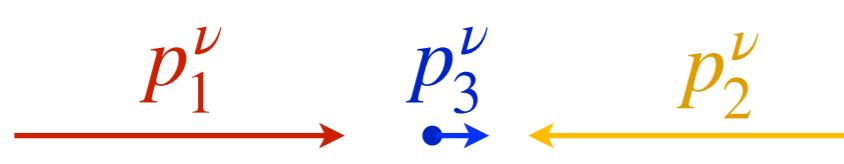
For particles with $m = 0$, the first expression is not useful, but the other two are still valid

~ Particle decay



$$\begin{bmatrix} m_\mu c \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_e^2 + m_e^2 c^2} \\ p_e \end{bmatrix} + \begin{bmatrix} \sqrt{p_\gamma^2 + m_\gamma^2 c^2} \\ p_\gamma \end{bmatrix}$$

~ Particles collide to create a third particle



$$\begin{bmatrix} \sqrt{p_1^2 + m_1^2 c^2} \\ p_1 \end{bmatrix} + \begin{bmatrix} \sqrt{p_2^2 + m_2^2 c^2} \\ p_2 \end{bmatrix} = \begin{bmatrix} \sqrt{p_3^2 + m_3^2 c^2} \\ p_3 \end{bmatrix}$$

4-momentum practice

~ Relationship between $\gamma(\beta)$, m, E, and p in Relativity

$$p^\nu = \begin{bmatrix} m\gamma c \\ m\gamma \vec{v} \end{bmatrix} = \begin{bmatrix} \frac{E}{c} \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \sqrt{p^2 + m^2 c^2} \\ \vec{p} \end{bmatrix} \quad \text{with} \quad \vec{p} = m\gamma \vec{v}$$

~ A proton ($m_p = 1 \text{ GeV}/c^2$) has a momentum of $10 \text{ GeV}/c$

→ What is its energy?

$$E = \sqrt{p^2 c^2 + m^2 c^4} = \sqrt{(10 \text{ GeV})^2 + (1 \text{ GeV})^2} = 10.05 \text{ GeV}$$

→ How would you calculate its speed?

$$\gamma = \frac{E}{mc^2} = 10.05 = \frac{1}{\sqrt{1 - \beta^2}} \rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.995$$

~ A proton ($m_p = 1 \text{ GeV}/c^2$) has an energy of $7 \text{ TeV}/c$

→ What is its speed and momentum? $\gamma = 7,000 \rightarrow \beta = 0.9999999898$

$$p = \sqrt{E^2/c^2 - m^2 c^4} = \sqrt{(7 \text{ TeV})^2 + (0.001 \text{ TeV})^2} = 7 \text{ TeV}/c$$

Particle colliders



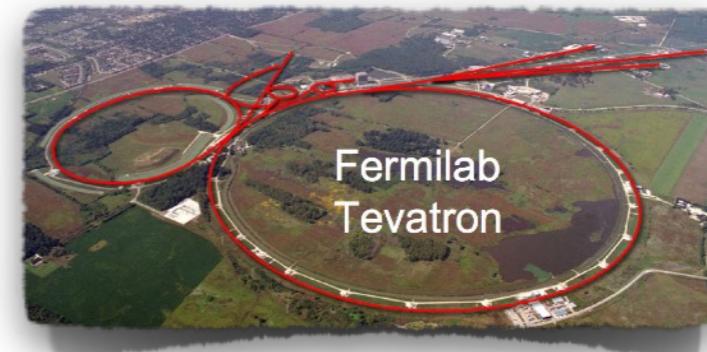
Particle accelerator types

Type of collision

Fixed target
NA62



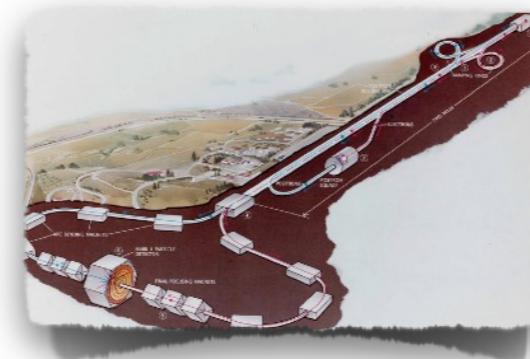
VS



Collider
Tevatron

Geometry

Linear collider
Stanford Linear Collider (SLC)



VS



Circular collider
Large Electron-Positron collider (LEP)

Type of particle

Lepton collider
Positron-Electron Project II (PEP-II)



