Numerical Methods

Module 4
Probability, Random Processes and Numerical Methods (MAT 204)

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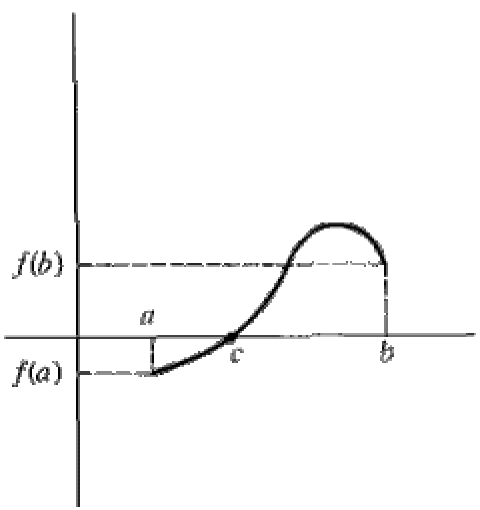
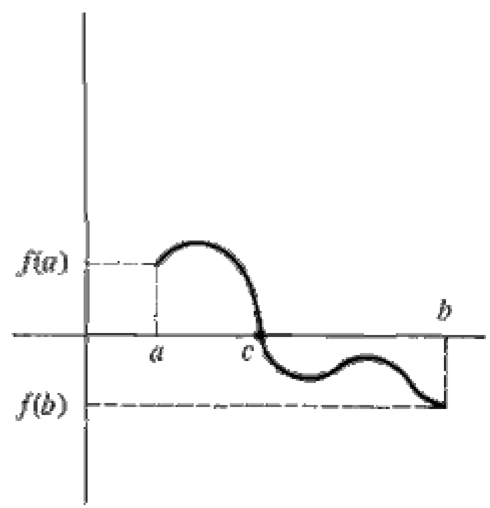
Intermediate value Theorem: If a function f(x) is continuous in closed interval [a,b] and satisfies f(a)>0 and f(b) < 0 then there exists at least one real root of the equation f(x) = 0 in open interval (a,b).





Find the real root of the equation $\cos x = 3x - 1$, using iteration method.

$$\begin{aligned}\text{Let } f(x) &= \cos x - 3x + 1 \\ f(0) &= \cos 0 - 0 + 1 = 2 = (+\text{ ve }) \\ f\left(\frac{\pi}{2}\right) &= 0 - 3\frac{\pi}{2} + 1 = (-\text{ ve })\end{aligned}$$

$\therefore$ A root lies between 0 and $\frac{\pi}{2}$. The given equation can be written as

$$x = \frac{1}{3}(1 + \cos x)$$

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$$x = \frac{1}{3}(1 + \cos x)$$

$$x_{n+1} = \frac{1}{3}(1 + \cos x_n)$$

$$\text{Let } g(x) = \tfrac{1}{3}(1 + \cos x)$$
$$g'(x) = -\tfrac{\sin x}{3}$$

Clearly,

$$|g'(x)| = \frac{|\sin x|}{3} < 1 \text{ in } \left[0, \frac{\pi}{2}\right]$$

Hence iteration method can be applied. Let the initial approximation be $x_0 = 0$
The successive approximation are as follows:

$$x_1 = \phi(x_0) = \frac{1}{3}(1 + \cos x_0) = \frac{1}{3}(1 + \cos 0)$$
$$= 0.66667$$

$$\text{Let } g(x) = \tfrac{1}{3}(1+\cos x)$$
$$g'(x) = -\tfrac{\sin x}{3}$$

$$x_{n+1} = g(x_n) \quad n \geq 0$$

Clearly,

$$|g'(x)| = \frac{|\sin x|}{3} < 1 \text{ in } \left[0, \frac{\pi}{2}\right]$$

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$$= 0.66667$$

$$x_{n+1} = \frac{1}{3}(1 + \cos x_n)$$

$n=1$

$$\begin{aligned} x_2 = \phi(x_1) = \frac{1}{3}(1+\cos x_1) &= \frac{1}{3}(1+\cos 0.66667) \\ &= 0.59529 \end{aligned}$$

$n=2$

$$\begin{aligned} x_3 = \phi(x_2) = \frac{1}{3}(1+\cos x_2) &= \frac{1}{3}(1+\cos 0.59529) \\ &= 0.60933 \end{aligned}$$

$n=3$

$$\begin{aligned} x_4 = \phi(x_3) = \frac{1}{3}(1+\cos x_3) &= \frac{1}{3}(1+\cos 0.60933) \\ &= 0.60668 \end{aligned}$$

$$
\begin{aligned}
x_5 = \phi(x_4) = \frac{1}{3}(1+\cos x_4) &= \frac{1}{3}(1+\cos 0.60668) \\
&= 0.60718 \\
x_6 = \phi(x_5) = \frac{1}{3}(1+\cos x_5) &= \frac{1}{3}(1+\cos 60718) \\
&= 0.60709 \\
x_7 = \phi(x_6) = \frac{1}{3}(1+\cos x_6) &= \frac{1}{3}(1+\cos 0.60709) \\
&= 0.60710 \\
&= 0.60710
\end{aligned}
$$

since the values of x_7 and x_8 are equal, the required root is 0.60710 .

Solve x^3- sin x -1 = 0 by fixed point iteration method correct up to 2 decimal places.

Solution: x^3- sin x -1 =0........ ....(1)
Let f(x) = x^3- sin x -1

f(0) = -1, f(1)= - 0.8415, f(2)=6.0907

As f(1)f(2)< 0 by Intermediate value Theorem the root of real root of the equation f(x) = 0 lies between 1 and 2

Let us rewrite the equation f(x) = 0 of the form x = g(x)
$x= (1 + \text{Sin } x_n)^{1/3} = g_1(x)$

We see that $| g_1'(x) | < 1$ in interval (1,2) containing the root for all values of x.

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We see that $|\, g_1'(x) \,| < 1$ in interval (1,2) containing the root for all values of x.

$x^3 - \sin x - 1 = 0 \;\; \Rightarrow \qquad x = (1 + \text{Sin } x)^{1/3} \qquad x_{n+1} = (1 + \sin x_n)^{1/3}$

We use $x_{n+1} = (1 + \text{Sin } x_n)^{1/3}$ as the successive formula to find approximate solution (root) of the equation (1).

Let $x_0 = 1.5$ be initial guess to the equation (1).

