DEPARTMENT OF COMPUTER SCIENCE

HSPC 2024: Solution Sketches

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P9	Bowser’s Diabolical Parentheses

$P1$: Koopa Rescue Mission

Problem: Given a string S, and strings S1, S2:

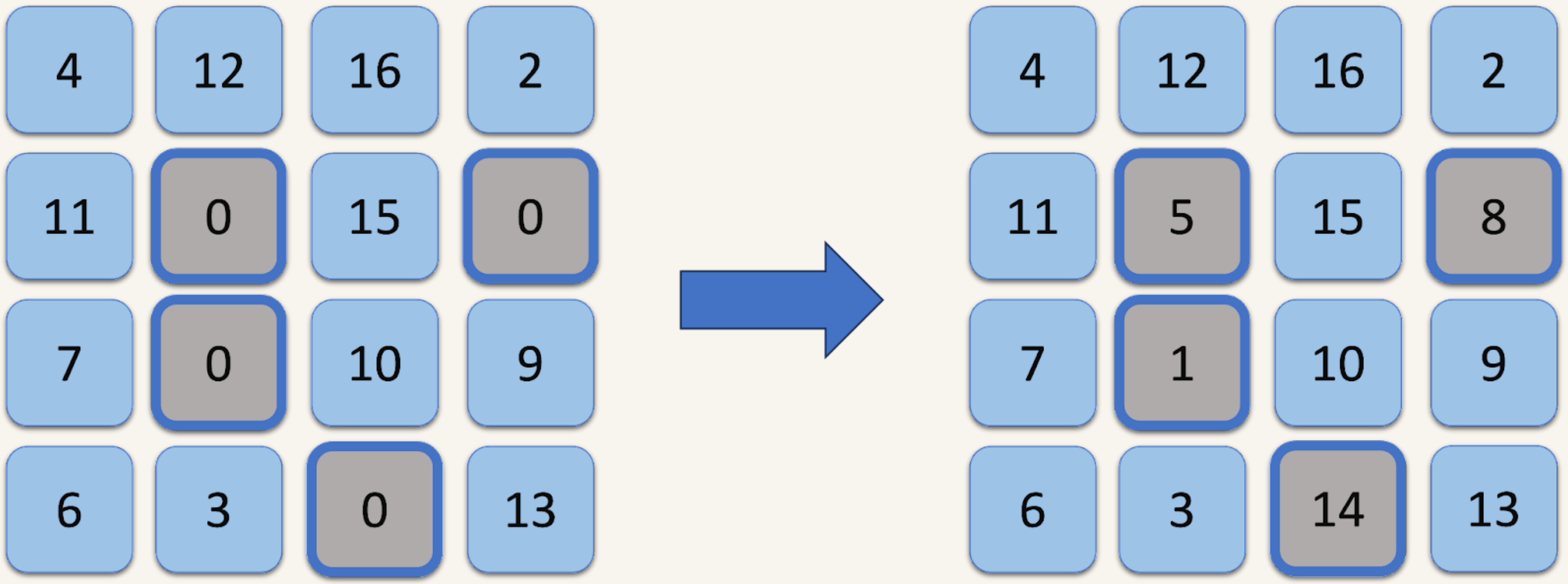
- If S2 is present in S, compute the number of times a string S1 appears in S, and

Solution: First check if S2 is present using S.contains(S2). If it does, repeatedly call S.indexOf(S1, offset) to count all occurrences of S1 in S.

```
private static int lookForKoopa(String str) {
  int result = -1;
  if (str.contains("Peach") ) {
    //count the Koopas
    result = 0;
    int searchPos = 0;
    int resultPos;
    while ((resultPos=str.indexOf("Koopa",searchPos)) != -1) {
      result++;
      searchPos=resultPos+5;
    }
  }
  return result;
}
```

$P2$: Mario's Magic Matrix

Problem: Given a 4 x 4 grid containing the numbers [1, 16] (each once). 4 cells are erased (along with 4 numbers in [1, 16]). Find the best way to place the numbers to maximize the sum of row products:

