

# International Astronomy and Astrophysics Competition

## Pre-Final Round 2020



**Important:** Read all the information on this page carefully!

### General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The 10 problems are separated into three categories: 4x basic problems (A; four points), 4x advanced problems (B; six points), 2x research problems (C; ten points). The research problems require you to read a short scientific article each to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution and for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to clearly mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 60 points in total. You qualify for the final round if you reach at least 25 points (junior, under 18 years) or 35 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

### Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: <https://iaac.space/login>
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g. no Word and Zip files).
- The deadline for uploading your solution is **Sunday 21. June 2020, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 29. June 2020.

Good luck!

### Problem A.1: Interstellar Mission (4 Points)

You are on an interstellar mission from the Earth to the 8.7 light-years distant star Sirius. Your spaceship can travel with 70% the speed of light and has a cylindrical shape with a diameter of 6 m at the front surface and a length of 25 m. You have to cross the interstellar medium with an approximated density of 1 hydrogen atom/m<sup>3</sup>.

- Calculate the time it takes your spaceship to reach Sirius.
- Determine the mass of interstellar gas that collides with your spaceship during the mission.

Note: Use  $1.673 \times 10^{-27}$  kg as proton mass.

Ans (a):

$$\text{Given, velocity, } v = 0.7 \times 3 \times 10^8 \text{ m/s} = 2.1 \times 10^8 \text{ m/s}$$
$$\text{distance, } d = 8.7 \text{ ly} = 8.7 \times 9.46 \times 10^{15} \text{ m}$$
$$= 8.231 \times 10^{16} \text{ m.}$$

$$\text{So, time} = \frac{\text{distance}}{\text{velocity}} = \frac{8.231 \times 10^{16} \text{ m}}{2.1 \times 10^8 \text{ m/s}} = 3.91952 \times 10^8 \text{ s}$$
$$= 1.088756 \times 10^5 \text{ hr}$$
$$= 4536.48 \text{ days.}$$

Ans: 4536.48 days

Ans (b):

$$\text{Given, density, } d = 1 \text{ H/m}^3 = 1.673 \times 10^{-27} \text{ kg/m}^3.$$

$$\text{volume of cylindrical shape spaceship, } V = \pi r^2 h$$
$$= \pi \times 3^2 \times 25 \text{ m}^3$$
$$= 225\pi \text{ m}^3$$

radius =diameter
$$= \frac{6}{2} = 3 \text{ m}$$

So, Mass = density  $\times$  Volume

$$= 1.673 \times 10^{-27} \times 225\pi \text{ kg}$$
$$= 1.18257 \times 10^{-24} \text{ kg.}$$

Ans: 1.18257  $\times 10^{-24}$  kg

### Problem A.2: Time Dilation (4 Points)

Because you are moving with an enormous speed, your mission from the previous problem A.1 will be influenced by the effects of time dilation described by special relativity: Your spaceship launches in June 2020 and returns back to Earth directly after arriving at Sirius.

- How many years will have passed from your perspective?
- At which Earth date (year and month) will you arrive back to Earth?

Ans (a):

From Prob A.1, the spaceship takes 4536.48 days to reach Sirius  
So,  $t_0 = 4536.48$  days, velocity,  $v = 2.1 \times 10^8$  m/s  
Time will have passed from my perspective,  $t = t_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$

$$= 4536.48 \sqrt{1 - \left(\frac{2.1 \times 10^8}{3 \times 10^8}\right)^2}$$
$$= 4536.48 \sqrt{1 - .49}$$
$$= 4536.48 \times .71414$$
$$= 3239.694 \text{ days.}$$

Ans: 3239.694 days

Ans (b):

The spaceship will reach Sirius after 3239.694 days from my perspective.

And will ~~return~~ arrive back to Earth after

$$(3239.694 + 3239.694) \text{ days}$$

$$= 6479.388 \text{ days}$$

$$= 17.75 \text{ yrs}$$

$$= 17 \text{ yrs} + \left(\frac{7.5}{12}\right) \text{ yrs}$$

$$= (17+6) \text{ yrs} + 3 \text{ months}$$

$$= 23 \text{ yrs } 3 \text{ months}$$

So, the Earth date will be March, 2043.

