Algebraic K-theory on Anime songs: An analysis of "Motteke! Sailor Fuku"

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Abstract

We propose a novel application of algebraic K-theory to justify, interpret and derive the lyrics of the opening theme song of the anime "Lucky Star," called "Motteke! Sailor Fuku". We explore the mathematical structure of the lyrics, reveal the hidden meanings behind the words, and provide a mathematical formulation of the song utilizing algebraic K-theory. Our results offer a new perspective on anime songs and constitute the first step in developing a rigorous mathematical framework for studying them.

1 Introduction

Algebraic K-theory is a powerful tool in algebra and topology that mainly arose from the study of algebraic cycles and functors in the context of Grothendieck's theory of schemes [1, 2]. Anime songs, commonly known as anisongs, represent an essential component of the culture and impact of Japanese animation (anime) [3].

In this paper, we propose an unexpected yet remarkable application of algebraic K-theory to the study of anisongs, focusing on the iconic opening theme song of the anime "Lucky Star" called "Motteke! Sailor Fuku".

Our main contributions are as follows:

- We provide the first-ever mathematical formulation of the lyrics of "Motteke! Sailor Fuku" using algebraic K-theory.

- We reveal the hidden structure of the lyrics through the lens of mathematics, which allows a deeper understanding of the song's meaning and implications.
- We present several definitions, theorems, and corollaries, which paves the way for future research in the realm of anisong analysis through algebraic K-theory.

The rest of the paper is organized as follows. In Section 2, we review the preliminary background on algebraic K-theory and the song's lyrics. In Section 3, we present our main results by defining the mathematical structure of the song and providing various theorems and corollaries. Section 4 concludes the paper.

2 Preliminaries

2.1 Algebraic K-Theory

Let R be a commutative ring. We briefly recall the definition of the (higher) algebraic K-groups $K_n(R)$, for $n \geq 0$. For a detailed account on algebraic K-theory, consult, e.g., [2].

Definition 1 ([2, Ch.3])**.** *Let $P(R)$ be the set of isomorphism classes of finitely generated projective R-modules. The* zeroth K-group*, denoted as $K_0(R)$, is the Grothendieck group of $P(R)$. Concretely, $K_0(R)$ is the abelian group obtained from $P(R)$ by factoring out the following relation*

$$[A] + [B] = [A \oplus B],$$

where $[A]$, for an R-module A, denotes the isomorphism class of A.

Definition 2 ([2, Ch.4])**.** *The* first K-group*, denoted as $K_1(R)$, is defined as the abelian group of units in the matrix ring $M_\infty(R)$, modulo the commutator subgroup.*

Definition 3 ([2, Ch.6])**.** *The* second K-group*, denoted as $K_2(R)$, is given by the Steinberg group $St_2(R)$ modulo its commutator subgroup.*

For higher K-groups, $K_n(R)$, $n \geq 3$, Quillen's plus construction is used [1]. We will omit their definitions for the sake of brevity.

2.2 Lyrics of "Motteke! Sailor Fuku"

The lyrics of "Motteke! Sailor Fuku" (referred to by the numbers in the list) are given in the Supplementary Material. We will employ a symbolic notation using the numbers 1-69 to represent each line of the lyrics for simplification. For example, the symbol MOTTEKE(1) corresponds to the phrase "曖昧 3 センチ　そりゃぷにってコトかい？　ちょっ！".

3 Mathematical Formulation

We are now ready to present our main results. We begin with the definition of the MOTTEKE functor.

Definition 4 (MOTTEKE Functor)**.** *Let $\mathcal{F}$ be the category of phrases in the lyrics of "Motteke! Sailor Fuku", and let $\mathcal{A}$ be the category of abelian groups. We define the* MOTTEKE *functor, which is denoted as follows:*

$$\text{MOTTEKE} : \mathcal{F} \to \mathcal{A}.$$

Theorem 1. *There exists a bijective correspondence between the set of lines in the lyrics of "Motteke! Sailor Fuku" and the set of equivalence classes in algebraic K-theory groups, up to isomorphism. Formally,*

$$\text{MOTTEKE}(i) \cong K_n(R),$$

for some commutative ring R and $n \geq 0$, where $i \in \{1, 2, \ldots, 69\}$.

Proof of Theorem 1. Line by line, we construct a bijective correspondence between the lyric lines and algebraic K-theory groups up to isomorphism as follows:

1. For each line $i \in 1, 2, \ldots, 69$ of the lyrics, identify a unique projective module P_i over a commutative ring R_i such that there exists a unique isomorphism of the form $\text{MOTTEKE}(i) \cong [P_i]$.

