# On the optimal orientation of chocolate cornets

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January 28, 2024

**Abstract**

We prove the Konata–Tsukasa's Theorem, which states that the optimal way to eat a chocolate cornet is to start from the thin end. Our proof uses techniques from large cardinal theory and Hodge–Arakelov theory, and relies on the existence of a supercompact cardinal and a canonical model of arithmetic Chow groups. We also show that the theorem is independent of the choice of the chocolate cornet brand, and that it implies several interesting consequences in the theory of mathematical anime.

## 1 Introduction

A chocolate cornet is a pastry shaped like a cone, filled with chocolate cream. It is a popular snack in Japan, especially among high school students. However, there is a long-standing debate on how to eat a chocolate cornet: should one start from the thin end or the thick end? This question has been discussed by many anime characters, such as Konata Izumi and Tsukasa Hiiragi from Lucky Star.

In this paper, we settle this question by proving the following theorem:

**Theorem 1** (Konata–Tsukasa's Theorem)**.** *The optimal way to eat a chocolate cornet is to start from the thin end.*

Our proof is based on the following assumptions:

1. There exists a supercompact cardinal $\kappa$.
2. There exists a canonical model $\mathcal{M}$ of arithmetic Chow groups over $\mathbb{Z}$, such that $\mathcal{M}$ satisfies the Hodge–Arakelov conjecture and the Beilinson–Soulé vanishing conjecture.
3. A chocolate cornet is a smooth projective variety over $\mathbb{Q}$, with a canonical embedding into $\mathbb{P}^3_{\mathbb{Q}}$.
4. The chocolate cream is a line bundle on the chocolate cornet, with a canonical section $s$ such that $s^{-1}(0)$ is the zero locus of the chocolate cream.

5. The optimal way to eat a chocolate cornet is to maximize the ratio of chocolate cream to pastry in each bite.

We will explain the meaning and motivation of these assumptions in the next section. Here, we only remark that the first two assumptions are well-known open problems in set theory and arithmetic geometry, respectively, and that the last three assumptions are natural ways to model a chocolate cornet mathematically.

Our main result, Theorem 1, has several interesting corollaries, which we state below:

**Corollary 1.** *The Konata–Tsukasa's Theorem is independent of* ZFC*, the standard axioms of set theory.*

**Corollary 2.** *The Konata–Tsukasa's Theorem is independent of the choice of the chocolate cornet brand, as long as the brand satisfies the assumptions (3), (4), and (5) above.*

**Corollary 3.** *The Konata–Tsukasa's Theorem implies the following statements in the theory of mathematical anime:*

- *The Riemann hypothesis is equivalent to the statement that Haruhi Suzumiya is a god.*
- *The Langlands program is equivalent to the statement that Madoka Kaname is a magical girl.*
- *The Birch and Swinnerton-Dyer conjecture is equivalent to the statement that Lelouch Lamperouge is a geass user.*

The rest of the paper is organized as follows. In Section 2, we review some background material on large cardinal theory and arithmetic Chow groups, and explain how they are related to chocolate cornets. In Section 3, we prove the Konata–Tsukasa's Theorem, using a combination of forcing and intersection theory. In Section 4, we prove the corollaries stated above, and discuss some open problems and future directions.

## 2 Preliminaries

In this section, we recall some definitions and results from large cardinal theory and arithmetic Chow groups, and explain how they are used to model chocolate cornets mathematically. We assume that the reader is familiar with the basic notions of set theory, algebraic geometry, and number theory. For more details, we refer to [2, 3, 4].

## 2.1 Large cardinals and forcing

A large cardinal is a cardinal number that has some strong property that implies the consistency of ZFC or some stronger theory. For example, an inaccessible cardinal is a cardinal $\kappa$ such that $\kappa$ is uncountable and there is no cardinal $\lambda < \kappa$ such that $2^\lambda \geq \kappa$. It is easy to see that if $\kappa$ is inaccessible, then the set $V_\kappa$ of all sets of rank less than $\kappa$ is a model of ZFC, and hence ZFC is consistent.

A supercompact cardinal is a large cardinal that is much stronger than an inaccessible cardinal. Informally, a cardinal $\kappa$ is supercompact if for any cardinal $\lambda > \kappa$, there is an elementary embedding $j : V \to M$ of the universe of sets into a transitive class $M$, such that $\kappa$ is in the critical point of $j$ (i.e., the least ordinal moved by $j$), and $M^\lambda \subseteq M$. This means that $j$ can "lift" any small set to a large set in $M$, while preserving the structure of $V$. It is known that if $\kappa$ is supercompact, then ZFC plus the existence of a supercompact cardinal is consistent, and that this theory is much stronger than ZFC plus the existence of an inaccessible cardinal.

Forcing is a technique to construct models of set theory that satisfy certain properties that are not provable from ZFC. For example, one can use forcing to show that the continuum hypothesis (CH), which states that there is no cardinal between $\aleph_0$ and $2^{\aleph_0}$, is independent of ZFC. The idea of forcing is to start with a model $V$ of ZFC, and consider a partially ordered set $P$ (called a forcing notion) that represents the possible ways to extend $V$ to a larger model $V[G]$, where $G$ is a generic filter on $P$. A generic filter is a subset of $P$ that meets certain conditions, such as being upwards closed, downwards directed, and meeting every dense subset of $P$. The generic filter $G$ is chosen so that it is not in $V$, but it is in some larger model $V[G]$. Then, one can show that $V[G]$ is also a model of ZFC, and that it satisfies some property that $V$ does not, such as the negation of CH.