Then $x_1 = (1 + \text{Sin } x_0)^{1/3} = (1 + \text{Sin } 1.5)^{1/3} = 1.963154$
$x_2 = (1 + \text{Sin } x_1)^{1/3} = (1 + \text{Sin } 1.963154)^{1/3} = 1.460827$
$x_3 = (1 + \text{Sin } x_2)^{1/3} = (1 + \text{Sin } 1.460827)^{1/3} = 1.440751$
$x_4 = (1 + \text{Sin } x_3)^{1/3} = (1 + \text{Sin } 1.440751)^{1/3} = 1.441289$

which is the root of equation (1) correct to two decimal places.

$x^3 - \sin x - 1 = 0 \quad \Rightarrow \quad x = (1 + \text{Sin } x)^{1/3} \qquad x_{n+1} = (1 + \sin x_n)^{1/3}$

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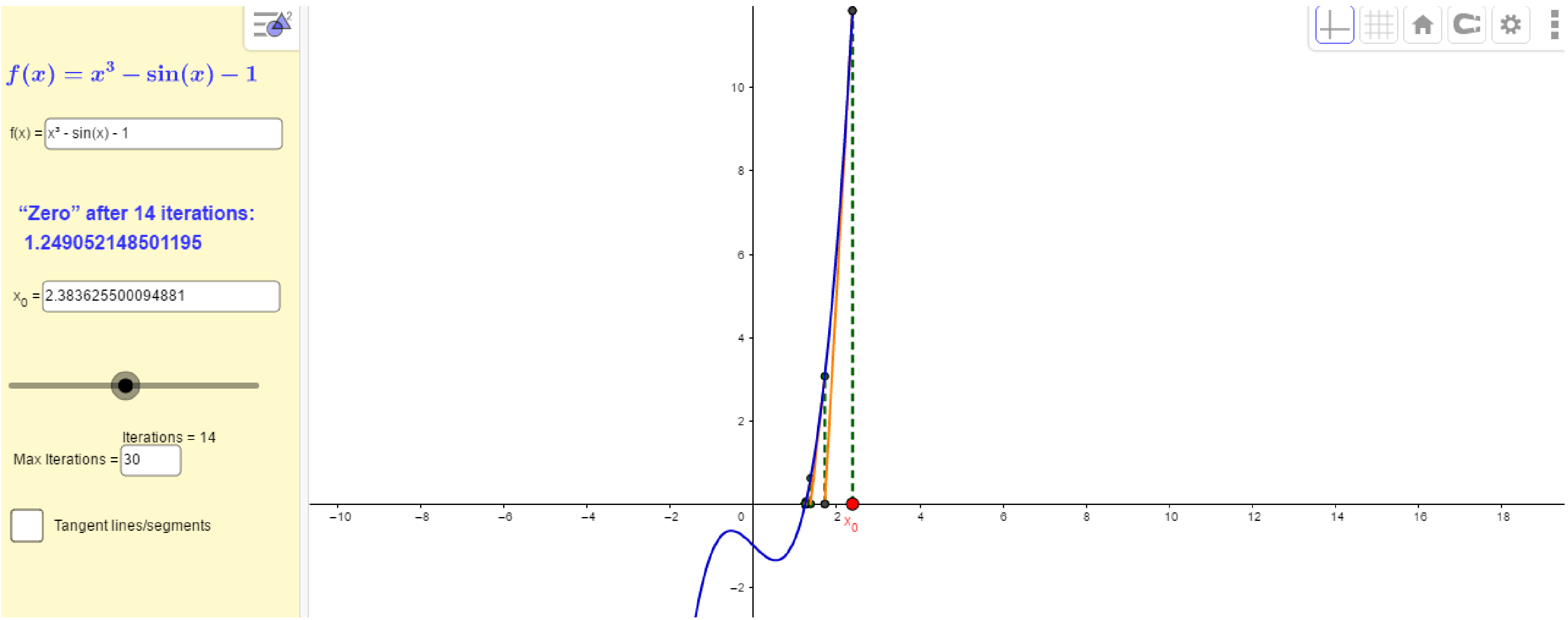

$f(x) = x^3 - \sin(x) - 1$
f(x) = x³ - sin(x) - 1
“Zero” after 14 iterations:
1.249052148501195
x0 = 2.383625500094881
Iterations = 14
Max Iterations = 30
Tangent lines/segments
x0

Newton method / Newton- Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton method / Newton- Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Q. Find the root of $x^3 - 4x - 9 = 0$ correct to 4 decimal places?

Ans:

Here,

$$f(x) = x^3 - 4x - 9 = 0$$
$$f(0) = -9$$
$$f(1) = -12$$
$$f(1) = -12$$

Consider, $|f(2)| \& |f(3)|$

$$|-9| \qquad\qquad |6|$$
$$9 \qquad\qquad\qquad 6.$$
$$9 \qquad > 6 \to f(3)$$

So choose $x_0 = 3$.

Also $f(x) = x^3 - 4x - 9$ and $\qquad f^{/}(x) = 3x^2 - 4$

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Also $f(x) = x^3 - 4x - 9$ and $f^{/}(x) = 3x^2 - 4$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_{n+1} = x_n - \frac{x_n^3 - 1x_n - 9}{3x_n^2 - 4}$$

Put $x_0 = 3$,

Finding

$$x_1 = 2.739130435$$
$$x_2 = 2.706997997$$
$$x_3 = 2.706528054.$$
$$x_4 = 2.706527954$$

ie $x = 2.7065$

Solve $x^4 - x - 7 = 0$ Newton- Raphson method correct up to 6 significant digits.

Solution: $\mathbf{x^4 - x - 7 = 0}$ ....(4)

Let $f(x) = \mathbf{x^4 - x - 7 = 0}$

$f(0) = -7$
$f(1)= -7$
$f(2)= 5$

As $f(1)<0$ and $f(2)> 0$ by the Intermediate value Theorem the root of the real root of equation $f(x) = 0$ lies between 1 and 2

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let x_0 = 1.5 be the initial guess to the equation (2).

Then $x_1 = x_0 - [f(x_0) / f'(x_0)] = 1.5 - f(1.5)/ f'(1.5) = 1.78541$

$x_2 = x_1 - [f(x_1) / f'(x_1)] = 1.7854- f1.7854)/ f'(1.7854) = 1.85876$

$x_3 = x_2 - [f(x_2) / f'(x_2)] = 1.85643$

$x_4 = x_3 - [f(x_3) / f'(x_3)] = 1.85632$

which is the root of equation (2) correct to 6S.

Find the value of $(24)^{1/3}$ using Newton-Raphson method.