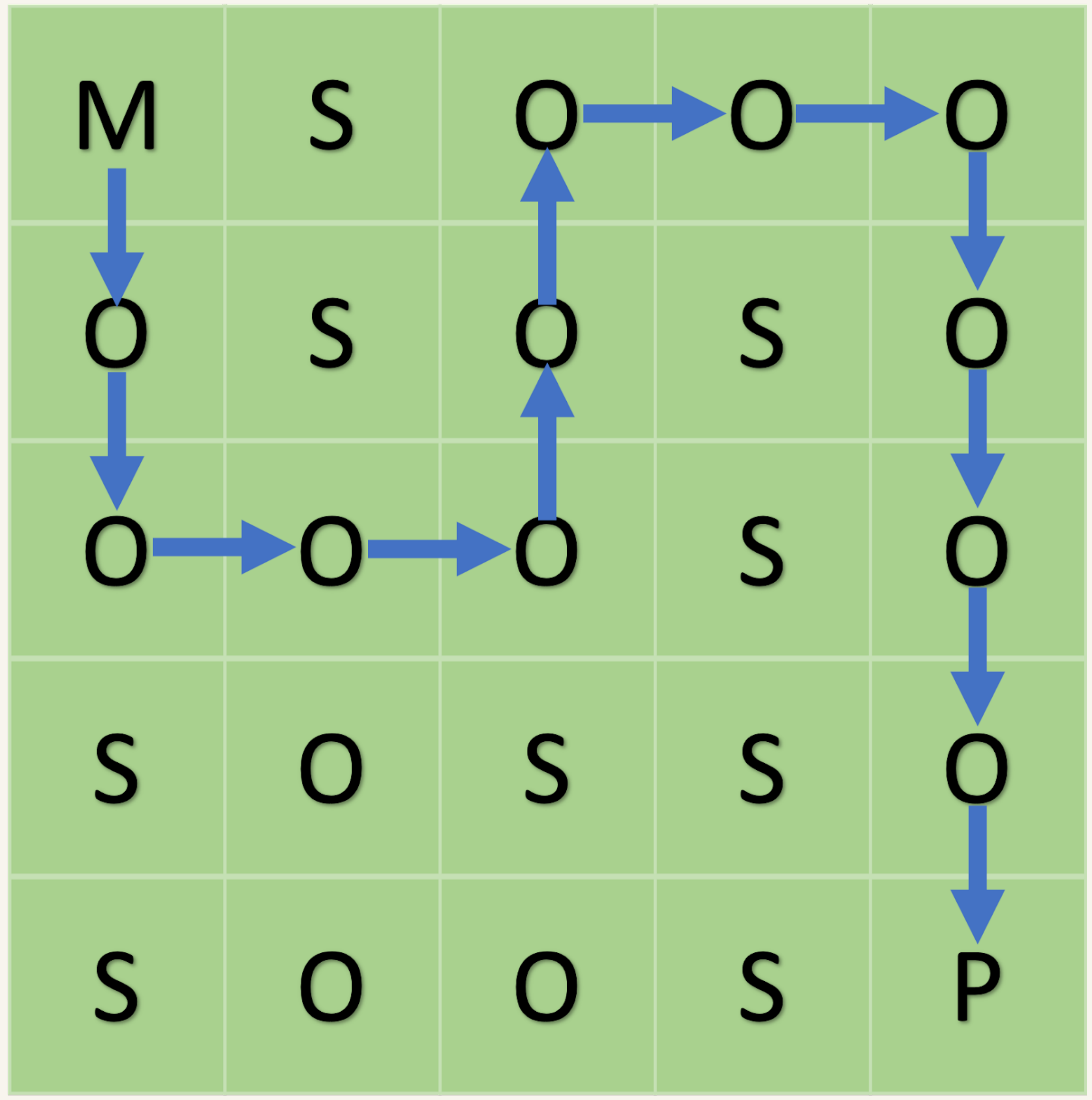


Solution: Try all 4! ways of placing the missing numbers, compute the score for each placement and take the best one.

