COMPUTER SCIENCE
UNIVERSITY OF MARYLAND

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Solution Sketches

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P1 – Relatively Prime Floating Mountains

Problem: All pairs of elements of a given set relatively prime?

Solution: Two nested loops to iterate/test all pairs.

```
boolean allPairsRelativelyPrime(int[] d) {
   for (int i = 0; i < d.length; i++)
      for (int j = i+1; j < d.length; j++)
         if (gcd(d [i], d [j]) != 1) return false
   return true;
}
```

P2 – Tsireya's Number Play

Problem: Given a number and 2 digits, make the largest number

Solution: Iterate through the digits. Find first digit smaller than new digit and insert before it. Otherwise insert at end

Example: 987654321 and 7, 0: Answer: 9877654321<u>0</u>

Java notes:

- Convert integer into a string of digits: `Integer.toString(n)`
- Substring of first i symbols: `str.substring(0, i)`
- Remaining substring: `str.substring(i)`
- Insert new digit into string after index i:

```
str.substring(0, i) + Integer.toString(digit) + str.substring(i)
```

P2 – Tsireya's Number Play

Find first digit of n smaller than digit, and insert digit before it:

```
String str = Integer.toString(n) // convert n to string form
boolean inserted = false
for(int i = 0; i < str.length(); i++) {
    if( (str.charAt(i) - '0') < digit) { // i-th digit of number < new digit?
        str = str.substring(0, i) + Integer.toString(digit) + str.substring(i)
        inserted = true; break
    }
}
if (!inserted) str = str + Integer.toString(digit) // insert at end
```

Repeat for both digits

P3 – Interleaved Omangi Messages

Problem: Decrypt string of length n by “unshuffling” every n/k-th character

Solution: Let off = n/k be the offset. Nest two loops. The outer gives the starting character i=0...(off-1) and the inner generates offsets i+0, i+off, i+2·off, …

```
String result = ""
    int offset = s.length()/k // offset size
    for (int i = 0; i < offset; i++) { // start of each group
        for (int j = i; j < s.length(); j += offset)
            result += s.charAt(j) // take every offset character
    }
}
```

P4 – Woodsprites' Patterns

Problem: Compute $\sum_{i=1}^{n} \lfloor n/i \rfloor$ for **very large** n

Observation:

- Takes a long time if n is very large
- When i is large ($i > \sqrt{n}$), there are many repeats.
- For example, if $n = 100$, then $3 = \lfloor n/26 \rfloor = \lfloor n/27 \rfloor = \ldots = \lfloor n/33 \rfloor$
- Compute $\lfloor n/a \rfloor$ directly for small $a \leq \sqrt{n}$ and in groups for large a

How many repeats?

- $\lfloor n/a \rfloor = k \Rightarrow \frac{n}{k+1} < a \leq \frac{n}{k} \Rightarrow \left\lfloor \frac{n}{k} \right\rfloor - \left\lfloor \frac{n}{k+1} \right\rfloor$ repeats

P4 – Woodsprites' Patterns

Compute $k = \lfloor n/a \rfloor$ based on size of a

```
long count = 0
long sqrt = Math.sqrt(n)
for (long k = 1; k <= sqrt; k++) { // n/a for large a in groups
    long count1 = n/k - n/(k+1)
    count += count1 * k
}
for (long a = n/(sqrt+1); a >= 1; a--) { // n/a for small a
    count += n/a
}
return count
```

P5 – Unstable Floating Mountains

Problem: Determine whether a sequence comes from a subprime modular (1007) Fibonacci series, allowing skips of up to 9 items

Strategy:

- The first two elements determine the entire sequence, but we only have one (that is, $seq[0]$) since $seq[1]$ may have been skipped. So we need to try all possible choices
- To test a sequence, we try to match an entry against the first 10. For each that match, recursively check the remaining elements.
- Tests all sequences of the form $\mathrm{f}(1) = x$, $\mathrm{f}(2) = seq[0]$, for $0 \leq x < 1007$.
  - If none work, return null
  - If any work, take the one that minimizes the second element

P5 – Unstable Floating Mountains

Test whether a portion of a sequence (seq) matches the sequence f(1)=a, f(2)=b, …

```
boolean satisfied(int a, int b, int[] seq) { // does seq match Fibonacci with skips?
     if(seq.length == 0) return true // empty sequence trivially matches
     for(int i = 0; i < 10; i++) { // try all possible skips
          int next = largestFactors[(a + b) % 1007] // next element of Fibonacci series
          if (next == seq[0]) { // do with match the next item in Fibonacci?
               if(satisfied(b, next, seq[1..seq.length-1])) { // yes...recursively check remainder
                    return true
               }
          }
          a = b; b = next // didn't match - try next in Fibonacci series
     }
     return false
}
```

Precompute largest factors

P5 – Unstable Floating Mountains

Overall solution:

```
int first = seq[0] // fix f(1) to first in element of sequence
int lowestSecond = ∞
for(int zeroth = 0; zeroth < PRIME; zeroth++) { // try all possible 0th elements
    // 0th and 1st define the sequence – need to check that rest of sequence matches
    if (satisfied(zeroth, first, seq[1..seq.length])) { // do we match?
        int second = largestFactors[(zeroth + first) % PRIME] // yes..generate 2nd
        if (second < lowestSecond) // best 2nd seend so far?
            lowestSecond = second // save it
    }
}
// Generate the final answer from first and lowestSecond
```

Problem 6 (Path-Breaker)