Let $x = (24)^{\frac{1}{3}}$; $x^3 = 24$; $x^3 - 24 = 0.$

$$f(x) = x^3 - 24 \qquad f(0) = -24.$$
$$f(1) = 1 - 24 = -23$$
$$f(2) = 8 - 24 = -16$$
$$f(3) = 27 - 24 = 3.$$

$|f(2)| = |-16| = 16$, and $|f(3)| = |3| = 3$ we have $|f(3)| < |f(2)|$

So choose $x_0 = 3$

$$f'(x) = 3x^2; \qquad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_{n+1} = x_n - \left[\frac{x_n^3 - 24}{3x_n^2}\right]$$

Put $x_0 = 3$;

$$x_1 = 2.888888889$$
$$x_2 = 2.884505808$$
$$x_3 = 2.884499141$$
$$x_4 = 2.884499141$$
$$\therefore\ x = 2.884499141$$

Regula Falsi Method.

Consider the equation $f(x) = 0$. Find two numbers a and b with $a < b$, such that $f(a)\&f(b)$ are of opposite sign. The positive real root in the first approximation is

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

The root in Second approximation is

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)}$$

Similarly steps follows for the evaluation of $x_3, x_4, \ldots$

Use Regula-Falsi method to solve the equation $x\log_{10} x = 1.2$ correct to four decimal places.

Answer

$$x\log_{10} x = 1.2$$

$$f(x) = x\log_{10} x - 1.2$$

Hence $f(1)$ = -1.2

$f(2) = 0.231363$

$f(3)$ =-0.59794

Choose a=2 and b=3

Here $a = 2, b = 3$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2f(3) - 3f(2)}{f(3) - f(2)}$$

$$x_1 = 2.721015118$$

$$f(x_1) = f(2.7210151) = -0.017090515.$$

Hence the root lies b/w 2.7210151 & 3 .

Then $a = 2.7210151, b = 3.$

$$x_2 = \frac{2.7210151f(3) - 3f(2.7210151)}{f(2.7210151) - f(3) - f(2.7210151)}$$

$$f(x_2) = f(2.7402058) \quad = 0 - 3.839920232 \times 10^{-4}$$

the root lies between 2.7402058 & 3 .

choose $a = 2.7402058, b = 3.$

$$x_3 = \frac{2.7402058f(3) - 3f(2.7402058)}{f(3) - f(2.7402058)}$$

$$= 2.740636265.$$

Next step choose $a = 2.740636265, b = 3$

$$x_4 \quad = \frac{2.740636265f(3) - 3f(2.740636265)}{f(3) - f(2.740636265)}$$
$$= 2.740645876$$

$$= 2.740205702 \approx 2.7402058$$

Use Regula-falsi method to find an approx. real root of the equation

$x^3 - 2x - 5 = 0.$

We have

$$f(x) = x^3 - 2x - 5$$

;

$f(0) = -5$, $f(1) = -6$; $f(2) = -1$; $f(3) = 16$

Root lies btw 2&3, because $f(2) f f(3)$ are of opposite sign. choose $a = 2$ &$b = 3$

$$x_1 = \frac{2f(3) - 3f(2)}{f(3) - f(2)} = \frac{2 \times 16 - 3 \times -1}{16 - -1} = 2.058823529.$$

$$2.05882.$$

$$f(x_1) = f(2.05882) = -0.3908377406 \approx -0.39084.$$

Hence the soot lies btw 2.0588243 and 3.

choose $a = 2.05882$ and $b = 3.$

$$x_2 = \frac{2.05882f(3) - 3f(2.05882)}{f(3) - f(2.05882)} = 2.081262461$$

$$\approx 2.08126.$$

$$f(x_2) = f(2.08126) = -0.14724$$

is root lies between 2.08126 and 3.
choose $a = 2.08126$ & $b = 3$

$$x_3 = \frac{2.08126f(3) - 3f(2.08126)}{f(3) - f(2.08126)} = 2.08964$$

$$f(x_3) = -0.0.08964) = -0.05467 \ldots$$

The root lies bl 2.08964 and 3.
choose $a = 2.08964$ & $b = 3$.

$$x_4 = \frac{2.08964f(3) - 3f(2.08964)}{f(3) - f(2.08964)} = 2.09274$$

$$f(x_4) = f(2.09274) = -0.02019$$

is root lies between 2.09274 & 3

choose $a = 2.09274$ & $b = 3.$

$$x_5 = \frac{2.09274 f(3) - 3f(2.09274)}{f(3) - f(2.09274)} = 2.09388$$

$$f(x_5) = f(2.09388) = -7.49187 \times 10^{-3}$$

Root lies bl 2.09388 & 3.
choose $a = 2.09388$ & $b = 3$

$$x_6 = \frac{2.09388 f(3) - 3f(2.09388)}{f(3) - f(2.09388)} = 2.094304$$

∴ The root of the eau, $x = 2.094304$

Set up a Newton iteration for computing the square root x of a given positive number c and apply it to $c = 2$.

Solution. We have $x = \sqrt{c}$, hence $f(x) = x^2 - c = 0$, $f'(x) = 2x$, and (5) takes the form

$$x_{n+1} = x_n - \frac{x_n^2 - c}{2x_n} = \frac{1}{2}\left(x_n + \frac{c}{x_n}\right).$$

For $c = 2$, choosing $x_0 = 1$, we obtain

$$x_1 = 1.500000, \qquad x_2 = 1.416667, \qquad x_3 = 1.414216, \qquad x_4 = 1.414214, \cdots.$$

x_4 is exact to 6D.

Interpolation

Suppose were given the following values of $y = f(x)$ for a set of values of

$$\begin{array}{llllll} x: & x_0 & x_1 & x_2 & \cdots & x_n \\ y: & y_0 & y_1 & y_2 & \cdots & y_n. \end{array}$$

Then the process of finding the values of y, corresponding to any value of $x = x_i$ between x_0 & x_n is called interpolation.

Interpolation with unequal interval

Lagrange interpolation

Interpolation (Lagrange)

•For Unequal intervals

interpolation is a type of estimation, a method of constructing new data points within the range of a discrete set of known data points

$$x: \quad x_o \quad x_1 \quad x_2 \quad x_3 \quad - \quad - \quad - \quad x_n$$

$$y: \quad y_o \quad y_1 \quad y_2 \quad y_3 \quad - \quad - \quad - \quad y_n$$

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$$x: \quad x_o \quad x_1 \quad x_2 \quad x_3 \quad - \quad - \quad - \quad x_n$$

$$y: \quad y_o \quad y_1 \quad y_2 \quad y_3 \quad - \quad - \quad - \quad y_n$$

Day	Height (mm)
1	0
3	4
5	8
7	12
9	16

Lagrange interpolation

Let $y = f(x)$ takes $(n+1)$ pairs of values $(x_0, y_0), (x_1, y_1), \cdots (x_n, y_n)$ for $x + y$.

