

Le variabili aleatorie

Massimo Borelli

Maggio 2024



1 Concetti di base sulle variabili aleatorie

Variabili aleatorie finite

Un esempio di variabile aleatoria finita:

- accessi ambulatorio / paziente / anno

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0.129 & 0.264 & 0.271 & 0.185 & 0.095 & 0.039 & 0.017 \end{pmatrix}$$

Parole da ricordare:

- funzione **probabilità di massa / densità discreta**
- funzione **di ripartizione / probabilità cumulativa**

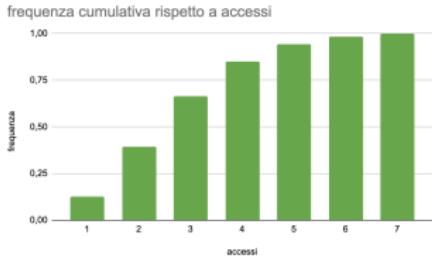
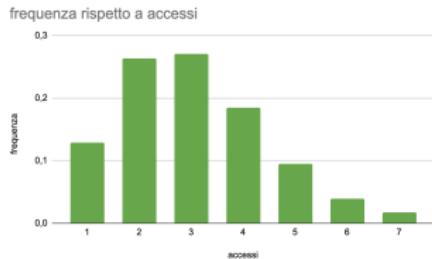
Variabili aleatorie finite

The screenshot shows an Excel spreadsheet titled "accessi". The table has three columns: "accessi" (number of accesses), "frequenza" (frequency), and "cumulativa" (cumulative frequency). The data is as follows:

	A	B	C
1	accessi	frequenza	cumulativa
2	1	0,129	
3	2	0,264	
4	3	0,271	
5	4	0,185	
6	5	0,095	
7	6	0,039	
9	7	0,017	
10			(somma)
11			

A purple arrow points from the value 0,017 in row 9 to the cell containing "(somma)" in row 10.

<https://bit.ly/3QcxHOQ>



Un esempio concreto



- the Reference Data Set
 - dal 2003 al 2014, King's College London Dental Institute
- LL8Gf
 - Lower Left Third Molar
 - Stage G female
 - 18 years old
 - caucasian UK

Un esempio concreto

<https://www.dentalage.co.uk/rds-uk-caucasian/>

	TDS	n	mean	sd	0th%ile	0.5%ile	5%ile	10%ile
					25%ile	50%ile	75%ile	90%ile
LL8Gf	114	18.25	1.63		14.96	15.73	15.97	16.32
					17.07	18.13	18.99	20.64
					95%ile	99.5%ile	100th%ile	
					21.35	21.91	22.78	

Un esempio concreto

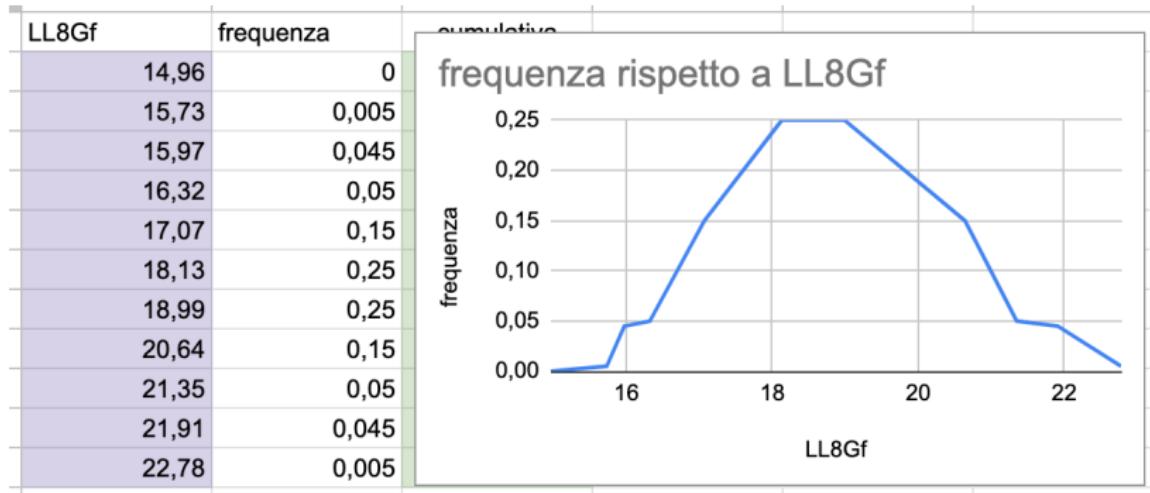
<https://www.dentalage.co.uk/rds-uk-caucasian/>