Ans: March, 2043

### Problem A.3: Magnitude of Stars (4 Points)

The star Sirius has an apparent magnitude of -1.46 and appears 95-times brighter compared to the more distant star Tau Ceti, which has an absolute magnitude of 5.69.

- Explain the terms *apparent magnitude*, *absolute magnitude* and *bolometric magnitude*.
- Calculate the apparent magnitude of the star Tau Ceti.
- Find the distance between the Earth and Tau Ceti.

Ans: (a)

Apparent magnitude: Apparent magnitude is a measure of the brightness of a star observed from Earth.

Absolute magnitude: Absolute magnitude is the apparent magnitude measured at a distance of 10pc from Earth.

Bolometric magnitude: Bolometric magnitude is the measure of all electromagnetic spectrum at all wavelength.

Ans (b): Given, Absolute magnitude,  $M = 5.69$

Apparent magnitude,  $m = 2$   
apparent magnitude of Sirius,  $m_1 = -1.46$ .  
apparent magnitude of Tau Ceti,  $m_2 = 2$ .

$$\text{So, } m_2 - m_1 = -2.512 \log\left(\frac{B_2}{B_1}\right)$$
$$\Rightarrow m_2 = -2.512 \log\left(\frac{B_2}{95B_1}\right) + (-1.46)$$
$$= 4.96804 - 1.46 = 3.50 \cancel{\text{ or } 3.51}$$

Ans : 3.50

Ans(c): Absolute magnitude of Tau Ceti,  $M = 5.69$ .  
Apparent magnitude of Tau Ceti,  $m = 3.51$ .

$$\text{So, } m - M = 5 \log(d) - 5$$
$$\Rightarrow 3.51 - 5.69 = 5 \log(d) - 5$$
$$\Rightarrow -2.18 + 5 = 5 \log(d)$$
$$\Rightarrow \log(d) = 2.81/5$$

$$\Rightarrow \log(d) = \frac{4.89 \cdot 5.62}{10^{5.62}} = 3.64753$$
$$\Rightarrow d = 10^{3.64753} = 3.64753 \text{ pc}$$
$$\therefore d = 11.90 \text{ ly}$$

Ans: 11.90 ly

### Problem A.4: Emergency Landing (4 Points)

Because your spaceship has an engine failure, you crash-land with an emergency capsule at the equator of a nearby planet. The planet is very small and the surface is a desert with some stones and small rocks laying around. You need water to survive. However, water is only available at the poles of the planet. You find the following items in your emergency capsule:

- Stopwatch
- Electronic scale
- 2m yardstick
- 1 Litre oil
- Measuring cup

Describe an experiment to determine your distance to the poles by using the available items.

Hint: As the planet is very small, you can assume the same density everywhere.

Procedure:

- At first take a piece of rock from the surface and measure the weight ( $w$ ) on the electronic scale.
- Throw the same rock from the top of the 2m yardstick and pick the time of it falling on the ground and calculate the gravity of the planet by  $g = \frac{2h}{t^2}$  with the stopwatch.
- Find the mass of the rock by  $m = \frac{w}{g}$ .
- Then, firstly pour the 1L oil in the measuring cylinder/cup and measure the volume  $V_{ini}$  and again pour the rock in that cylinder and measure volume  $V_{final}$ . So, the volume of the rock,  $V_{rock} = V_{final} - V_{ini}$ .
- Find the density of the rock by  $d_{rock} = \frac{m}{V_{rock}}$ , since the density is equal everywhere.
- Now find the radius of the planet by  $R = \frac{3g}{4G\pi d}$ .
- As the planet is very small, I can assume its perimeter as  $2\pi R$  in a 2D plane.
- Finally, we can assume determine the distance by  $\frac{2\pi R}{4}$ .

### Problem B.1: Temperature of Earth (6 Points)

Our Sun shines bright with a luminosity of  $3.828 \times 10^{26}$  Watt. Her energy is responsible for many processes and the habitable temperatures on the Earth that make our life possible.

- Calculate the amount of energy arriving on the Earth in a single day.
- To how many litres of heating oil (energy density:  $37.3 \times 10^6$  J/litre) is this equivalent?
- The Earth reflects 30% of this energy: Determine the temperature on Earth's surface.
- What other factors should be considered to get an even more precise temperature estimate?