Here $f(x)$ can be expressed as

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} y_2 \pm ---------$$

Given $f(2) = 5, f(2,5) = 6$. Find the linear interpolating. polynomial. using Lagranger's formula & so find $f(2,2)$

$$f(2) = 5; f(2.5) = 6$$
$$x_0 = 2; y_0 = 5$$
$$x_1 = 2.5; y_1 = 6$$

$$?f(x) = \frac{(x-x_1)}{(x_0-x_1)}y_0 + \frac{(x-x_0)}{(x_1-x_0)}y_1 = 5 \times \frac{(x-2.5)}{(2-2.5)} + \frac{(x-2)}{(2.5-2)} \times 6$$
$$= \frac{5x-12.5}{-0.5} + \frac{6x-12}{0.5} = \frac{12.5-5x+6x-12}{0.5}$$
$$= \frac{x+0.5}{0.5} = 2x + 1$$
$$f(x) = 2x + 1$$

$$f(x) = 2x + 1$$
$$f(2 \cdot 2) = (2 \times 2 \cdot 2) + 1 = 5.4$$

Using Lagranges Interpolation method find value of y at $x = 10$, from the following data.

$x:$	5	6	9	11
$y:$	12	13	14	16

$$P_n(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(2-9)(x-1)}{(6-5)(6-9)(6-11)} \times 13 + \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)\cdot(9-11)} \times 14 = \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(2-9)(x-1)}{(6-5)(6-9)(6-11)} \times 13$$

$$+ \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)\cdot(9-11)} \times 14 = \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \times 16$$

at $x = 10$,

$$P_n(10) = \Bigg(\frac{10-6)(10-9)(10-11)}{-24} \times 12 + \frac{(10-5)(10-6)(10.9)}{15} \times 13$$

$$+ \frac{(10-5)(10-6)(10-11)}{-24} \times 14 + \frac{(10-5)(10-6)(10-9)}{60} \times 16$$

=14.6

$$f(x) = 2x + 1$$

$$f(2 \cdot 2) = (2 \times 2 \cdot 2) + 1 = 5.4$$

Find the unique polynomial $P_3(x)$ of degree 3 or less, the graph of which passes through the points $(-1,3), (0,-4)$, 1,5), $(2,-6)$.

$$\begin{array}{ll} x_0 = -1; & y_0 = 3 \\ x_1 = 0; & y_1 = -4 \\ x_2 = 1; & y_2 = 5 \\ x_3 = 2 & ; y_3 = -6 \end{array}$$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$\frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} \times 3 + \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} \times -4$$

$$=$$

$$+ \frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)} \times 5 + \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)} \times -6$$

$$= \frac{-1}{2}\left[x^3 - 3x^2 + 2x\right] - 2\left[x^3 - x - 2x^2 + 2\right]$$

$$- \frac{5}{2}\left[x^3 - x^2 - 2x\right] - \left[x^3 - x\right].$$

$$= x^3\left[\frac{-1}{2} - 2 - \frac{5}{2} - 1\right] + x^2\left[\frac{3}{2} + 4 + \frac{5}{2}\right]$$

$$+x[-1 + 2 + 5 + 1] + (-4)$$

$$= -6x^3 + 8x^2 + 7x - 4$$

Apply Lagrangar's interpolation retd to find the value of y when x=10. Given the following table of values of x and y

	x_0	x_1	x_2	x_3
$\boldsymbol{x}$	5	6	9	11
	y_0	y_1	y_2	y_3
$\boldsymbol{y}$	13	14	16	16

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(x-9)(x-11)}{1 \times -3 \times -5} \times 13$$
$$+ \frac{(x-5)(x-6)(x-11)}{4 \times 3 \times -2} \times 14 + \frac{(x-5)(x-6)(x-9)}{6 \times 5 \times 2} \times 16$$

$$= \frac{[4 \times 1 \times -1]}{-1 \times -4 \times -6} \times 12 + \frac{5 \times 1 \times -1}{+15} \times 13 + \frac{[5 \times 4 \times -1] \quad \times \quad 7}{-12}$$

$$+ \frac{[5 \times 4 \times 1] \quad \times \ 8}{30}$$

$$== 14.67$$

Newton's divided difference interpolation.

Newton's divided difference formula can be applied for arbitrarily

positioned values of inept variable ' x '.

Newtons divided difference formula is

$$\begin{aligned} f(x) = \quad & y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] \\ & + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3] + \cdots . \end{aligned}$$

Using Newton's divided difference interpolating polynomial estimate, $f(7)$ from the following data

$$\begin{array}{lcccc} x: & 5 & 6 & 9 & 11 \\ y=f(x): & 12 & 13 & 14 & 16 \end{array}$$

x	y	$[x_0,x_1]$	$[x_0,x_1,x_2]$	$[x_0,x_1,x_2,x_3]$
5	12			
		$\frac{13-12}{6-5}=1$		
6	13		$\frac{\frac{1}{3}-1}{9-5}=-\frac{1}{6}$	
		$\frac{14-13}{9-6}=\frac{1}{3}$		$\frac{\frac{2}{15}-\frac{-1}{6}}{11-5}=\frac{1}{20}$
9	14		$\frac{1-\frac{1}{3}}{11-6}=\frac{2}{15}$	
		$\frac{16-14}{11-9}=1$		
11	16			

$$\begin{aligned} f(x) = \; & y_0 + (x-x_0)[x_0,x_1] + (x-x_0)(x-x_1)[x_0,x_1,x_2] \\ & +(x-x_0)(x-x_1)(x-x_2)[x_0,x_1,x_2,x_3]. \end{aligned}$$

x	y	$[x_0, x_1]$	$[x_0, x_1, x_2]$	$[x_0, x_1, x_2, x_3]$
5	**12**			
		$\frac{13-12}{6-5} = 1$		
6	**13**		$\frac{\frac{1}{3}-1}{9-5} = -\frac{1}{6}$	
		$\frac{14-13}{9-6} = \frac{1}{3}$		$\frac{\frac{2}{15}-\frac{-1}{6}}{11-5} = \frac{1}{20}$
9	**14**		$\frac{1-\frac{1}{3}}{11-6} = \frac{2}{15}$	
		$\frac{16-14}{11-9} = 1$		
11	**16**			

$$f(x) = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3]$$

$$= 12 + (x - 5)[1] + (x - 5)(x - 6)\left[\frac{-1}{6}\right] + (x - 5)(x - 6)(x - 9)\left[\frac{1}{20}\right]$$

$$f(7) = 12 + 2 + \frac{-1}{3} + \frac{-1}{5} = 13{,}467$$

Using Newtons divided difference interpolating formulas find the missing value from the following data.

x:	1	2	3	4	5	6
y:	14	15	–	5	6	19.