P3 : Mario’s Perilous Path

- 'M': Mario's starting position
- 'P': Princess Peach's location
- 'S': Spikes obstacles Mario cannot pass through
- 'O': Unobstructed path

Mario can only move up/down/left/right on the grid.
Cannot move through spikes or step outside grid.
Problem: find min dist path from Mario to Peach.



Solution: Build a grid graph. For any cell marked “O”, connect it to adjacent cells marked “O”, but not if they are marked “S”. Run a breadth-first search (BFS) from the cell corresponding to M, and store the distance to each reachable cell from M.

$P4$: Princess Peach's Garden Party

Problem: Given an 8 vertex graph with M edges, determine:

- If there is a 4-set of vertices where all 4 vertices are pairwise connected (i.e., there is a 4-clique on these vertices)
- If there is a 3-set of vertices where there are no edges between any of the 3.

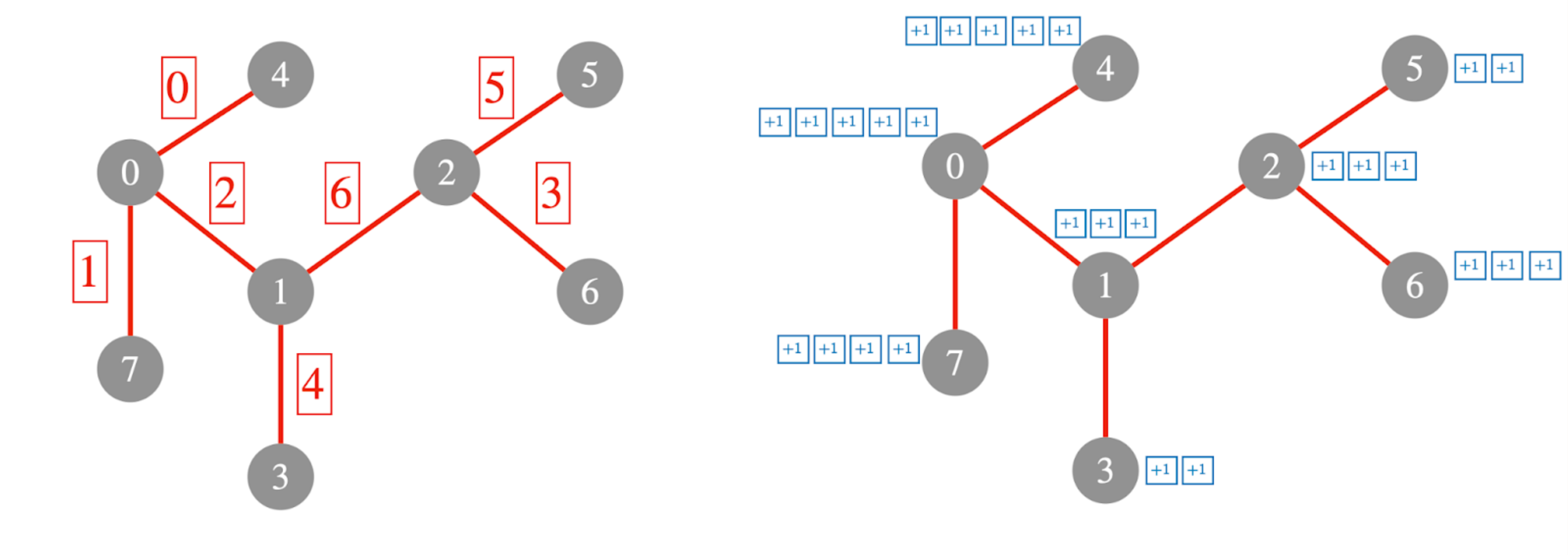
Solution:

- Check all sets of 4 vertices, and determine if they are fully connected. Otherwise return null for this part.
- Check all sets of 3 vertices, and determine if no edges exist among the 3. Otherwise, return null.

P5 : Maximizing Harmony

Problem:

Merge the edges in the tree in order of increasing weight, and determine the maximum number of merges "visible" to a vertex in the tree.



