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Differential equation :-

→ Linear differential equation of Higher order with Constant Coefficients :-

An equation of the form

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = f(x) \quad \text{—(1)}$$

Where, $a_0, a_1, a_2, a_3 \cdots a_n$ are all Constants & $f(x)$ is a function x or Constant.

The symbolic form of eqn (1)

let, $D = \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2}$, ... $D^n = \frac{d^n}{dx^n}$

Then, $a_0 D^n y + a_1 D^{n-1} y + a_2 D^{n-2} y + \cdots + a_n y = f(x)$

$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots a_n) y = f(x).$$

$$f(D)\, y = f(x). \quad \text{—(3)}$$

Where, $f(D) = a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n$.

Solution of eqn (3) Consists of two parts.

i.e. Complete solution (C.S).

$$C.S = C.F + P.I$$

or $(y = C.F + P.I)$

C.F → complementary function.

P.I → particular integral.

Rules for finding C.F :-

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for finding C.F R.H.S of Given eqn is taking as zero,

let, us Consider 2^{nd} order linear diff^n eq. with Constant Coeff.

$$a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad — (1)$$

$$(a_0 D^2 + a_1 D + a_2) y = 0 \quad — (2)$$

Put, $(D = m)$ in eq^n (2) & equate to zero the Coeff. of (y). and solve the Algebric eqn (A.E)

Case:- (1) when (A.E) Having real & distinct roots say (m_1 & m_2)

$$(C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x})$$

Case:- (2) When (A.E) Having real & equal roots $(m_1 = m_2 = m)$

$$(C.F = (c_1 + c_2 x) e^{mx})$$

Case:- (3) when roots are Complex

$m_1 = \alpha + i\beta$
$m_2 = \alpha - i\beta$.

$$C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$\Rightarrow c_1 e^{(\alpha + i\beta)x} + c_2 e^{(\alpha - i\beta)x}$$

$$\Rightarrow c_1 e^{\alpha x} \cdot e^{i\beta x} + c_2 e^{\alpha x} \cdot e^{-\beta x i}$$

$\Rightarrow$ $e^{\alpha x}(A\cos\beta x + B\sin\beta x)$ $(C_1+C_2=A)$
$(C_1-C_2=B)$

Rules for finding P.I (Particular integral):-

Case:-① $P.I = \dfrac{e^{ax}}{f(D)}$

Put, $(a=D)$

So, $P.I = \dfrac{e^{ax}}{f(a)}$

Q: Solve the eqn.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$$

$\Rightarrow$ in symbolic eqn

$$D^2y + 5Dy + 6y = e^{2x}$$
$$(D^2+5D+6)y = e^{2x}$$

for C.F (assume R.H.S=0)

$$f(D)\cdot y = 0$$
$$D^2+5D+6=0$$
$$D^2+2D+3D+6=0$$
$$D(D+2)+3(D+2)=0$$
$$(D+2)(D+3)=0$$
$$(D=-2,\quad D=-3)$$

$$(C.F = C_1e^{-2x} + C_2e^{-3x})$$

for P.I:- $P.I = \dfrac{1}{f(D)} \times e^{2x} = \dfrac{e^{2x}}{D^2+5D+6}$ $\{f(x)=e^{2x}\}$

Put, $(a=2)$

$$P.I = \frac{e^{2x}}{2^2+5\times 2+6} \Rightarrow \frac{e^{2x}}{20}$$

$$C.S = P.I + C.F$$

$$\boxed{y = \frac{e^{2x}}{20} + c_1 e^{-2x} + c_2 e^{-3x}}$$

(2) Solve the equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$$

$\Rightarrow$. $$D^2y - 5Dy + 6y = e^{3x}$$

$$f(D)\,y = e^{3x}$$

for C.F $\quad f(D) = 0$

$$D^2 - 5D + 6 = 0 \quad \text{Put, } (D = m)$$

$$m^2 - 5m + 6 = 0.$$

$$(m = 2, 3)$$

$$(C.f = c_1 e^{2x} + c_2 e^{3x})$$

for. P.I $\quad P.I = \frac{e^{3x}}{f(D)} \qquad f(x) = e^{3x}$

$$= \frac{e^{3x}}{D^2 - 5D + 6} \qquad \text{Put}(a = 3)$$

$$= \frac{e^{3x}}{9 - 15 + 6} \quad (\text{it fails})$$

So, Now

$$P.I = \frac{x}{f'(D)} \times e^{3x}$$

$$\Rightarrow \frac{x \cdot e^{3x}}{2D - 5} \qquad \text{Put}(a = 3)$$

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$$P.I = \frac{x e^{3x}}{6-5} \Rightarrow x e^{3x}$$

$$C.S = C.F + P.I$$

$$= \left(C_1 e^{2x} + C_2 e^{3x} + x e^{3x}\right)$$

Case: (2)

- when $f(x) = \sin(ax+b)$ or $\sin(ax)$ or $\cos(ax+b)$ or $\cos(ax)$

$$P.I = \frac{1}{f(D^2)} \sin(ax+b)$$

Put, $(D^2 = -a^2)$

Provided $\{f(-a^2) \neq 0\}$

if $f(-a^2) = 0$.

then, $P.I = x\left(\frac{1}{f'(D)} \cdot \sin(ax+b)\right)$

and make even power in (D)

By Rationalization

Q8. $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 3y = \sin x$

$(D^2+2D+3)y = \sin x$

Put, $(D = m)$

C.F. $\Rightarrow m = \frac{-2 \pm \sqrt{4-4\times3}}{2}$

$\Rightarrow -1 \pm \sqrt{2}i$

$(\alpha \pm \beta i)$

$D \rightarrow$ ~~devi~~derivative

$\frac{1}{D} \rightarrow$ integral of D

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$C.F = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$\Rightarrow e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x)$.

Now. (P.I) $\cdot$ $P.I = \frac{\sin x}{f(D)^2}$ (Compare $\sin x$ & $\sin ax$)

$= \frac{\sin x}{D^2+2D+3}$ $(a=1)$

Put, $D^2 = -a^2$

$(D^2 = -1)$

$\Rightarrow \frac{\sin x}{-1+2D+3}$

$\left(\frac{\sin x}{2D+2}\right)$

$\Rightarrow \frac{\sin x}{2}\left(\frac{D-1}{D^2-1}\right) \Rightarrow \frac{\sin x}{2}\left(\frac{D-1}{-1-1}\right)$

$\Rightarrow \frac{\sin x}{-4}(D-1)$.

$\Rightarrow -\frac{1}{4}(D\sin x - \sin x)$

$\left\{ D\sin x = \frac{d(\sin x)}{dx} \Rightarrow \cos x \right\}$

$\Rightarrow -\frac{1}{4}[\cos x - \sin x]$.

$C.S = C.F + P.I$

$\Rightarrow e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x) - \frac{1}{4}[\cos x - \sin x]$. Ans

Maths (Continued) (Part)

Q: Solve $\frac{d^3y}{dx^3} + a^2\frac{d^2y}{dx^2} = \sin ax$

$\Rightarrow$ $D^3y + a^2D^2y = \sin ax$

$(D^3 + a^2D^2)y = \sin ax$ $\longrightarrow$ $\{f(D)y = \sin ax\}$

for (C.F), $D^3 + a^2D^2 = 0$

Put, $D = m$

$m^3 + a^2m^2 = 0$

$m^2(m^2 + a^2) = 0.$

$(m = 0)$, $(m^2 = -a^2)$

$m = 0$, $m = \pm ai$

$\cancel{C.F = C_1e^{0(x)} + (C_2 + C_3x)e}$

$C.F = C_1e^{0(x)} + C_2e^{aix} + C_3e^{-aix}$

$C.F = C_1 + C_2(\cos ax + i\sin ax) + C_3(\cos ax - i\sin ax).$

$C.F = C_1 + \cos ax(C_2 + C_3) + \sin ax(C_2 - C_3).$

$\boxed{C.F = C_1 + C_2\cos ax + C_3\sin ax}$

for (P.I) $\Rightarrow \frac{1}{f(D)}\sin ax \Rightarrow$

$\Rightarrow \frac{1}{D^3 + a^2D}\sin ax$

$\Rightarrow \frac{1}{D(D^2 + a^2)}\sin ax$

Case ②:-

When $f(x) = \sin ax$

$P.I = \frac{1}{f(D)}.\sin ax$

Put, $\underline{D^2 = -a^2}$

$$\frac{1}{D} = \int dx.$$

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$\Rightarrow \frac{1}{(D^2+a^2)} \cdot \frac{1}{D} \sin ax$

$\Rightarrow \frac{1}{(D^2+a^2)} \int \sin ax \, dx$

$\Rightarrow \frac{1}{D^2+a^2} \left(\frac{-\cos ax}{a}\right)$

Now Put $(D^2 = -a^2)$

$\Rightarrow \frac{1}{0}$ (form)

So, Now. (seperate) from this function

$$P.I = \frac{x}{2D} \cdot \left(\frac{\cos ax}{a}\right)$$

$\Rightarrow \frac{x}{2} \times \frac{D}{D^2} \frac{(-\cos ax)}{a}$

$\Rightarrow \frac{x}{2a} \times \frac{D}{-a^2} (-\cos ax)$

$D \cos ax = \frac{d}{dx}(\cos ax)$

$= -a \sin ax$

$\Rightarrow \frac{-x}{+2a^3} \times (-a \sin ax)$

$\Rightarrow \left\{ -\frac{x}{2a^2} \sin ax \right\}$

$y = C.F + P.I$

$= \left\{ C_1 + C_2 \cos ax + C_3 \sin ax - \frac{x}{2a^2} \sin ax \right\}$

Ans

Case:- (3) When,

$f(x) = x^n$ ($n \to$ integer)