VS

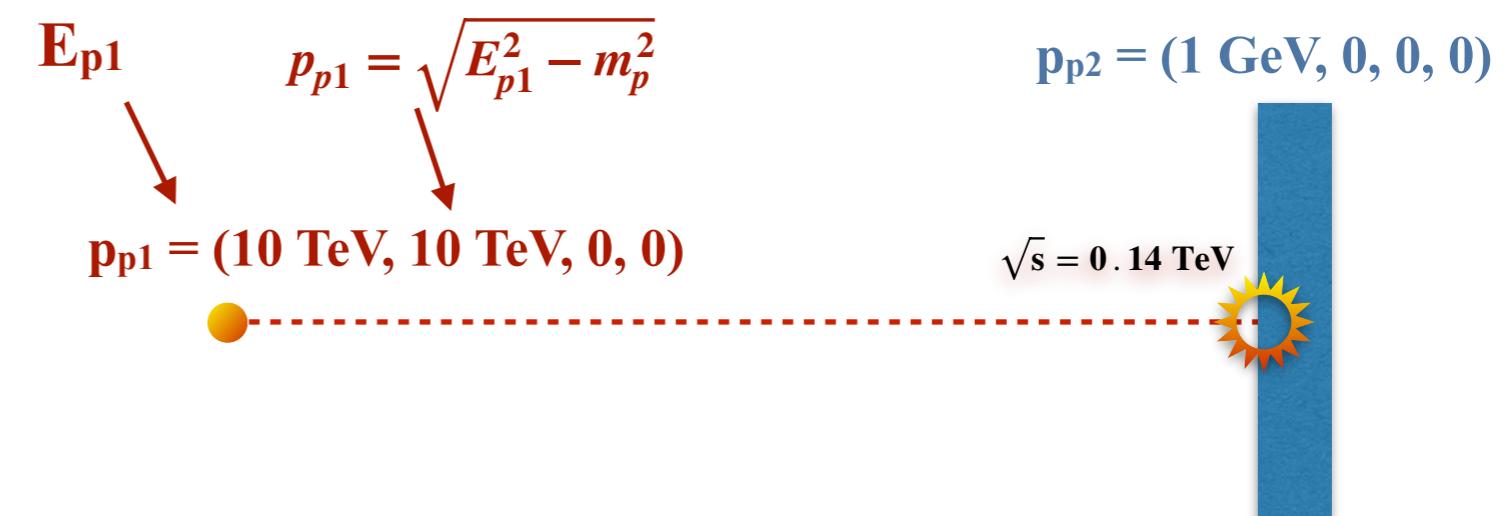


Hadron collider
Large Hadron Collider (LHC)

Fixed target vs Colliders

~ In **Fixed target experiments** a beam of high-energy particles collides against a large and stationary target

- Easy, cheap
- Inefficient → difficult to reach high energies



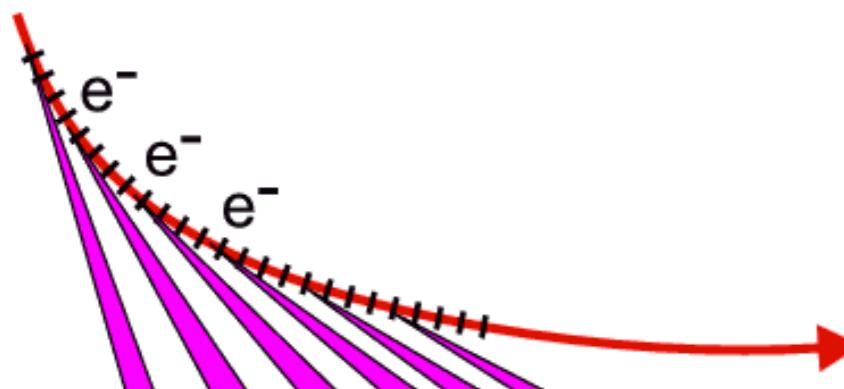
~ **Colliders** crash two narrow beams of particles

- Difficult but efficient

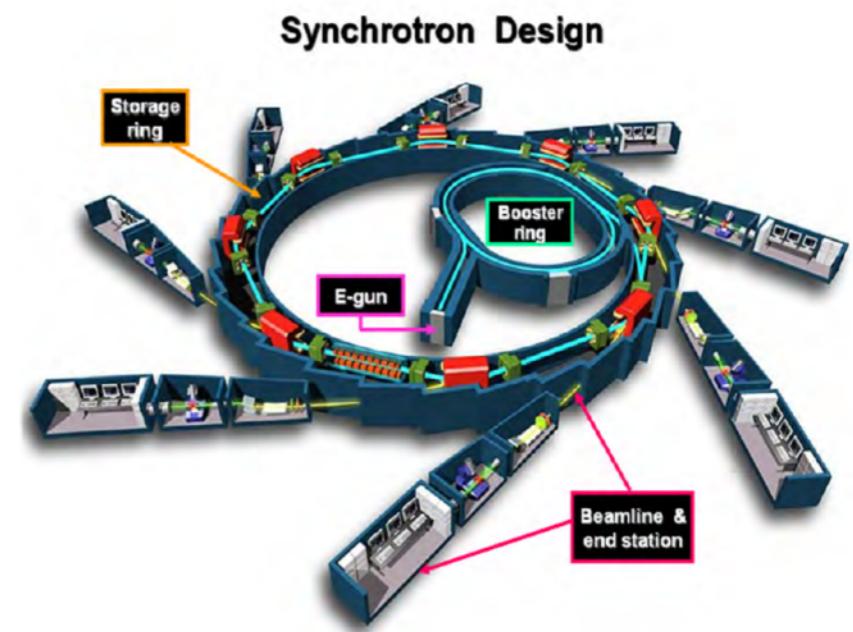


Linear vs Circular colliders

- ~ **Circular** geometry has **key advantages** as particles circle around
 - Can have **higher energies** as particles are accelerated at each cycle
 - Can have **higher collision rates** as particle bunches have more opportunities to collide
- ~ However, charged particles emit **synchrotron radiation when changing direction**
 - **Power loss** grows very **quickly with energy**
 - Circular geometry **not feasible for light particles** (eg, electrons) for $\sqrt{s} \geq \sim 250$ GeV
 - ♦ Use protons ($m_p \sim 2000m_e$) for higher energies



