

T-Test: l'approccio bayesiano

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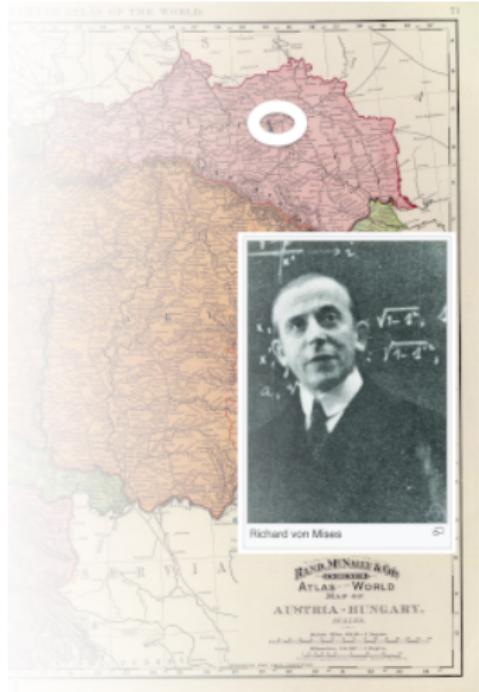


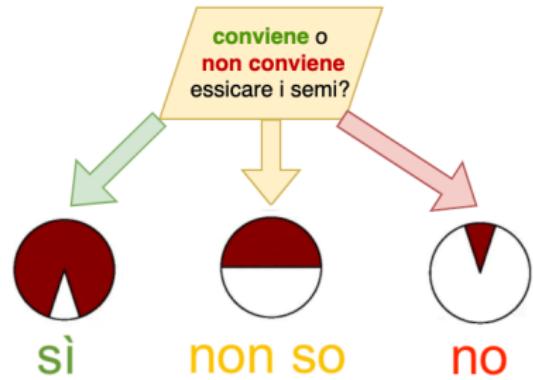
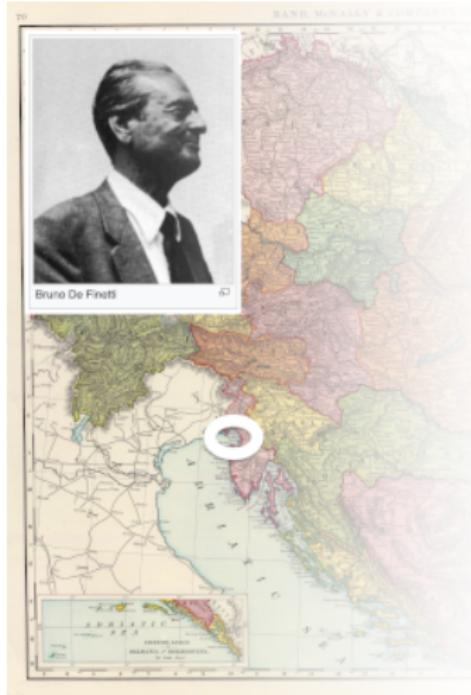
inferenza: il concetto generale





| | t | df | p |
|------------|-------|----|--------------|
| difference | 1.690 | 10 | 0.122 |





ricordate? un cenno al teorema di Bayes

| odontoiatra | dolore | | Total |
|-------------|--------|----|-------|
| | no | si | |
| carie | 19 | 53 | 72 |
| no | 26 | 7 | 33 |
| Total | 45 | 60 | 105 |

Qual è la probabilità che l* mi* paziente abbia una carie?

Qual è la probabilità che l* mi* paziente abbia una carie, sapendo che l* fa male il dente?