Answer

x:	1	2	4	5	6
y:	14	15	5	6	19

Since value of 3 is not given

so its not taken. unequal interval → so using Newtons interpolation

x	**y**	$[x_0, x_1]$	$[x_0, x_1, x_2]$	$[x_0, x_1, x_2, x_3]$	$[x_0, x_1, x_2, x_3, x_4]$
1	14				
		$\frac{15-14}{2-1} = 1$			
2	15		$\frac{-5-1}{4-1} = -2$		
		$\frac{5-15}{4-2} = -5$		$\frac{2--2}{5-1} = 1$	
4	5		$\frac{1--5}{5-2} = 2$		$\frac{1-1}{6-1} = 0$
		$\frac{6-5}{5-4} = 1$		$\frac{6-2}{6-2} = 1$	
5	6		$\frac{13-1}{6-4} = 6$		
		$\frac{19-6}{6-5} = 13$			
6	19				

$$
\begin{aligned}
f(x) = \quad & y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] \\
& +(x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3] + \\
& (x - x_0)(x - x_1)(x - x_2)(x - x_3)[x_0, x_1, x_2, x_3, x_4]. \\
f(3) = \quad & 14 + (3 - 1)[1] + (3 - 1)(3 - 2)[-2] + (3 - 1)(3 - 2)(3 - 4)[1] \\
= \quad & 10
\end{aligned}
$$

Newton’s forward interpolation

Equal intervals

- **First forward difference**

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

$$\cdots\cdots$$

$$\cdots\cdots\cdots$$

$$\Delta y_{n-1} = y_n - y_{n-1}$$

- **Second forward difference**

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

Newton's forward interpolation formula

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + --------$$

Where

$$u = \frac{x - x_0}{h}$$

For the following data calculate value of y when x=9

x	8	10	12	14	16	18
y	10	19	32.5	54	89.5	154

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
8	10					
		9				
10	19		4.5			
		13.5		3.5		
12	32.5		8		2.5	
		21.5		6		6.5
14	54		14		9	
		35.5		15		
16	89.5		29			
		64.5				
18	154					

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12	32.5		8		2.5	
		21.5		6		6.5
14	54		14		9	
		35.5		15		
16	89.5		29			
		64.5				
18	154					

$$u = \frac{x - x_0}{h} = (9-8)/2 = 0.5$$

For the following data calculate value of y when x=9

x	8	10	12	14	16	18
y	10	19	32.5	54	89.5	154

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
8	10					
		Δy_0 9				
10	19		$\Delta^2 y_0$ 4.5			
		13.5		$\Delta^3 y_0$ 3.5		
12	32.5		8		$\Delta^4 y_0$ 2.5	
		21.5		6		$\Delta^5 y_0$ 6.5
14	54		14		9	
		35.5		15		
16	89.5		29			
		64.5				
18	154					

$$u = \frac{x - x_0}{h} \quad = (9-8)/2 = 0.5$$

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + --------$$

For the following data calculate value of y when x=9

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
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		Δy_0 9				
10	19		$\Delta^2 y_0$ 4.5			
		13.5		$\Delta^3 y_0$ 3.5		
12	32.5		8		$\Delta^4 y_0$ 2.5	
		21.5		6		$\Delta^5 y_0$ 6.5
14	54		14		9	
		35.5		15		
16	89.5		29			
		64.5				
18	154					

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0$$

$$y(9) = 10 + 0.5\times 9 + \frac{0.5(0.5-1)}{2!}\times 4.5 + \frac{0.5(0.5-1)(0.5-2)}{3!}3.5 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}\times 2.5 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5!}\times 6.5$$

= 14.2323

Newton's Backward interpolation formula

Backward difference

$$\nabla y_1 = y_1 - y_0$$

$$\nabla y_2 = y_2 - y_1$$

$$\ldots\ldots$$

$$\ldots\ldots\ldots$$

$$\nabla y_{n-1} = y_n - y_{n-1}$$

Second Backward difference

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$$

Newton’s backward interpolation formula

$$y(x) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n$$

$$-----------------------$$

$$v = \frac{x - x_n}{h}$$

Use the following Data to calculate value for 1996 ?

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66				
			-5		
		15			
1981	81			2	-3
			-3		
		12			
1991	93			-1	
			-4		
		8			
2001	101				

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66		-5		
		15		2	
1981	81		-3		-3
		12		-1	
1991	93		-4		
		8			
2001	101				

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
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			-5		
		15			
1981	81			2	-3
			-3		
		12			
1991	93			-1	
			-4		
		8			
2001	101				

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	**y**	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66				
			-5		
		15			
				2	
1981	81				-3
			-3		
		12			
1991	93			-1	
			-4		
		8			
2001	101				

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Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66				
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		8			
2001	101				

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x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66		-5		
		15		2	
1981	81		-3		
		12			-3
1991	93			-1	
			-4		
		8			
2001	101				

$$v = \frac{x - x_n}{h} = \frac{1996 - 2001}{10} = \mathbf{-0.5}$$

$$y(x) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n$$

$$y(x) = y_4 + v\nabla y_4 + \frac{v(v+1)}{2!}\nabla^2 y_4 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_4 + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^3 y_4$$

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66		-5		
		15		2	
1981	81		-3		
		12			-3
1991	93			-1	
			-4		
2001	101	8			
		∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	

$$v = \frac{x - x_n}{h} = \frac{1996 - 2001}{10} = \mathbf{-0.5}$$

$$y(x) = y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n$$

$$y(x) = y_4 + v\nabla y_4 + \frac{v(v+1)}{2!}\nabla^2 y_4 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_4 + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^3 y_4$$

Year	1961	1971	1981	1991	2001
Population	46	66	81	93	101

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1961	46				
		20			
1971	66		-5		
		15		2	
1981	81		-3		-3
		12		-1	
1991	93		-4		
		8			
2001	101				