Input: $seed$, $N = 10$, $K = 2$

We generate a permutation, P, using the seed

$$P = [1, 3, 0, 2, 5, 4, 6, 9, 8, 7]$$

Place defense systems on 0 and 7

Output: 7 habitats can be saved

```
PriorityQueue<Integer> PQ = new PriorityQueue<>(Collections.reverseOrder());

for (int i = 0; i < N; ++i) {
  if (!visited[i]) {
    visited[i] = true;
    int cur = P[i];
    int length = 1;
    while (cur != i) {
      visited[cur] = true;
      cur = P[cur];
      ++length;
    }
    PQ.add(length);
  }
}

int num_saved = 0;
for (int j = 0; j < K && PQ.size() > 0; ++j) {
  num_saved += PQ.poll();
}
```

4 cycles in the permutation above

Problem is to compute the length of each cycle in the permutation

Greedy solution of placing defense units on the largest cycles is optimal

P7 – Akula Topple

Problem: A game with 2 players, 5 objects [A B C D E], assignment of objects to players (e.g., 1 ⟵ {A,C,D}, 2 ⟵ {A,B,E}), and each player has 6 actions. Determine the winner.

Solution:

- Classical “complete information” game.
- Game state:
  - char[5] indicating the current positions of objects
  - int[3] indicating the remaining moves by player 1 (1-up, 2-up, or Topple)
  - int[3] indicating the remaining moves by player 2
  - Brute force: $5!\,(3^3)(3^3) = 87{,}480$ possibilities
- Each game state is winning either for player-1 or player-2

P7 - Akula Topple

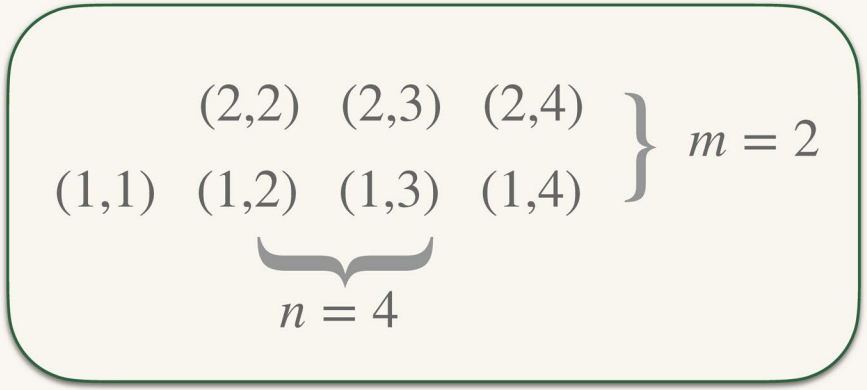
Solution:

- Each game state is winning either for player-1 or player-2
- There are 12 plays in total (6 apiece for players 1 and 2)
- Simulate the game tree, using the set of winning/losing configurations conditioned upon the number of plays remaining in the game.
- Use dynamic programming to “memoize” the results, avoiding the exponential blowup of states

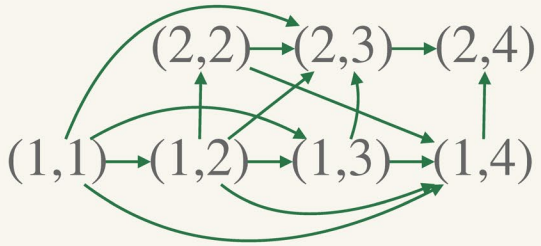
Problem 8 (Hidden Pathways)

Had a small interpretation issue; fixed during clarifiactions in the contest

Input: $n = 4, m = 2$



Should have had $(2,2) \to (1,4)$!



Edge between $(1,3)$ and $(2,3)$ since

$gcd(1+3, 2+3) = gcd(4,5) = 1$

8	6	2	1	1	num_paths
1	1	2	2	1	counts
2	3	4	5	6	values

(1,1) (1,2) (2,2) (1,4) (2,4)
(1,3) (2,3)

num_paths[i] represents paths from one of the (potentially multiple) nodes with sum = i

```
for (int k=m+n; k>1; --k) {
  for (int i = k+1; i <= m+n; ++i) {
    if (gcd(k, i) == 1) {
      num_paths[k] += counts[i] * num_paths[i];
    }
  }
}
```

P9 – Starfish

Problem: Given points X on the line, capacity (cap), length (len), and gap size (gap), compute disjoint intervals that cover a maximum number of points of X, such that no interval has more than cap points, no interval is longer than len, and no two intervals are closer than gap

Solution: Based on dynamic programming

- Precomputed helper indices: Suppose an interval ends at X[i]
  - cap[i] = i-cap+1 // earliest start to interval contains at most cap points
  - len[i] = $\min_{1 \le j \le i} (x[i] - x[j] \le len)$ // earliest start so interval of length at most len
- Suppose interval starts at X[i]
  - gap[i] = $\max_{0 \le j < i} (x[i] - x[j] \le gap)$ // latest end for an interval for gap bound

P9 – Starfish

- Define: $M[i]$ = largest number of points covered by valid intervals ending at $X[i]$
- How to compute $M[i]$?
  - Smallest starting index s for the interval is $\max(len[i], cap[i])$
    $$\Rightarrow \max(len[i], cap[i]) \leq s \leq i$$
  - Largest ending index t for prior interval is $gap[s]$
    $$\Rightarrow 0 \leq t \leq gap[s]$$
  - This covers $(i - s + 1)$ starfish plus prior starfish up to X[t]

$$M[i] = \max_{\max(len[i],cap[i]) \leq s \leq i} \; \max_{0 \leq t \leq gap[s]} ((i - s + 1) + M[t])$$