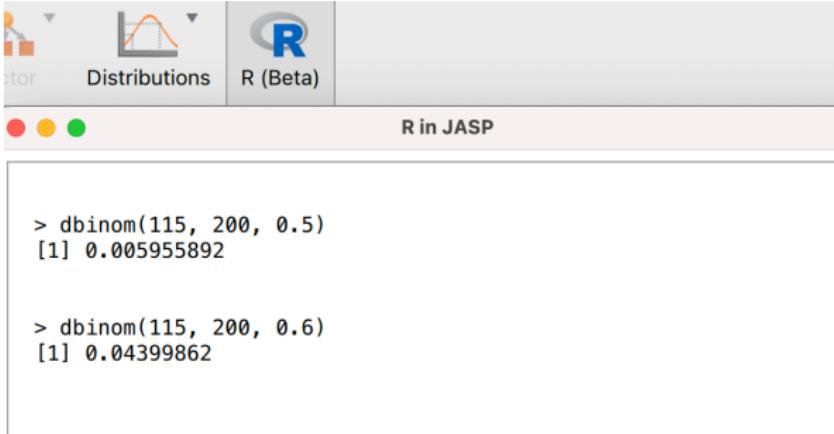
un nuovo ingrediente: il fattore di Bayes



- urna di Alice equa ($p = 0.5$)
- urna di Bob truccata ($p = 0.6$).
(schema binomiale, estrazioni con
rimpiazzo)
- osserviamo 115 successi su 200
estrazioni

? era l'urna di Alice o di Bob ?

un nuovo ingrediente: il fattore di Bayes



The screenshot shows the JASP software interface. At the top, there are three tabs: "Factor" (with a bar chart icon), "Distributions" (with a histogram icon), and "R (Beta)" (with an R logo icon). Below the tabs is a window titled "R in JASP". Inside the window, two R code snippets are displayed:

```
> dbinom(115, 200, 0.5)
[1] 0.005955892

> dbinom(115, 200, 0.6)
[1] 0.04399862
```

$$\frac{P(X = 115 \mid Bob)}{P(X = 115 \mid Alice)} \approx \frac{.044}{.006} \approx 7.4$$

un nuovo ingrediente: il **fattore di Bayes**



il **fattore di Bayes**:

$$\frac{P(D|M_1)}{P(D|M_2)} = \frac{P(M_1|D)}{P(M_2|D)} \cdot \frac{P(M_2)}{P(M_1)}$$

it is much more likely that the balls have been drawn by Bob's urn: about seven times higher

| Not Kiln-Dried | Kiln-Dried | Difference |
|-----------------------|-------------------|-------------------|
| 1903 | 2009 | +106 |
| 1935 | 1915 | -20 |
| 1910 | 2011 | +101 |
| 2496 | 2463 | -33 |
| 2108 | 2180 | +72 |
| 1961 | 1925 | -36 |
| 2060 | 2122 | +62 |
| 1444 | 1482 | +38 |
| 1612 | 1542 | -70 |
| 1316 | 1443 | +127 |
| 1511 | 1535 | +24 |

.. dunque, ecco il **risultato** del test T di Student bayesiano



JASP: **Bayesian One Sample T-Test**

| | BF_{10} | error % |
|------------|-----------|---------|
| difference | 0.885 | 0.021 |

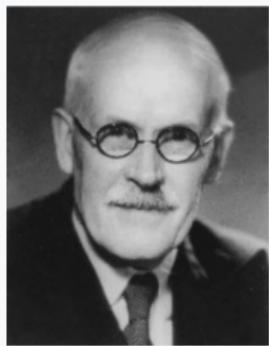
Note. For all tests, the alternative hypothesis specifies that the population mean differs from 0.

ci rimane da capire: **come si interpreta questo risultato?** Conviene o non conviene essiccare i semi?

What does JASP stand for?

In recognition of Bayesian pioneer Sir Harold Jeffreys, JASP stands for Jeffreys's Amazing Statistics Program.