	A	B	C
1	accessi	frequenza	cumulativa
2	14,96		0
3	15,73		0,005
4	15,97		0,05
5	16,32		0,1
6	17,07		0,25
7	18,13		0,5
8	18,99		0,75
9	20,64		0,9
10	21,35		0,95
11	21,91		0,995
12	22,78		1



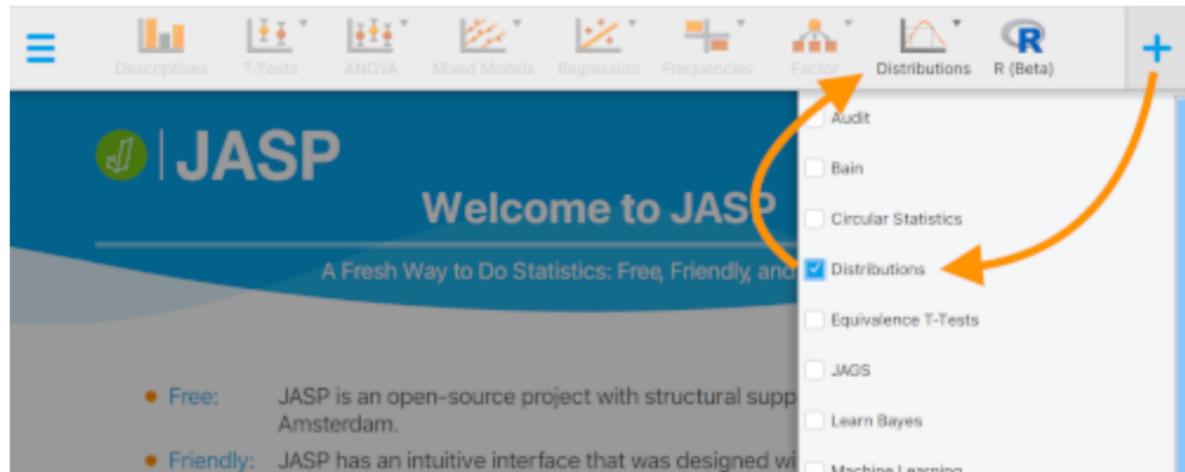
0th%ile	0.5%ile	5%ile	10%ile
14.96	15.73	15.97	16.32
<hr/>			
25%ile	50%ile	75%ile	90%ile
17.07	18.13	18.99	20.64
<hr/>			
95%ile	99.5%ile	100th%ile	
21.35	21.91	22.78	

Un esempio concreto



	TDS	n	mean	sd
LL8Gf	114	18.25	1.63	

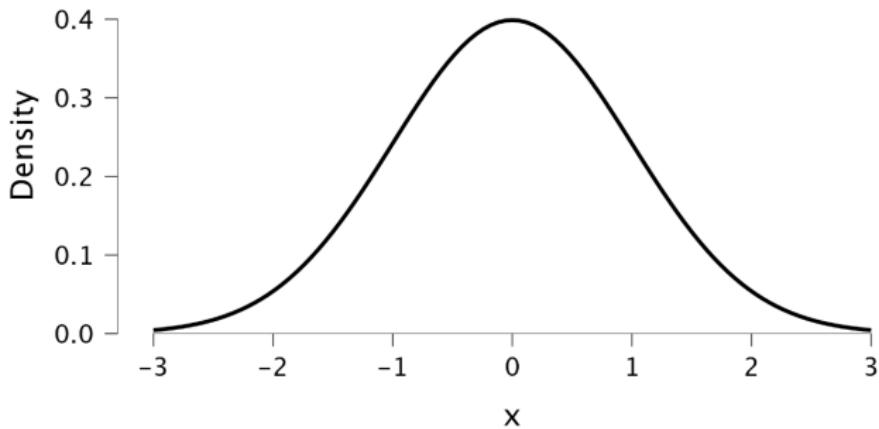
La variabili aleatorie con JASP



la distribuzione Normale

Probability Density Function

Density Plot



la distribuzione Normale

▼ Show Distribution

Parameters μ, σ^2 ▾

Mean: μ 0

Variance: σ^2 1

Display

- Explanatory text
- Parameters, support, and moments
- Probability density function
- Cumulative distribution function
- Quantile function

