Approximation Algorithms for Stochastic Inventory Control Models

A periodic-review stochastic inventory control problem

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Github:github.com/blakeapm/stochastic-inventory

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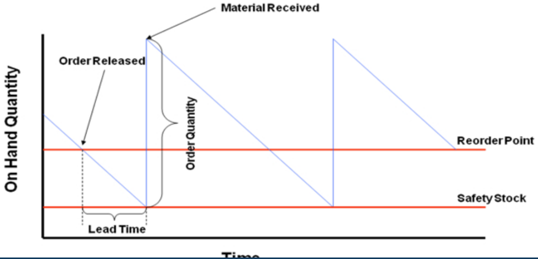
Minimizing Expected Cost

Our goal is to supply each unit of demand without ordering too early or too late. This is difficult because:

- Oftentimes demand and lead times (time between order and receipt of product) are unpredictable.
- Inaccuracies can be extremely costly, so it is important that approximations are provably accurate within a certain range of error

To illustrate the problem, a different, and simpler algorithm:

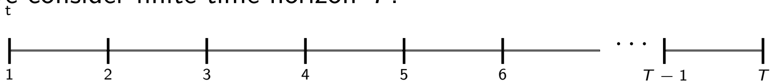


The Periodic-Review Stochastic Inventory Control Problem

- Computing a provably efficient inventory control policies is difficult because:
  - demand per time-period is correlated
  - demand is non-stationary (time-dependent, distribution changes with time)
  - cost is hard to predict due to the nature of demand
- Important for any company that seeks to minimize cost of holding excess inventory while minimizing backorder costs (unmet demand)
- Particularly important in industries where the demand environment is highly dynamic. (i.e. Apple's supply-chain for solid state drives, seasonal products such as rock salt)
- These environments experience high correlation between demands in different periods (difficult to compute optimal inventory policy).

Model

We consider finite time horizon T.



At each time period $t = 1, \cdots, T$, the following cost are incurred

$$L_t(x_t, d_t, q_t) := c_t q_t + h_t(x_t + q_t - d_t)^+ + p_t(x_t + q_t - d_t)_t^-$$

(Local Cost = Ordering Cost + Holding Cost + Backlogging Cost)

- c_t: per-unit ordering cost at time t.
- h_t: per-unit holding cost from t to $t+1$.
- p_t: per-unit backlogging penalty at time t.
- x_t: net inventory at time t. (our state)
- q_t: ordering quantity at time t. (our control)
- d_t: demand quantity at time t. (we observe d_t after decide q_t)

Goal: At time t, choose q_t to minimize the expected total cost from period t to T.
Let V_t be the minimum expected cost from period t to T. Then

$$\begin{aligned} V_T(x_T, d_{1..T-1}) &= \min_{q_T \geq 0} E[L_T(x_T, D_T, q_T)|d_{1..T-1}] \\ V_t(x_t, d_{1..t-1}) &= \min_{q_t \geq 0} E[L_t(x_t, D_t, q_t) \\ &\quad + V_{t+1}(x_t + q_t - D_t, d_{1..t-1}, D_t)|d_{1..t-1}] \end{aligned}$$

This gives us two natural ways to "solve" the problem:

- Dynamic Programming: solve for q_t backwards using the formula above.
- Myopic: At time t, ignore V_{t+1}, we only minimize $E[L_t(x_t, D_t, q_t)|d_{1..t-1}]$.

Recall:

$$V_T(x_T, d_{1..T-1}) = \min_{q_T \geq 0} E[L_T(x_T, D_T, q_T)|d_{1..T-1}]$$

$$V_t(x_t, d_{1..t-1}) = \min_{q_t \geq 0} E[L_t(x_t, D_t, q_t) + V_{t+1}(x_t + q_t - D_t, d_{1..t-1}, D_t)|d_{1..t-1}]$$

- At period $s > t$, D_s is a random variable that depends on perivous demands $d_1, d_2, ..., d_{t+1}, ..., d_{s-1}$.
  To compute the above conditional expectation, we need to work on $q_t, ..., q_T$ which are are our future decision.
- The myopic approach works extremely well on many cases[1], but may perform extremely poorly on other cases. (We will explain a poorly-performing example at the end of the talk.)