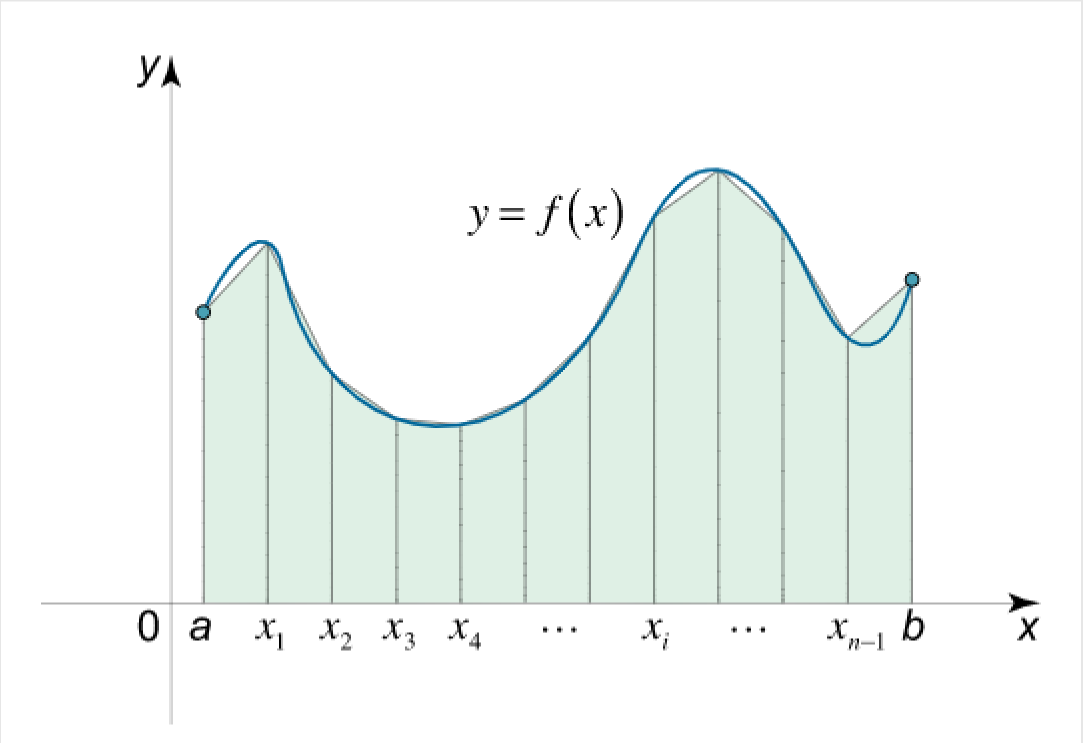
$$v = \frac{x - x_n}{h} = \frac{1996 - 2001}{10} = \mathbf{-0.5}$$

$$y(x) = y_4 + v\nabla y_4 + \frac{v(v+1)}{2!}\nabla^2 y_4 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_4 + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^3 y_4$$

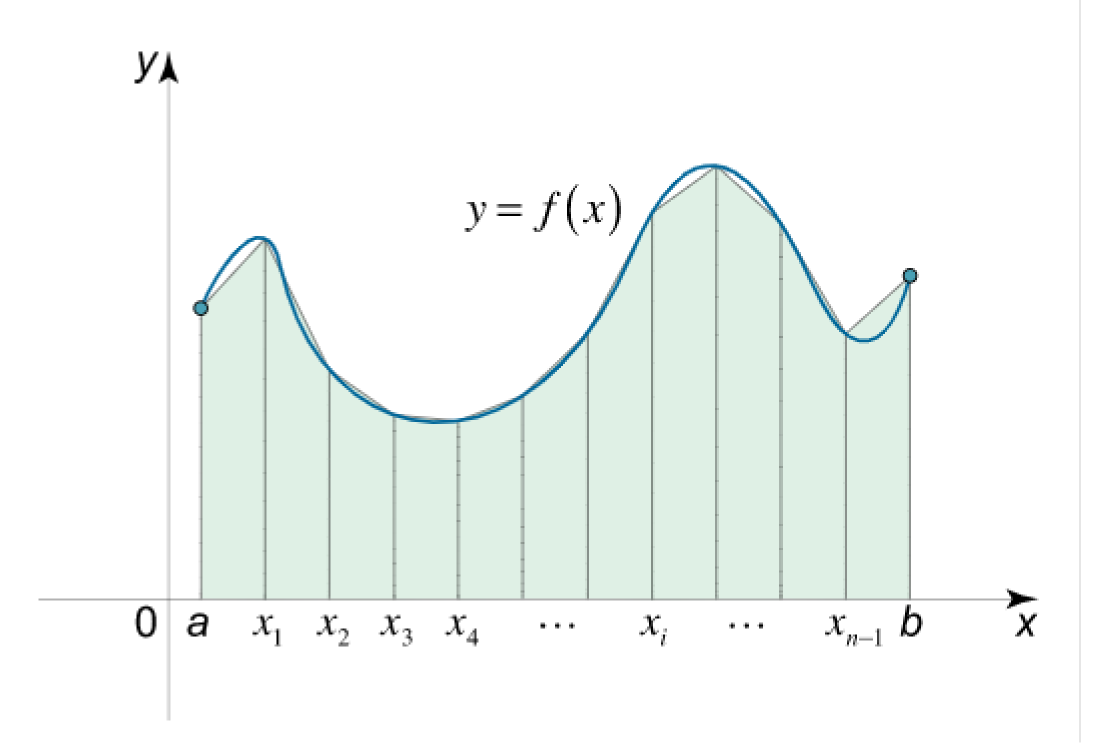
$$101 + (-0.5)\times 8 + \frac{(-0.5)(-0.5-1)}{2!}(-4) + \frac{(-0.5)(-0.5-1)(-0.5-2)}{3!}(-1) + \frac{(-0.5)(-0.5-1)(-0.5-2)(-0.5-3)}{4!}(-3)$$