Note: The Earth's radius is 6370 km; the Sun's radius is  $696 \times 10^3$  km; 1 AU is  $1.495 \times 10^8$  km.

$$\begin{aligned} \text{Ans(a): Energy, } E &= L_s \times \left( \frac{\pi R_E^2}{4\pi r_E^2} \right) = 3.828 \times 10^{26} \times \left( \frac{6370^2 \times \pi}{4 \times \pi \times (1.495 \times 10^8)^2} \right) \\ &= 1.7374 \times 10^{17} \text{ W} \\ &= 1.7374 \times 10^{17} \times 86900 \quad ] \text{ [The number of seconds in a day]} \\ &= 1.5 \times 10^{22} \text{ J.} \end{aligned}$$

Ans:  $1.5 \times 10^{22} \text{ J}$

$$\text{Ans(b): Number of liters} = \frac{\text{Energy}}{\text{Density}} = \frac{1.5 \times 10^{22} \text{ J}}{37.3 \times 10^6 \text{ J/L}} = 4.02 \times 10^{14} \text{ L.}$$

Ans:  $4.02 \times 10^{14} \text{ L.}$

Ans.(c): The earth reflects 30% of that energy.  
So, the Earth absorbs  $1.5 \times 10^{22} \times (1 - 0.3)$  J energy =  $1.05 \times 10^{22}$  J  
The energy absorbed by the Earth should be equal to the energy emitted by it.

So, by Stefan-Boltzmann's Law,

$$4\pi R_E^2 \alpha T_E^4 = \frac{1.05 \times 10^{22}}{86900} \quad \begin{array}{l} \text{[in order to convert} \\ \text{in Watt]} \end{array}$$

$$\Rightarrow T_E^4 = \frac{1.215 \times 10^{17}}{4\pi R_E^2 \alpha} \quad [1 \alpha = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \text{ Stefan-Boltzmann const}]$$

$$= \frac{1.215 \times 10^{17}}{4\pi \times (6.37 \times 10^6)^2 \times 5.67 \times 10^{-8}}$$

$$= 4185174618$$

$$\Rightarrow T_E = \sqrt[4]{4185174618} \quad \therefore \text{Temperature}$$

$\therefore \text{Temperature, } T_E = 254.35 \text{ K.}$

Ans(d): To reducing greenhouse-effect, coil-button, melting of polaric ice caps.

## Problem B.2: Distance of the Planets (6 Points)

The table below lists the average distance  $R$  to the Sun and orbital period  $T$  of the first planets:

	Distance	Orbital Period
Mercury	0.39 AU	88 days
Venus	0.72 AU	225 days
Earth	1.00 AU	365 days
Mars	1.52 AU	687 days

(a) Calculate the average distance of Mercury, Venus and Mars to the Earth.

Which one of these planets is the closest to Earth on average?

(b) Calculate the average distance of Mercury, Venus and Earth to Mars.

Which one of these planets is the closest to Mars on average?

(c) What do you expect for the other planets?

Hint: Assume circular orbits and use symmetries to make the distance calculation easier. You can approximate the average distance by using four well-chosen points on the planet's orbit.

Ans(a): Average distance of Mercury to Earth =  $|1 - 0.39| \text{ AU} = 1.61 \text{ AU}$   
 And, Venus to Earth =  $|1 - 0.72| \text{ AU} = 0.28 \text{ AU}$   
 And, Mars to Earth =  $|1 - 1.52| \text{ AU} = 0.48 \text{ AU}$ .