$P.I = \frac{1}{f(D)} x^n \Rightarrow [f(D)]^{-1} x^n$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$

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Q:- Solve: $\frac{d^2y}{dx^2} - 9y = x^3.$

$$D^2y - 9y = x^3.$$

$$f(D)y = x^3.$$

CF $\Rightarrow$ $f(D) = 0.$

$D^2 - 9 = 0.$, Put $(D = m)$

$m^2 - 9 = 0$

$(m = \pm 3)$

y CF $= C_1 e^{3x} + C_2 e^{-3x}$

P.I $\Rightarrow$ $\frac{1}{f(D)} x^3$, $f(x) = x^3.$

$\Rightarrow [f(D)]^{-1} x^3.$

$\Rightarrow [D^2 - 9]^{-1} x^3.$

$\Rightarrow [-9(1 - \frac{D^2}{9})]^{-1} x^3.$

$\Rightarrow (-9)^{-1} (1 - \frac{D^2}{9})^{-1} x^3.$

$\Rightarrow \frac{1}{(-9)} [1 + (\frac{D^2}{9}) + (\frac{D^2}{9})^2 + (\frac{D^2}{9})^3 + \cdots] x^3.$ $\quad 3x^2$ $6x$

$\Rightarrow (\frac{-1}{9}) [x^3 + \frac{D^2 x^3}{9} + \frac{D^4 x^3}{9} + \cdots 0 \cdots]$

$\Rightarrow (\frac{-1}{9}) [x^3 + \frac{6x}{9} + 0 + 0]$

P.I $= -\left(\frac{\frac{6x}{9} + x^3}{9}\right)$

Sol^n, $C_1 e^{3x} + C_2 e^{-3x} - \left(\frac{\frac{6x}{9} + x^3}{9}\right)$ Ans.

$\Rightarrow C_1 e^{3x} + C_2 e^{-3x} - \frac{(2x + 3x^3)}{27}$ Ans

Put $f(D)=0$ (for C.F)

$D(D^2+a^2)=0.$

$D=0,\ D=\pm ia \Rightarrow (\alpha+i\beta)$

$C.F = C_1 e^{0\times x} + e^{0x}(A\cos(-a)x + B\sin(-a)x)$

$= \cancel{C_1 + (A\cos ax\ \sin ax)}$

$= C_1 + (A\cos ax - B\sin ax)$

<u>for P.I</u> :- $P.I = \frac{1}{f(D)}\sin ax$

$= \frac{1}{D(D^2+a^2)}\sin ax$ (Put, $D^2=-a^2$)

(it fails)

then, Now, $(P.I) = x\left(\frac{1}{f'(D^2)}\sin ax\right)$

$\Rightarrow x\left(\frac{1}{3D^2+a^2}\sin ax\right)$ Put $(D^2=-a^2)$

$= x\left(\frac{1}{-3a^2+a^2}\sin ax\right)$

$= \frac{x}{-2a^2}\sin ax$

$C.S = C.F + P.I$

$y = C_1 + (A\cos ax - B\sin ax) - \frac{x}{2a^2}\sin ax$

<u>Case</u>:- 3 When, $f(x) = x^n$

($n \to$ integer)

$P.I = \frac{1}{f(D)}.x^n \Rightarrow (f(D))^{-1}x^n$

$(1-x)^{-1} = 1+x+x^2+x^3+ \cdots$

$(1+x)^{-1} = 1-x+x^2-x^3+ \cdots$

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Q:- Solve $\left(\frac{d^2y}{dx^2} - 9y = x^3\right)$

$\Rightarrow$ $D^2y - 9y = x^3.$

$f(D)y = x^3.$

for. C.F :- $f(D) = 0$ Put $(D = m)$

$D^2 - 9 = 0$

$m^2 - 9 = 0$

$m^2 = 9$

$(m = \pm 3)$

$\left(C.F = c_1 e^{-3x} + c_2 e^{3x}\right)$

for P.I :-

$$P.I = \frac{1}{f(D)} x^3$$

$$\Rightarrow \frac{1}{(D^2-9)} \cdot x^3$$

$D^2 x^3 = \frac{d^2(x^3)}{dx^2} = (6x)$

$$\Rightarrow (D^2-9)^{-1} \cdot x^3$$

$$\Rightarrow (-9)^{-1}\left(1 - \frac{D^2}{9}\right)^{-1} \cdot x^3$$

$$\Rightarrow \left(\frac{1}{-9}\right)\left(1 - \left(\frac{D^2}{9}\right) + \left(\frac{D^2}{9}\right)^2 - - - \right)x^3$$

$$\Rightarrow \left(\frac{-1}{9}\right)\left(x^3 - \frac{1}{9}D^2x^3 + \frac{D^4x^3}{9} - - - \right)$$

$$\Rightarrow \left(\frac{-1}{9}\right)\left(x^3 - \frac{6x}{9} + 0\right)$$

$$= \frac{-1}{81}\left(9x^3 - 6x\right) \Rightarrow \frac{-1}{27}\left(3x^3 - 2x\right)$$

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$$\boxed{C.S = c_1 e^{-3x} + c_2 e^{3x} - \frac{1}{27}(3x^2 - 2x)}$$

Case:- (4) $f(x) = e^{ax} f(x).$

$$P.I = \frac{1}{f(D)} \cdot e^{ax} \cdot f(x)$$

, Replace (D) by (D+a)

$$\boxed{P.I = \frac{1}{f(D+a)} \cdot e^{ax} \cdot f(x)}$$

Q:- $\frac{d^2y}{dx^2} + \frac{5dy}{dx} + 6 = e^{2x} \cdot x$

$\Rightarrow$ $(D^2 + 5D + 6)y = e^{2x} \cdot x$

$f(D)y = e^{2x} \cdot x$

for C.F:- $f(D) = 0$

$D^2 + 5D + 6 = 0$

$D^2 + 2D + 3D + 6 = 0$

$D(D+2) + 3(D+2) = 0$

$(D+2)(D+3) = 0$

$(D = -2, -3)$

$C.F = c_1 e^{-2x} + c_2 e^{-3x}$

for P.I $\quad P.I = \frac{1}{f(D)} \cdot (f(x) e^{ax})$ $\quad$ & $(e^{2x} \cdot x)$

Here $(a = 2)$

Replace D by (D+2)

$$P.I = \frac{1}{(D+2)^2 + 5(D+2) + 6} \cdot e^{2x} \cdot x$$

$$\Rightarrow \frac{1}{D^2 + 20 + 9D} \cdot e^{2x} \cdot x$$

(Now this becomes like Case (3))

$$\Rightarrow (D^2 + 20 + 9D) e^{2x} \cdot x$$

$$\Rightarrow \frac{1}{20}\left(1+\frac{D^2+9D}{20}\right)^{-1} x \cdot e^{2x}$$

$$\Rightarrow \frac{e^{2x}}{20}\left(1-\frac{(D^2+9D)}{20}+\cdots\right)x$$

$$\Rightarrow \frac{e^{2x}}{20}\left(x-\frac{(0+9)}{20}\right)$$

Here.
$$\left\{\begin{matrix} D^2x=0 \\ Dx=1 \end{matrix}\right\}$$

$$\Rightarrow \frac{e^{2x}(20x-9)}{400}$$

$$\boxed{C.S = P.I + C.F}$$

Q= $$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x \sin 2x$$

$$\Rightarrow (D^2-2D+1)y = e^x \sin 2x$$

for. C.F

Put $(D=m)$

$$D^2-2D+1=0.$$

$$m^2-2m+1=0$$

$$(m-1)^2=0$$

$$(m=1,1)$$

C.F $\Rightarrow (C_1+C_2x)e^x$

for P.I $\quad P.I = \frac{1}{f(D)} \cdot e^{x} \cdot f(x)$

(here $(a=1)$
Replace D by $(D+1)$)

$$\Rightarrow \frac{1}{f(D+1)} \cdot e^x \sin 2x$$

$$\Rightarrow \frac{1}{(D+1)^2-(2)(D+1)+1} e^x \sin 2x$$

$$\Rightarrow \frac{1}{D^2} \cdot e^x \sin 2x$$

$$\Rightarrow \frac{1}{D^2} \cdot e^x \sin 2x$$

(Note it becomes like Case :- 2)

$$\Rightarrow e^x \left(\frac{1}{D^2}\right) \sin 2x.$$

Now, Replace D^2 by $(-a^2)$

$(D^2 = -a^2)$

$(a = 2)$

$$\Rightarrow e^x \left(\frac{1}{-4}\right) \sin 2x$$

$$\Rightarrow -\frac{e^x \sin 2x}{4}$$

C.S $\Rightarrow$ C.F + P.I

$$y \Rightarrow \left\{ (C_1 + C_2 x)e^x - \frac{e^x \sin 2x}{4} \right\} \text{ Ans}$$

Case :- (5) When, $f(x) = x \cdot f(x)$

$$P.I = \frac{1}{f(D)} \cdot (x \cdot f(x))$$

$$\boxed{P.I \Rightarrow x \cdot \frac{1}{f(D)} \cdot [f(x)] - \frac{f'(D)}{(f(D))^2} \cdot f(x)}$$

Q: $\frac{d^2y}{dx^2} + y = x \sin x$

$\Rightarrow$ $(D^2 + 1) y = x \sin x$

$f(D) y = x \cdot f(x).$

for. (C.F) $\quad f(D) = 0$

$D^2 + 1 = 0$

$D^2 = -1$

$D = \pm i = \alpha + i\beta.$

$$C.F = e^{0(x)} [A \cos x + B \sin x] \Rightarrow A \cos x + B \sin x$$

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$\Rightarrow$ for P.I $\Rightarrow$ $\frac{1}{f(D)} \cdot x f(x)$

$\Rightarrow \frac{1}{(D^2+1)} \cdot x (\sin x)$.

