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Lecture 8 Logistic Regression II

STAT 441/505: Applied Statistical Methods in Data Mining

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Outline

Multiple Logistic Regression

Case-control sampling and logistic regression

Multiclass Logistic Regression

Summary and Remark

Logistic Regression with indicator variable

- We can predict if an individual default by checking if she is a student or not. Thus we can use a qualitative variable `Student` coded as (Student $=1$, Non-student $=0$).

```
> glm.fit=glm(default~student,data=defaultData,family=binomial)
> summary(glm.fit)
Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         -3.50413    0.07071  -49.55  < 2e-16 ***
factor(student)Yes   0.40489    0.11502    3.52 0.000431 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

- β_1 is positive. This indicates students tend to have higher default probabilities than non-students.

```
> predict(glm.fit, list(student = c('Yes','No')),type="response")
         1          2
0.04313859 0.02919501
```

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Multiple logistic Regression

- Logistic Regression with several covariates

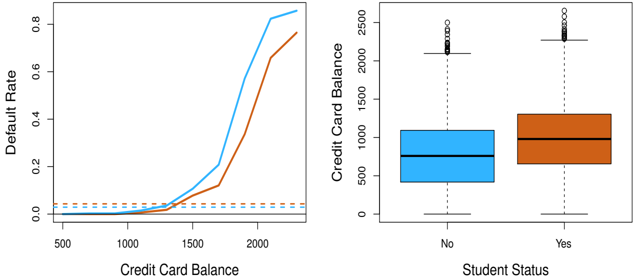
$$p(X) = \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}.$$

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p.$$

```
> glm.fit=glm(default~balance+income+student,data=defaultData,fam
> summary(glm.fit)
Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.087e+01  4.923e-01 -22.080  < 2e-16 ***
balance      5.737e-03  2.319e-04  24.738  < 2e-16 ***
income       3.033e-06  8.203e-06   0.370  0.71152
studentYes  -6.468e-01  2.363e-01  -2.738  0.00619 **
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

- Why is coefficient for `student` negative, while it was positive before?

Confounding



- Students tend to have higher balances than non-students, so their marginal default rate is higher than for non-students.
- But for each level of balance, students default less than non-students.
- Multiple logistic regression can tease this out.

South African Heart Disease

- 160 cases of MI (myocardial infarction) and 302 controls (all male in age range $15-64$), from Western Cape, South Africa in early 80s.
- Overall prevalence very high in this region: 5.1%.
- Measurements on seven predictors (risk factors), shown in scatterplot matrix.
- Goal is to identify relative strengths and directions of risk factors.
- This was part of an intervention study aimed at educating the public on healthier diets.

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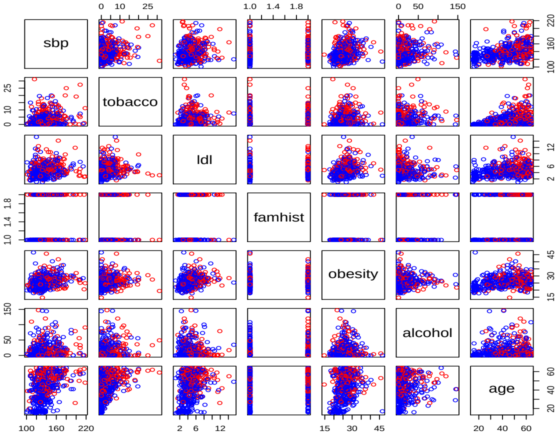
Pair Plots



Scatterplot matrix of the South African Heart Disease data. The response is color coded. The cases (MI) are red, the controls blue. `famhist` is a binary variable, with 1 indicating family history of MI.