- In fact we also need to consider leading time $L \neq 0$ (i.e. it takes L periods to receive our order.) Then we need to modify:

$$x_t := Net\ \ Inventory + Undelived\ \ orders$$

$$:= Net\ \ Inventory + \sum_{j=t-L+1}^{t-1} q_j$$

$$L_t(x_t, d_{t..t+L}, q_t) := c_t q_t + h_{t+L}(x_t + q_t - \sum d_{t..t+L})^+$$
$$+ p_{t+L}(x_t + q_T - \sum d_{t..t+L})^-$$

$***\sum d_{t..t+L} := d_t + ... + d_{t+L}$

- We can assume $c_t = 0$ by doing the transformation:

$$\hat{c}_t := 0;$$
$$\hat{h}_t := h_t + c_t - c_{t+1}$$
$$\hat{p}_t := p_t - c_t + c_{t+1}$$
$$c_{T+1} := 0$$

Dual-Balancing Algorithm [2]

Recall *Local Backlogging Cost*:

$$\Pi_t(x_t, q_t) := p_{t+L}(x_t + q_T - \sum D_{t..t+L})^-$$

Now, we define *Marginal Holding Cost*:

$$H_t(x_t, q_t) := \sum_{j=t+L}^{T} h_j(q_t - (\sum D_{t..j} - x_t)^+)^+$$

. Dual-Balancing Algorithm:

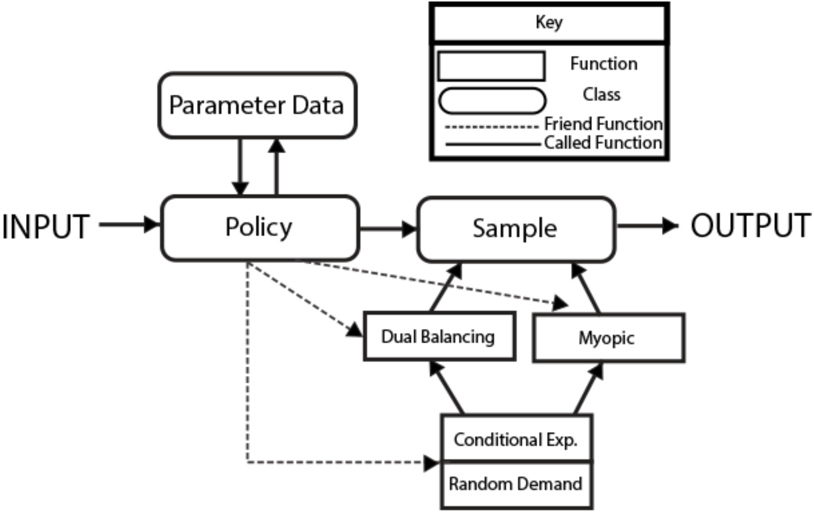
- Do transformation to make the ordering cost 0.
- Solve the convex optimization problem:

$$q_t^* = argmin_{q_t > 0} \max E[H_t(x_t, q_t)|d_{1..t-1}], E[\Pi_t(x_t, q_t)|d_{1..t-1}]$$

in order to find out q_t^* such that

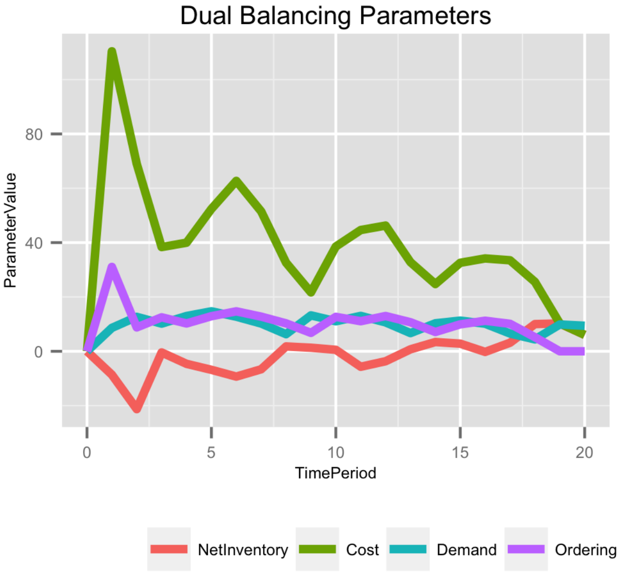
$$E[H_t(x_t, q_t^*)|d_{1..t-1}] = E[\Pi_t(x_t, q_t^*)|d_{1..t-1}]$$