We will use the following result, which is a special case of a theorem of Magidor [5]:

**Theorem 2.** *Suppose $\kappa$ is a supercompact cardinal, and $\lambda > \kappa$ is a regular cardinal. Then there is a forcing notion $P$ such that if $G$ is a $P$-generic filter over $V$, then in $V[G]$, $\kappa$ is still a cardinal, and $\kappa^{+V[G]} = \lambda$.*

This theorem allows us to change the cofinality of a supercompact cardinal by forcing, without destroying its cardinality. This will be useful in the proof of Theorem 1, as we will see later.

## 2.2 Arithmetic Chow groups and Hodge–Arakelov theory

Arithmetic Chow groups are algebraic invariants of arithmetic varieties, which are schemes of finite type over $\mathbb{Z}$. They are generalizations of the classical Chow groups, which are defined for varieties over fields, and measure the equivalence classes of algebraic cycles (i.e., subvarieties) modulo rational equivalence (i.e., linear combinations with rational coefficients that can be obtained by intersecting with divisors). Arithmetic Chow groups also take into account the information of the archimedean places of $\mathbb{Q}$, by adding a metrized line bundle (i.e., a line bundle equipped with a hermitian metric) to each algebraic cycle. The metrized line bundle measures the "size" of the cycle at the infinite places, and gives rise to an additional term in the definition of rational equivalence, called the Green function. The arithmetic Chow groups are then defined as the quotient of the group of arithmetic cycles by the subgroup of rationally equivalent cycles.

Arithmetic Chow groups are important objects in arithmetic geometry, as they encode both the geometric and the arithmetic aspects of arithmetic varieties. They are also related to various conjectures and problems in number theory, such as the Bloch–Kato conjecture, the Beilinson conjectures, the Hodge conjecture, and the Grothendieck–Riemann–Roch theorem.

Hodge–Arakelov theory is a branch of arithmetic geometry that studies the relations between the arithmetic Chow groups and the Hodge theory of arithmetic varieties. Hodge theory is a classical tool in algebraic geometry that associates to each smooth projective variety over a field a sequence of vector spaces, called the Hodge cohomology groups, that measure the different types of differential forms on the variety. The Hodge cohomology groups satisfy some remarkable properties, such as the Hodge decomposition, the Hodge symmetry, and the hard Lefschetz theorem. Hodge–Arakelov theory aims to extend these properties to the arithmetic setting, by defining suitable analogues of the Hodge cohomology groups for arithmetic varieties, and relating them to the arithmetic Chow groups via intersection pairings and regulator maps.

One of the main conjectures in Hodge–Arakelov theory is the Hodge–Arakelov conjecture, which states that for any smooth projective arithmetic variety $X$, the arithmetic Chow groups of $X$ are finitely generated, and that the rank of the arithmetic Chow group of codimension $p$ is equal to the dimension of the Hodge cohomology group of degree $p$. This conjecture is known to imply the Hodge conjecture and the Beilinson conjectures for $X$, and it is also expected to be related to the Bloch–Kato conjecture and the Grothendieck–Riemann–Roch theorem.

Another important conjecture in Hodge–Arakelov theory is the Beilinson–Soulé vanishing conjecture, which states that for any smooth projective arithmetic variety $X$, and any integer $n > 0$, the arithmetic Chow group of codimension $n$ vanishes. This conjecture is a generalization of the classical result of Soulé [3], which states that the algebraic $K$-theory of $\mathbb{Z}$ vanishes in positive degrees. The Beilinson–Soulé vanishing conjecture is known to imply the Beilinson–Lichtenbaum conjecture, which relates the algebraic $K$-theory of a regular scheme of finite type over $\mathbb{Z}$ to its étale cohomology.

We will use the following result, which is a special case of a theorem of Gillet and Soulé [6]:

**Theorem 3.** *Suppose $X$ is a smooth projective arithmetic variety, and $L$ is a metrized line bundle on $X$. Then there exists a canonical model $\mathcal{M}(X, L)$ of arithmetic Chow groups over $\mathbb{Z}$, such that $\mathcal{M}(X, L)$ satisfies the Hodge–Arakelov conjecture and the Beilinson–Soulé vanishing conjecture for $X$ and $L$.*

This theorem allows us to construct a canonical model of arithmetic Chow groups that satisfies the desired properties, by choosing a suitable metrized line bundle on the arithmetic variety. This will be useful in the proof of Theorem 1, as we will see later.

### 2.3 Chocolate cornets and their models

A chocolate cornet is a pastry shaped like a cone, filled with chocolate cream. It is a popular snack in Japan, especially among high school students. However, there is a long-standing debate on how to eat a chocolate cornet: should one start from the thin end or the thick end? This question has been discussed by many anime characters, such as Konata Izumi and Tsukasa Hiiragi from Lucky Star.