Options

Range of x from -3 to 3

Highlight

Density Probability

Interval

- from 0 to 1
- from $-\infty$ to 0
- from 0 to ∞

la distribuzione Normale

> Scand J Dent Res. 1990 Apr;98(2):149-58. doi: 10.1111/j.1600-0722.1990.tb00954.x.

Unilateral, isometric bite force in 8-68-year-old women and men related to occlusal factors

M Bakke¹, B Holm, B L Jensen, L Michler, E Möller

Affiliations + expand

PMID: 2343274 DOI: [10.1111/j.1600-0722.1990.tb00954.x](https://doi.org/10.1111/j.1600-0722.1990.tb00954.x)

Results

Bite force versus sex, age, and occlusion – Force recordings were normally distributed. As biteforce values did not differ significantly between right and left side (right side: 479 N, SD 130; left side: 480 N, SD 127; P of difference: 0.89), data were averaged in each subject for further calculations. Bite force in women was lower than in men (women: 441 N, SD 113; men: 522 N, SD 123; P: 0.006; Table 1).

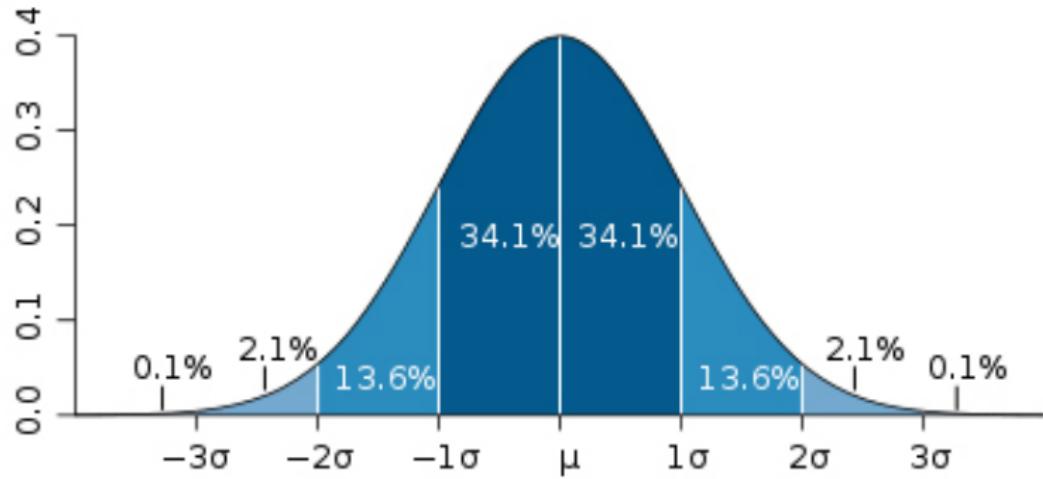
Attività di laboratorio

Massimo Borelli

Maggio 2024

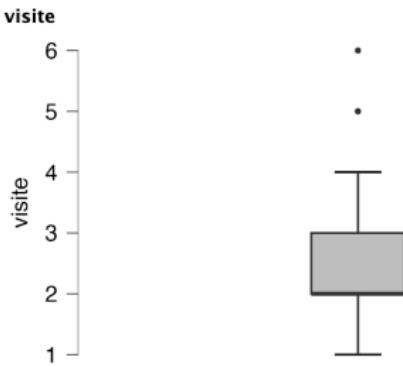
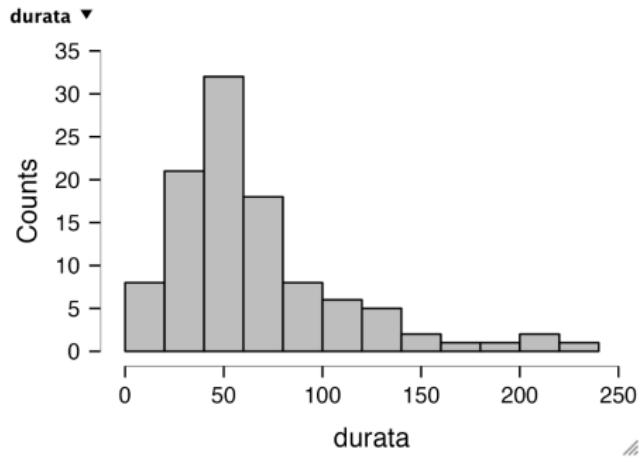