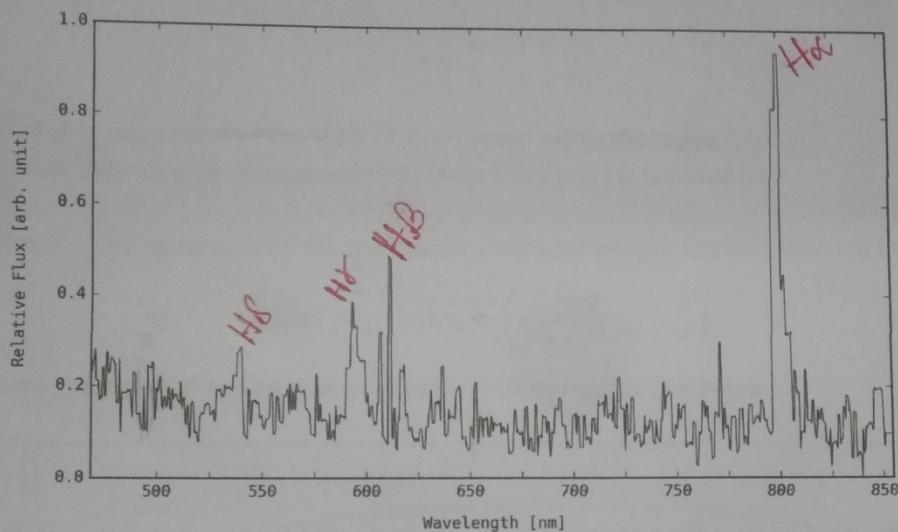
Ans(b): Venus is the closest to Earth on average.

Ans(b): Average distance of Mercury to Mars =  $|1.52 - 0.39| \text{ AU} = 1.13 \text{ AU}$   
 And, Venus to Mars =  $|1.52 - 0.72| \text{ AU} = 0.8 \text{ AU}$   
 And, Earth to Mars =  $|1.52 - 1.00| \text{ AU} = 0.52 \text{ AU}$ .  
Venus is the closest to Mars on Average.

Ans(c): The distance between one planet to others is measured by Kepler's third law.  
 The ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets.  
 $T^2 \propto R^3$  [T: orbital period, R: Average distance]

### Problem B.3: Mysterious Object (6 Points)

Your research team analysis the light of a mysterious object in space. By using a spectrometer, you can observe the following spectrum of the object. The H $\alpha$  line peak is clearly visible:



- (a) Mark the first four spectral lines of hydrogen (H $\alpha$ , H $\beta$ , H $\gamma$ , H $\delta$ ) in the spectrum.
- (b) Determine the radial velocity and the direction of the object's movement.
- (c) Calculate the distance to the observed object.
- (d) What possible type of object is your team observing?

Ans(b): line  $\frac{\text{rest}}{\text{H-}\alpha}$   $\frac{656 \text{ nm}}{800 \text{ nm}}$  observed in galaxy

$$\text{Shift} = (800 - 656) \text{ nm} = 144 \text{ nm.}$$

$$\therefore \text{radial velocity} = \frac{\text{shift}}{\text{observed wavelength}} \times \text{velocity of light}$$

$$= \left( \frac{144}{800} \times 3 \times 10^8 \right) \text{ m/s} = \boxed{5.4 \times 10^7 \text{ m/s.}} \\ = \boxed{5.4 \times 10^9 \text{ km/s.}}$$

So, the object was moving away from us at about  $5.4 \times 10^9 \text{ km/s}$ , when the spectrum was taken.

Ans(c): From Hubble Law,

$$\text{radial velocity} = H \times \text{distance} \quad [H: \text{Hubble constant: } 65 \text{ km/s Mpc}]$$

$$\therefore \text{distance} = \frac{\text{radial velocity}}{H} = \frac{5.4 \times 10^9}{65} = 830.769 \text{ Mpc.}$$

Ans : 830.769 Mpc.

Ans(d) : Electron.