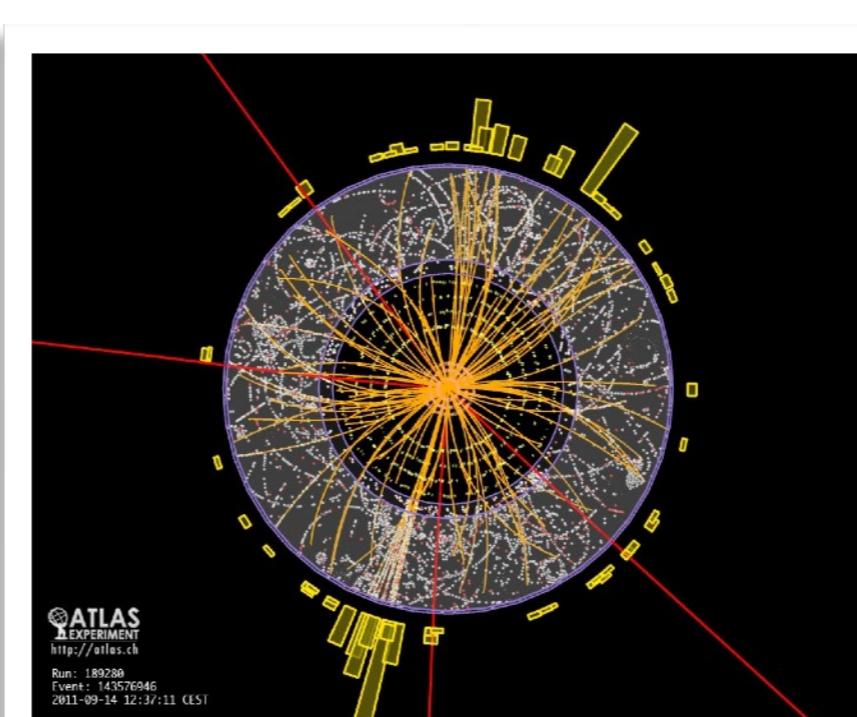
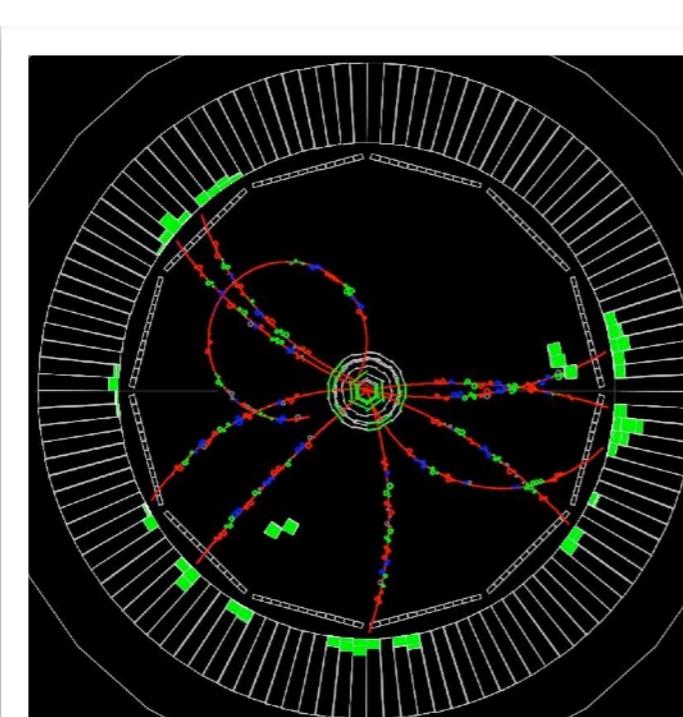
$$P_{\text{sync}}^{\text{loss}} \propto \frac{E^4}{m^4 R^2}$$



Lepton vs Hadron colliders

- ~ Easier to make **precision measurements** in lepton colliders
 - Known collision energy because leptons are elementary particles
 - Cleaner environment

Clean lepton-lepton collision
BaBar at SLAC
PEP-II



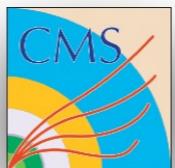
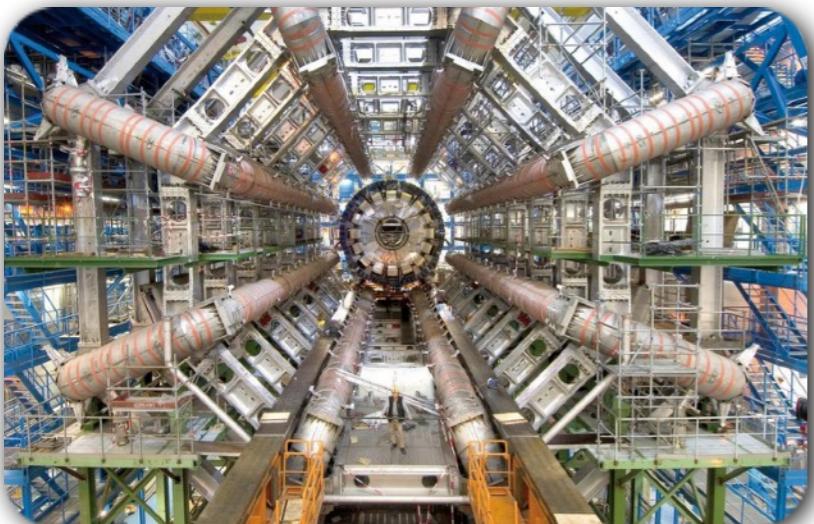
Messy hadron-hadron collision
ATLAS at CERN LHC



- ~ However, **hadron** colliders can achieve **higher energies**
 - **Higher statistics too** due to higher production cross sections