cerchiamo di non dimenticarlo



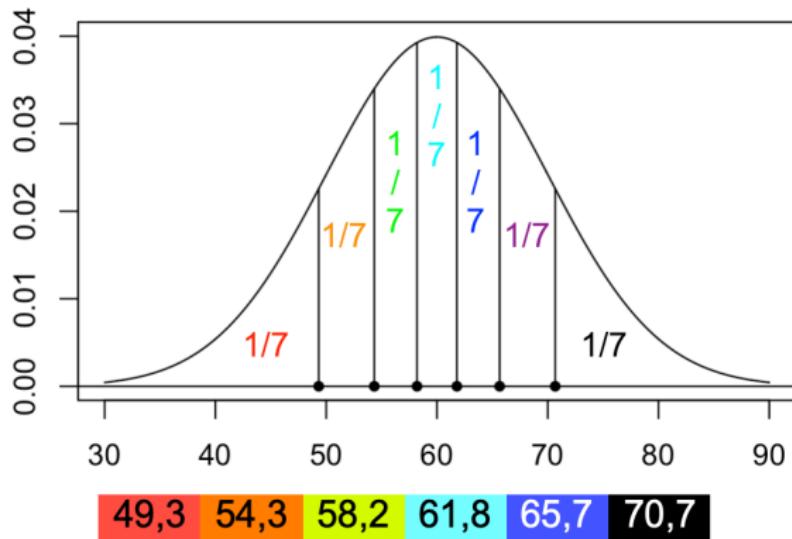
come prendere una decisione? la distribuzione è normale?

il dataset carie.ods



il QQplot: spiegazione

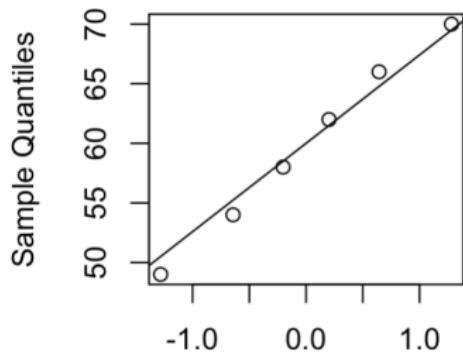
49, 54, 58, 62, 66, 70
'sono normali' ?



il QQplot: spiegazione

49, 54, 58, 62, 66, 70
'sono normali'?

Normal Q-Q Plot

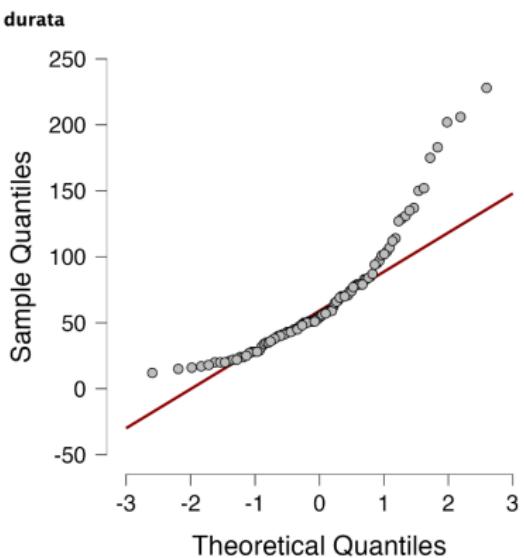
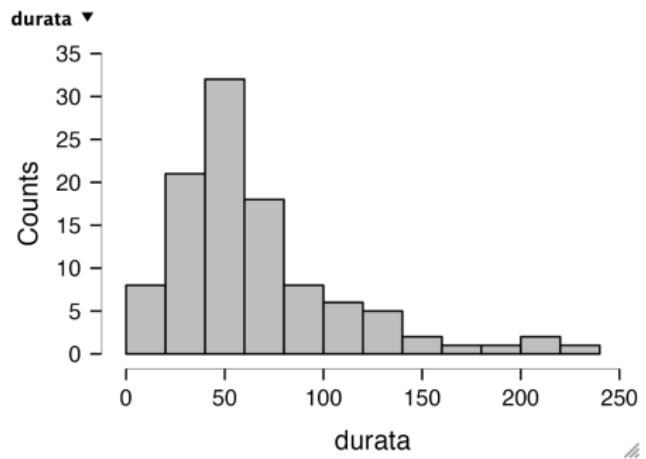