=97.6796

Numerical integration



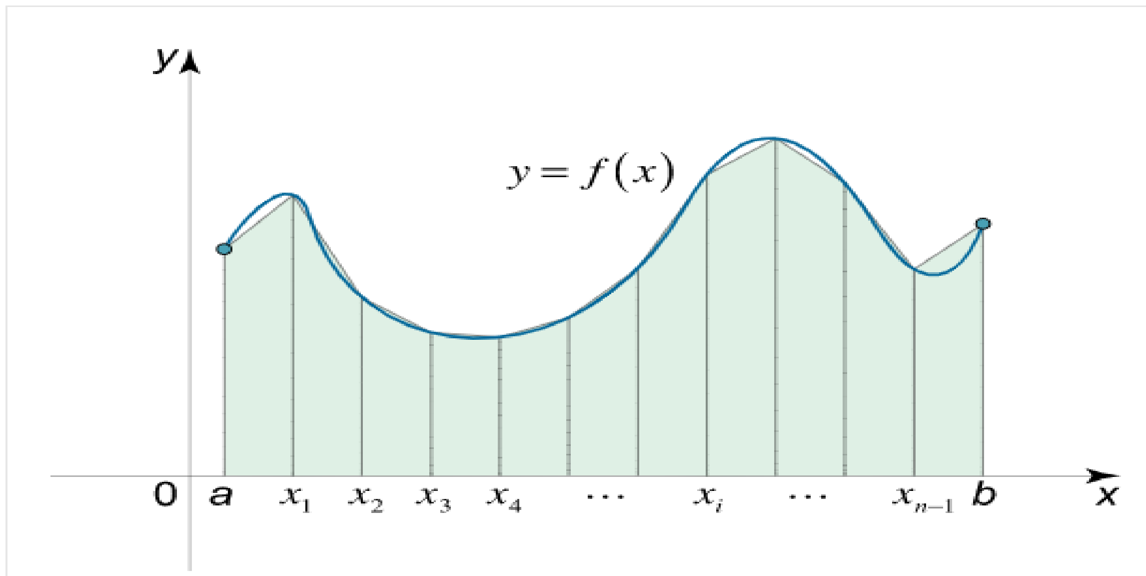
Numerical integration



Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + -------- + y_{n-1}\right)\right]$$

$$h = \frac{b-a}{n}$$



$$x_0 = a < x_1 < x_2 -------- < x_n = b$$

$$y_n = f(b) \qquad y_0 = f(a) \qquad y_i = f(x_i)$$

Find $\int_{-3}^{3} x^4 dx$ Using Trapezoidal rule

$x_0 = -3$ $\qquad$ $x_n = 3$

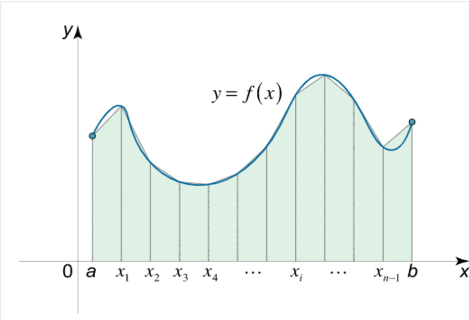
x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$y = x^4$	81	16	1	0	1	16	81

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + -------- + y_{n-1}\right)\right]$$

Take h=1, and n=6

$$\int_{-3}^{3} x^4 dx = \frac{1}{2}\left[81 + 81 + 2\left(16 + 1 + 0 + 1 + 16\right)\right] = 115$$

Find $\int_{-3}^{3} x^4 dx$ Using Trapezoidal rule



$x_0 = -3$ $\qquad\qquad$ $x_n = 3$

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$y = x^4$	81	16	1	0	1	16	81

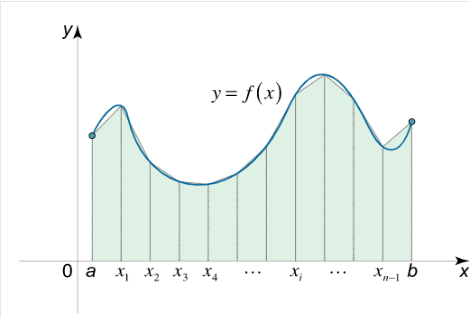
$$h = \frac{b-a}{n} = \frac{3--3}{6} = 1$$

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + \text{--------} + y_{n-1}\right)\right]$$

Take h=1, and n=6

$$\int_{-3}^{3} x^4 dx = \frac{1}{2}\left[81 + 81 + 2\left(16 + 1 + 0 + 1 + 16\right)\right] = 115$$

Find $\int_{-3}^{3} x^4 dx$ Using Trapezoidal rule



$x_0 = -3$ $\qquad$ $x_n = 3$

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$y = x^4$	81	16	1	0	1	16	81

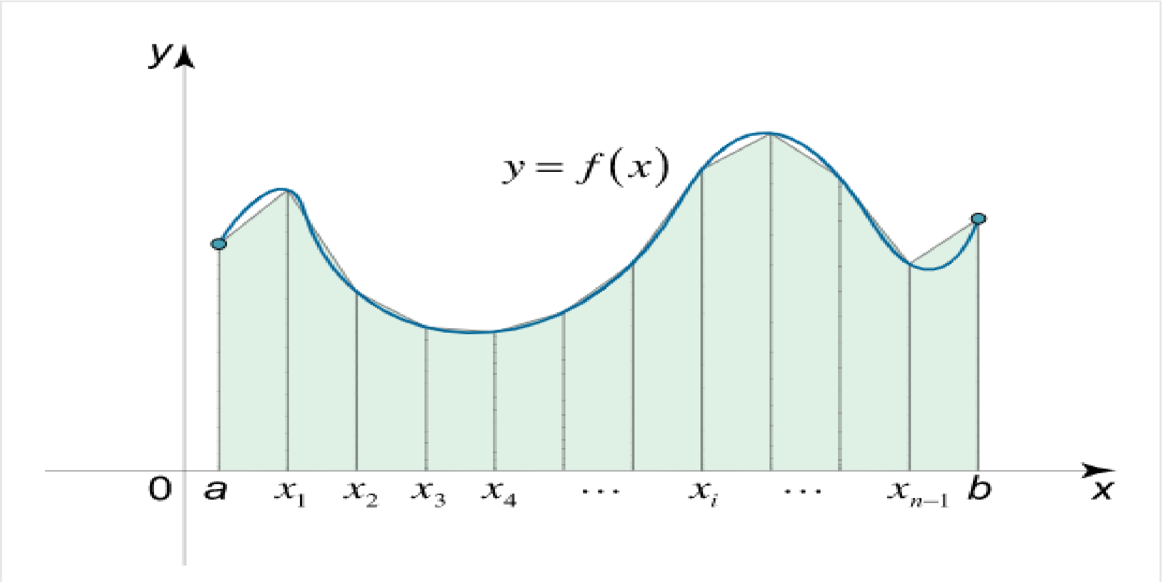
$$h = \frac{b-a}{n} = \frac{3--3}{6} = 1$$

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + -------+ y_{n-1}\right)\right]$$

Take h=1, and n=6

$$\int_{-3}^{3} x^4 dx = \frac{1}{2}\left[81 + 81 + 2\left(16 + 1 + 0 + 1 + 16\right)\right] = 115$$

Simpson's 1/3 rule



$$\int_a^b f(x)dx \approx \frac{h}{3}\left[y_0 + y_n + 4\left(y_1 + y_3 + - - - + y_{n-1}\right) + 2\left(y_2 + y_4 + - - - + y_{n-2}\right)\right]$$