### Problem B.4: Distribution of Dark Matter (6 Points)

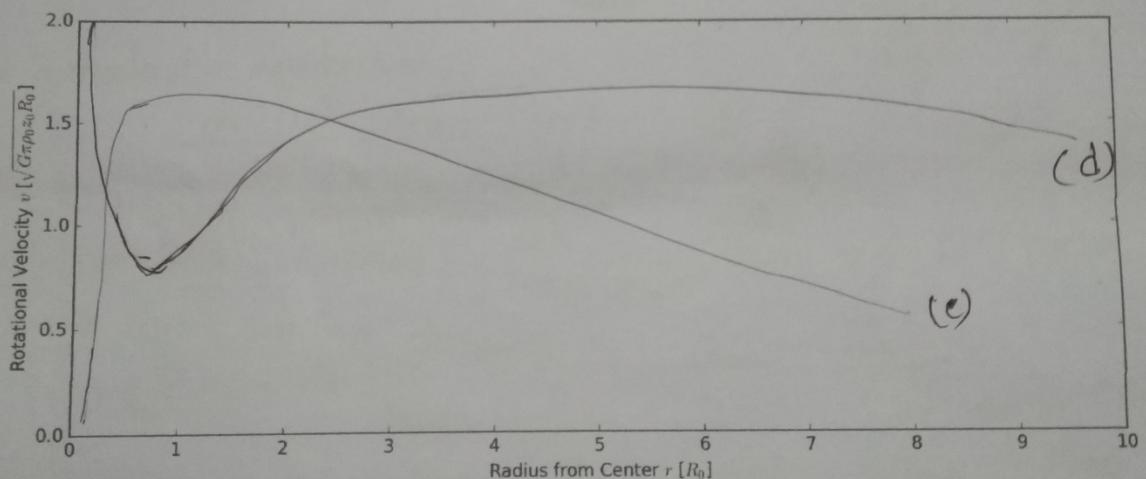
The most mass of our Milky Way is contained in an inner region close to the core with radius  $R_0$ . Because the mass outside this inner region is almost constant, the density distribution can be written as following (assume a flat Milky Way with height  $z_0$ ):

$$\rho(r) = \begin{cases} \rho_0, & r \leq R_0 \\ 0, & r > R_0 \end{cases}$$

- (a) Derive an expression for the mass  $M(r)$  enclosed within the radius  $r$ .
- (b) Derive the expected rotational velocity of the Milky Way  $v(r)$  at a radius  $r$ .
- (c) Astronomical observations indicate that the rotational velocity follows a different behaviour:

$$v_{obs}(r) = \sqrt{G\pi\rho_0 z_0 R_0} \left( \frac{5/2}{1 + e^{-4r/R_0}} - \frac{5}{4} \right)$$

Draw the expected and observed rotational velocity into the plot below:



- (d) Scientists believe the reasons for the difference to be *dark matter*: Determine the rotational velocity due to dark matter  $v_{DM}(r)$  from  $R_0$  and draw it into the plot above.
- (e) Derive the dark matter mass  $M_{DM}(r)$  enclosed in  $r$  and explain its distributed.
- (f) Explain briefly three theories that provide explanations for *dark matter*.

Ans (a): Volume,  $V(r) = \frac{4}{3}\pi r^3$   
 $\therefore$  Mass,  $M(r) = \rho \times V(r) = \rho \times \frac{4}{3}\pi r^3$   
 $= \boxed{\frac{4}{3}\pi \rho r^3}$

Ans (b): Rotational velocity,  $v(r) = \sqrt{\frac{GM(r)}{r}}$  (P.E.O.)

(extra page for problem B.4: Distribution of Dark Matter)

$$\therefore V(r) = \sqrt{\frac{G}{\pi} \times \frac{4}{3} \pi r^3}$$

Ans (d): Ans (e):

The balance of forces on any mass  $m$  in the galaxy is:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \text{--- (1), giving, } v^2 = \frac{GM(r)}{r} \quad \text{--- (4)}$$

The avg kinetic energy of the dark matter can be written as,

$$K = \frac{1}{2} m_d v^2 \quad (v \rightarrow \text{velocity}, m_d \rightarrow \text{mass of dark matter particle})$$

Then the total mass within the radius  $r$  is,  $M(r) = \frac{v^2 r}{G} = \frac{2K r}{m_d G} \quad \text{--- (iv)}$

$$\Rightarrow \frac{dM(r)}{dr} = \frac{2K}{m_d G} \quad \text{--- (v).}$$

For hydrostatic equilibrium,

$$\frac{dM(r)}{dr} = 4\pi r^2 P(r) \quad \text{--- (vi)}$$

$$\text{From (v) } \frac{2K}{m_d G} = 4\pi r^2 P(r) \quad \text{--- (vii)}$$

$$\text{From (vii) } P(r) = \frac{2K}{4\pi r^2 m_d G} \quad \text{--- (viii) that implies } P(r) \propto \frac{1}{r^2}.$$

