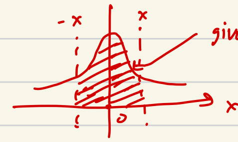
220412

Error function

We use Erf of the following form:

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

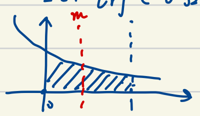
which is defined for a Gauss distribution of $\mu=0$ and $\sigma = 1/\sqrt{2}$



given by Erf(x)

For a generic Gaussian w/ μ and σ, it's cdf is:

$$\frac{1}{2}\left(1+\mathrm{Erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$$



Shaded area: $\frac{1}{2}\left(\mathrm{Erf}\left(\frac{1-\mu}{\sigma\sqrt{2}}\right)-\mathrm{Erf}\left(\frac{0-\mu}{\sigma\sqrt{2}}\right)\right)$

We need to find the value $m \in [0,1]$

s.t. $\mathrm{Erf}\left(\frac{m-\mu}{\sigma\sqrt{2}}\right)-\mathrm{Erf}\left(\frac{0-\mu}{\sigma\sqrt{2}}\right) = \mathrm{Erf}\left(\frac{1-\mu}{\sigma\sqrt{2}}\right)-\mathrm{Erf}\left(\frac{m-\mu}{\sigma\sqrt{2}}\right)$

$\rightarrow \mathrm{Erf}\left(\frac{m-\mu}{\sigma\sqrt{2}}\right) = \underbrace{\frac{1}{2}\left(\mathrm{Erf}\left(\frac{0-\mu}{\sigma\sqrt{2}}\right)+\mathrm{Erf}\left(\frac{1-\mu}{\sigma\sqrt{2}}\right)\right)}_{\text{denote this as } k}$ ← this is directly computable

$\rightarrow \frac{m-\mu}{\sigma\sqrt{2}} = \mathrm{Erf}^{-1}(k)$

$\rightarrow m = \sigma\sqrt{2}\,\mathrm{Erf}^{-1}(k)+\mu$