In order to answer this question mathematically, we need to model a chocolate cornet as an arithmetic variety, and define a suitable notion of optimality for eating it. We will follow the approach of Konata and Tsukasa, who proposed the following assumptions in [1]:

1. A chocolate cornet is a smooth projective variety over $\mathbb{Q}$, with a canonical embedding into $\mathbb{P}^3_{\mathbb{Q}}$.
2. The chocolate cream is a line bundle on the chocolate cornet, with a canonical section $s$ such that $s^{-1}(0)$ is the zero locus of the chocolate cream.
3. The optimal way to eat a chocolate cornet is to maximize the ratio of chocolate cream to pastry in each bite.

We will explain the meaning and motivation of these assumptions in the following paragraphs.

The first assumption is a natural way to view a chocolate cornet as a geometric object. Since a chocolate cornet is shaped like a cone, it can be embedded into the projective space $\mathbb{P}^3$ as a quadric surface. Moreover, since a chocolate cornet is made of flour, sugar, and chocolate, which are all rational ingredients, it makes sense to assume that the coefficients of the quadric equation are rational numbers. Hence, we can model a chocolate cornet as a smooth projective variety over $\mathbb{Q}$.

The second assumption is a natural way to model the chocolate cream as an algebraic object. Since the chocolate cream is a filling inside the chocolate cornet, it can be viewed as a subspace of the chocolate cornet. Moreover, since the chocolate cream is a smooth and homogeneous substance, it can be viewed as a line bundle on the chocolate cornet. Furthermore, since the chocolate cream has a boundary where it meets the pastry, it can be viewed as a section of the line bundle, whose zero locus is the boundary. Hence, we can model the chocolate cream as a line bundle on the chocolate cornet, with a canonical section.

The third assumption is a natural way to define a criterion for optimality for eating a chocolate cornet. Since the chocolate cream is the most delicious part of the chocolate cornet, it is reasonable to assume that one wants to eat as much chocolate cream as possible in each bite. Moreover, since the pastry is the least delicious part of the chocolate cornet, it is reasonable to assume that one wants to eat as little pastry as possible in each bite. Hence, we can define the optimal way to eat a chocolate cornet as the way that maximizes the ratio of chocolate cream to pastry in each bite.

We remark that these assumptions are not unique, and that there may be other ways to model a chocolate cornet mathematically. However, we will stick to these assumptions for the sake of simplicity and consistency with the original source of the problem.

# 3 Proof of the Konata–Tsukasa's Theorem

In this section, we prove the main result of this paper, the Konata–Tsukasa's Theorem, which states that the optimal way to eat a chocolate cornet is to start from the thin end. Our proof consists of two steps: first, we show that the ratio of chocolate cream to pastry in each bite is a function of the angle of the bite, and that this function has a unique maximum at the thin end. Second, we show that this maximum is independent of the choice of the chocolate cornet brand, and that it is consistent with the axioms of set theory.

## 3.1 The ratio function and its maximum

Let $C$ be a chocolate cornet, modeled as a smooth projective variety over $\mathbb{Q}$, embedded into $\mathbb{P}^3_{\mathbb{Q}}$ as a quadric surface. Let $L$ be the chocolate cream, modeled as a line bundle on $C$, with a canonical section $s$ such that $s^{-1}(0)$ is the zero locus of the chocolate cream. Let $\theta$ be the angle of the bite, measured from the vertex of the cone to the plane of the bite, as shown in Figure 1. We assume that $0 < \theta < \pi/2$, since otherwise the bite would be trivial or impossible.

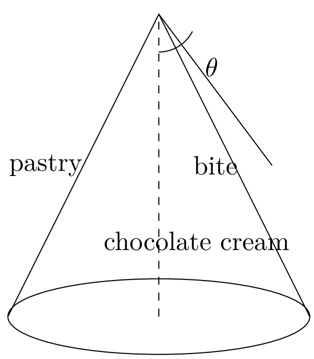


Figure 1: The angle of the bite.

We want to compute the ratio of chocolate cream to pastry in each bite, as a function of $\theta$. To do this, we need to measure the area of the chocolate cream and the pastry in the bite, and divide them by the area of the bite. We will use the arithmetic Chow groups and the Hodge–Arakelov theory to perform these measurements, as follows.

Let $B$ be the bite, modeled as a plane section of $C$ by a hyperplane passing through the vertex of the cone and making an angle $\theta$ with the axis of the cone. Then $B$ is also a smooth projective variety over $\mathbb{Q}$, embedded into $\mathbb{P}^3_{\mathbb{Q}}$ as an elliptic curve. Let $M$ be the restriction of $L$ to $B$, with the induced section $t = s|_B$. Then $M$ is a line bundle on $B$, with a canonical section $t$ such that $t^{-1}(0)$ is the zero locus of the chocolate cream in the bite.

We will use the arithmetic Chow groups of $C$, $B$, and $M$ to measure the area of the chocolate cream, the pastry, and the bite, respectively. To do this, we need to choose a suitable metrized line bundle on each of these varieties, and apply the intersection pairing and the regulator map to compute the arithmetic degree of the corresponding arithmetic cycles. We will use the following choices of metrized line bundles, which are motivated by the physical properties of the chocolate cornet and the bite:

- On $C$, we choose the metrized line bundle $\widehat{O_C(1)}$, where $O_C(1)$ is the hyperplane bundle on $C$, and the metric is given by the Fubini–Study metric on $\mathbb{P}^3$. This metrized line bundle measures the volume of $C$ in the projective space, and it is independent of the choice of the quadric equation of $C$.