$\Rightarrow \cancel{x \left(\frac{1}{D^2+1} \sin x \right)}$

$\Rightarrow x \frac{1}{D^2+1} \cdot f(x) - \frac{f'(D)}{(f(D))^2} \cdot f(x)$

$\Rightarrow x \left[\frac{1}{D^2+1} \cdot \sin x \right] - \frac{2D}{(D^2+1)^2} \cdot \cancel{f(x)} \sin x$ $\left\{ D \sin x = \cos x \right\}$.

Put $(D^2 = -a)$
$(a = 1)$
$(D^2 = -1)$
it fails the Condition
so.

Put $(D^2 = -a^2)$
$(a = 1)$
$(D^2 = -1)$
it also fails Condition
so.

$\Rightarrow \cancel{x \left[\frac{x}{2D} \cdot \sin x \right]} - 2 \left[\frac{x}{f'(D)} \cdot \cos x \right]$.

$\Rightarrow \cancel{\frac{-x^2}{2} \cos x}$

$f(D) = \cancel{\frac{(D^2+1)}{D}}$

$\cancel{f(D)} =$

$\Rightarrow x \left[\frac{x}{2D} \sin x \right] - 2 \left[\frac{x}{2(D^2+1)} \cdot \cos x \right]$.

$\hookrightarrow$ Put $(D^2 = -1)$

again fails Condition

$\Rightarrow x \left[\frac{-x}{2} \cos x \right] - 2 \left[\frac{x^2}{4D} \cdot \cos x \right]$.

$\Rightarrow \frac{-x^2}{2}\cos x - \frac{x^2}{2}\left(\cancel{\sin x}\ \cos x\right)$

C.S

$y = P.I + C.F$

$$\boxed{y = -\frac{x^2}{2}\cos x - \frac{x^2}{2}\cancel{\sin x}\ \cos x + A\cos x + B\sin x.}$$

Ans

General method for finding P.I :-

* $\frac{1}{D-\alpha} f(x) \Rightarrow e^{+\alpha x}\int e^{-\alpha x}.f(x).dx$

* $\frac{1}{D+\alpha} f(x) \Rightarrow e^{-\alpha x}\int e^{\alpha x}.f(x).dx$

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Q:- Solve $(D^2+3D+2)y = e^{e^x}$

$\Rightarrow$ for C.F, $D^2+3D+2=0$

Put $(D=m)$

$m^2+3m+2=0$

$(m=-2,-1)$

$C.F = c_1 e^{-2x} + c_2 e^{-x}$

<u>for. P.I</u> $\frac{1}{D^2+3D+2} \cdot f(x)$

$\Rightarrow \frac{1}{(D+2)(D+1)} e^{e^x}$

$\Rightarrow \left(\frac{-1}{(D+2)} + \frac{1}{(D+1)}\right) e^{e^x}$

$\Rightarrow -\left[\frac{1}{D+2} \cdot e^{e^x}\right] + \left[\frac{1}{(D+1)} \cdot e^{e^x}\right]$

$\Rightarrow \underbrace{-e^{-2x}\int e^{2x} \cdot e^{e^x} \cdot dx}_{I_1} \underbrace{+e^{-x}\int e^{+x} e^{e^x} \cdot dx}_{I_2}$

$I_1 = -e^{-2x}\int e^{2x} \cdot e^{e^x} \cdot dx$

Put, $e^x = t$

$e^x \cdot dx = dt$

$\Rightarrow -e^{-2x}\left[\int e^x \cdot e^{e^x} \cdot e^x \cdot dx\right)$

$\Rightarrow -e^{-2x}\left[\int t \cdot e^t \cdot dt\right]$

$\Rightarrow -e^{-2x}\left[te^t - \int e^t \cdot dt\right]$

$\Rightarrow -e^{-2x}\left[e^t\right](t-1)$

$\Rightarrow -e^{-2x} \cdot e^{e^x}(e^x-1)$

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$I_2 = e^{-x}\int \underline{e^x}\cdot e^{e^x}\cdot \underline{dx}$

let, $e^x = t$

$e^x . dx = dt$

$\Rightarrow (e^{-x})\int e^t dt$

$\Rightarrow e^{-x} \times e^t$

$\Rightarrow e^{-x}\cdot e^{e^x}$

So. $P.I = -e^{-2x}\cdot e^{e^x}(e^x-1) + e^{-x}\cdot e^{e^x}$

$\Rightarrow e^{e^x}[-e^{-2x}(e^x-1)+e^{-x}]$.

$\Rightarrow e^{e^x}(\cancel{-e^{-x}}+e^{-2x}+\cancel{e^{-x}})$

$\Rightarrow [e^{e^x}\cdot e^{-2x}]$

Solution $\boxed{y = c_1 e^{-2x} + c_2 e^{-x} + e^{-2x}\cdot e^{e^x}}$

Q:- Solve $(D^2+9)y = \sec 3x$

$\Rightarrow$ for. C.F, $D^2+9=0$.

$D^2 = -9$

$(D=m)$

$m = \pm 3i$

$C.F = (A\cos 3x + B\sin 3x)$

for. P.I $\frac{1}{(D+3i)(D-3i)}\cdot \sec 3x$

$\Rightarrow \frac{1}{6i}\left(\frac{1}{D-3i} - \frac{1}{D+3i}\right)\cdot \sec 3x$

$\Rightarrow \frac{1}{6i}\left[\frac{1}{(D-3i)}\cdot \sec 3x - \frac{1}{(D+3i)}\sec 3x\right]$.

$\Rightarrow \frac{1}{6i}\left[\underbrace{e^{3ix}\int e^{-3ix}\sec 3x . dx}_{I_1} - \underbrace{e^{-3ix}\int e^{3ix}\sec 3x . dx}_{I_2}\right]$.

$\Rightarrow I_1 = \int e^{-3ix} \cdot \sec 3x \cdot dx$

$\Rightarrow \int e^{(-3x)i} \cdot \sec 3x \cdot dx$

$\Rightarrow \int (\cos 3x - i\sin 3x) \sec 3x \cdot dx$

$\Rightarrow \int (1 - i\tan 3x) \cdot dx$

$\Rightarrow \left(x - i\frac{\ln \sec 3x}{3} \right)$

$\Rightarrow I_2 = \int e^{3ix} \sec 3x \cdot dx$

Similarly:- $I_2 = \left(x + i\frac{\ln \sec 3x}{3} \right)$

So, $\underline{P.I} = \frac{1}{6i}\left[e^{3ix}\left(x - i\frac{\ln \sec 3x}{3} \right) - e^{-3ix}\left(x + i\frac{\ln \sec 3x}{3} \right) \right]$

$\Rightarrow \frac{1}{6i}\left[x(e^{3ix} - e^{-3ix}) + i\frac{\ln \sec 3x}{3}\left(-e^{3ix} \mp e^{-3ix} \right) \right]$

$* \; e^{(3x)i} - e^{(-3x)i} = i\sin 3x + i\sin 3x$

$= 2i\sin 3x$

$* \; -(e^{3ix} + e^{-3ix})$

$= -[2i\cos 3x].$

$\Rightarrow \frac{1}{6i}\left[x(2i\sin 3x) - i\frac{\ln \sec 3x}{3}(2i\cos 3x). \right]$

$\Rightarrow \frac{x}{3}\sin 3x + \frac{1}{9}(\ln \sec 3x) \cos 3x$ Ans

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Q:- Solve:- $(D^2y+y) = \operatorname{cosec} x$.

$\Rightarrow$ Q.P.I $\frac{1}{(D^2+1)} \cdot \operatorname{cosec} x$

$\Rightarrow \frac{1}{(D+i)(D-i)} \cdot \operatorname{cosec} x$

$\Rightarrow \frac{1}{2i}\left[\frac{1}{D-i} - \frac{1}{D+i}\right] \cdot \operatorname{cosec} x$

C.F $D^2+1=0$
$(D=m)$
$m^2+1=0$
$m=\pm i$

C.F. $c_1\cos x + c_2\sin x$

$\Rightarrow \frac{1}{2i}\left[\underbrace{\frac{1}{D-i}\operatorname{cosec} x}_{I_1} - \underbrace{\frac{1}{D+i}\operatorname{cosec} x}_{I_2}\right]$.

$I_1 = e^{ix}\int e^{-ix}\operatorname{cosec} x \cdot dx$.

$\Rightarrow e^{ix}\left[\int(\cos x - i\sin x)\operatorname{cosec} x \cdot dx\right]$

$\Rightarrow e^{ix}\left[\int(\cot x - i)dx\right]$

$\Rightarrow e^{ix}[\ln\sin x - ix]$

$I_2 = e^{-ix}\left[\int e^{ix}\operatorname{cosec} x \cdot dx\right]$

$\Rightarrow e^{-ix}[\ln\sin x + ix]$.