Now the dark matter density is:  $\rho_{\text{halo}} = \frac{\rho_0 R_0^2}{R_0^2 + r^2}$

$$\text{From (iv) and (viii), } \rho_{\text{halo}} = \frac{1}{4\pi r^2} \times M(r) \quad \text{--- (ix)}$$

$$\Rightarrow M(r) = \rho_{\text{halo}} \times 4\pi r^2.$$

So, now,

$$M(r) = \int_0^r \rho_{\text{halo}} \times 4\pi r^2$$

$$= \int_0^r \frac{\rho_0 R_0^2}{R_0^2 + r^2} \times 4\pi r^2 dr$$

$$= 4\pi \rho_0 \int_0^r \frac{R_0^2 r^2 dr}{R_0^2 + r^2} = 4\pi \rho_0 R_0^2 \int_0^r \frac{R_0^2 + r^2 - R_0^2}{R_0^2 + r^2} dr$$

$$= 4\pi \rho_0 R_0^2 \left( \frac{R_0^2 + r^2}{R_0^2 + r^2} \right) dr - 4\pi \rho_0 R_0^4 \int_0^r \frac{dr}{R_0^2 + r^2}$$

$$= \left[ 4\pi \rho_0 R_0^2 \right]_0^r - 4\pi \rho_0 R_0^4 \left[ \frac{1}{R_0} \tan^{-1} \frac{r}{R_0} \right]_0^r$$

$P(r)$  can be written as  $\rho_{\text{halo}}$  as  $\rho_{\text{halo}}$  is the density of dark matter and beyond halo is a function of  $r$ .

(P.t.o)

After the integration,

$$M(r) = \left[ \frac{4\pi \rho_0 R^3}{3} \left( -2 - 3 \tan^{-1} \left( \frac{r-R}{R} \right) \right) + 4\pi r \rho_0 R^2 \right].$$

Ans. no(d):

$$\begin{aligned} v^2 &= \frac{GM(r)}{r} = \frac{G}{r} \left[ \frac{4\pi \rho_0 R^3}{3} \left( -2 - 3 \tan^{-1} \left( \frac{r-R}{R} \right) \right) + 4\pi r \rho_0 R^2 \right] \\ &= \left[ \frac{4\pi G \rho_0 R^3}{3r} \left( -2 - 3 \tan^{-1} \left( \frac{r-R}{R} \right) \right) + \frac{4\pi G r \rho_0 R^2}{r} \right] \\ &= \left[ \frac{4\pi G \rho_0 R^3}{3r} \left( -2 - 3 \tan^{-1} \left( \frac{r-R}{R} \right) \right) + 4\pi G r \rho_0 R^2 \right] \\ \therefore v &= \sqrt{\frac{4\pi G \rho_0 R^3}{3r} \left( -2 - 3 \tan^{-1} \left( \frac{r-R}{R} \right) \right) + 4\pi G r \rho_0 R^2}. \end{aligned}$$

Ans. no(f):

(i) X-ray radiation produced by the Annihilation:

The plasma researchers proposed a scenario in which two dark matter particles collide, resulting in their mutual annihilation. The annihilation of dark matter is a two-step process. During the initial step, an intermediate state is formed, which later disintegrates into the observed X-ray photons.

(ii) Velocity dispersion:

Velocity dispersion estimates the of elliptical galaxies. The relationship between velocity dispersion and dark matter takes several forms based on the object being observed. It is correlated with dark matter halo mass.

(iii) Red-shift distortions:

Galaxies in front of a supercluster have excess radial velocity towards it and have redshift slightly higher than their distance would imply, while galaxies behind the supercluster have redshifts slightly low for their distance.

## Problem C.1 : Detection of Gravitational Waves (10 Points)

This problem requires you to read the following recently published scientific article:

**Observation of Gravitational Waves from a Binary Black Hole Merger.**  
B. P. Abbott et al., LIGO Scientific Collaboration and Virgo Collaboration  
arXiv:1602.03837, (2016). Link: <https://arxiv.org/pdf/1602.03837.pdf>

Answer following questions related to this article:

- (a) How was the existence of gravitational waves first shown?

In 2015, Laser Interferometer Gravitational-Wave Observatory first detected gravitational waves. The gravitational waves were first shown as binary black hole system merging to form a single black hole.

- (b) Which detectors exist around the world? Why did only LIGO detect GW150914?