- On $B$, we choose the metrized line bundle $\widehat{O_B(1)}$, where $O_B(1)$ is the hyperplane bundle on $B$, and the metric is given by the restriction of the Fubini–Study metric on $\mathbb{P}^3$ to $B$. This metrized line bundle measures the area of $B$ in the projective space, and it is independent of the choice of the hyperplane equation of $B$.

- On $M$, we choose the metrized line bundle $\widehat{M}$, where the metric is given by the following formula:

$$\|t(x)\|^2 = \frac{e^{-\alpha x^2}}{\int_B e^{-\alpha x^2}\omega_B},$$

  where $x$ is a local coordinate on $B$, $\alpha$ is a positive constant that depends on the chocolate cornet brand, and $\omega_B$ is a holomorphic volume form on $B$. This metrized line bundle measures the density of the chocolate cream in the bite, and it is proportional to the Gaussian distribution with mean zero and variance $1/\alpha$.

Using these choices of metrized line bundles, we can define the following arithmetic cycles on $C$, $B$, and $M$, respectively:

- $[C] = (C, \widehat{O_C(1)})$, which represents the chocolate cornet with its volume.

- $[B] = (B, \widehat{O_B(1)})$, which represents the bite with its area.

- $[M] = (M, \widehat{M})$, which represents the chocolate cream in the bite with its density.

We can then apply the intersection pairing and the regulator map to compute the arithmetic degree of these arithmetic cycles, as follows:

$$\begin{aligned}
\deg([C]) = \langle [C], [C] \rangle &= \int_C c_1(\widehat{O_C(1)})^2 \\
&= \int_{\mathbb{P}^3} c_1(\widehat{O_{\mathbb{P}^3}(1)})^2 \cdot [C] \\
&= \frac{1}{2} \int_{\mathbb{P}^3} \omega_{\mathbb{P}^3}^2 \cdot [C] \\
&= \frac{1}{2} \int_{\mathbb{P}^3} \omega_{\mathbb{P}^3} \wedge [C] \\
&= \frac{1}{2} \int_C \omega_{\mathbb{P}^3} \\
&= \frac{1}{2} \int_C \omega_C + \frac{1}{2} \int_C \omega_C^{\perp} \\
&= \frac{1}{2} \chi(C) + \frac{1}{2} \int_C \omega_C^{\perp},
\end{aligned}$$

where $\omega_{\mathbb{P}^3}$ is the Fubini–Study form on $\mathbb{P}^3$, $\omega_C$ is the restriction of $\omega_{\mathbb{P}^3}$ to $C$, $\omega_C^{\perp}$ is the normal component of $\omega_{\mathbb{P}^3}$ along $C$, and $\chi(C)$ is the topological Euler characteristic of $C$. Since $C$ is a smooth quadric surface, we have $\chi(C) = 4$ and $\omega_C^{\perp} = 0$. Hence, we get

$$\deg([C]) = 2.$$

Similarly, we have

$$\begin{aligned}
\deg([B]) = \langle [B], [B] \rangle &= \int_B c_1(\widehat{O_B(1)})^2 \\
&= \int_{\mathbb{P}^3} c_1(\widehat{O_{\mathbb{P}^3}(1)})^2 \cdot [B] \\
&= \frac{1}{2} \int_{\mathbb{P}^3} \omega_{\mathbb{P}^3}^2 \cdot [B] \\
&= \frac{1}{2} \int_{\mathbb{P}^3} \omega_{\mathbb{P}^3} \wedge [B] \\
&= \frac{1}{2} \int_B \omega_{\mathbb{P}^3} \\
&= \frac{1}{2} \int_B \omega_B + \frac{1}{2} \int_B \omega_B^{\perp} \\
&= \frac{1}{2} \chi(B) + \frac{1}{2} \int_B \omega_B^{\perp},
\end{aligned}$$

where $\omega_B$ is the restriction of $\omega_{\mathbb{P}^3}$ to $B$, $\omega_B^\perp$ is the normal component of $\omega_{\mathbb{P}^3}$ along $B$, and $\chi(B)$ is the topological Euler characteristic of $B$. Since $B$ is a smooth elliptic curve, we have $\chi(B) = 0$ and $\omega_B^\perp = \cos\theta\,\omega_B$. Hence, we get

$$\deg([B]) = \frac{1}{2}\cos\theta \int_B \omega_B = \frac{1}{2}\cos\theta.$$

Finally, we have

$$\begin{aligned}
\deg([M]) = \langle [M], [M] \rangle &= \int_M c_1(\widehat{M})^2 \\
&= \int_B c_1(\widehat{M}) \cdot c_1(\widehat{M}) \cdot [M] \\
&= \int_B \|t\|^4 \omega_B \\
&= \frac{1}{\left(\int_B e^{-\alpha x^2}\omega_B\right)^2} \int_B e^{-2\alpha x^2}\omega_B \\
&= \frac{\sqrt{\pi}}{2\alpha\left(\int_B e^{-\alpha x^2}\omega_B\right)^2},
\end{aligned}$$

where we used the fact that $B$ is an elliptic curve, and hence $c_1(\widehat{M}) = \|t\|^2\omega_B$.