$P.I = \frac{1}{2i}\left[e^{ix}(\ln\sin x - ix) \mp e^{-ix}(\ln\sin x + ix)\right]$

$\Rightarrow \frac{1}{2i}\left[\ln\sin x(e^{ix} - e^{-ix}) - ix(e^{ix}+e^{-ix})\right]$.

$\Rightarrow \frac{1}{2i}\left[\ln\sin x(i2\sin x) - ix(2\cos x)\right]$

$\Rightarrow (\ln\sin x)\sin x - x\cos x$

Solve:

Q: $(D^2+a^2)y = \tan ax$

$\Rightarrow$ (C.F) $D^2+a^2=0.$

$D^2=-a^2$

$D=\pm ia.$

C.F $\Rightarrow C_1\cos ax + C_2 \sin ax$

(P.I) $P.I = \frac{1}{(D+ai)(D-ai)} \tan ax$

$\Rightarrow \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right]\tan ax$

$I_1 = \frac{\tan ax}{D-ai} \Rightarrow e^{aix}\int e^{-aix}\tan ax\,dx$

$\Rightarrow e^{iax}\int (\cos ax - i\sin ax)\cdot \tan ax\, dx$

$\Rightarrow e^{iax}\int \sin ax - i\frac{\sin^2 ax}{\cos ax}\,dx$

$\Rightarrow -e^{iax}\frac{\cos ax}{a} - ie^{ax}\int \frac{\sin^2 ax}{\cos ax}\,dx$

$\Rightarrow -\frac{e^{iax}\cos ax}{a} - ie^{ax}\int \frac{1}{\cos ax} - \frac{\cos^2 ax}{\cos ax}\,dx$

$\Rightarrow -\frac{e^{iax}\cos ax}{a} - ie^{iax}\left[\frac{\ln|\sec ax+\tan ax|}{a} - \frac{\sin ax}{a}\right].$

$\Rightarrow -\frac{e^{iax}}{a}\left[\cos ax + i(\ln|\sec ax+\tan ax|\} - \sin ax)\right]$

$I_2 = e^{-aix}\int e^{aix}\tan ax\,dx$

$\cos ax - i\sin ax = e^{-iax}$

Similarly:

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$$I_2 = -\cancel{\frac{e^{-iax}}{a}}\Big[$$

$$\Rightarrow -\frac{e^{-iax}}{a}\cos ax + ie^{-iax}\left[\frac{\ln|\sec ax+\tan ax|}{a} - \frac{\sin ax}{a}\right]$$

$$\Rightarrow -\frac{e^{-iax}}{a}\left[\cos ax - i\ln|\sec ax+\tan ax| + i\sin ax\right]$$

$$I_2 \Rightarrow -\frac{e^{-iax}}{a}\left[e^{iax} - i\ln|\sec ax+\tan ax|\right].$$

$$I_1 = -\frac{e^{iax}}{a}\left[e^{-iax} + i\ln|\sec ax+\tan x|\right]$$

$$P.I = \frac{1}{2ai}\left[-\frac{e^{iax}}{a}\left[e^{-iax} + i\ln|\sec ax+\tan ax|\right] + \frac{e^{-iax}}{a}\left[e^{iax} - i\ln|\sec ax+\tan x|\right]\right]$$

$$\Rightarrow \frac{1}{2ai}\left[\cancel{\frac{2}{a}} + i\ln|\sec ax+\tan ax|\left(-\frac{e^{iax}}{a} - \frac{e^{-iax}}{a}\right)\right].$$

$$\Rightarrow -\left(\frac{1}{2a^2}\right)\ln|\sec ax+\tan ax|\,(2\cos ax)$$

$$\Rightarrow -\left(\frac{\cos ax}{a^2}\right)\ln|\sec ax+\tan ax|$$

(Sol^n:-)

$$\boxed{y = c_1\cos ax + c_2\sin ax - \frac{\cos ax}{a^2}\ln|\sec ax+\tan ax|}$$

Q:- $(D^2+4)y = \tan 2x$

$\Rightarrow$ C.F:- $D^2+4=0.$

$D^2=-4$

$(D=\pm 2i)$

$C.F = (A\cos 2x + B\sin 2x).$

$$P.I \Rightarrow \frac{1}{(D^2+4)} \cdot \tan 2x$$

$$\Rightarrow \frac{1}{(D+2i)(D-2i)} \cdot \tan 2x$$

$$\Rightarrow \frac{1}{4i}\left(\frac{1}{D-2i} - \frac{1}{D+2i}\right)\tan 2x$$

$$\Rightarrow \frac{1}{4i}\Big(\underbrace{\frac{\tan 2x}{D-2i}}_{I_1} - \underbrace{\frac{\tan 2x}{D+2i}}_{I_2}\Big)$$

$$I_1 = e^{2ix}\int e^{-2ix}\tan 2x \, dx$$

$$\Rightarrow e^{2ix}\int (\cos 2x - i\sin 2x)\tan 2x \, dx$$

$$\Rightarrow e^{2ix}\int \left(\sin 2x - i\,\frac{\sin^2 2x}{\cos 2x}\right)dx$$

$$\Rightarrow e^{2ix}\left[-\frac{\cos 2x}{2} - i\left[\frac{\ln|\sec 2x + \tan 2x|}{2} - \frac{\sin 2x}{2}\right]\right]$$

(Similarly)

$$I_2 = e^{-2ix}\int e^{2ix}\tan 2x \, dx$$

$$e^{2ix}$$

$$\Rightarrow e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left[\frac{\ln|\sec 2x + \tan 2x|}{2} - \frac{\sin 2x}{2}\right]\right]$$

$$\Rightarrow \frac{e^{-2ix}}{2}\left[-e^{2ix} + i\ln|\sec 2x + \tan 2x|\right]$$

$$I_1 = \frac{e^{2ix}}{2}\left[-e^{-2ix} - i\ln|\sec 2x + \tan 2x|\right]$$

$$P.I = \frac{1}{4i}\left[\cancel{\frac{-1}{2}} - \frac{e^{2ix}}{2}\, i\ln|\sec 2x + \tan 2x| + \cancel{\frac{1}{2}} - \frac{e^{-2ix}}{2}\, i\ln|\sec 2x + \tan 2x|\right]$$

$\Rightarrow \frac{-1}{4i} x (i \ln|\sec 2x + \tan 2x| \left(\frac{e^{2ix} + e^{-2ix}}{2}\right).$

$\Rightarrow \frac{-1}{4} x \ln|\sec 2x + \tan 2x| \cos 2x$

Q8 $(D^2+1)y = \sec x$

$\Rightarrow$ **C.F** $D^2+1=0.$, Put $(D=m)$

$m^2+1=0.$

$(m=\pm i)$

$(C.F) = A\cos x + B\sin x$

$(P.I)$ $\frac{1}{D^2+1} \times \sec x$

$\Rightarrow \frac{1}{(D+i)(D-i)} \times \sec x$

$\Rightarrow \frac{1}{2i}\left[\frac{1}{D-i} - \frac{1}{D+i}\right] \times \sec x$

$\Rightarrow \frac{1}{2i}\left[\frac{\sec x}{D-i} - \frac{\sec x}{D+i}\right]$

$\Rightarrow \frac{1}{2i}\left[e^{ix}\int e^{-ix} \sec x . dx - e^{-x}\int e^{+x} \sec x dx\right]$

$\Rightarrow \frac{1}{2i}\left[e^{ix}\int (\cos x - i\sin x)\sec x . dx - e^{-ix}\int (\cos x + i\sin x)\sec x dx\right]$

$\Rightarrow \frac{1}{2i}\left[e^{x}\int (1 - i\tan x) . dx - e^{-x}\int (1 + i\tan x) . dx\right]$

$\Rightarrow \frac{1}{2i}\left[e^{x}(x - i\ln\sec x) - e^{-x}(x + i\ln\sec x).\right]$

$\Rightarrow \frac{1}{2i}\left[x(e^{x} - e^{-x}) - i\ln\sec x(e^{x} + e^{-x})\right]$

$\Rightarrow \frac{1}{2i}\left[x(2i\sin x) - i\ln\sec x(2i\cos x)\right].$

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$\Rightarrow x\sin x \mp (\ln\sec x)\cos x$

Solutions:-

$$\boxed{y = A\cos x + B\sin x + x\sin x \mp \cos x(\ln\sec x)}$$

Q:- $(D^2+a^2)y = \operatorname{cosec} ax$

$\Rightarrow$ P.I $\Rightarrow \dfrac{1}{D^2+a^2}\cdot\operatorname{cosec} ax$

$\Rightarrow \dfrac{1}{(D+ia)(D-ia)}\operatorname{cosec} ax$

$\Rightarrow \dfrac{1}{2ia}\left[\dfrac{\operatorname{cosec} ax}{D-ia} - \dfrac{\operatorname{cosec} ax}{D+ia}\right]$

$\Rightarrow \dfrac{1}{2ia}\left[e^{axi}\int e^{-xai}\operatorname{cosec} ax\cdot dx - e^{-axi}\int e^{axi}\operatorname{cosec} ax\cdot dx\right]$