Find $\int_{-3}^{3} x^4 dx$ **Using Simpson's 1/3 rule**

$x_0 = -3$ $\qquad\qquad\qquad\qquad$ $x_n = 3$

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$y = x^4$	81	16	1	0	1	16	81

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[y_0 + y_n + 4\left(y_1 + y_3 + ---- + y_{n-1}\right) + 2\left(y_2 + y_4 + ---- + y_{n-2}\right)\right]$$

Take h=1, and n=6

$$\int_{-3}^{3} x^4 dx = \frac{1}{3}\left[81 + 81 + 4\left(16 + 0 + 16\right) + 2(1 + 1)\right] = 98$$

Euler's Method

Given the initial value problem,

$$y' = f(x, y) \qquad y(x_0) = y_0$$

,

$$x_1 = x_0 + h \qquad x_2 = x_1 + h$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$n = 0,1,2----$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$ and h=0.02

$$x_0 = 0,\ \mathrm{y}_0 = 0 \qquad f(x, y) = 2xy + 1$$

take h=0.02 then find y(0.1)
Given

Step 1 n=0, $x_1 = x_0 + h$ = 0+0.02 = 0.02

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = y(x_1) = y(0.02)$$

$$y_1 = 0 + 0.02 f(0,0) = 0 + 0.02[2 \times 0 \times 0 + 1] = 0.02$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$ and h=0.02

$$x_0 = 0, \ \mathrm{y}_0 = 0 \qquad f(x, y) = 2xy + 1$$

Step 1 n=0,

$x_1 = x_0 + h$ = 0+0.02 = 0.02

take h=0.02 then find y(0.1)
Given

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = y(x_1) = y(0.02)$$

$$y_1 = 0 + 0.02 f(0,0) = 0 + 0.02[2 \times 0 \times 0 + 1] = 0.02$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$ and h=0.02

$$f(x, y) = 2xy + 1$$

$$x_0 = 0,\ \mathrm{y}_0 = 0$$
$$x_1 = 0.02,\ \mathrm{y}_1 = 0.02$$

Step 2 n=1,

$x_2 = x_1 + h$ = 0.02+0.02 = 0.04

$$y_2 = y_1 + hf(x_1, y_1)$$
$$y_2 = y(x_2) = y(0.04)$$
$$= 0.02 + 0.02 f(0.02, 0.02)$$
$$= 0.02 + 0.02[2 \times 0.02 \times 0.02 + 1] = 0.04$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$

$$x_0 = 0,\ \mathrm{y}_0 = 0$$
$$x_1 = 0.02,\ \mathrm{y}_1 = 0.02$$
$$x_2 = 0.04,\ \mathrm{y}_2 = 0.04$$

$$f(x, y) = 2xy + 1$$

Step 3 n=2,

$x_3 = x_2 + h$ = 0.04+0.02 = 0.06

$$y_3 = y_2 + hf(x_2, y_2)$$
$$y_3 = 0.04 + 0.02 f(0.02, 0.02)$$
$$= 0.04 + 0.02[2 \times 0.04 \times 0.04 + 1] = 0.06$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$

$$x_0 = 0,\ \mathrm{y}_0 = 0$$
$$x_1 = 0.02,\ \mathrm{y}_1 = 0.02$$
$$x_2 = 0.04,\ \mathrm{y}_2 = 0.04$$
$$x_3 = 0.06,\ \mathrm{y}_3 = 0.06$$

$$f(x, y) = 2xy + 1$$

Step 4 n=3, $x_4 = x_3 + h$ = 0.06+0.02 = 0.08

$$y_4 = y_3 + hf(x_3, y_3)$$
$$y_4 = 0.06 + 0.02 f(0.06, 0.06)$$
$$= 0.06 + 0.02\left[2 \times 0.06 \times 0.06 + 1\right] = 0.08$$

Solve the Differential equation $y' = 2xy + 1$ with $y(0) = 0$

$$x_0 = 0,\ \mathrm{y}_0 = 0$$
$$x_1 = 0.02,\ \mathrm{y}_1 = 0.02$$
$$x_2 = 0.04,\ \mathrm{y}_2 = 0.04$$
$$x_3 = 0.06,\ \mathrm{y}_3 = 0.06$$
$$x_4 = 0.08,\ \mathrm{y}_4 = 0.08$$

$$f(x, y) = 2xy + 1$$

Step 5 n=4, $x_5 = x_4 + h$ = 0.08+0.02 = 0.1 y(0.1)

$$y_5 = y_4 + hf(x_4, y_4)$$
$$y_5 = y(0.1) = 0.08 + 0.02 f(0.08,0.08)$$
$$= 0.08 + 0.02\left[2 \times 0.08 \times 0.08 + 1\right] = 0.1$$

Runge–Kutta method

$$\frac{dy}{dx} = f(x, y) \qquad y(x_0) = y_0$$

The approximate values of

$$y_1, y_2, y_3 ----$$

$$x_1 = x_0 + h$$

$$x_2 = x_1 + h$$

The solution y(x) is computed by

$$y_{n+1} = y_n + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$K_1 = hf\left(x_n, y_n\right)$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$

$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$

$$K_4 = hf\left(x_n + h, y_n + K_3\right)$$

Using Rage Kutta method with h=0.1 to find y(0.2) given $\frac{dy}{dx} = e^x + y$ with y(0)=0

Given $f(x,y) = e^x + y \qquad x_0 = 0, \quad y_0 = 0$

,

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$K_1 = hf\left(x_0, y_0\right) = 0.1 \times e^0 + 0 = 0.1$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.1 \times f\left(0 + \frac{0.1}{2}, 0 + \frac{0.1}{2}\right)$$

$$= 0.1 \times f(0.5, 0.5) = 0.1 \times e^{0.5} + 0.5 = 0.1101$$

Using Rage Kutta method with h=0.1 to find y(0.2) given $\frac{dy}{dx}=e^x+y$ with y(0)=0

Given $f(x,y)=e^x+y \qquad x_0=0, \quad y_0=0$

,

$$K_3=hf\left(x_0+\frac{h}{2},y_0+\frac{K_2}{2}\right)=0.1\times f\left(0+\frac{0.1}{2},0+\frac{0.1101}{2}\right)$$

$$=0.1\times f(0.5,0.055)=0.1\times e^{0.5}+0.055=0.1106$$

$$K_4=hf(x_0+h,y_0+K_3)=0.1\times f(0+0.1,\ 0.1106)$$

$$=0.1\times f(0.1,\ 0.1106)=0.1\times e^{0.1}+0.1106=0.1216$$

$$y_1 = y(x_1) = y(0.1)$$

$$y_1 = y_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 0 + \frac{1}{6}\left[0.1_1 + 2 \times 0.1101 + 2 \times 0.1106 + 0.1216\right]$$

= 0.1105

Thank You !!!