Overview of the LHC experiments



General purpose

Discovered Higgs boson in 2012
Direct searches for new physics



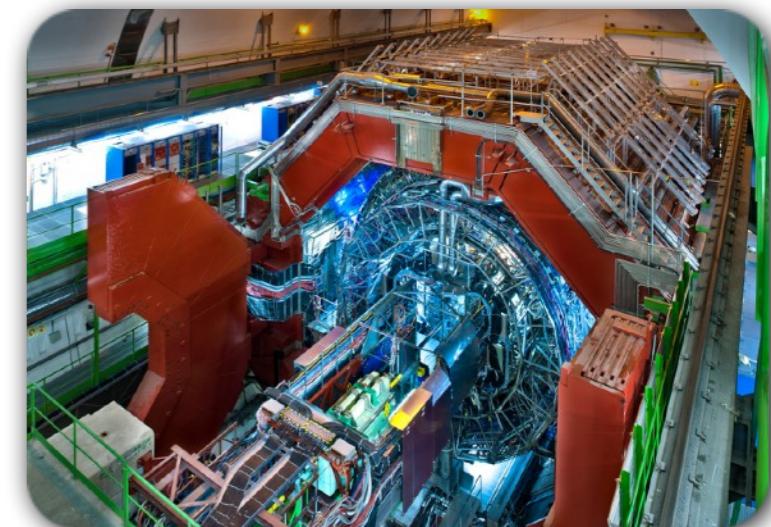
Precision flavor measurements

Covers 4% solid angle
(25% of b decays)
Indirect searches for
new physics

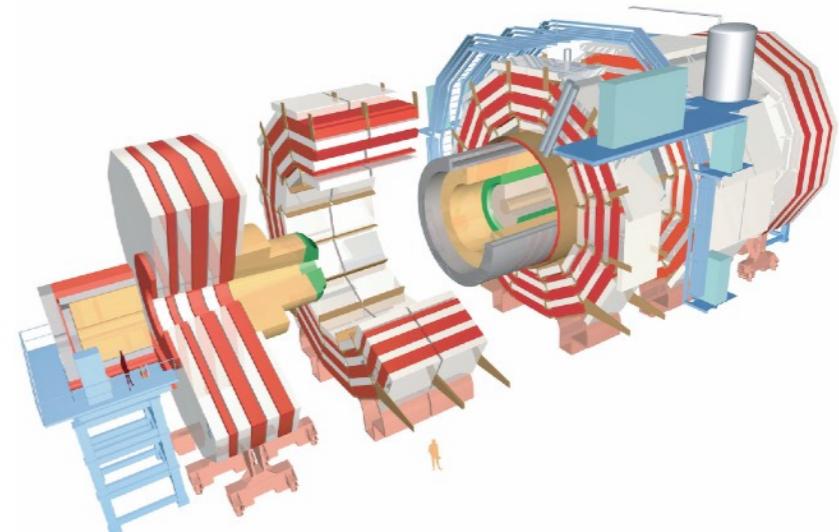
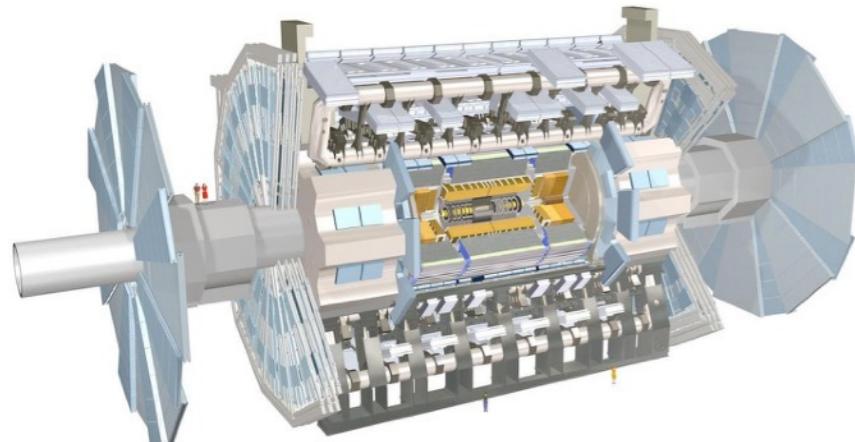