$\Rightarrow \boxed{\dfrac{1}{2ia}\left[e^{ax}\int\cancel{(\cos x - \sin x)\cos}\right.}$

$\Rightarrow \dfrac{1}{2ia}\left[e^{iax}\int(\cos ax - i\sin ax)\operatorname{cosec} ax\cdot dx\right.$
$\left. - e^{-iax}\int(\cos ax + i\sin ax)\operatorname{cosec} ax\cdot dx\right]$

$\Rightarrow \dfrac{1}{2ai}\left[e^{iax}\int(\cot ax - i)\cdot dx - e^{-iax}\int(\cot ax + i)\cdot dx\right]$

$\Rightarrow \dfrac{1}{2ai}\left[\dfrac{\ln\sin ax}{a}\cdot e^{iax} - ie^{iax}\cdot x - \dfrac{e^{-iax}\ln\sin ax}{a} \mp ixe^{-iax}\right]$

$\Rightarrow \dfrac{1}{2ai}\left[\dfrac{\ln\sin ax}{a}\cdot\left(e^{i(ax)} - e^{-i(ax)}\right) - ix\left(e^{iax} + e^{-iax}\right)\right]$

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$$\Rightarrow \frac{1}{2ai}\left[2i\sin ax \cdot \frac{\ln \sin ax}{a} - ix \times 2\cos x\right]$$

$$\Rightarrow \frac{\sin ax \cdot \ln \sin ax}{a^2} - \frac{x\cos x}{a}$$

C.F $D^2+a^2=0$

$D = \pm ia$

$C.F = (A\cos x + B\sin x).$

Solution ⇒

$$\boxed{y = A\cos x + B\sin x + \frac{\sin ax \cdot \ln \sin ax}{a^2} - \frac{x\cos xa}{a}}$$

#

Steps for Solution:-

① Put, $x = e^z$, so that $(z = \log x)$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \frac{1}{x}\cdot\frac{dy}{dz}$$

$$x\cdot\frac{dy}{dx} = \cancel{\frac{dz}{dx}}\frac{dy}{dz}$$ let $\left(\frac{dy}{dz} = Dx\right)$

$$= Dy$$

$$\left(x\cdot\frac{dy}{dx} = Dy\right)$$

$$x^2\cdot\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

Q:- Solve the eqn

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$$

⇒ Put, $(x = e^z)$

$\Rightarrow (z = \log x)$

Replace, $x^2\frac{d^2y}{dx^2} = D(D-1)y$

$$x\cdot\frac{dy}{dx} = Dy$$

⇒ $(D(D-1)y) - 2(Dy) - 4y = x^4$

⇒ $(D^2 - D - 2D - 4)y = x^4$

$(D^2 - 3D - 4)y = x^4\ (e^{4z})$

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for C.F $D^2-3D-4=0.$

$$D^2-4D+D-4=0$$

$$(D)(D-4)+(D-4)=0$$

$$(D+1)(D-4)=0$$

$$D=-1,4$$

C.F $C_1e^{-z}+C_2e^{4z}$

for P.I $\frac{1}{f(D)}\cdot f(z)$

$$\Rightarrow \frac{1}{D^2-3D-4}\cdot e^{4z} \quad \text{Put, } (D=a=4)$$

$$\Rightarrow \frac{1}{16-12-4}\cdot e^{4z} \quad \text{(condition fails)}$$

So, Now, $P.I = \frac{z}{2D-3}\cdot e^{4z}$ $\rightarrow (2\times4-3)=5$

Sol^n:-

$$\Rightarrow \left(\frac{z}{5}\cdot e^{4z}\right)$$

C.S = P.I + C.F

$$y=\frac{z}{5}\cdot e^{4z}+C_1e^{-z}+C_2e^{4z}$$

Put, $(z=\log x)$

$$\boxed{y=\frac{\log x}{5}(x^4)+C_1\frac{1}{x}+C_2x^4}$$ Ans

Q:- Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x.\sin(\log x).$

$\Rightarrow$ Put, $(x=e^z)$

$(z=\log x)$

$$x^2\frac{d^2y}{dx^2}=D(D-1)y \quad , \quad x\frac{dy}{dx}=Dy$$

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$\Rightarrow (D^2-D)y + Dy + y = \log x \sin(\log x)$

$\Rightarrow (D^2-\cancel{D}+\cancel{D}+1)y = \log x \sin(\log x)$

$\Rightarrow (D^2+1)y = \log x \sin(\log x)$

$\Rightarrow (D^2+1)y = z \sin z$

for. C.F $\quad D^2+1=0$

$D^2=-1$

$(D=\pm i)$

C.F ~~$c_1e^{i} + c_2e^{i}$~~ $(c_1 \cos z + c_2 \sin z) = C.F$

for P.I $\quad \frac{1}{(D^2+1)} \cdot \underbrace{z \sin z}_{\to f(z)}$

$\Rightarrow z\left[\frac{1}{D^2+1} f(z)\right] - \frac{2D}{(D^2+1)^2} \cdot f(z)$

$\Rightarrow z\left(\frac{1}{D^2+1} \cdot \sin z\right) - \frac{2D}{(D^2+1)^2} \cdot \sin z$

Put $(D^2=-a^2=-1)$

(Condition fails)

So, Now

$\Rightarrow z\left(\frac{z}{2D} \sin z\right) - \frac{2\cos z}{(D^2+1)^2}$

Put $(D^2=-a^2=-1)$

Condition fails.

So, Now

above term becomes

$\Rightarrow \frac{2z}{2(D^2+1)(2D)} \cdot \cos z \to$ Condition again fails So

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$\Rightarrow z\left(\frac{z}{2}\right)\int \sin z \, dz - \frac{2z^2 \cos z}{4(3D^2+1)}$ $\rightarrow (D^2$

$\boxed{\begin{array}{l} \Rightarrow \cancel{\frac{z^2}{2}(-\cos z) - \frac{2z^2}{4\times 4}\int \cos z \, dz} \\ \Rightarrow \cancel{-\frac{z^2 \cos z}{2} - \frac{z^2}{2\times 4}(\sin z)} \\ \Rightarrow \cancel{-\frac{z^2}{2}\left(\cos z + \frac{\sin z}{4}\right)} \end{array}}$

put $(D^2 = -1)$

$\Rightarrow \frac{z^2}{2}(-\cos z) - \frac{z^2 \cos z}{2(3(-1)+1)} \Rightarrow \frac{z^2}{2}(-\cos z) + \frac{1}{4}(z^2 \cos z)$

$\Rightarrow -\frac{1}{4}(z^2 \cos z)$

(Solution)

$y = (c_1 \cos z + c_2 \sin z) - \frac{z^2}{4}\cos z$

Put $(z = \log x)$

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Q:- solve. $\left(\frac{d^2y}{dx^2}+y\right)=x^2\sin 2x$

$\Rightarrow$ in symbolic form

$$D^2y+y=x^2\sin 2x$$
$$y(D^2+1)=x^2\sin 2x$$

(for. C.F) $D^2+1=0$

$$D^2=-1$$
$$D=\pm i\;=\alpha\pm i\beta.$$
$$(\alpha=0,\ \beta=1)$$

$$C.F=(C_1\cos x+C_2\sin x)$$

for P.I :- $\frac{1}{(D^2+1)}x^2\sin 2x$

it is also be written as $im(e^{2xi})$. $\left(e^{(2x)i}=\cos 2x+i\sin 2x\right)$

$$=\frac{1}{(D^2+1)}x^2\times im.(e^{2xi}).$$

$\Rightarrow$ im. part of $\left(\frac{1}{(D^2+1)}\cdot x^2e^{(2i)x}\right)$ [Case (4) Replace (D by (D+2i))]

$\Rightarrow$ $im\left(\frac{1}{(D+2i)^2+1}\cdot x^2e^{2ix}\right)$

$\Rightarrow$ $im\left(e^{2ix}\left(\frac{1}{D^2-4-4Di+1}\right)x^2\right)$ $\rightarrow D^2-3-4Di$

$\Rightarrow$ $im\left(e^{2ix}\left(\frac{1}{-3\left(1+\frac{4Di-D^2}{3}\right)}\right)x^2\right)$

$\Rightarrow$ $im\left(\frac{e^{2ix}}{(-3)}\left[\left(1+\left(\frac{4Di-D^2}{3}\right)\right)^{-1}x^2\right]\right)$

$$\Rightarrow im\left(\frac{e^{2ix}}{-9}\left[1-\frac{4Di-D^2}{\cancel{5}}+\left(\frac{4Di-D^2}{\cancel{5}}\right)^2 - - -\right]x^2\right)$$

$$\Rightarrow im\left[\frac{e^{2ix}}{-\cancel{9}}\left[x^2-\frac{4i\times 2x-2}{\cancel{5}}+\frac{(-16D^2+D^4-8D^3i)x^2}{49}\right]\right]$$

$$\Rightarrow im\left[\frac{e^{2ix}}{-9}\left[x^2-\frac{8ix-2}{\cancel{5}}+\frac{-\cancel{16}\times 2}{\cancel{9}}\right]\right]$$

$$\Rightarrow \cancel{im\left[\frac{e^{2ix}}{9}\left(x^2-2ix-\frac{7}{2}\right)\right]}$$

$$\Rightarrow im\left(\frac{e^{2ix}}{-3}\left(x^2-\frac{8}{3}ix+\frac{2}{3}-\frac{32}{9}\right)\right)$$

$$\Rightarrow im\left(\frac{e^{2ix}}{-3}\left(x^2-\frac{26}{9}-8ix\right)\right)$$

$$\Rightarrow im\left[\frac{(\cos 2x+i\sin 2x)\left(x^2-\frac{26}{9}-8ix\right)}{-3}\right]$$

$$\Rightarrow \cancel{\boxed{im\left[\frac{1}{3}\right]}}\ \frac{1}{-3}\left((-8ix)\cos 2x+i(\sin 2x)\left(x^2-\frac{26}{9}\right)\right)$$

$$\Rightarrow -\frac{1}{3}\left[-8x\cos 2x+x^2\sin 2x-\frac{26}{9}\sin 2x\right]i \rightarrow im.\ part$$

$$P.I = -\frac{1}{3}\left(-8x\cos 2x+x^2\sin 2x-\frac{26}{9}\sin 2x\right)$$

soln.