Theoretical Quantiles

-1.1	-0.6	-0.2	0.2	0.6	1.1
49,3	54,3	58,2	61,8	65,7	70,7

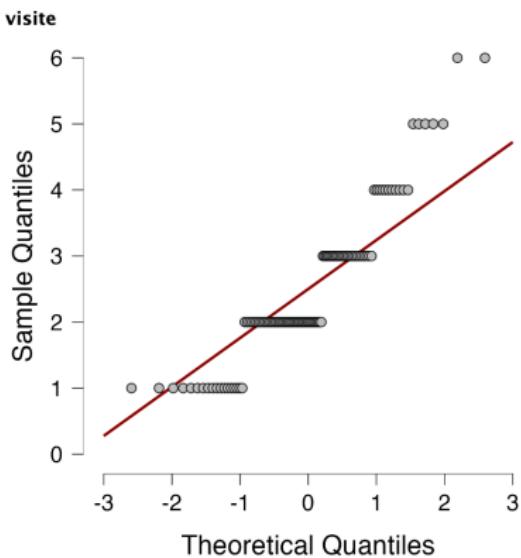
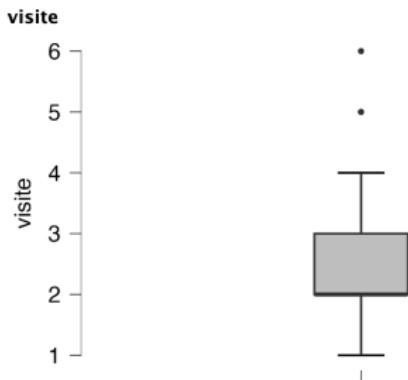
come prendere una decisione? la distribuzione è normale?

... no!



come prendere una decisione? la distribuzione è normale?

... no!



attenzione: un errore importante

un cammello è la somma di due dromedari?



attenzione: un errore frequente

un camello è la somma di due dromedari?

Bernard Rosner

.. linear combination of normal random variables are often of specific concern. It can be shown that any linear combination of normal random variables is itself normally distributed.

Martin Bland:

... If we add two variables from Normal distributions together, even with different means and variances, the sum follows a Normal distribution.

attenzione: un errore frequente

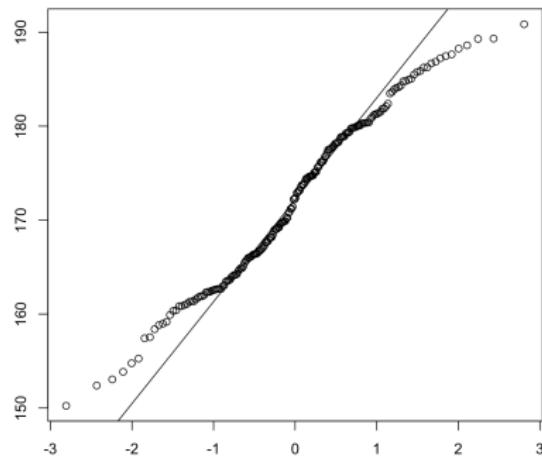
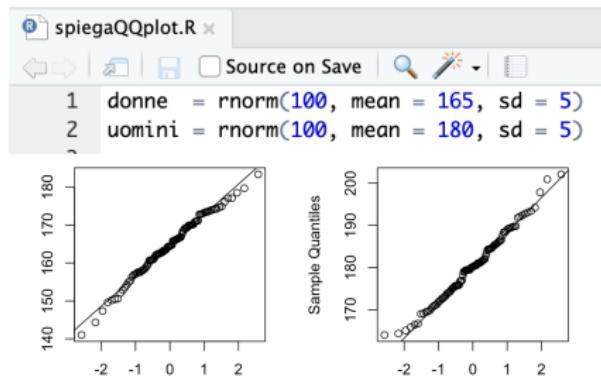
un camello è la somma di due dromedari?