LIGO detectors exist around the world.

- LIGO can respond proportionally to gravitational-wave amplitude, at low red-shift volume of space to which they're sensitive, increasing in the cube of strain sensitivity.

- (c) Explain the components of the LIGO detectors.

Each LIGO Detectors consists of two arms, each 4km long. Each arm contains a resonant optical cavity, formed by its two main mirrors. A partially transmissive power-recycling recycling mirror at the input. A partially transmissive signal-recycling mirror at the output, 1064 nm wavelength Nd: YAG laser.

- (d) Describe the different sources of noise. How was their impact reduced?

Photon shot noise, low displacement noise, seismic noise, thermal noise, quantum noise, oscillator noise, electronic noise, beam jitter. Thermal noises are reduced by using low-mechanical-loss materials in test mass that are low-loss dielectric coating. Optical noises are reduced by using 1.2m diameter tubes, other stages in ultrahigh vacuum.

- (e) What indicates that the gravitational wave originated from the merger of a black hole?

(i) The GRU150914 detection-event statistic value of  $S_c = 23.6$  which is larger than any background event.

(ii) false alarm rate of 1 per 2.3 yrs and poissonian false alarm probability of '02.

- (f) Which are the methods used to search for gravitational wave signals in the detector data?

(a) Recover signals from the coalescence of compact objects.

(b) Target a broad range of generic transient signals with minimal assumptions about waveforms.

- (g) How were the source parameters (mass, distance, etc.) determined from the data?

The source parameters (mass, distance) were determined by compact binary signals from simulated data from the advanced detectors. The three compact binary signals are - a binary neutron star, a neutron-star-black hole binary and a binary black hole.

## Problem C.2 : First Image of a Black Hole (10 Points)

This problem requires you to read the following recently published scientific article:

**First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole.**  
The Event Horizon Telescope Collaboration, arXiv:1906.11238, (2019). Link: <https://arxiv.org/pdf/1906.11238.pdf>

Answer following questions related to this article:

- (a) Calculate the photon capture radius and the Schwarzschild radius of M87\* (in AU).

$$\text{Photon capture radius, } R_c = \sqrt{27} \times \frac{GM}{c^2}$$
$$= \sqrt{27} \times \frac{6.673 \times 10^{-11} \times 10 \times 1.989 \times 10^{30}}{(3 \times 10^8)^2}$$
$$= 7.663 \times 10^4 \text{ m}$$

$$\text{Schwarzschild radius of M87 is}$$
$$2GM/c^2 = \frac{2 \times 6.673 \times 10^{-11} \times 1.989 \times 10^{30}}{(3 \times 10^8)^2}$$
$$= 1.927 \times 10^{13} \text{ m}$$

- (b) Why was it not possible for previous telescopes to take such a picture of the black hole?

Because of shorter wavelength including increased noise in radio receiver electronics, higher atmospheric opacity, increased noise phase fluctuations and decreased efficiency.

- (c) Describe the components and functionality of the event horizon telescope.

Radio telescopes are used to receive radio waves from astronomical radio sources in the sky, studies the radio frequency of electromagnetic spectrum. Through very-long-baseline interferometry (VLBI), many independent radio antennas separated by hundreds of kilometers can act as a phased array.

- (d) Explain the two algorithms used to reconstruct the image from the telescope data.

"CLEAN": CLEAN is an inverse-modelling approach that deconvolves the interferometric point-spread function from the Fourier-transformed visibilities.

"RML": RML is a forward-modelling approach that searches for an image that is not only consistent with the observed data but also favours specified image properties.

- (e) What parameters were required for the GRMHD simulations to generate an image?

The dimensionless spin and the net dimensionless magnetic flux over the event horizon.

- (f) Explain the physical origins of the features in Figure 3 (central dark region, ring, shadow).

The central dark region arises from the black-hole's rotation and relativistic beaming.

The image is dominated by a ring with an asymmetric azimuthal profile that is oriented at a position angle that increases to  $\sim 170^\circ$  east of north.

The central funnel depression is the observational signature of the black-hole shadow.

- (g) How can the image resolution be increased in future observations?

The image resolution can be increased by decreasing the wavelength, i.e. increasing the frequency, angular resolution.