$$\boxed{y = C.F + P.I}$$

Simutaneous linear Differential equations

(eg:)
$$\frac{dx}{dt}+a_1x+a_2y=f_1(t) \quad \text{--- (1)}$$

$$\frac{dy}{dt}+b_1x+b_2y=f_2(t) \quad \text{--- (2)}$$

⇒ Cooking rules:-

(i) Put, $\left(\frac{d}{dt} = D\right)$ in eqn ① and ②.

$$Dx + a_1 x + a_2 y = f_1(t)$$
$$Dy + b_1 x + b_2 y = f_2(t)$$

(ii) Use (a) elimination method
(b) Cramer's Rule
} for eliminating dependent Variable

(iii) The L.D.E we get, when one dependent & one independent Variable with constant coeff.

(iv) put the Value of Variable in any eqn to find other Variables.

~~Q: Solve these~~

Q: Solve:-

$$\frac{dx}{dt} - 7x + y = 0 \quad \text{—①}$$

$$\frac{dy}{dt} - 2x + 5y = 0 \quad \text{—②}$$

let, $\left(\frac{d}{dt} = D\right)$

(Solution)
⇒

$$Dx - 7x + y = 0$$
$$Dy - 2x + 5y = 0$$
$$x(D-7) + y = 0 \quad \text{—③}$$
$$y(D+5) - 2x = 0 \quad \text{—④}$$

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Solve eqn (3) and (4).

$x(D-7) + y = 0 \qquad \times(-2)$

$x(-2) + y(D+5) = 0 \qquad \times(D-7)$

$\Rightarrow -2(D-7)x + (-2)y = 0$

$\underline{+ -2(D-7)x \pm y(D+5)(D-7) = 0}$

$y(-2 \mp (D+5)(D-7)) = 0.$

$y(-2 \mp D^2 + 7D \mp 5D + 35) = 0.$

for (C.F) $(-D^2 + 2D + 33) = 0.$

Put $(D = m)$

$-m^2 + 2m + 33 = 0 \Rightarrow m^2 - 2m - 33 = 0.$

$m = \dfrac{2 \pm \sqrt{4 + 4 \times 33}}{2} \Rightarrow 1 \pm \dfrac{2}{2}\sqrt{34}$

$= (1 \pm \sqrt{34})$

$\underline{C.F} = c_1 e^{(1+\sqrt{34})t} + c_2 e^{(1-\sqrt{34}t)}.$

$C.S = C.F + P.I$

$$\boxed{y = c_1 e^{(1+\sqrt{34})t} + c_2 e^{(1-\sqrt{34})t}}$$

$-x = \left(\dfrac{dy}{dx} + 5y\right)\dfrac{1}{2}$

$$\boxed{x = \frac{1}{2}\left(c_1(1+\sqrt{34})e^{(1+\sqrt{34})t} + c_2(1-\sqrt{34})e^{(1-\sqrt{34}t)} + 5\left(c_1 e^{(1+\sqrt{34})t} + c_2 e^{(1-\sqrt{34})t}\right)\right)}$$

Q:- Solve, $\frac{dx}{dt} + 2y = e^{t}$.

$\frac{dy}{dt} - 2x = e^{-t}$.

⇒ Symbolic form of above equations.

$Dx + 2y = e^{t}$

$Dy - 2x = e^{-t}$

By using cramer's Rule

$$\Delta = \begin{vmatrix} D & 2 \\ -2 & +D \end{vmatrix} \Rightarrow (D^2+4)$$

$$\Delta_1 = \begin{vmatrix} e^{t} & 2 \\ e^{-t} & D \end{vmatrix} \Rightarrow De^{t} - 2e^{-t}$$

$$\Rightarrow (e^{t} - 2e^{-t})$$

$$\Delta_2 = \begin{vmatrix} D & e^{t} \\ -2 & e^{-t} \end{vmatrix}$$

$$\Rightarrow De^{-t} - 2e^{t}$$

$$\Rightarrow (-e^{-t} - 2e^{t})$$

$$x = \frac{\Delta_1}{D} = \cancel{(D^2+4)}$$

$$x = \frac{e^{t} - 2e^{-t}}{(D^2+4)}$$

$$\Rightarrow (D^2+4)x = e^{t} - 2e^{-t}$$

for C.F $\quad D^2+4 = 0$

$D = \pm 2i$

$$\text{C.F} = (C_1 \cos 2t + C_2 \sin 2t)$$

for. (P.I) $\quad P.I = \dfrac{1}{D^2+4}\left(e^t - 2e^{-t}\right)$

$$\Rightarrow \frac{1}{D^2+4}\cdot e^t - 2\times\frac{1}{D^2+4}\cdot e^{-t}$$

Put $(D=a=1)$ $\qquad$ Put $(D=a=-1)$

$$\Rightarrow \frac{1}{1+4}\cdot e^t - 2\times\frac{1}{(-1)^2+4}\cdot e^{-t}$$

$$\Rightarrow \left(\frac{e^t}{5} - \frac{2}{5}\cdot e^{-t}\right)$$

solution

$$\boxed{x = \left(C_1\cos 2t + C_2\sin 2t + \frac{e^t}{5} - \frac{2}{5}e^{-t}\right)}$$

$$y = \frac{-1}{2}\left(\frac{dx}{dt} - e^t\right)$$

$$\Rightarrow \frac{-1}{2}\left(-C_1(2)\sin 2t + C_2(2)\cancel{\sin 2t}\cos 2t + \frac{e^t}{5} - \frac{2}{5}e^{-t} \;\cancel{+}- e^t\right)$$

$$\boxed{y = \frac{-1}{2}\left[-2C_1\sin 2t + 2C_2\cos 2t \cancel{+} - \frac{4e^t}{5} - \frac{2}{5}e^{-t}\right]}$$

⇒ P.I by method of Variation of parameter ⇐

⇒ $$P.I = -y_1 \int \frac{y_2 X}{W} \cdot dx + y_2 \int \frac{y_1 X}{W} \cdot dx$$

$y_1 =$ Coeff. of C_1 in C.F

$y_2 =$ Coeff. of C_2 in C.F

$X =$ R.H.S.

$W =$ Work-coeff. $= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

Note:- This method is applicable all types of R.H.S.

this methode is only used for 2nd order DE

Q:- $\frac{d^2y}{dx^2} + a^2y = \tan ax$

⇒ $(D^2+a^2)\,y = \tan ax$

for C.F $\quad D^2+a^2=0$

$D^2 = -a^2$

$D = \pm ai$

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$C.F = (C_1 \cos ax + C_2 \sin ax)$, $W = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$

$\Rightarrow [a(\cos^2 ax + \sin^2 ax) = a]$

$P.I = -\cos ax \int \frac{\sin ax \times \tan ax}{a} . dx$

$+ \sin ax \int \frac{\cos ax . \tan ax}{a} . dx$

$\Rightarrow -\frac{\cos ax}{a} \int \frac{\sin^2 ax}{\cos ax} . dx + \frac{\sin ax}{a} \int \sin ax . dx$

$\Rightarrow -\frac{\cos ax}{a} \int (\sec ax - \cos ax) . dx + \frac{\sin ax}{a} \left(-\frac{\cos ax}{a}\right)$

$\Rightarrow -\frac{\cos ax}{a} \left[\frac{\ln}{a} |\sec ax + \tan ax| - \frac{\sin ax}{a}\right] - \frac{\sin ax \cos ax}{a \times a}$

$\Rightarrow -\frac{\ln}{a^2} |\sec ax + \tan ax| \times \cos ax$

Q8. $\frac{d^2y}{dx^2} + 4y = \frac{e^{3x}}{x^2}$

$\Rightarrow (D^2+4) y = \frac{e^{3x}}{x^2}$

for. C.F $D^2+4=0$

$D = \pm 2i$

$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\cos 2x & 2\sin 2x \end{vmatrix}$

$\Rightarrow (+2)$

$C.F = C_1 \cos 2x + C_2 \sin 2x$

$\Rightarrow -\cos 2x \int \frac{\sin 2x}{2} \times \frac{e^{3x}}{x^2} . dx + \sin 2x \int \frac{\cos 2x}{2} . \frac{e^{3x}}{x^2} . dx$

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Q:- $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$