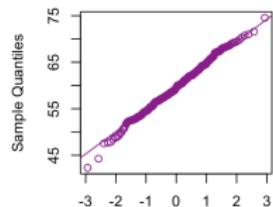
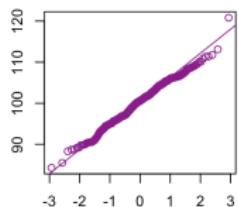
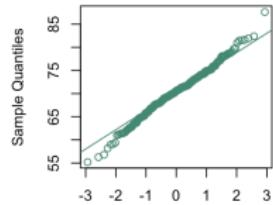
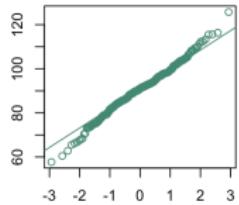
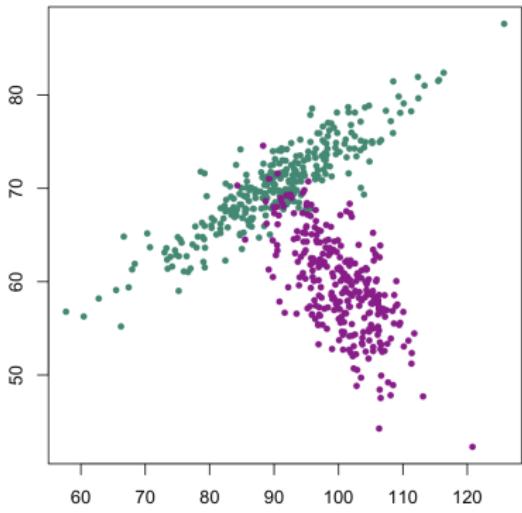
Living histogram, <https://www.jstor.org/stable/1403117>

attenzione: un errore frequente

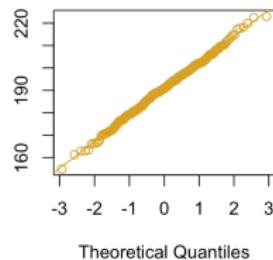
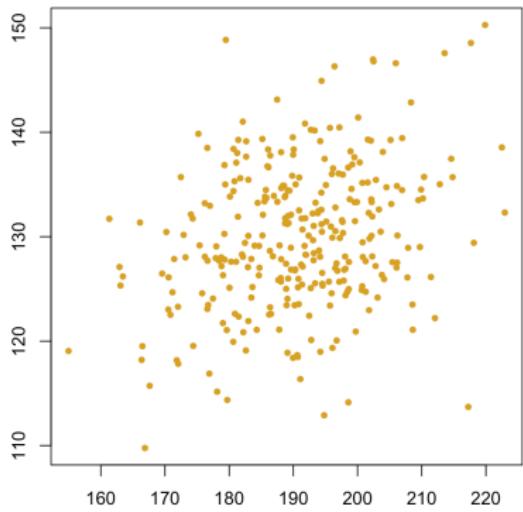
un cammello è la somma di due dromedari?



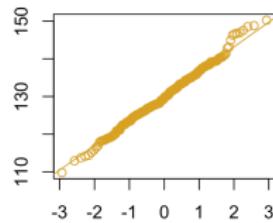
la spiegazione



la spiegazione



Normal Q-Q Plot



Edward L. Melnick, Aaron Tenenbein. 2012
Misspecifications of the Normal Distribution
<https://www.tandfonline.com/doi/pdf/10.1080/00031305.1982.10483052>