Solution:

Sort the edges by weight. Initially each vertex is in its own cluster.

Store the vertices in a Disjoint Set Union (DSU) data structure (i.e., Union-Find).

Merge each (u,v) edge by:

(1) finding the clusters corresponding to u, v: c(u) and c(v)

(2) merge c(u) into c(v)

(3) set harmony(c(v)) = max(harmony(c(u)), harmony(c(v)) + 1

Return the max harmony we see over the course of all merges.

P5 : Maximizing Harmony

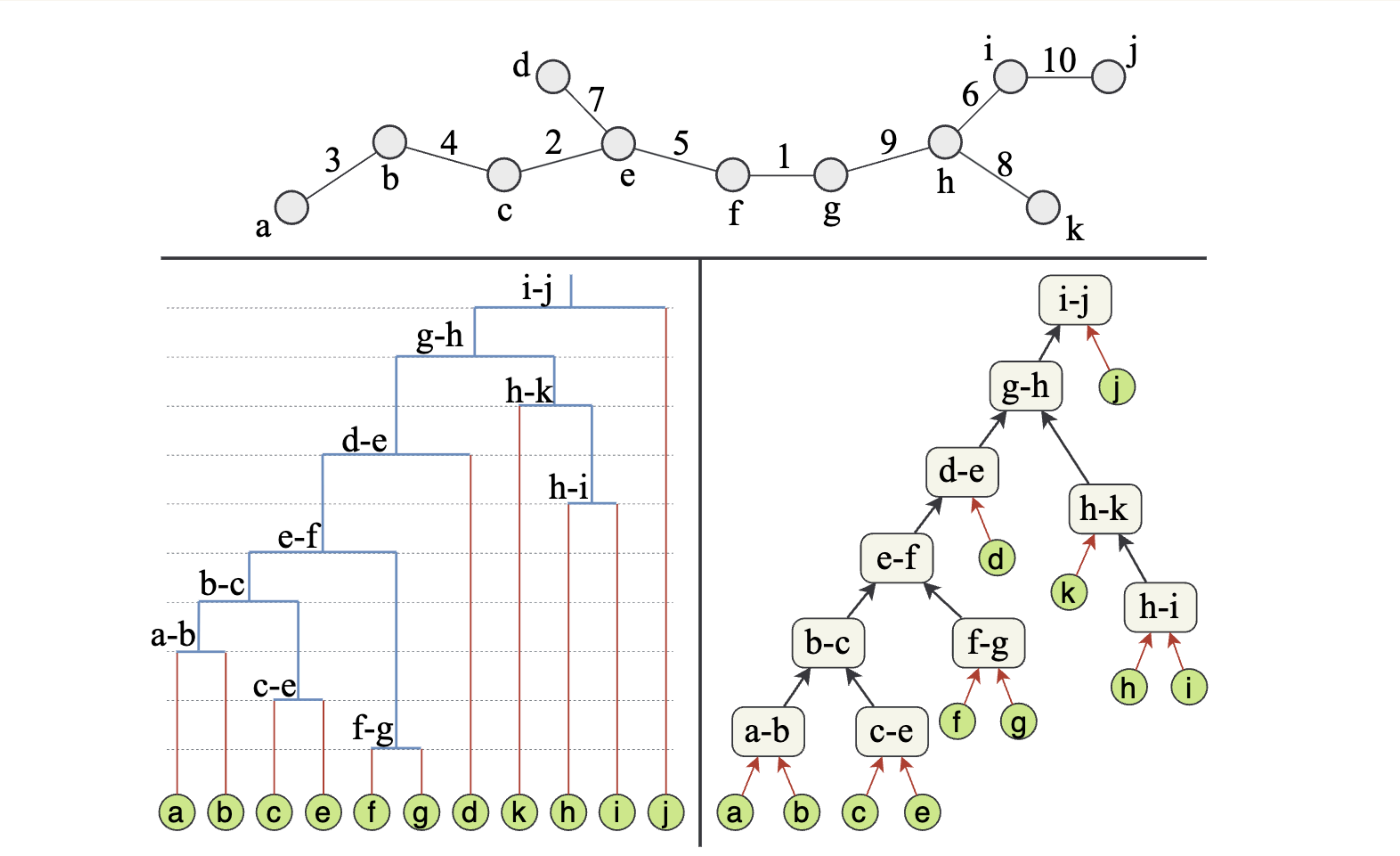
Problem:
Merge the edges in the tree in order of increasing weight, and determine the maximum number of merges "visible" to a vertex in the tree.