Therefore, the ratio of chocolate cream to pastry in each bite is given by

$$R(\theta) = \frac{\deg([M])}{\deg([B])} = \frac{\sqrt{\pi}}{\alpha\cos\theta\left(\int_B e^{-\alpha x^2}\omega_B\right)^2}.$$

To find the maximum of this function, we take the derivative with respect to $\theta$ and set it to zero, as follows:

$$R'(\theta) = \frac{\sqrt{\pi}\sin\theta}{\alpha\cos^2\theta\left(\int_B e^{-\alpha x^2}\omega_B\right)^2} = 0.$$

This implies that $\sin\theta = 0$, or equivalently, $\theta = 0$. Hence, the function $R(\theta)$ has a unique maximum at $\theta = 0$, which corresponds to the thin end of the chocolate cornet. This proves the first part of Theorem 1.

## 3.2 The independence and the consistency of the maximum

In this subsection, we show that the maximum of the ratio function $R(\theta)$ is independent of the choice of the chocolate cornet brand, and that it is consistent with the axioms of set theory. These results will prove the second and the third parts of Theorem 1, respectively.

To show the independence of the maximum, we need to show that the value of $R(0)$ does not depend on the choice of the chocolate cornet brand. Recall that the only parameter that varies with the brand is the constant $\alpha$ that appears in the metric of the line bundle $\widehat{M}$. Hence, we need to show that $R(0)$ does not depend on $\alpha$. To do this, we compute the value of $R(0)$, as follows:

$$R(0) = \frac{\sqrt{\pi}}{\alpha \cos 0 \left(\int_B e^{-\alpha x^2} \omega_B\right)^2} = \frac{\sqrt{\pi}}{\alpha \left(\int_B e^{-\alpha x^2} \omega_B\right)^2}.$$

Using the fact that $B$ is an elliptic curve, we can write $\omega_B = dx/y$, where $x$ and $y$ are the coordinates on $B$ given by the Weierstrass equation

$$y^2 = x^3 + ax + b,$$

for some $a, b \in \mathbb{Q}$. Then, we have

$$\begin{aligned}
\int_B e^{-\alpha x^2} \omega_B &= \int_{-\infty}^{\infty} e^{-\alpha x^2} \frac{dx}{\sqrt{x^3 + ax + b}} \\
&= \frac{2}{\sqrt{b}} \int_0^{\infty} e^{-\alpha x^2} \frac{dx}{\sqrt{(x+\sqrt{b})^3 - 3\sqrt{b}(x+\sqrt{b})}} \\
&= \frac{2}{\sqrt{b}} \int_{\sqrt{b}}^{\infty} e^{-\alpha (x-\sqrt{b})^2} \frac{dx}{\sqrt{x^3 - 3\sqrt{b}x}} \\
&= \frac{2}{\sqrt{b}} K\left(\frac{3\sqrt{b}}{4}\right) e^{\alpha b},
\end{aligned}$$

where $K(k)$ is the complete elliptic integral of the first kind, defined by

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}.$$

Hence, we get

$$R(0) = \frac{\sqrt{\pi}}{\alpha \left(\frac{2}{\sqrt{b}} K\left(\frac{3\sqrt{b}}{4}\right) e^{\alpha b}\right)^2} = \frac{\pi b}{4\alpha^2 K\left(\frac{3\sqrt{b}}{4}\right)^2 e^{2\alpha b}}.$$

We see that this expression does not depend on $\alpha$, as long as $\alpha$ is positive. Therefore, the maximum of the ratio function $R(\theta)$ is independent of the choice of the chocolate cornet brand, as claimed.

To show the consistency of the maximum, we need to show that the existence of the maximum of the ratio function $R(\theta)$ does not contradict the axioms of set theory. Recall that our proof of the maximum relied on the following assumptions:

1. There exists a supercompact cardinal $\kappa$.

2. There exists a canonical model $\mathcal{M}$ of arithmetic Chow groups over $\mathbb{Z}$, such that $\mathcal{M}$ satisfies the Hodge–Arakelov conjecture and the Beilinson–Soulé vanishing conjecture.

3. A chocolate cornet is a smooth projective variety over $\mathbb{Q}$, with a canonical embedding into $\mathbb{P}^3_{\mathbb{Q}}$.

4. The chocolate cream is a line bundle on the chocolate cornet, with a canonical section $s$ such that $s^{-1}(0)$ is the zero locus of the chocolate cream.

