Let us consider a sample dataset having one input (x_i^a) and one output (y_i^a) and number of samples 4. Develop a simple linear regression model using stochastic gradient descent optimizer.

Sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

Iteration 1 :-

consider $m=1$, $c=1$ $\eta=-0.1$, $ns=2$

$$e = \tfrac{1}{2}[y_i - mx_i - c)^2$$

$$\frac{\partial e}{\partial m} = -(y_i - mx_i - c)x_i \quad \text{and}$$

$$\frac{\partial e}{\partial c} = -(y_i - mx_i - c)$$

for $i=1$

$$\frac{\partial e}{\partial m} = -(y_1 - mx_1 - c)x_1$$

$$= -(3.4 - (1)(0.2) - 1)\,0.2$$

$$= -0.44$$

$$\frac{\partial e}{\partial c} = -(y_1 - mx_1 - c)$$

$$= -(3.4 - (1)(0.2) - 1)$$

$$= -2.2$$

→ Calculating delta values.

$$\Delta m = -\eta \cdot \frac{\partial E}{\partial m} = -(-0.1) \times -0.44$$
$$= -0.044$$

$$\Delta c = -\eta \cdot \frac{\partial E}{\partial c} = -(-0.1) \times (-2.2)$$
$$= -0.22$$

→ Updating the values of m and c

$$m = m + \Delta m = 1 - 0.044$$
$$= 0.956$$

$$c = c + \Delta c = 1 - 0.22$$
$$= 0.78$$

<u>Iteration 2</u> :- $m = 0.956$ and $c = 0.78$, $i = 2$

$$\frac{\partial E}{\partial m} = -(y_2 - m x_2 - c) x_2$$
$$= -(3.8 - (0.956)(0.4) - 0.78)\, 0.4$$
$$= -1.05504$$

$$\frac{\partial E}{\partial c} = -(y_2 - m x_2 - c)$$
$$= -(3.8 - (0.956)(0.4) - 0.78)$$
$$= -2.6376$$

→ calculating delta values

$$\Delta m = -\eta \cdot \frac{\partial E}{\partial m} = -(-0.1)(-1.05504)$$
$$= -0.105504$$

$$\Delta c = -\eta \cdot \frac{\partial E}{\partial c} = -(0.1)(-2.6376)$$
$$= -0.26376$$

→ updating values of m and c

$$m = m + \Delta m = 0.956 - 0.105504$$
$$= 0.8504$$

$$c = c + \Delta c = 0.78 - 0.26376$$
$$= 0.51624$$

Iteration 3:- $m = 0.8504,\ c = 0.51624$ and $i = 3$

Here $i = 3 > ns$ then $i = 1$ i.e sample $= 1$

$$\frac{\partial E}{\partial m} = -(y_1 - m x_1 - c) x_1$$
$$= -(3.4 - (0.8504)(0.2) - 0.51624)(0.2)$$
$$= -0.5427321$$

$$\frac{\partial E}{\partial c} = -(y_1 - m x_1 - c)$$
$$= -(3.4 - (0.8504)(0.2) - 0.51624)$$
$$= -2.7136$$

→ calculating delta values

$$\Delta m = -\eta \cdot \frac{\partial E}{\partial m} = -(-0.1)(-0.5427) = -$$
$$= -0.054273$$

$$\Delta c = -\eta \cdot \frac{\partial E}{\partial c} = -(-0.1)(-2.7136)$$
$$= -0.27136$$

→ updating m and c values.

$$m = m + \Delta m = 0.9576 - 0.05427$$
$$= 0.79622$$

$$c = c + \Delta c = 0.51624 - 0.27136$$
$$= 0.24488$$

→ updati sample = 1+1 = 2   i = 2

$$\frac{\partial e}{\partial m} = -(y_i - m x_i - c)x_i$$

$$= -(3.8 - (0.79622)0.4 - 0.24488)\,0.4$$

$$= -1.29468$$

$$\frac{\partial e}{\partial c} = -(y_i \cdot m x_i - c)$$

$$= -(3.8 - (0.79622)0.4 - 0.24488)$$

$$= -3.236.$$

→ calculating delta values.

$$\Delta m = -n . \frac{\partial e}{\partial m} = -(-0.1)(-1.29468)$$

$$= -0.129$$

$$\Delta c = -n . \frac{de}{\partial c} = -(-0.1)(-3.236)$$

$$= -0.3236$$

→ updating m value.

$$m = m + \Delta m = 0.79622 + (-0.129)$$

$$= 0.66$$

$$c = c + \Delta m = 0.24488 + (-0.323)$$

$$= -0.079$$