https://en.wikipedia.org/wiki/Single-linkage_clustering

Equal to the *height* of the single-linkage dendrogram.

Time complexity is $O(n \log n)$ due to sorting.

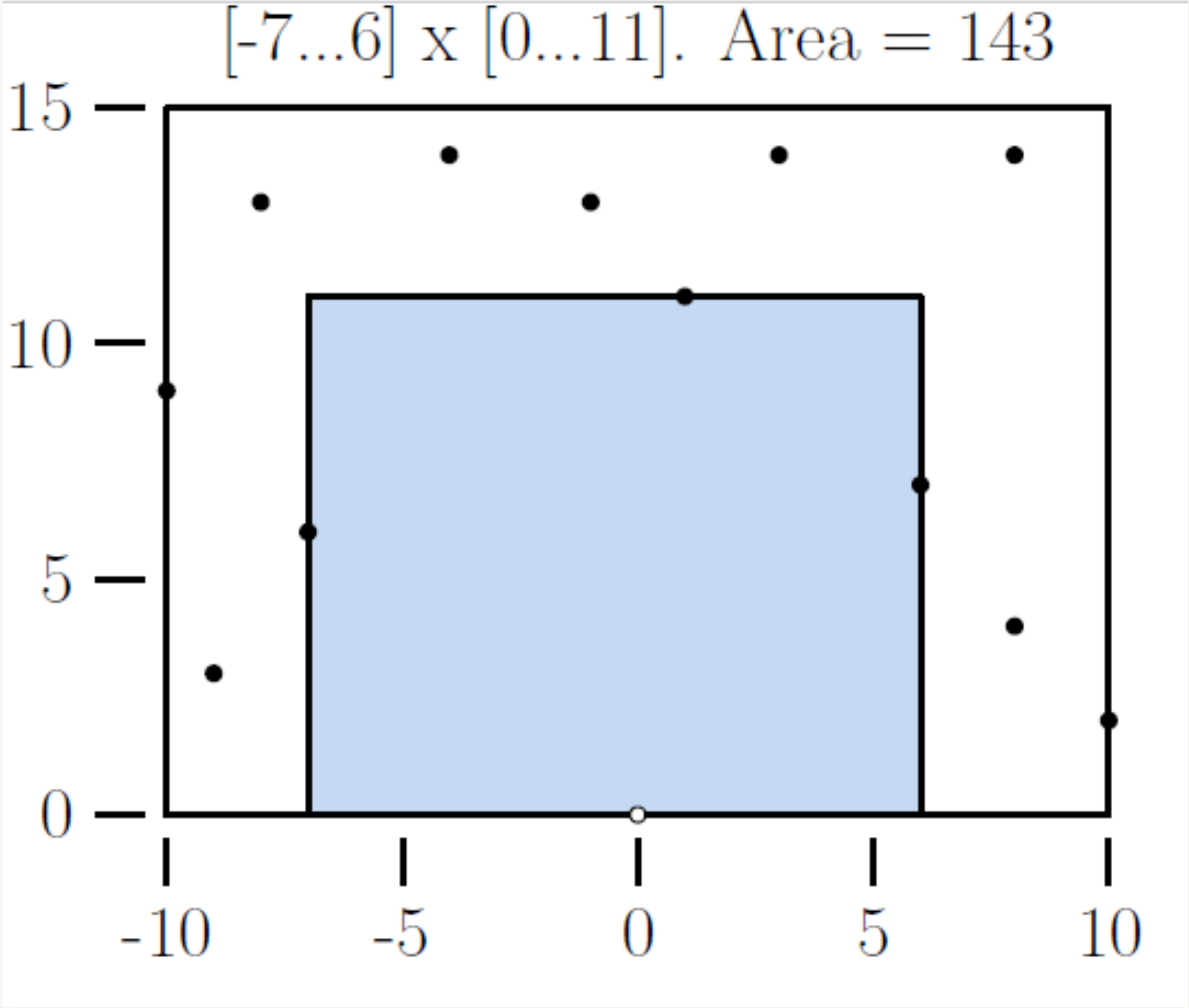
Turns out if the weights are pre-sorted, $O(n)$ is possible using decremental tree connectivity! See Demaine et al. *"On Cartesian trees and range minimum queries"*