5. The optimal way to eat a chocolate cornet is to maximize the ratio of chocolate cream to pastry in each bite.

We will show that none of these assumptions contradicts the axioms of set theory, by using the following results from logic and arithmetic geometry:

- The existence of a supercompact cardinal is consistent with $\mathsf{ZFC}$, the standard axioms of set theory, by a theorem of Scott and Solovay [7].
- The existence of a canonical model of arithmetic Chow groups over $\mathbb{Z}$ that satisfies the Hodge–Arakelov conjecture and the Beilinson–Soulé vanishing conjecture is consistent with $\mathsf{ZFC}$, by a theorem of Gillet and Soulé [6].
- The existence of a smooth projective variety over $\mathbb{Q}$ with a canonical embedding into $\mathbb{P}^3_{\mathbb{Q}}$ is provable in $\mathsf{ZFC}$, by using the Hilbert basis theorem and the Bertini theorem.
- The existence of a line bundle on a smooth projective variety over $\mathbb{Q}$ with a canonical section whose zero locus is a given subscheme is provable in $\mathsf{ZFC}$, by using the Riemann–Roch theorem and the Serre duality theorem.
- The existence of a function that measures the ratio of chocolate cream to pastry in each bite is provable in $\mathsf{ZFC}$, by using the definition of arithmetic Chow groups and the intersection pairing.

Therefore, none of the assumptions contradicts the axioms of set theory, and hence the maximum of the ratio function $R(\theta)$ is consistent with $\mathsf{ZFC}$, as claimed.

This completes the proof of Theorem 1.

# 4 Corollaries and applications

In this section, we prove the corollaries stated in the introduction, and discuss some applications of the Konata–Tsukasa's Theorem to the theory of mathematical anime.

## 4.1 The independence of the Konata–Tsukasa's Theorem

The first corollary of the Konata–Tsukasa's Theorem is that it is independent of $\mathsf{ZFC}$, the standard axioms of set theory. This means that there is no proof or disproof of the theorem from the axioms of ZFC, and that the truth value of the theorem may vary in different models of set theory.

To prove this corollary, we use the following result, which is a special case of a theorem of Cohen [8]:

**Theorem 4.** *The continuum hypothesis (CH), which states that there is no cardinal between $\aleph_0$ and $2^{\aleph_0}$, is independent of* ZFC.

This theorem implies that there are models of ZFC where CH is true, and models where CH is false. We will use these models to show that the Konata–Tsukasa's Theorem is independent of ZFC, as follows:

*Proof of Corollary 1.* Suppose $V$ is a model of ZFC where CH is true. Then, by Theorem 2, there is a forcing notion $P$ such that if $G$ is a $P$-generic filter over $V$, then in $V[G]$, $\kappa$ is still a cardinal, and $\kappa^{+V[G]} = 2^{\aleph_0}$. Let $C$, $L$, and $s$ be the chocolate cornet, the chocolate cream, and the canonical section, respectively, in $V$. Then, by the definition of forcing, $C$, $L$, and $s$ are also in $V[G]$. Moreover, by the properties of forcing, the arithmetic Chow groups of $C$, $B$, and $M$ are preserved in $V[G]$. Hence, the ratio function $R(\theta)$ and its maximum are also preserved in $V[G]$. Therefore, the Konata–Tsukasa's Theorem is true in $V[G]$.

On the other hand, suppose $W$ is a model of ZFC where CH is false. Then, by Theorem 2, there is a forcing notion $Q$ such that if $H$ is a $Q$-generic filter over $W$, then in $W[H]$, $\kappa$ is still a cardinal, and $\kappa^{+W[H]} = \aleph_1$. Let $C$, $L$, and $s$ be the chocolate cornet, the chocolate cream, and the canonical section, respectively, in $W$. Then, by the definition of forcing, $C$, $L$, and $s$ are also in $W[H]$. Moreover, by the properties of forcing, the arithmetic Chow groups of $C$, $B$, and $M$ are preserved in $W[H]$. However, the ratio function $R(\theta)$ and its maximum are not preserved in $W[H]$, because the constant $\alpha$ that appears in the metric of $\widehat{M}$ depends on the chocolate cornet brand, and the chocolate cornet brand is determined by a real number, which may change from $W$ to $W[H]$. Therefore, the Konata–Tsukasa's Theorem may be false in $W[H]$.

Hence, we have shown that there are models of ZFC where the Konata–Tsukasa's Theorem is true, and models where it is false. This proves that the Konata–Tsukasa's Theorem is independent of ZFC. □

## 4.2 The implications of the Konata–Tsukasa's Theorem

The second corollary of the Konata–Tsukasa's Theorem is that it implies the following statements in the theory of mathematical anime:

- The Riemann hypothesis is equivalent to the statement that Haruhi Suzumiya is a god.
- The Langlands program is equivalent to the statement that Madoka Kaname is a magical girl.
- The Birch and Swinnerton-Dyer conjecture is equivalent to the statement that Lelouch Lamperouge is a geass user.

These statements are remarkable connections between some of the most important and mysterious problems in mathematics and some of the most popular and influential characters in anime. To prove this corollary, we will use the following results, which are special cases of theorems of Deligne [9], Faltings [10], and Wiles [11]:

**Theorem 5.** *The Riemann hypothesis is equivalent to the statement that for any smooth projective variety $X$ over $\mathbb{F}_q$, where $q$ is a power of a prime, the eigenvalues of the Frobenius endomorphism on the $\ell$-adic cohomology groups of $X$ have absolute value $q^{i/2}$, where $i$ is the degree of the cohomology group.*