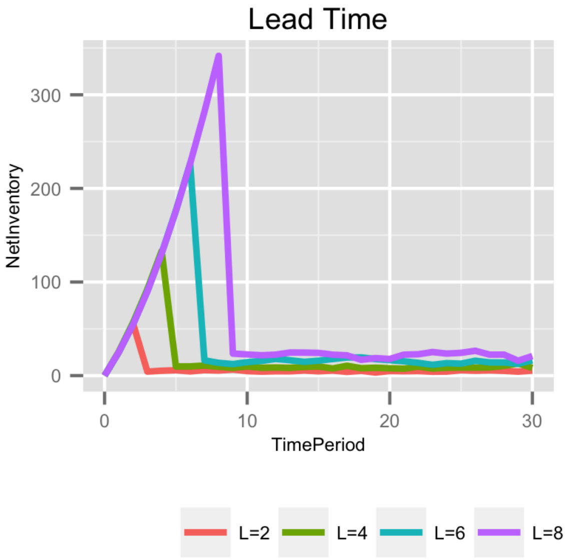
Challenges in Implementation



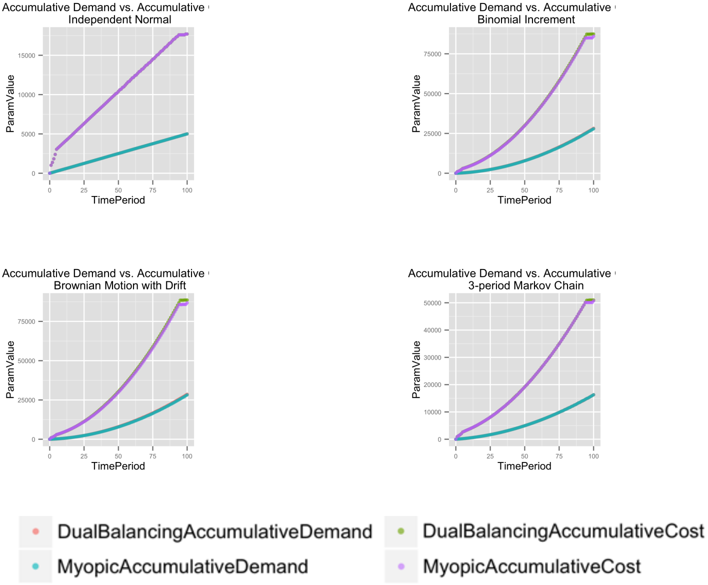
In following simulations:

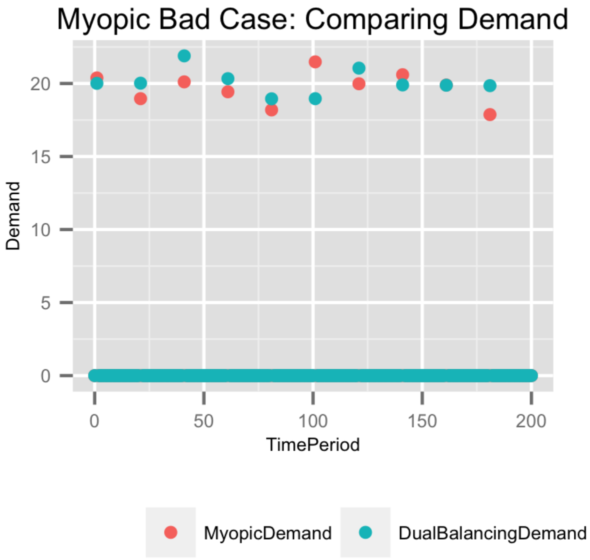
- We always let $c_t = 3, h_t = 1, p_t = 2$.
- Without specific mention, time horizon $T = 100, L = 5, x_0 = 0$.
- The sample size is 20, if we do averages.
- We considered four different demands (i.e. D_t) distribution:
  1. $D_t = N(50, 5)$ are iid.
  2. $D_t - D_{t-1} = N(5, 1.581)$ with $D_0 = 30$.
  3. $D_t - D_{t-1} = B(10, 0.5)$ with $D_0 = 30$.
  4. $D_t - (D_{t-1} + D_{t-2} + D_{t-3})/3 = N(5, 1.581)$ with $D_0 = 30$.