$P6$: Magical Platform Planning for the Star Festival

Problem: Find the largest rectangle (by area) that contains the origin, but does not contain any flowers (points on the the 2D plane). The output rectangle should satisfy:

- -maxWidth <= xMin < 0 < xMax <= maxWidth
- 0 < yMax <= maxHeight



xMin = -7, xMax = 6, and yMax = 11

Solution:

- Key idea: the solution is determined by the flowers, since points (flowers) will be on the boundary.
- Use a *sweepline* technique:
  - sort the points by y-coordinate and sweep upward
  - maintain the largest xMin coordinate (left wall) and the smallest xMax coordinate (right wall) with height smaller than the current y coordinate
  - if the current point's x coordinate is "inside" the current rectangle, shrink the left (or right) wall for the next iteration.
- Time complexity: $O(n \log n)$ (for sorting by y)

$P7$: Chompy Chain Clusters

Problem: Vertices are on the 2D grid. Some of the grid cells have "Chompy Chains" that grow clusters starting at those locations. Clusters start at their *start time* and grow at unit rate. If a cluster is already covered when it is supposed to start it doesn't grow. The first cluster that reaches a vertex captures it in its cluster. The problem is to output the **final size of each cluster.**

Solution:

- Build a grid graph on W^2 vertices with unit-weight edges.
- Create a super-source src, and connect to each chain vertex with weight equal to the start time of the chain
- Run Dijkstra, and keep track of the last cluster that reduced the distance to a vertex. When we process a vertex, increase the size of the cluster corresponding to the last visitor.

Cost is $O(W^2 \log W)$ to run Dijkstra. With a little care, $O(W^2)$ is possible.

Related to the *low-diameter decomposition problem*—a very useful primitive for designing fast parallel and distributed graph algorithms: see

[Submitted on 14 Jul 2013]

