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Learning to solve Sliding Puzzles using Reinforcement Learning

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Report submitted in partial fulfillment of the requirements of the module
Project (E) 448 for the degree Baccalaureus in Engineering in the Department of
Electrical and Electronic Engineering at Stellenbosch University.

Supervisor: JC Schoeman

August 11, 2020

Acknowledgements

I would like to thank my dog, Muffin. I also would like to thank the inventor of the incubator; without him/her, I would not be here. Finally, I would like to thank Dr Herman Kamper for this amazing report template.



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Abstract

English

The English abstract.

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Nomenclature

Variables and functions

$p(x)$ Probability density function with respect to variable x .

Acronyms and abbreviations

AE Afrikaans English

Chapter 1

Introduction

Generally in robotics a manipulator (eg. an arm) is used to manipulate an object in the environment. It is normally easier to first simulate the robots behavior in a more simple environment. In this project we try to solve a sliding puzzle using reinforcement learning, where the same algorithm can then later be applied to the robotics problem.

1.1. Section heading

This is some section with two table in it: Table 1.1 and Table 1.2.

Table 1.1: Performance of the unconstrained segmental Bayesian model on TIDigits1 over iterations in which the reference set is refined.

Metric	1	2	3	4	5
WER (%)	35.4	23.5	21.5	21.2	22.9
Average cluster purity (%)	86.5	89.7	89.2	88.5	86.6
Word boundary F -score (%)	70.6	72.2	71.8	70.9	69.4
Clusters covering 90% of data	20	13	13	13	13

Table 1.2: A table with an example of using multiple columns.

Model	Accuracy (%)		Bitrate
	Intermediate	Output	
Baseline	27.5	26.4	116
VQ-VAE	26.0	22.1	190
CatVAE	28.7	24.3	215

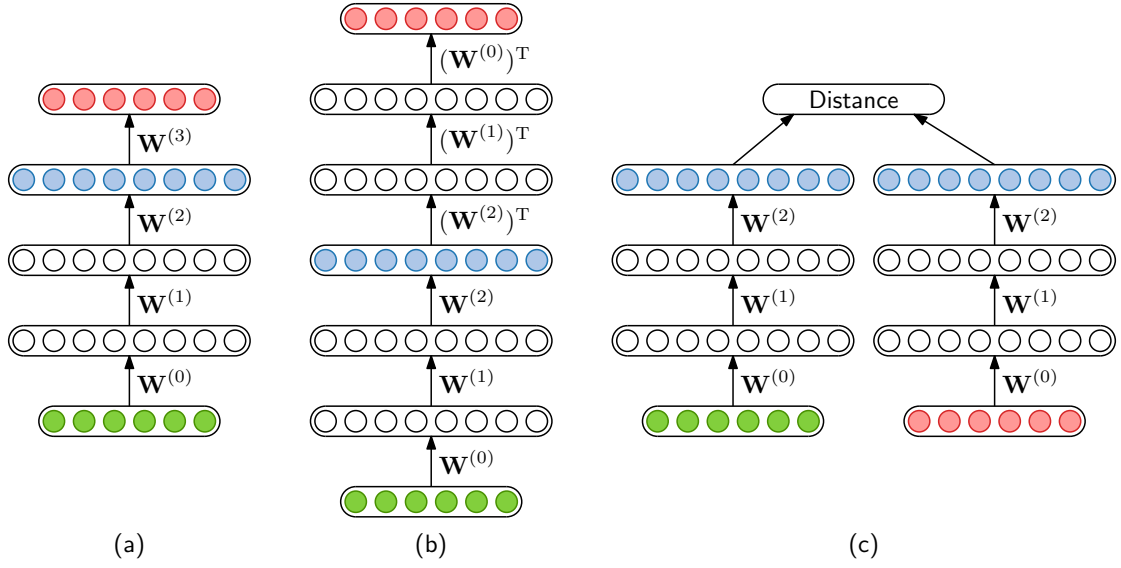


Figure 1.1: (a) The cAE as used in this chapter. The encoding layer (blue) is chosen based on performance on a development set. (b) The cAE with symmetrical tied weights. The encoding from the middle layer (blue) is always used. (c) The siamese DNN. The cosine distance between aligned frames (green and red) is either minimized or maximized depending on whether the frames belong to the same (discovered) word or not.

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Chapter 2

Solvability of a NxN sliding puzzle

2.1. General puzzle description

Let us assume that we have an NxN puzzle, then we have NxN number of blocks. We can represent the puzzle as an NxN array, then we stack the array into a one dimensional array of $1 \times (N \times N)$. For example see the 4x4 puzzle in Figure 2.1 we have a 1×16 array as: $\text{Array} = (12, 7, 8, 13, 4, 9, 2, 11, 3, 6, 15, 14, 5, 1, 10)$. Before we describe the conditions for a sliding puzzle to be solvable, we first define the term “inversion”. Assuming the the first index of the $1 \times N^2$ array starts at the left top corner (valued 12) in Figure 2.1, and that it runs from $[0, (N \times N) - 1]$. Then an inversion occurs when $\text{Array}[\text{index}] > \text{Array}[\text{index} + 1]$ where index is an arbitrary integer between 0 and $N \times N - 1$. Hence in Figure 2.1 we have a total: $\text{sum of inversions}(\text{Array}) = 11 + 6 + 6 + 8 + 3 + 5 + 1 + 5 + 1 + 2 + 4 + 3 + 1 + 0 = 56$.