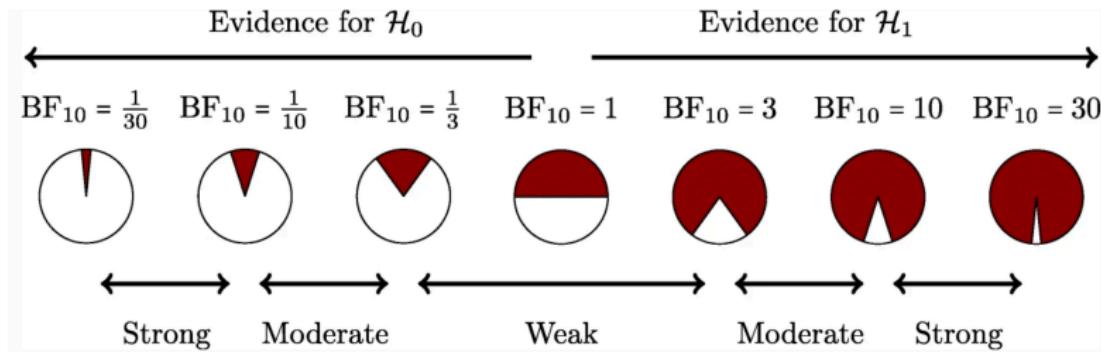
Sir Harold Jeffreys



| K | dHart | bits | Strength of evidence |
|-----------------------------|-------|------|----------------------------|
| $< 10^0$ | | | Negative (supports M_2) |
| $10^0 \text{ to } 10^{1/2}$ | | | Barely worth mentioning |
| $10^{1/2} \text{ to } 10^1$ | | 3 | Substantial |
| $10^1 \text{ to } 10^{3/2}$ | | 6 | Strong |
| $10^{3/2} \text{ to } 10^2$ | | 9 | Very strong |
| $> 10^2$ | | | Decisive |



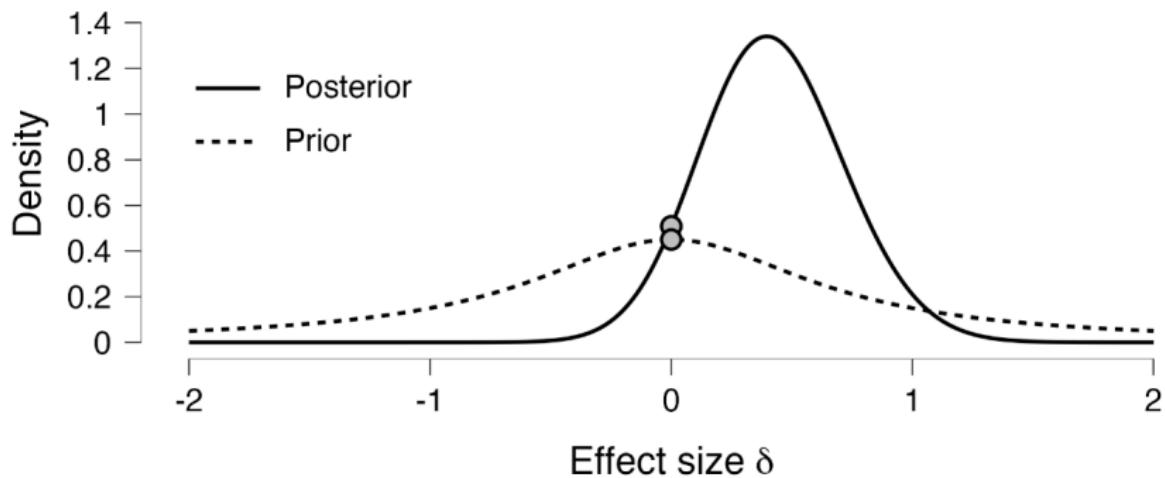
| | BF_{10} | error % |
|------------|-----------|---------|
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van Doorn et al, PMID: 33037582



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| difference | 0.885 | 0.021 |



In conclusione

A 2-sided one-sample t-test comparing the sample population difference ($m = 33.7$) to the null mean ($\mu = 0$) returns a p-value = .122, not significant according an α level of 0.10. The BF_{01} of 0.885 suggests anecdotal evidence in favour of the alternative hypothesis: therefore the observed data are 1.13 times more likely to have occurred under the null than under the alternative hypothesis.

ci poniamo alcune domande ..

il dataset carie

- c'è differenza di genere rispetto alla diagnosi dell'odontoiatra?
- possiamo affidarci al dolore per predire la diagnosi dell'odontoiatra?
- c'è differenza di punteggio scoreAI rispetto alla diagnosi dell'odontoiatra?
- il numero di visite in ambulatorio varia in base al genere?