Heavy ion measurements

Focus on quark-gluon plasma

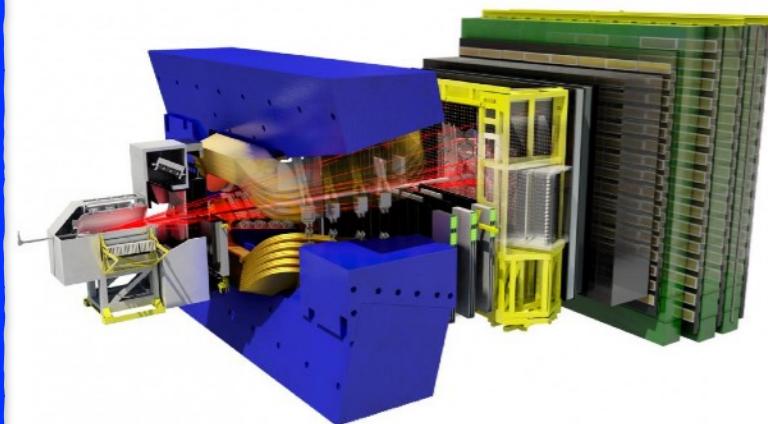


Overview of the LHC experiments



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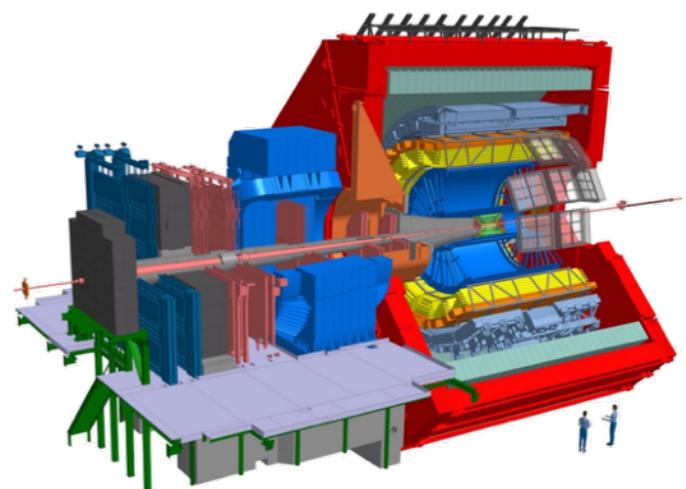
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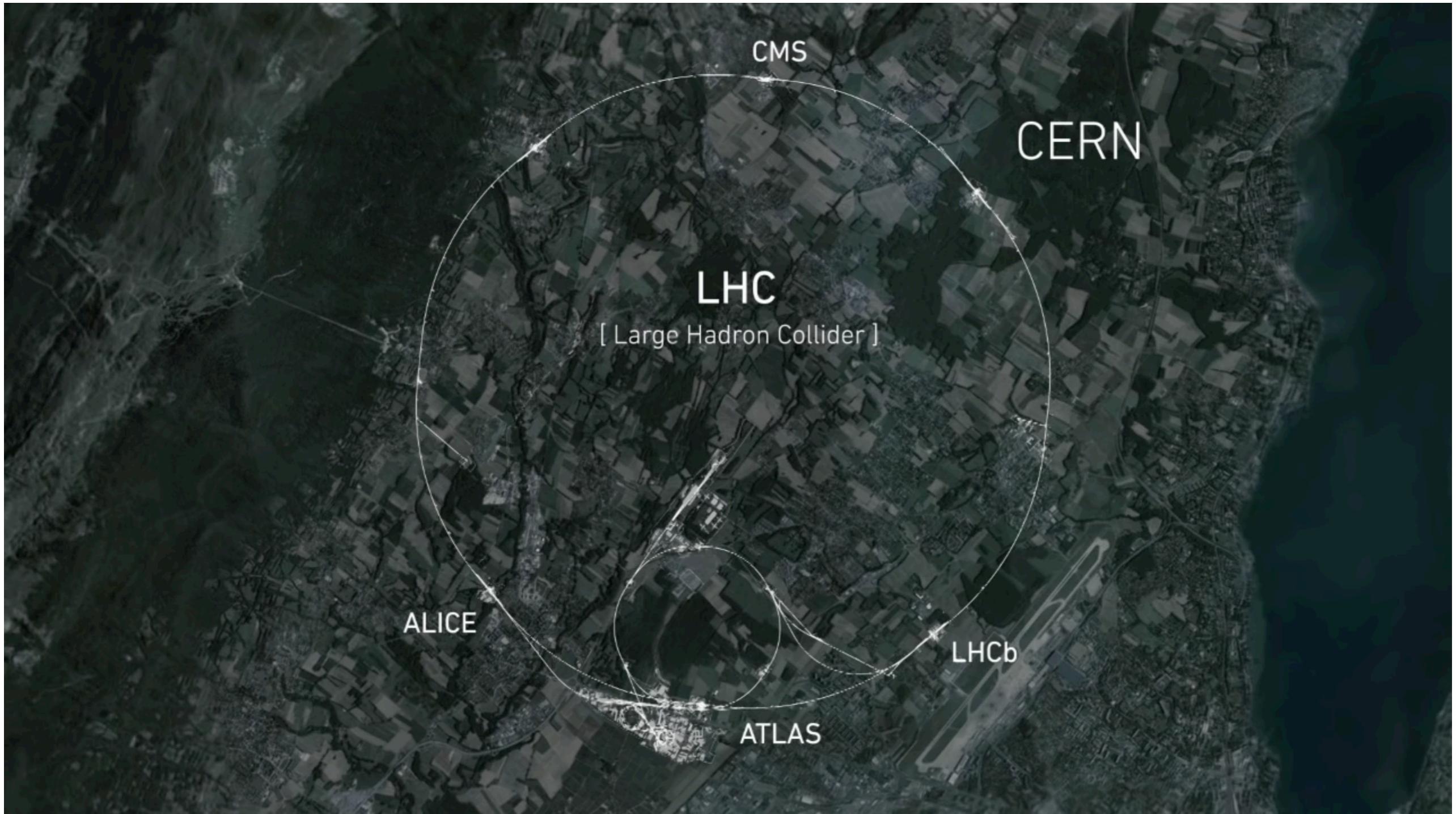