12	7	8	13
4	9	2	11
3	6	15	14
5	1		10

Figure 2.1: Example of a sliding puzzle

2.2. Conditions for solvability

Even and odd sized boards are analysed separately (where $\text{size} = N$).

For odd sized boards where N is odd we have the puzzle only being solvable if and only if the boards has an even number of inversions. The proof for this can be deduced by looking at Figure 2 and noting that for every switch of the blank block we have an even change in the sum of inversions of the board. [1]

For even sized boards where N is even we have the board solvable if and only if the

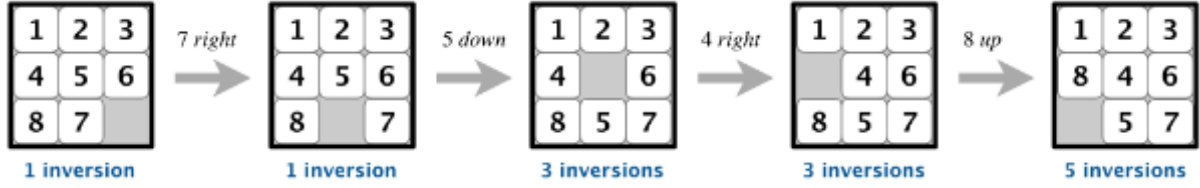


Figure 2.2: Odd boards with change in blank piece only having even inversion change [1]

number of inversions plus the row of the blank square is odd. This is illustrated in Figure 3.

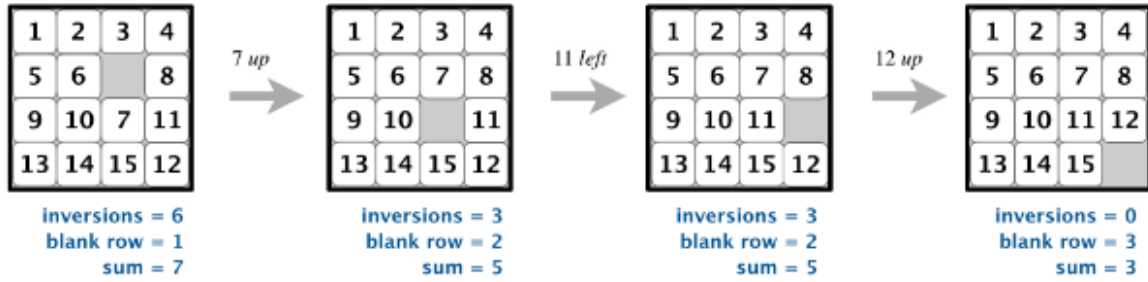


Figure 2.3: Even board solvability [1]

Half of all puzzle configurations are unsolvable. [2] This means that we only have $N! / 2$ configurations that are solvable for an $N \times N$ board. This was proven using parity in the paper in [2]. Sliding puzzles can be solved relatively quickly with today's processing of computers for puzzles for example an 5×5 puzzle was solved in 205 tile moves in 2016. [3]

The issue more so lies in finding the shortest path to solving a puzzle. This specific problem of solving with the least amount of tile moves of a sliding puzzle has been defined as NP (non-deterministic polynomial-time) hard. NP hardness is are problems that are at least as hard as NP. Where in computational complexity theory NP (non-deterministic polynomial-time) is a has a solution with a proof variable to be in polynomial time by a deterministic Turing Machine. A Turing machine is a mathematical model defining an abstract machine which manipulates symbols according to a set of rules. [4]

In simpler terms a problem is NP if it can be solved within a time that is a polynomial function of the input. For instance if we define the time to solve a problem as 'T' and the input data as 'D'. Then as long as $T = \text{polynomial function}(D)$ then a problem is NP.

Chapter 3

Summary and Conclusion

Bibliography

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- [2] W. E. Johnson, Wm. Woolsey; Story, “Notes on the ”15” puzzle,” *American Journal of Mathematics*, vol. 29, no. 2(4), p. 397–404, 1879.
- [3] D. of the Cube Forum, “5x5 sliding puzzle can be solved in 205 moves,” 2016. [Online]. Available: <http://cubezzz.dyndns.org/drupal/?q=node/view/559>
- [4] Minsky, *Computation: finite and infinite machines*. Prentice Hall, 1967.

Appendix A

Project Planning Schedule

This is an appendix.

Appendix B

Outcomes Compliance



This is another appendix.

Appendix C

Student and Supervisor agreement

Agreement between skripsie student and study leader regarding mutual responsibilities

Project (E) 448, Department of Electrical and Electronic Engineering, Stellenbosch University

Student name and SU#:	Umr Barends 18199313
Study leader:	Mr JC Schoeman
Project title:	Learning to solve Sliding Puzzles using Reinforcement Learning
Project aims:	Generally in robotics a manipulator (eg. an arm) is used to manipulate an object in the environment. It is normally easier to first simulate the robots behavior in a more simple environment. In this project we try to solve a sliding puzzle using reinforcement learning, where the same algorithm can then later be applied to the robotics problem.
<ol style="list-style-type: none">1. It is the responsibility of the student to clarify aspects such as the definition and scope of the project, the place of study, research methodology, reporting opportunities and -methods (e.g. progress reports, internal presentations and conferences) with the study leader.2. It is the responsibility of the study leader to give regular guidance and feedback with regard to the literature, methodology and progress.3. The rules regarding handing in and evaluation of the project is outlined in the Study Guide/website and will be strictly adhered to.4. The project leader conveyed the departmental view on plagiarism to the student, and the student acknowledges the seriousness of such an offence.	
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Figure C.1: Student and Supervisor agreement