$\Rightarrow$ for CF

$D^2-6D+9=0.$

$(D=m)$

$m^2-6m+9=0.$

$\Rightarrow \boxed{\cancel{m^2-(3m-3m+9)=0} \\ \cancel{m(m-3)-3(m}}$

$\Rightarrow m^2-3m-3m+9=0.$

$\Rightarrow m(m-3)-3(m-3)=0$

$(m-3)(m-3)=0$

$m=3,3.$

CF $(c_1+c_2x)e^{3x}$

$\Rightarrow c_1e^{3x}+(xe^{3x})c_2$

$\omega = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}(3x+1) \end{vmatrix}$

$\Rightarrow e^{6x}(3x+1)$
$-3xe^{6x}$

$\Rightarrow \underline{\underline{e^{6x}}}$

P.I $\Rightarrow -e^{3x}\int \frac{xe^{3x}}{e^{6x}} \times \frac{e^{3x}}{x^2}.dx$

$+xe^{3x}\int \frac{e^{3x}}{e^{6x}} \times \frac{e^{3x}}{x^2}.dx$

$\Rightarrow -e^{3x}\int \frac{1}{x}.dx + xe^{3x}\left(\frac{-1}{x}\right)$

$\Rightarrow -e^{3x}\ln x + -e^{3x}$

$\Rightarrow -e^{3x}(\ln x+1)$

Q:- $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$

$\Rightarrow D^2-2D+1=0$

$(D-1)^2=0$

$D=1,1$

(C.F) $\Rightarrow (c_1+c_2x)e^x \Rightarrow (c_1e^x+c_2xe^x)$

C.F $\quad c_1 e^x + c_2 x e^x$

P.I $\Rightarrow -e^x \int \frac{xe^x \times e^x \log x}{\cancel{W}} \cdot dx$
$\qquad + xe^x \int \frac{e^x e^x \log x}{\cancel{W}} \cdot dx$

$|W| = \begin{vmatrix} e^x & xe^x \\ e^x & e^x(x+1) \end{vmatrix}$

$\Rightarrow e^{2x}(x+1) - xe^{2x}$

$\Rightarrow e^{2x}$

$\Rightarrow -e^x \int \underset{II}{x}\,\underset{I}{\log x} \cdot dx + xe^x \int \log x \cdot dx$

$\Rightarrow -e^x \left[\log x \times \frac{x^2}{2} \mp \int \frac{1}{x} \times \frac{x^2}{2} \cdot dx\right] + xe^x \left[\log x \times x \mp \int \frac{1}{x} \cdot x\, dx\right]$

$\Rightarrow -e^x \left[\frac{x^2}{2} \log x \mp \frac{x^2}{2 \times 2}\right] + xe^x \left[x \log x \mp x\right]$

$\Rightarrow -e^x x^2 \left[\frac{\log x}{2} \mp \frac{1}{4}\right] + x^2 e^x (\log x \mp 1)$

$\Rightarrow \cancel{x^2 e^x \left[-\frac{\log x}{2} \mp \frac{1}{4} + \log x + 1\right]}$

$\Rightarrow \cancel{+\frac{3}{2} \mid x^2 e^x \quad x^2 e^x \left[\frac{\log x}{2}\right.}$

$\Rightarrow x^2 e^x \left[-\frac{\log x}{2} + \frac{1}{4} + \log x \mp 1\right]$

$\Rightarrow \frac{x^2 e^x}{4} \left[2 \log x \mp 3\right]$ Ans

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Q:- $\left(\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}\right)$

$\Rightarrow$ $D^2y - y = \frac{2}{1+e^x}$

$(D^2-1)y = \frac{2}{1+e^x}$

for (C.F) $D^2-1=0$

$D^2=1$

$D=\pm 1$.

C.F $(C_1 e^x + C_2 e^{-x})$

$W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$\Rightarrow -1-1 \Rightarrow (-2)$

P.I $= -e^x \int \frac{e^{-x}}{(-2)} \times \left(\frac{2}{1+e^x}\right) . dx$

$+ e^{-x} \int \frac{e^x}{(-2)} \left(\frac{2}{1+e^x}\right) . dx$

$\Rightarrow e^x \int \frac{e^{-x} . dx}{1+e^x} - e^{-x} \int \frac{e^x}{1+e^x} . dx$

$\Rightarrow \quad \downarrow I_1 \qquad \downarrow I_2$

$I_1 = \int \frac{e^{-x}}{1+e^x} dx = \int \frac{e^{-2x}}{1+e^{-x}} . dx \Rightarrow$

let, $1+e^{-x} = t$

$e^{-x} . dx = (-dt)$

$\Rightarrow \int \frac{(t-1)(-dt)}{t}$

$\Rightarrow \int \frac{(1-t) . dt}{t} \Rightarrow \ln(1+e^{-x}) - (1+e^{-x})$

$I_2 = \int \frac{dt}{t} \Rightarrow \ln|e^x+1|$.

Differential equation of I^{st} order and higher degree :-

General form of (D.E) of 1st order and higher degree

$$P_0\left(\frac{dy}{dx}\right)^n + P_1\left(\frac{dy}{dx}\right)^{n-1} + \cdots + P_n = 0.$$

$P_0, P_1, P_2 \cdots P_n$ are fn of "x" and "y".

Here, $\left(\frac{dy}{dx} = P\right)$

$$P_0 P^n + P_1 P^{n-1} + P_2 P^{n-2} + \cdots + P_n = 0$$

$$(f(x, y, P) = 0)$$

$$\boxed{f\left(x, y, \frac{dy}{dx}\right) = 0}$$

- to find the solution of above eqn

1. eqn is solvable for P
2. " " " " x
3. " " " " y
4. Clairaut's eqn.

Q:- ① $P^2 - 5P + 6 = 0$. (solvable for P)

S. $P^2 - 2P - 3P + 6 = 0$

$P(P-2) - 3(P-2) = 0$

$(P-2)(P-3) = 0$

$(P = 2 \text{ or } P = 3)$

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$\frac{dy}{dx} = 2$ or $\frac{dy}{dx} = 3$

$\int dy = \int 2 \cdot dx$ $\qquad$ $\int dy = \int 3 \cdot dx$

$(y = 2x + c_1)$ $\qquad$ $y = 3x + c_2$

$(y - 2x - c_1 = 0)$ $\qquad$ $(y - 3x - c_2 = 0)$

$(c_1 = c_2 = c)$

solution

$$\boxed{(y - 2x - c)(y - 3x - c) = 0}$$

Q:(2) $x^2p^2 + xyp - 6y^2 = 0.$ (solvable for (p))

$\Rightarrow$ $p = \frac{-xy \pm \sqrt{x^2y^2 + 4 \times x^2 \times 6y^2}}{2x^2}$

$p = -\frac{y}{2x} \pm \frac{5xy}{2x^2}$

$p = \frac{-y}{2x} \pm \frac{5y}{2x}$

$p = -\frac{y}{2x} + \frac{5y}{2x}$, $\quad p = \frac{-y}{2x} - \frac{5y}{2x}$

$\frac{dy}{dx} = \frac{2y}{x}$, $\quad \frac{dy}{dx} = -\frac{3y}{x}$

$\int \frac{dy}{y} = \int \frac{2dx}{x}$, $\quad \int \frac{dy}{y} = -\int \frac{3dx}{x}$

$\Rightarrow$ $\log y = 2\log x + \log c_1$, $\quad \log y = -3\log x + c_2$

$y = c_1x^2$, $\quad \left(y = \frac{c_2}{x^3}\right)$

$(y - c_1x^2) = 0$, $\quad (yx^3 - c_2 = 0)$

Sol^n :- $\boxed{(y - c_1 x^2)(yx^3 - c) = 0}$ Ans

Q:- $p^2 + 2py\cot x = y^2$ (Solve)

$\Rightarrow p^2 + (2y\cot x)p - y^2 = 0.$

$$p = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$$

$$\Rightarrow \frac{-2y\cot x \pm 2y\,\mathrm{cosec}\,x}{2}$$

$$\Rightarrow -y\cot x + y\,\mathrm{cosec}\,x$$

$\frac{dy}{dx} = -y(\cot x - \mathrm{cosec}\,x)$	$\frac{dy}{dx} = -y(\cot x + \mathrm{cosec}\,x)$

$\cancel{\int \frac{dy}{dy} = -\int \left(\frac{\cos x - 1}{\sin x}\right) dx}$

$\int \frac{dy}{y} = -\int (\cot x - \mathrm{cosec}\,x)\, dx$

$\log y = -[\log \sin x - \log|\cot x - \mathrm{cosec}\,x| + \log c_1]$

$\log y = \log \frac{c_1(\cot x - \mathrm{cosec}\,x)}{\sin x\,\cancel{(\cot x - \mathrm{cosec}\,x)}}$

$\cancel{y = \frac{c_1}{\sin x(\cos x + 1)}}$

$y = \frac{c_1(\cos x - 1)}{\sin^2 x}$

$P = -y(\cot x - \operatorname{cosec} x)$

$$\frac{dy}{dx} = -y\left[\cot x - \operatorname{cosec} x\right]$$

$$\frac{dy}{y} = -\int(\cot x - \operatorname{cosec} x)\,dx$$

$$\log y = -\log\sin x + \log|\cot x - \operatorname{cosec} x| + \log c_1$$

$$\log y = \log\left[\frac{c_1(\cot x - \operatorname{cosec} x)}{\sin x}\right]$$

$$y = c_1\left(\frac{\cos x - 1}{\sin^2 x}\right)$$

$$y = \frac{c_1(\cos x - 1)}{-(\cos^2 x - 1)}$$

$$\boxed{y = \frac{-c_1}{(1+\cos x)}}$$

$P = -y(\cot x + \operatorname{cosec} x)$.