Heavy ion measurements

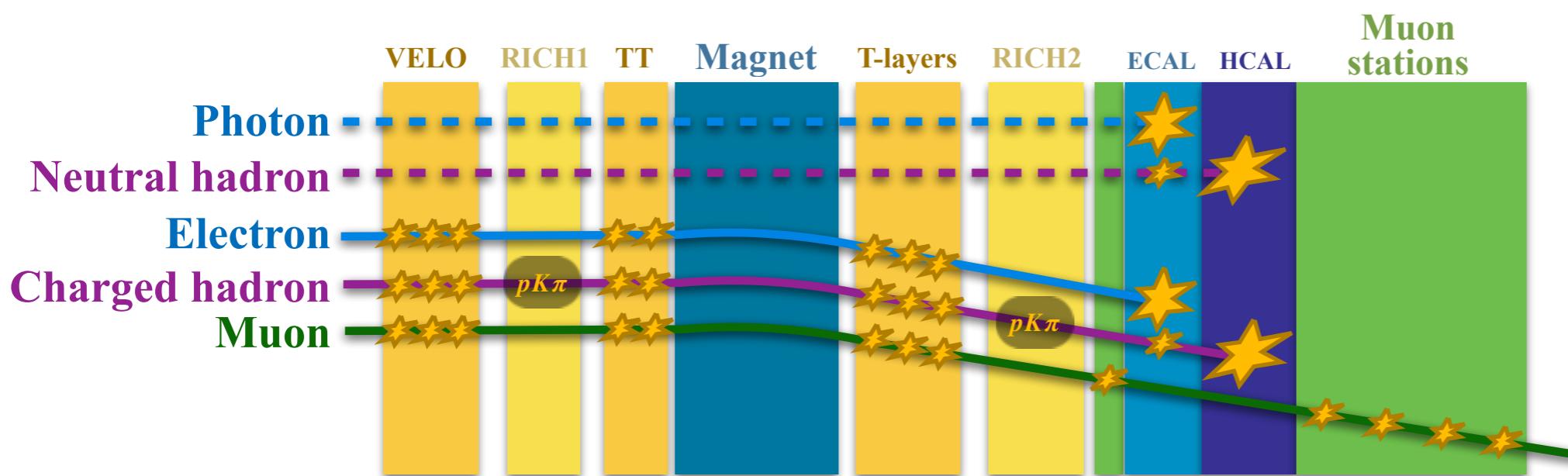
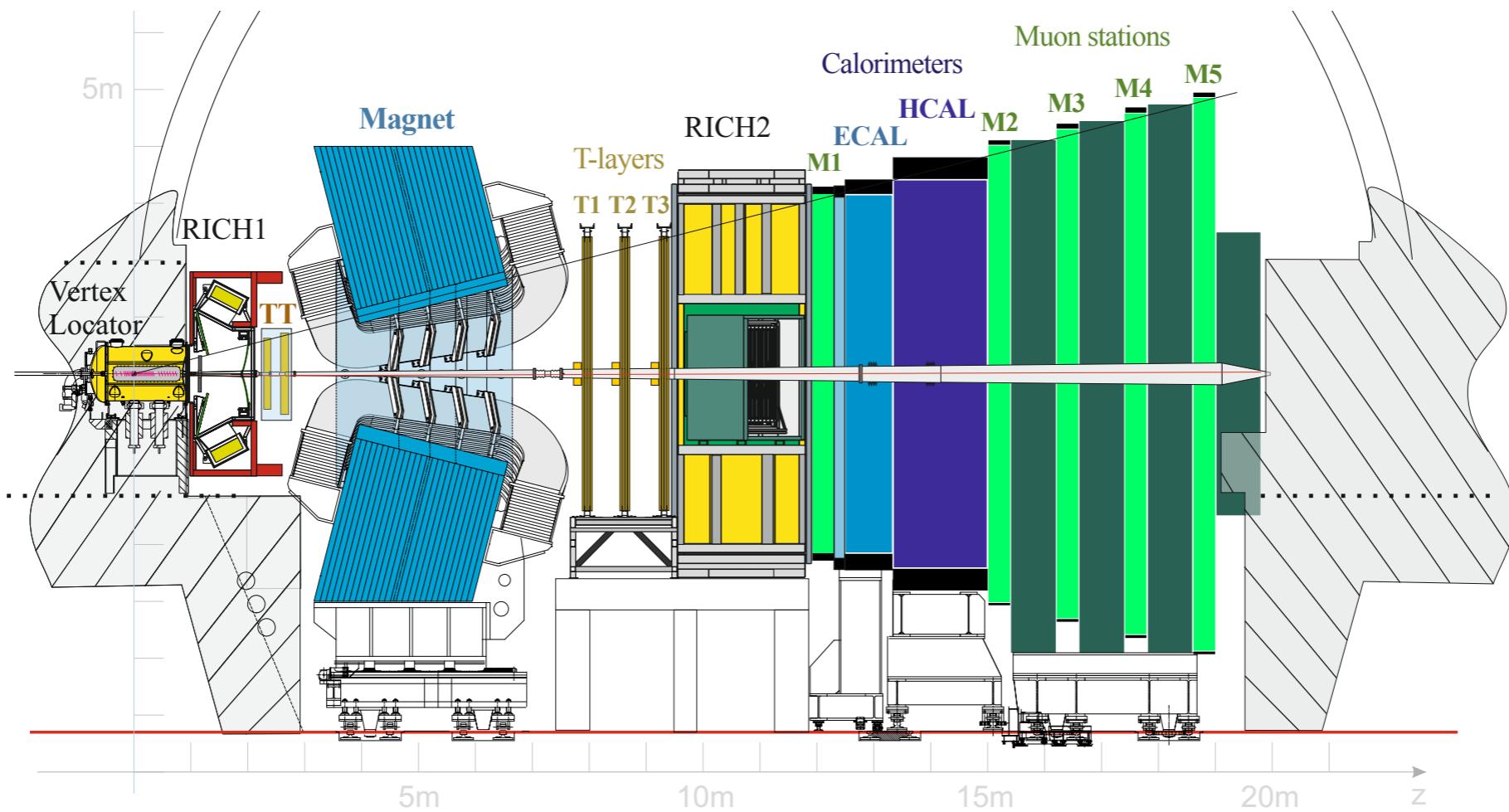
Focus on quark-gluon plasma