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South African Heart Disease

```
> glm.fit=glm(chd~.,data=heartData1,family=binomial)
>
> summary(glm.fit)

Call:
glm(formula = chd ~ ., family = binomial, data = heartData1)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7517  -0.8378  -0.4552   0.9292   2.4434

Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)    -4.1295997  0.9641558  -4.283 1.84e-05 ***
sbp             0.0057607  0.0056326   1.023  0.30643
tobacco         0.0795256  0.0262150   3.034  0.00242 **
ldl             0.1847793  0.0574115   3.219  0.00129 **
famhistPresent  0.9391855  0.2248691   4.177 2.96e-05 ***
obesity        -0.0345434  0.0291053  -1.187  0.23529
alcohol         0.0006065  0.0044550   0.136  0.89171
age             0.0425412  0.0101749   4.181 2.90e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 596.11  on 461  degrees of freedom
Residual deviance: 483.17  on 454  degrees of freedom
AIC: 499.17
```

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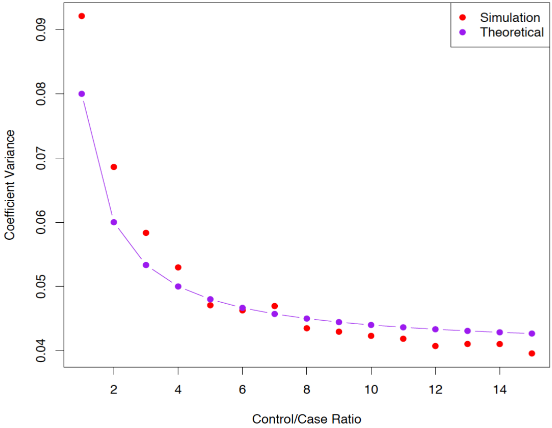
Case-control sampling and logistic regression

- In South African data, there are 160 cases, 302 controls — $\tilde{\pi} = 0.35$ are cases. Yet the prevalence of MI in this region is $\pi = 0.05$.
- With case-control samples, we can estimate the regression parameters β_j accurately (if our model is correct); the constant term β_0 is incorrect.
- We can correct the estimated intercept by a simple transformation

$$\hat{\beta}_0^* = \hat{\beta}_0 + \log\left(\frac{\pi}{1-\pi}\right) - \log\left(\frac{\tilde{\pi}}{1-\tilde{\pi}}\right).$$

- Often cases are rare and we take them all; up to five times that number of controls is sufficient.

Diminishing returns in unbalanced binary data



Sampling more controls than cases reduces the variance of the parameter estimates. But after a ratio of about 5 to 1 the variance reduction flattens out.

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Logistic regression with more than two classes

- So far we have discussed logistic regression with two classes. It is easily generalized to more than two classes.
- One version (used in the `R` package `glmnet` or `nnet`) has the symmetric form.

$$\Pr(Y = k|X) = \frac{e^{\beta_{0k}+\beta_{1k}X_1+\cdots+\beta_{pk}X_p}}{\sum_{k=1}^{K}\left(e^{\beta_{0k}+\beta_{1k}X_1+\cdots+\beta_{pk}X_p}\right)}.$$

- Here there is a linear function for each class.
- only $K-1$ linear functions are needed as in 2-class logistic regression.
- Multiclass logistic regression is also referred to as multinomial regression.

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Simulated Example

```
> library(nnet)
> x=matrix(rnorm(100*5),100,5)
>
> y=rnorm(100)
> #multinomial
> g4=sample(1:4,100,replace=TRUE)
> fit3=multinom(g4~x)
# weights:  28 (18 variable)
initial  value 138.629436
iter  10 value 130.910132
iter  20 value 130.869074
final  value 130.868827
converged
> summary(fit3)
Call:
multinom(formula = g4 ~ x)

Coefficients:
  (Intercept)        x1          x2         x3        x4        x5
2 -0.09206107 0.7771141 -0.07521353 0.48808850 0.3695944 0.4197601
3  0.19450922 0.1198007  0.21709470 0.27615848 0.2629457 0.1542603
4 -0.14379965 0.2477509 -0.29897262 0.01837793 0.2444425 0.2160098

Std. Errors:
  (Intercept)        x1        x2        x3        x4        x5
2   0.3219613 0.3626664 0.3128671 0.3311019 0.3513568 0.3064434
3   0.2923117 0.3150980 0.2862657 0.3169007 0.3333487 0.2906013
4   0.3133926 0.3645049 0.3230845 0.3515986 0.3620371 0.3227726

Residual Deviance: 261.7377
AIC: 297.7377
>
```

Summary and Remark

- Multiple Logistic regression
- Case-control sampling and logistic regression
- Multiclass logistic regression
- Read textbook Chapter 4 and `R` code
- Do `R` lab