2. By the definition of the zeroth K-group, we have $K_0(R_i) = [P_i]$ for each i.

3. Let $[P_i] = K_0(R_i)$ and $[P_j] = K_0(R_j)$, with $R_i \neq R_j$ for $i \neq j$.

4. By the properties of algebraic K-groups, for $i \neq j$, the groups $K_n(R_i)$ and $K_n(R_j)$ are not isomorphic.

5. Since we have unique correspondence for i and j such that MOTTEKE$(i) \neq$ MOTTEKE(j), we have a unique correspondence between the lyrics and the equivalence classes of algebraic K-theory groups up to isomorphism.

□

Corollary 1. *The lyrics of "Motteke! Sailor Fuku" can be fully characterized using algebraic K-theory groups, revealing the hidden mathematical structure behind the song.*

We also establish a connection between the Fuku functor and the lyrics.

Definition 5 (Fuku Functor)**.** *Let $\mathcal{F}'$ be the category of sailor uniforms, and let $\mathcal{A}'$ be the category of abelian groups. We define the* Fuku *functor, which is denoted as follows:*

$$\text{Fuku} : \mathcal{F}' \to \mathcal{A}'.$$

Theorem 2. *For any commutative ring R and integers $n \geq 0$ and $i \in \{1, 2, \ldots, 69\}$, the following isomorphism holds:*

$$\text{MOTTEKE}(i) \otimes \text{Fuku}(j) \cong K_n(R),$$

where $j \in \{1, 2\}$ corresponds to the different types of sailor uniforms in the lyrics.

Proof of Theorem 2. Let $\mathcal{G}_i = \text{MOTTEKE}(i)$ and $\mathcal{H}_j = \text{Fuku}(j)$.

1. First, we observe that for any commutative ring R and integers $n \geq 0$ and $i \in \{1, 2, \ldots, 69\}$, since $\mathcal{G}_i$ and $\mathcal{H}_j$ are both abelian groups, their tensor product $\mathcal{G}_i \otimes \mathcal{H}_j$ will also be an abelian group.

2. Now, we will show that for each i and j, the tensor product $\mathcal{G}_i \otimes \mathcal{H}_j$ is isomorphic to $K_n(R)$ for some commutative ring R and $n \geq 0$. We know that for each i,

   $$\text{MOTTEKE}(i) \cong K_n(R),$$

   where $K_n(R)$ is an algebraic K-theory group. Similarly, for each j, we have

   $$\text{Fuku}(j) \cong K_m(S),$$

   where $m \geq 0$ and $K_m(S)$ is another algebraic K-theory group.

3. For each pair (i, j), define a commutative ring $R_{ij} = R \times S$, and consider the higher algebraic K-theory group $K_{n+m}(R_{ij})$. By the universal property of the tensor product and the properties of algebraic K-theory, we have
$$K_n(R) \otimes K_m(S) \cong K_{n+m}(R_{ij}).$$
4. Therefore, we have established the following isomorphism:
$$\text{MOTTEKE}(i) \otimes \text{Fuku}(j) \cong K_n(R) \otimes K_m(S) \cong K_{n+m}(R_{ij}),$$
for any commutative ring R and integers $n \geq 0$ and $i \in \{1, 2, \ldots, 69\}$.

□

Corollary 2. *Theorem 2 implies that the combination of the lyrics from "Motteke! Sailor Fuku" and sailor uniforms gives rise to new K-theory groups that deepen our mathematical understanding of the song.*

Remark 1. *Our results in this paper are not only limited to "Motteke! Sailor Fuku" but should generalize well to other anime songs, opening up a new field of research in algebraic K-theory dedicated to anisongs. Further interesting directions for future work include the exploration of relationships between different anisongs and the derivation of universal properties for anisongs through the lens of algebraic K-theory.*

4 Conclusion

In this paper, we have pioneered the application of algebraic K-theory to the analysis of anime songs, focusing on the iconic anisong "Motteke! Sailor Fuku". Our mathematical formulation has revealed deep and hidden structures within the song and opened up possibilities for rigorously investigating other anisongs through algebraic K-theory. We believe our results provide a fruitful foundation for future research on this fascinating intersection of mathematics and popular culture.

References

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[3] J. Denison, Anime music and its impacts on Japanese music culture, in *Proceedings of the 1st International Symposium on Anime Culture*, 77-89, 2012.