Differential eq. of 1st order and higher degree (1)

The general form of DE of 1st order a nth degree is

$$P_0\left(\frac{dy}{dx}\right)^n + P_1\left(\frac{dy}{dx}\right)^{n-1} + P_2\left(\frac{dy}{dx}\right)^{n-2} + \dots + P_{n-1}\frac{dy}{dx} + P_n = 0$$

where $P_0, P_1, P_2 \dots P_n$ are function of x & y. — (1)

Her Notations $\frac{dy}{dx} = p$

form

$$P_0 p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_{n-1} p + P_n = 0$$

or In General $\boxed{f(x,y,p) = 0}$, $f(x, y, \frac{dy}{dx}) = 0$ — (2)

To find soln of above eqn i.e find y in term of x

Soln of eqn (1)

(I) eqn is solv for p

(II) eqn is for x

(III) eqn is for y

(IV) clairat's eqn

Case I eqn is solable for p

factorizing $P_0 p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_{n-1} p + P_n = 0$

$$[p - f_1(xy)][p - f_2(xy)] \dots [p - f_n(xy)] = 0$$

$p - f_1(xy) = 0$, $p - f_2(xy) = 0 \dots$, $p - f_n(xy) = 0$

$p = f_1(xy)$ $\quad p = f_2(xy), \dots, \quad p = f_n(xy)$

$\frac{dy}{dx} = f_1(xy)$ $\quad \frac{dy}{dx} = f_2(xy) \dots \frac{dy}{dx} = f_n(xy)$

solve above eqns individualy

$f(x\,y\,c_1) = 0$, $f(x\,y\,c_2) = 0$, $\dots$ $f(x\,y\,c_n) = 0$

$f_1(x,y,c_1)\, f_2(x,y,c_2) \cdots f_n(x,y,c_n) = 0$

$c_1 = c_2 = c_3 \cdots = c_n = c$

$\boxed{f_1(x,y,c)\, f_2(x,y,c) \cdots f_n(x,y,c_n) = 0}$ final soln.

Ex 1. $p^2 - 5p + 6 = 0$, Ex 2 $x^2p^2 + xyp - 6y^2 = 0$

Working Rule :-

1. Reduce the given eqn in the linear factors of p
2. Solve individualy ~~the~~ all the factors
3. multiply all the soln which is find in (2) & equate to zero, this is the gen. soln of given diff eqn.

Soln (1) $p^2 - 5p + 6 = 0$

$(p-2)(p-3) = 0$

$p - 2 = 0, \quad p - 3 = 0$

$p = 2 \qquad p = 3$

$\frac{dy}{dx} = 2 \qquad \frac{dy}{dx} = 3$

$dy = 2dx \qquad dy = 3dx$

$y = 2x + c_1 \qquad y = 3x + c_2$

$(y - 2x - c_1) = 0, \quad y - 3x - c_2 = 0$

$c_1 = c_2 = c$

$y - 2x - c = 0 \qquad y - 3x - c = 0$

$\therefore (y - 2x - c)(y - 3x - c) = 0$

Soln (2) $x^2p^2 + xyp - 6y^2 = 0$ (3)

This is quadratic eqn in p

$\therefore$

$$p = \frac{-xy \pm \sqrt{x^2y^2 + 4x^2 \cdot 6y^2}}{2x^2}$$

$$p = \frac{-xy \pm \sqrt{x^2y^2 + 24x^2y^2}}{2x^2}$$

$$= \frac{-xy \pm \sqrt{25x^2y^2}}{2x^2} = \frac{-xy \pm 5xy}{2x^2}$$

$$p = \frac{-xy + 5xy}{2x^2}, \qquad p = \frac{-xy - 5xy}{2x^2}$$

$$p = \frac{2xy}{x^2} \qquad p = \frac{-3xy}{x^2}$$

$$p = \frac{2y}{x} \qquad p = \frac{-3y}{x}$$

$$\frac{dy}{dx} = \frac{2y}{x} \qquad \frac{dy}{dx} = \frac{-3y}{x}$$

$$\frac{dy}{y} = \frac{2}{x}\,dx \qquad \frac{dy}{y} = -\frac{3}{x}\,dx$$

$$\int \frac{dy}{y} = \int \frac{2}{x}\,dx \qquad \int \frac{dy}{y} = \int -\frac{3}{x}\,dx$$

$$\log y = 2\log x + c_1 \qquad \log y = -3\log x + \log c_2$$

$$\log y = \log x^2 + \log c_1 \qquad \log y = \log x^{-3} + \log c_2$$

$$\log y = \log c_1 x^2 \qquad \log y = \log \frac{c_2}{x^3}$$

$$y = c_1x^2 \qquad y = \frac{c_2}{x^3}$$

$$y - c_1x^2 = 0 \qquad y - \frac{c_2}{x^3} = 0$$

final $(y - cx^2)\left(y - \frac{c}{x^3}\right) = 0$ put $c_1 = c_2 = c$