Path of the protons

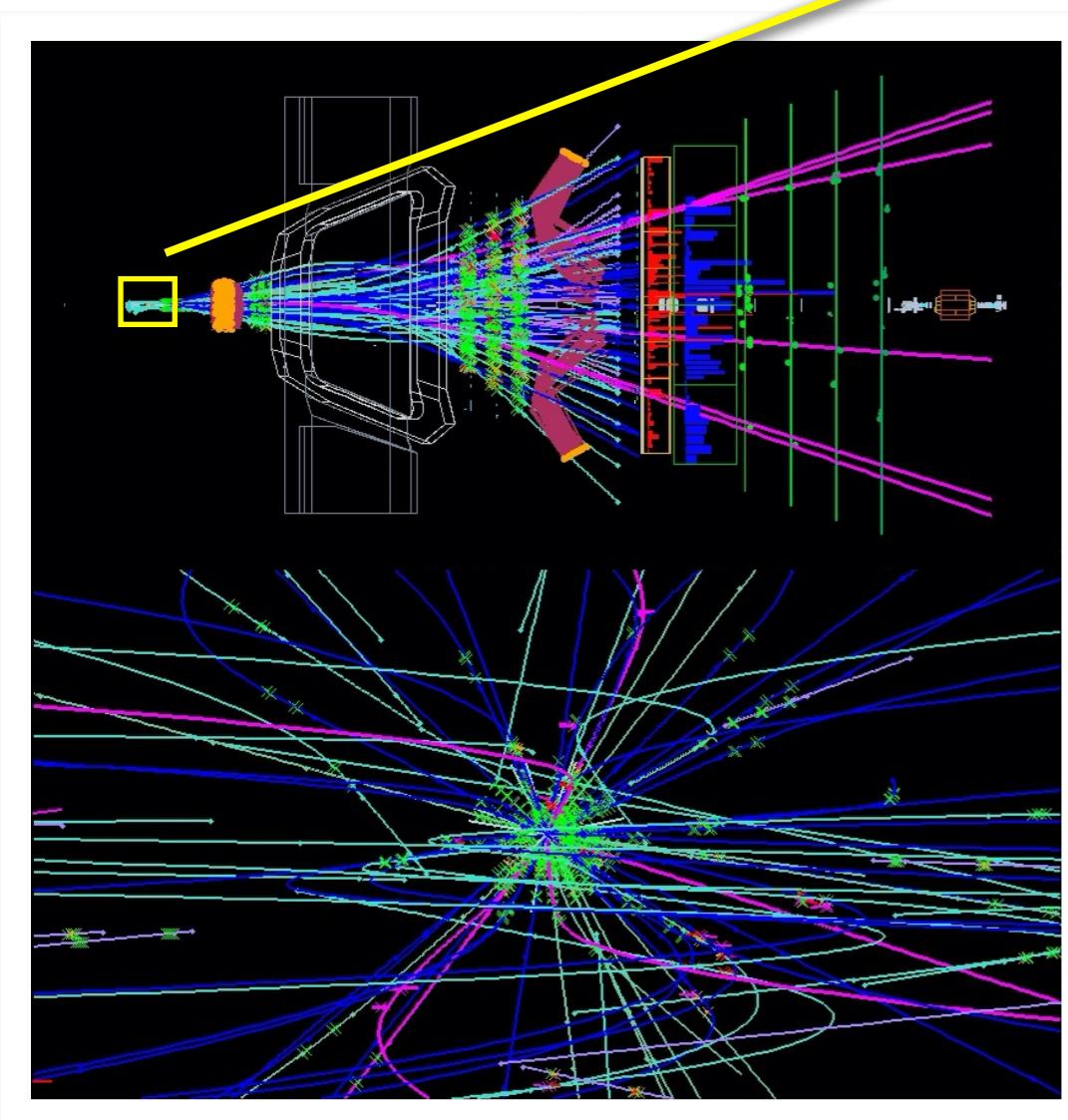


LHCb subdetectors



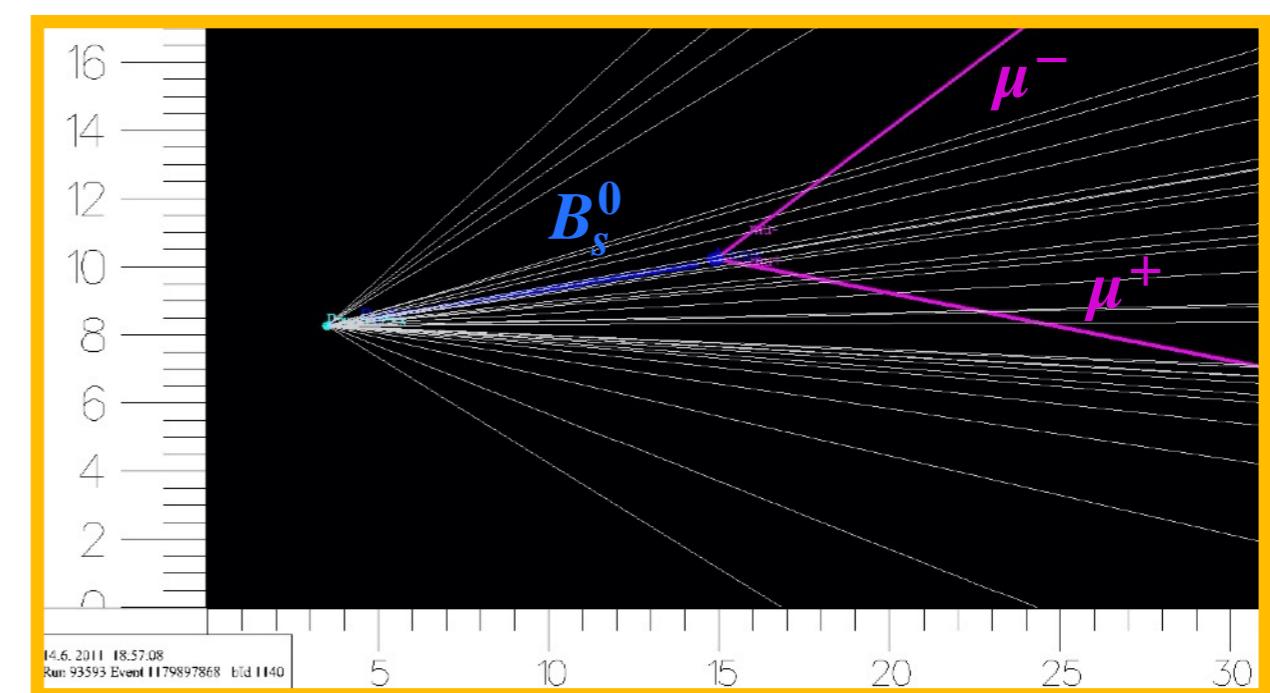
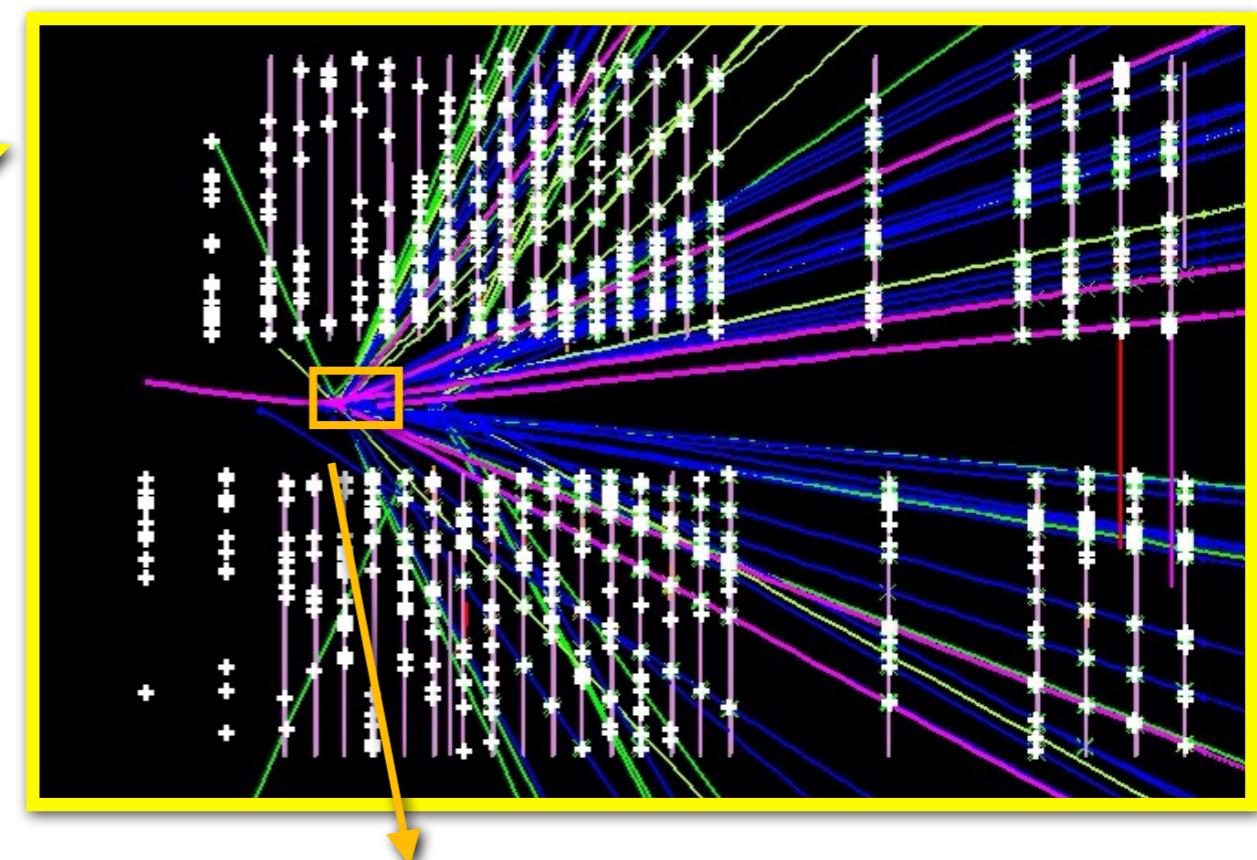
Vertexing is key