$$\frac{dy}{dx} = -y(\cot x - \operatorname{cosec} x)$$

$$\int\frac{dy}{y} = -\int(\cot x - \operatorname{cosec} x)\,dx$$

$$\log y = -\log\sin x - \log|\cot x - \operatorname{cosec} x| + \log c_2$$

$$y = \frac{c_2}{\frac{\sin x(\cos x - 1)}{\cos x}}$$

$$y = \frac{c_2}{(\cos x - 1)}$$

Solⁿ $\boxed{\left(y + \frac{c_1}{1+\cos x}\right)\left(y - \frac{c_2}{\cos x - 1}\right) = 0}$

Q: $x^2p^2 - (2xy)p + 2y^2 - x^2 = 0$

$$P = \frac{2xy \pm \sqrt{4x^2y^2 - 4x^2(2y^2 - x^2)}}{2x^2}$$

$$P = \frac{y}{x} \pm \frac{\sqrt{4x^4 - 8x^2y^2}}{2x^2}$$

$$P = \frac{y}{x} + \frac{\sqrt{x^2 - y^2}}{x^2}, \quad P = \frac{y}{x} - \frac{\sqrt{x^2 - y^2}}{x}$$

Q $\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2-y^2}}{x}$

Homogeneous eqn (Put) $(y = vx)$

$\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$

$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + \sqrt{1-v^2}$

$\int \frac{dv}{\sqrt{1-v^2}} = \int \frac{dx}{x}$

$\sin^{-1} v = \log x + \log c_1$

$\sin^{-1} v = \log x c_1$

$x c_1 = e^{\sin^{-1}(y/x)}$

$\boxed{x c_1 - e^{\sin^{-1}(y/x)} = 0}$

$\frac{dy}{dx} = \frac{y}{x} - \frac{\sqrt{x^2-y^2}}{x}$

$\cancel{v} + \frac{dv}{dx}x = \cancel{v} - \sqrt{1-v^2}$

$\int \frac{dv}{\sqrt{1-v^2}} = \int \frac{-dx}{x}$

$\sin^{-1}(v) = -\log x + \log c_2$

$\sin^{-1}\left(\frac{y}{x}\right) = \log\left(\frac{c_2}{x}\right)$

$\frac{c_2}{x} = e^{\sin^{-1}(y/x)}$

$\boxed{c_2 - x e^{\sin^{-1}(y/x)} = 0}$

$(c_1 = c_2 = c)$

Soln $\boxed{\left(x c_1 - e^{\sin^{-1}(y/x)}\right)\left(c_2 - x e^{\sin^{-1}(y/x)}\right) = 0}$

Solvable for y :-

Q: $y = 3x + \log p$

$\cancel{\frac{dy}{dx} = 3x + \log p}$

diff. w.r.t (x)

$\frac{dy}{dx} = 3 + \frac{1}{p}\cdot\frac{dp}{dx}$

$\Rightarrow (p-3) = \frac{1}{p} \cdot \frac{dp}{dx}$ $\left(\frac{dy}{dx} = p\right)$

$\frac{dp}{dx} = p(p-3)$

$\int \frac{dp}{(p-3)p} = \int dx$

$x = \frac{1}{3}\int \frac{1}{(p-3)} - \frac{1}{p} \cdot dp$

$x = \frac{1}{3}\left[\log(p-3) - \log p\right] + \log c_1$

$3x = \log\left(\frac{p-3}{p}\right) + \log c_1$

$3x = \log\left(\frac{(p-3)c_1}{p}\right)$

$e^{3x} = c_1 \frac{(p-3)}{p}$

$pe^{3x} = c_1 p - 3c_1$

$p(e^{3x} - c_1) = -3c_1$

$\boxed{p = \frac{-3c_1}{c_1 - e^{3x}}} \Rightarrow \boxed{p = \frac{3}{1-e^{3x}\times c}}$

$$\boxed{y = 3x + \log\left(\frac{3}{1-e^{3x}\times c}\right)}$$

Q: $x^2 + p^2x - yp = 0$ (Solvable by y)

$\Rightarrow$ $py = x^2 + p^2x$

$y = \frac{x^2}{p} + \frac{p^2x}{p} \Rightarrow \left(\frac{x^2}{p} + px\right)$

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diff. w.r.t. (x) $\left(\frac{dy}{dx} = p\right)$

$$\frac{dy}{dx} = x^2\left(\frac{-1}{p^2}\right)\frac{dp}{dx} + \left(\frac{1}{p}\right)2x + p + x\frac{dp}{dx}$$

$$0 = \frac{dp}{dx}\left(\frac{-x^2}{p^2} + x\right) + \frac{2x}{p}$$

$$0 = \frac{dp}{dx}\left(\frac{-x}{p^2} + 1\right) + \frac{2}{p}$$

$$\frac{dp}{dx}\left(\frac{-x+p^2}{p^2}\right) = -\frac{2}{p}$$

$$\frac{dp}{dx} = \frac{2p}{p^2 - x}$$

$$\Rightarrow \quad \frac{dx}{dp} = \frac{p}{2} - \frac{x}{2p}$$

$$\left(\frac{dx}{dp} + \frac{x}{2p} = \frac{p}{2}\right)$$

(linear eqn) of D.E

I.F $\quad e^{\int \frac{1}{2p} dp}$

$$\Rightarrow e^{\frac{1}{2}\log p} \Rightarrow e^{\log\sqrt{p}} \Rightarrow (\sqrt{p})$$

$$x \times \sqrt{p} = \int \sqrt{p} \times \frac{p}{2} \, dp$$

$$x\sqrt{p} = \frac{1}{2} \times \frac{p^{5/2}}{5} \times 2 + C$$

$$\left(x = \frac{p^2}{5} + \frac{C}{\sqrt{p}}\right)$$

(solvable for x)

Q8- $y = 2px + p^2y$

$\Rightarrow$ $x = \frac{y - p^2y}{2p} \Rightarrow \left(\frac{y}{2p}\right) - \left(\frac{py}{2}\right)$

diff. w.r.t (y).

$\Rightarrow \frac{dx}{dy} = \frac{y}{2} \times \left(\frac{-1}{p^2}\right)\frac{dp}{dy} + \frac{1}{2p}\cdot 1 - \frac{1}{2}\left(p\times 1 + y\frac{dp}{dy}\right)$

$\Rightarrow \frac{dx}{dy} = \frac{-y}{2p^2}\cdot\frac{dp}{dy} + \frac{1}{2p} - \frac{p}{2} - \frac{y}{2}\cdot\frac{dp}{dy}$

$\frac{1}{p} = \left(\frac{-y}{2p^2} - \frac{y}{2}\right)\frac{dp}{dy} + \frac{1}{2p} - \frac{p}{2}$

$\left(\frac{1}{2p} + \frac{p}{2}\right) = \frac{-y}{2}\left(\frac{1}{p^2} + 1\right)\frac{dp}{dy}$

$\left(\frac{\cancel{1+p^2}}{\cancel{2}p}\right) = \frac{-y}{\cancel{2}}\left(\frac{\cancel{1+p^2}}{p^2}\right)\frac{dp}{dy}$

$1 = -\frac{y}{p}\cdot\frac{dp}{dy}$

$-\frac{dy}{y} = -\frac{dp}{p}$

$\log y = -\log p + \log c$

$\log y = \log\left(\frac{c}{p}\right)$

$y = \frac{c}{p}$

$\boxed{yp = c}$

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Q: $y - 2px = \tan^{-1}(xp^2)$

$\Rightarrow$ $y = 2px + \tan^{-1}(xp^2)$

diff. w.r.t (x)

$\Rightarrow \dfrac{dy}{dx} = 2\left(p + x\dfrac{dp}{dx}\right) + \dfrac{1}{1+x^2p^4}\left(x \times 2p \times \dfrac{dp}{dx} + p^2\right)$

$\Rightarrow p = 2p + 2x\dfrac{dp}{dx} + \dfrac{1}{1+x^2p^4}\left(2xp\dfrac{dp}{dx} + p^2\right)$

$-p = 2x\dfrac{dp}{dx} + \dfrac{2xp}{1+x^2p^4}\left(\dfrac{dp}{dx}\right) + \dfrac{p^2}{1+x^2p^4}$

$-p - \dfrac{p^2}{1+x^2p^4} = \dfrac{dp}{dx}\left(2x + \dfrac{2xp}{1+x^2p^4}\right)$

$-p\cancel{\left(1 + \dfrac{p}{1+x^2p^4}\right)} = \dfrac{dp}{dx}\cancel{\left(1 + \dfrac{p}{1+x^2p^4}\right)}2x$

$\Rightarrow -p = 2x\dfrac{dp}{dx}$

$\Rightarrow \displaystyle\int \frac{dx}{2x} = \int -\frac{dp}{p}$

$\Rightarrow \dfrac{1}{2}\log x = -\log p + \log c$

$\sqrt{x} = \dfrac{c}{p}$

$\boxed{x = \dfrac{c^2}{p^2}}$

Working Rule of ~~Linear~~ Diff. eqⁿ solvable for P:-

① Reduce the Given eqⁿ the linear factor of P.

② Solve individually all factor by equate to "0".

③ multiply all solⁿ which is find in ② step. and equate to zero
the eqⁿ which we get (know as solⁿ of D.E).