Parallel Graph Decompositions Using Random Shifts

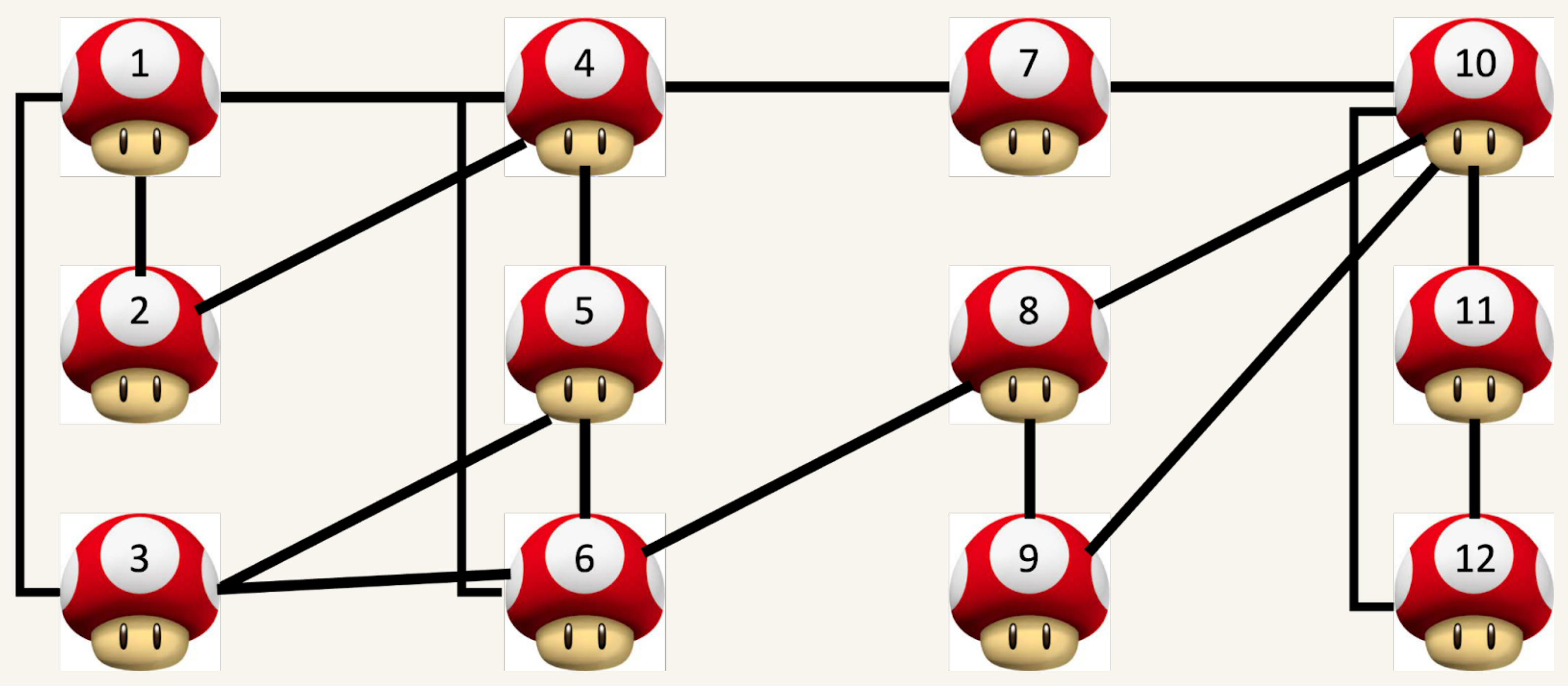
Gary L. Miller, Richard Peng, Shen Chen Xu

$P8$: Colorful Chaos in the Mushroom Kingdom!

Problem: Given a $3 \times W$ size grid (each column has 3 vertices). There can only be edges within a column, or between adjacent columns. The problem is to color the vertices using as few colors as possible (it is always possible to 6-color the vertices).

Solution:

- Fix the number of colors $C < 6$.
- Use dynamic programming. Check whether the i-th group (of the W groups) can be colored using some colors c_1, c_2, c_3 where each $c_i \leq C$.
  - ★ Check if the colors are valid within the group
  - ★ Check all valid possibilities for the previous group, and check if the coloring is consistent.
- Return true (for this guess for C) if and only if the last column can be successfully colored.



Cost (theoretically) is $O(W)$, but the constant factors are pretty large :-)

Problem 9: Balancing Parentheses

- Couple of observations:
  - Only swap consecutive ")("

    Instead of: ")))))(" → "()))))"

    Do: ")))))(" → "))))()" → ")))())" → … → "()))))"
  - Convert the string to an array of numbers:

    Total Sum: 2

) ( ) ( ) (( ) ( ) ( ) ( ) ( )

    -1 0 -1 0 -1 0 1 0 1 0 1 0 1 0 1 0
  - Every swap basically takes a "X X-1 X" triple and makes it "X X+1 X"

) (**)** **(**) (( ) ( ) (**)** **(**) ( )

    -1 **0** **-1** **0** -1 0 1 0 1 0 **1** **0** **1** 0 1 0

) (**(** **)**) (( ) ( ) (**(** **)**) ( )

    -1 **0** **1** **0** -1 0 1 0 1 0 **1** **2** **1** 0 1 0

    Total Sum: 6 → 2 moves
  - Can make moves until:

) (((((((( )))))))

    -1 0 1 2 3 4 5 6 7 6 5 4 3 2 1 0

    Total Sum: 48 → (48-2)/2 = 23 moves