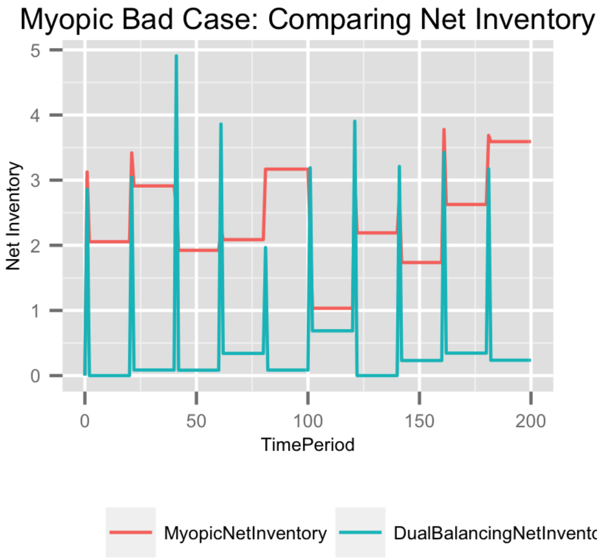

Dual Balancing Parameters
ParameterValue
80
40
0
0
5
10
15
20
TimePeriod
NetInventory
Cost
Demand
Ordering

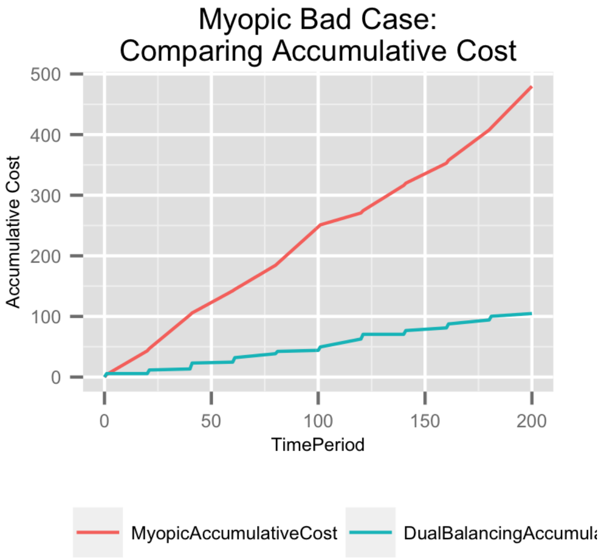

Lead Time
NetInventory
300
200
100
0
0
10
20
30
TimePeriod
L=2
L=4
L=6
L=8

Minimizing Expected Cost




Myopic Bad Case: Comparing Demand
Demand
20
15
10
5
0
0
50
100
150
200
TimePeriod
MyopicDemand
DualBalancingDemand


Myopic Bad Case: Comparing Net Inventory
Net Inventory
0
1
2
3
4
5
0
50
100
150
200
TimePeriod
MyopicNetInventory
DualBalancingNetInvento


Myopic Bad Case:
Comparing Accumulative Cost
Accumulative Cost
500
400
300
200
100
0
0
50
100
150
200
TimePeriod
MyopicAccumulativeCost
DualBalancingAccumul

References

[1] Iida, T., P. H. Zipkin. 2006. Approximate solutions of a dynamic forecast-inventory model. Manufacturing Service Oper. Management 8 407425.
[2] Retsef Levi, Martin Pal, Robin Roundy and David Shmoys, 2007. Approximation Algorithms for Stochastic Inventory Control Models, Mathematics of Operations Research, Volume 32 (2), pages 284-302,

Questions?