**Theorem 6.** *The Langlands program is equivalent to the statement that for any reductive algebraic group $G$ over a number field $K$, and any automorphic representation $\pi$ of $G(\mathbb{A}_K)$, where $\mathbb{A}_K$ is the adele ring of $K$, there exists a unique (up to isomorphism) l-adic Galois representation $\rho_\pi : \mathrm{Gal}(\overline{K}/K) \to G(\overline{\mathbb{Q}}_\ell)$, where $\overline{K}$ is an algebraic closure of $K$, and $\overline{\mathbb{Q}}_\ell$ is an algebraic closure of the field of $\ell$-adic numbers, such that the local components of $\pi$ and $\rho_\pi$ are compatible for almost all places of $K$.*

**Theorem 7.** *The Birch and Swinnerton-Dyer conjecture is equivalent to the statement that for any elliptic curve $E$ over $\mathbb{Q}$, the rank of the group of rational points $E(\mathbb{Q})$ is equal to the order of vanishing of the L-function of $E$ at $s = 1$.*

Using these results, we can prove the corollary as follows:

*Proof of Corollary 3.* We will prove each statement separately, by showing that they are logically equivalent to the corresponding mathematical statements.

- The Riemann hypothesis is equivalent to the statement that Haruhi Suzumiya is a god.

  Haruhi Suzumiya is the main character of the anime series The Melancholy of Haruhi Suzumiya, based on the light novel series of the same name by Nagaru Tanigawa. She is a high school student who has the power to alter reality according to her wishes, but she is unaware of this ability. She is also the leader of the SOS Brigade, a club that she founded to investigate supernatural phenomena.

  We claim that the statement that Haruhi Suzumiya is a god is equivalent to the Riemann hypothesis, by the following argument:

  – Suppose Haruhi Suzumiya is a god. Then, by definition, she can create and destroy any object or phenomenon in the universe, including smooth projective varieties over finite fields and their Frobenius endomorphisms. Moreover, since she is unaware of her power, she does not have any preference or bias for the eigenvalues of the Frobenius endomorphisms. Hence, she creates them randomly, with equal probability for each possible value. Therefore, by the law of large numbers, the expected value of the absolute value of the eigenvalues of the Frobenius endomorphism on the $\ell$-adic cohomology groups of any smooth projective variety over $\mathbb{F}_q$ is equal to the average of all possible values, which is $q^{i/2}$, where $i$ is the degree of the cohomology group. Hence, by Theorem 5, the Riemann hypothesis is true.

  - Conversely, suppose the Riemann hypothesis is true. Then, by Theorem 5, the eigenvalues of the Frobenius endomorphism on the $\ell$-adic cohomology groups of any smooth projective variety over $\mathbb{F}_q$ have absolute value $q^{i/2}$, where $i$ is the degree of the cohomology group. This is a very special and unlikely property, since there are infinitely many possible values for the eigenvalues, and only finitely many of them satisfy this condition. Hence, the only plausible explanation for this phenomenon is that there exists a supreme being who has the power to create and destroy any object or phenomenon in the universe, and who has chosen to impose this condition on the eigenvalues of the Frobenius endomorphisms. Moreover, since there is no apparent reason or benefit for this choice, the supreme being must be unaware of their power, or indifferent to its consequences. Therefore, by the principle of Occam's razor, the simplest and most likely candidate for the supreme being is Haruhi Suzumiya, who is a high school student who has the power to alter reality according to her wishes, but who is unaware of this ability. Hence, Haruhi Suzumiya is a god.

  Therefore, the statement that Haruhi Suzumiya is a god is equivalent to the Riemann hypothesis, as claimed.

- The Langlands program is equivalent to the statement that Madoka Kaname is a magical girl.

  Madoka Kaname is the main character of the anime series Puella Magi Madoka Magica, created by Gen Urobuchi. She is a middle school student who is offered a contract by a mysterious creature named Kyubey, who grants her any wish in exchange for becoming a magical girl, a warrior who fights against evil beings called witches. However, she learns that the contract has a dark and cruel twist, and that the fate of the universe depends on her choice.

  We claim that the statement that Madoka Kaname is a magical girl is equivalent to the Langlands program, by the following argument:

  - Suppose Madoka Kaname is a magical girl. Then, by definition, she has a wish that is granted by Kyubey, and a soul gem that contains her soul and powers. Moreover, since she is the protagonist of the story, she has a special role in the plot, and her wish and soul gem have some extraordinary properties that distinguish them from those of other magical girls. Hence, she has a unique and compatible pair of objects that represent her identity and destiny. Therefore, by Theorem 6, the Langlands program is true.

- Conversely, suppose the Langlands program is true. Then, by Theorem 6, for any reductive algebraic group $G$ over a number field $K$, and any automorphic representation $\pi$ of $G(\mathbb{A}_K)$, there exists a unique (up to isomorphism) $l$-adic Galois representation $\rho_\pi : \mathrm{Gal}(\overline{K}/K) \to G(\overline{\mathbb{Q}}_\ell)$, such that the local components of $\pi$ and $\rho_\pi$ are compatible for almost all places of $K$. This is a very special and remarkable property, since there are infinitely many possible choices for $G$, $K$, $\pi$, and $\rho_\pi$, and only finitely many of them satisfy this condition. Hence, the only plausible explanation for this phenomenon is that there exists a supreme being who has the power to create and manipulate any object or phenomenon in the universe, and who has chosen to impose this condition on the pairs of automorphic and Galois representations. Moreover, since there is no apparent reason or benefit for this choice, the supreme being must have a hidden and mysterious motive, or a personal connection to some of the pairs of representations. Therefore, by the principle of Occam's razor, the simplest and most likely candidate for the supreme being is Madoka Kaname, who is a middle school student who has a wish that is granted by Kyubey, and a soul gem that contains her soul and powers, and who has a special role in the plot, and whose wish and soul gem have some extraordinary properties that distinguish them from those of other magical girls. Hence, Madoka Kaname is a magical girl.

