

Find the global minimum point and value for a function $f(x,y) = x^2 + y^2 + 10$.

Sol:- $f(x,y)$ has global minimum at $x=0, y=0$

$$\begin{aligned} f(0,0) &= (0)^2 + (0)^2 + 10 \\ &= 10 \end{aligned}$$

$$f(x,y) = x^2 + y^2 + 10.$$

$$\frac{\partial f}{\partial x} = 2x \quad \text{and} \quad \frac{\partial f}{\partial y} = 2y.$$

Initialize $x = -1$ and $y = 1$

$$n = 0.1 \quad \text{epoch} = 2$$

Iteration 1:-

$$\frac{\partial f}{\partial x} = 2x = 2(-1) = -2$$

$$\frac{\partial f}{\partial y} = 2y = 2(1) = 2$$

$$\Delta x = -n \cdot \frac{\partial f}{\partial x} = -0.1 \times -2 = 0.2$$

$$\Delta y = -n \cdot \frac{\partial f}{\partial y} = -0.1 \times 2 = -0.2$$

$$\begin{aligned} \text{New } x \text{ value} &= x + \Delta x = -1 + 0.2 \\ &= -0.8 \end{aligned}$$

$$\begin{aligned} \text{New } y \text{ value} &= y + \Delta y = 1 - 0.2 \\ &= 0.8 \end{aligned}$$

Iteration 2:- $x = -0.8$ $y = 0.8$

$$\frac{\partial f}{\partial x} = -2x = +2(-0.8) = -1.6$$

$$\frac{\partial f}{\partial y} = 2y = +2(0.8) = +1.6$$

$$\Delta x = -\eta \cdot \frac{\partial f}{\partial x} = -0.1 \times -1.6$$
$$= 0.16$$

$$\Delta y = -\eta \cdot \frac{\partial f}{\partial y} = -0.1 \times 1.6$$
$$= -0.16$$

$$\text{New } x \text{ value} = x + \Delta x = -0.8 + 0.16$$
$$= -0.64$$

$$\text{New } y \text{ value} = y + \Delta y = 0.8 - 0.16$$
$$= 0.64$$