B mesons fly a few cm
thanks to LHC boost →
reconstruct decay vertex



$$pp \rightarrow X_b B_s^0 X$$

$$B_s^0 \rightarrow \mu^+ \mu^-$$



Invariant mass reconstruction

Find momenta of **two muon candidates**

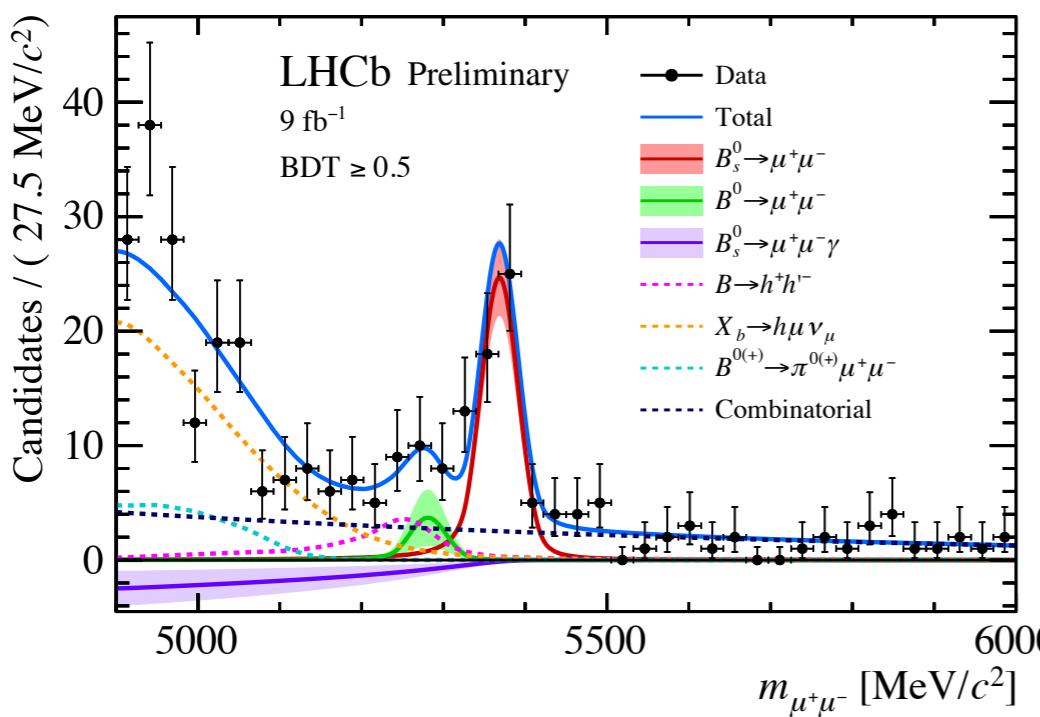
$$\vec{p}^{\mu 1} = (p_x^{\mu 1}, p_y^{\mu 1}, p_z^{\mu 1}) \text{ and } \vec{p}^{\mu 2} = (p_x^{\mu 2}, p_y^{\mu 2}, p_z^{\mu 2})$$

Calculate **invariant mass** $m(\mu^+ \mu^-)$ assuming they are muons
 $(m_\mu = 105.658 \text{ MeV from } \underline{\text{PDG}})$

Important

$$m(\mu^+ \mu^-) = \sqrt{(E^{\mu 1} + E^{\mu 2})^2 - (\vec{p}^{\mu 1} + \vec{p}^{\mu 2})^2} =$$

$$\sqrt{\left(\sqrt{(p^{\mu 1})^2 + m_\mu^2} + \sqrt{(p^{\mu 2})^2 + m_\mu^2} \right)^2 - \left(p_x^{\mu 1} + p_x^{\mu 2} \right)^2 - \left(p_y^{\mu 1} + p_y^{\mu 2} \right)^2 - \left(p_z^{\mu 1} + p_z^{\mu 2} \right)^2}$$



Signal events coming from $B_s^0 \rightarrow \mu^+ \mu^-$ **peak** at B_s^0 mass,
background events have different shapes