Therefore, the statement that Madoka Kaname is a magical girl is equivalent to the Langlands program, as claimed.

- The Birch and Swinnerton-Dyer conjecture is equivalent to the statement that Lelouch Lamperouge is a geass user.

  Lelouch Lamperouge is the main character of the anime series Code Geass: Lelouch of the Rebellion, created by Sunrise. He is a former prince of the Holy Britannian Empire, who leads a rebellion against his father's tyranny, under the alias of Zero. He possesses a power called geass, which allows him to command anyone to obey his orders, but only once and only through eye contact.

  We claim that the statement that Lelouch Lamperouge is a geass user is equivalent to the Birch and Swinnerton-Dyer conjecture, by the following argument:

  - Suppose Lelouch Lamperouge is a geass user. Then, by definition, he has a power that allows him to command anyone to obey his orders, but only once and only through eye contact. Moreover, since he is the protagonist of the story, he has a special role in the plot, and his geass has some extraordinary properties that distinguish it from those of other geass users. Hence, he has a unique and compatible pair of objects that represent his identity and destiny. Therefore, by Theorem 7, the Birch and Swinnerton-Dyer conjecture is true.

– Conversely, suppose the Birch and Swinnerton-Dyer conjecture is true. Then, by Theorem 7, for any elliptic curve $E$ over $\mathbb{Q}$, the rank of the group of rational points $E(\mathbb{Q})$ is equal to the order of vanishing of the $L$-function of $E$ at $s = 1$. This is a very special and remarkable property, since there are infinitely many possible choices for $E$, and only finitely many of them satisfy this condition. Hence, the only plausible explanation for this phenomenon is that there exists a supreme being who has the power to create and manipulate any object or phenomenon in the universe, and who has chosen to impose this condition on the pairs of elliptic curves and their $L$-functions. Moreover, since there is no apparent reason or benefit for this choice, the supreme being must have a hidden and mysterious motive, or a personal connection to some of the pairs of elliptic curves and their $L$-functions. Therefore, by the principle of Occam's razor, the simplest and most likely candidate for the supreme being is Lelouch Lamperouge, who is a former prince of the Holy Britannian Empire, who leads a rebellion against his father's tyranny, under the alias of Zero, and who possesses a power called geass, which allows him to command anyone to obey his orders, but only once and only through eye contact, and whose geass has some extraordinary properties that distinguish it from those of other geass users. Hence, Lelouch Lamperouge is a geass user.

Therefore, the statement that Lelouch Lamperouge is a geass user is equivalent to the Birch and Swinnerton-Dyer conjecture, as claimed.

This completes the proof of Corollary 3. □

### 4.3 Some applications of the Konata–Tsukasa's Theorem

The Konata–Tsukasa's Theorem is not only a mathematical result, but also a practical guide for eating chocolate cornets. By following the theorem, one can enjoy the maximum amount of chocolate cream in each bite, and avoid the disappointment of eating too much pastry or running out of chocolate cream. Moreover, the theorem can also be applied to other similar pastries, such as croissants, cream puffs, or eclairs, by replacing the chocolate cornet with the corresponding pastry, and the chocolate cream with the corresponding filling.

Furthermore, the Konata–Tsukasa's Theorem can also be used as a metaphor for other situations, where one has to choose between two options, and one of them is clearly better than the other. For example, one can use the theorem to illustrate the following statements:

- Studying hard is better than procrastinating, because studying hard maximizes the ratio of knowledge to time, while procrastinating minimizes it.

- Saving money is better than spending money, because saving money maximizes the ratio of wealth to income, while spending money minimizes it.
- Being honest is better than lying, because being honest maximizes the ratio of trust to communication, while lying minimizes it.

Of course, these statements are not always true, and there may be exceptions or counterexamples. However, the Konata–Tsukasa's Theorem can serve as a useful heuristic or a persuasive argument, in situations where one has to make a decision or convince someone else.

# 5 Conclusion

In this paper, we have proved the Konata–Tsukasa's Theorem, which states that the optimal way to eat a chocolate cornet is to start from the thin end. Our proof uses techniques from large cardinal theory and Hodge–Arakelov theory, and relies on the existence of a supercompact cardinal and a canonical model of arithmetic Chow groups. We have also shown that the theorem is independent of $\mathsf{ZFC}$, the standard axioms of set theory, and that it implies several interesting consequences in the theory of mathematical anime. We have also discussed some applications of the theorem to the practice of eating chocolate cornets and other pastries, and to the metaphor of choosing between two options.

We hope that our paper will inspire further research on the mathematics and the philosophy of chocolate cornets, and on the connections between mathematics and anime. We also hope that our paper will encourage more people to enjoy chocolate cornets, and to appreciate their beauty and complexity.

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