Energy-Efficient Data Dissemination in Ad Hoc Networks: Mechanism Design with Potential Game

Mahdi Mousavi†, Hussein Al-Shatri†, Matthias Wichtlhuber‡, David Hausheer‡, and Anja Klein†

† Communications Engineering Lab, TU Darmstadt, Germany, {m.mousavi, h.shatri, a.klein}@nt.tu-darmstadt.de
‡ Peer-to-Peer Systems Engineering, TU Darmstadt, Germany, {mwichtl, hausheer}@ps.tu-darmstadt.de

***Abstract*—In this paper, a mechanism is designed based on game theory which aims at minimizing the transmit power in a multi-hop wireless broadcast network. There are multiple nodes in a network and among them, there is a source node which has a common message for all other nodes. For the sake of energy efficiency, the source's message should be forwarded to all nodes by a collaboration between different nodes in a multi-hop manner. Minimizing the total transmit power in the network is the goal of this paper. To this end, the nodes in the network are modeled as rational players and a mechanism is designed based on a potential game model. In this game, the action set of each node changes during the game based on the action of other players. Besides, it is proposed to exploit the weakly dominant strategy at the nodes such that the nodes change their actions even if a new action with the the same cost exists. Simulation results show that the proposed decentralized mechanism significantly outperforms other conventional decentralized algorithms. Moreover, when the network is not dense, our algorithm can outperform centralized algorithms on average.**

I. Introduction

The number of mobile users has dramatically increased during the past decade and the problems related to infrastructureless networks e.g., topology control [1] or minimizing the transmit power [2] [3], attracted much attention. A wireless Ad Hoc network is a network composed of a set of wireless devices which communicate with each other in the absence of an infrastructure. Depending on the distance between a transmitter and its receiver, the communication in such networks could be done in a single-hop or multi-hop manner. In a multi-hop transmission, a message initiated by a source node and intended for a destination node is forwarded by the nodes located between the source node and the destination node. Since the wireless mobile devices are battery equipped with limited energy, minimizing the power consumption in such devices is an important issue. This issue is challenging especially when a high amount of data shall be transmitted by mobile devices, e.g., streaming a high quality video from a node to all other nodes in its geographical proximity.

The problem in this paper is to minimize the total transmit power required for distributing a message in a single source wireless Ad Hoc network. The transmission scheme in such networks is also called multi-hop broadcast. In a multi-hop broadcast transmission, the source node distributes its message to all nodes of the network by the help of some nodes which forward the source's message for other nodes. Minimizing the total transmit power in a multi-hop broadcast network is known to be an NP-hard problem [4]. The proposed algorithms for this problem are mainly categorized into centralized [2] [5] and decentralized [3] [6] algorithms.

One of the very first well-known centralized algorithms for minimizing the transmit power in a multi-hop broadcast network is the Dijkstra algorithm proposed in [5]. The Dijkstra algorithm connects the nodes to the source either directly or in a multi-hop manner by finding the shortest path (cost) from a node to the source. The Dijkstra algorithm is simple, but it is not suitable for wireless networks since it does not take the broadcast nature of wireless channels into account. In [2], the authors propose an algorithm called broadcast incremental power (BIP). This algorithm is iterative and exploits the broadcast nature of wireless channels in a centralized manner. It starts with the source node and at each iteration, it connects one of the nodes of the network to the source, either in a single-hop or multi-hop manner. Besides the BIP algorithm, the authors of [2] introduce a so-called sweep operation. The sweep operation improves the result of BIP by removing unnecessary transmissions when a node can be served by more than one transmitter. In this paper, the BIP algorithm along with the sweep operation is called as BIPSW. Although the performance of the centralized algorithms such as the BIP and the BIPSW are good, they rely on a central unit which may not be available in an Ad Hoc network.

Due to the lack of a central unit in Ad Hoc networks, finding a decentralized algorithm for minimizing the transmit power in such networks is important. In [7], the authors propose a decentralized implementation of the BIP algorithm, called DynaBIP, but the result of the algorithm is not good compared to the BIP algorithm. A decentralized algorithm called broadcast decremental power (BDP) is proposed by the authors of [6]. In BDP, a cost is defined for a node based on the required transmit power for a link between a transmitter and a receiver. The BDP algorithm outperforms the BIP algorithm, but its performance is not as good as for BIPSW. Recently, in our previous work in [3], a game theoretical model for this problem is suggested. In [3], a fixed action set is defined for the nodes by which the nodes can choose another node in the network to connect to and get the source's message. This action set will be determined prior to the start of the game and a node can connect to one of the pre-determined nodes during the game. Although the decentralized algorithms do not require a central unit and they are more suitable for Ad Hoc networks

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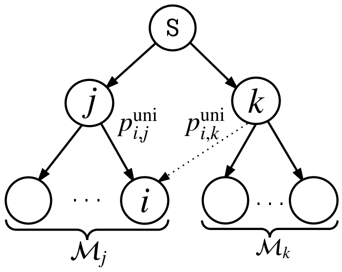

Fig. 1: A sample network. Solid and dashed arrows show current and possible connections a in broadcast tree, respectively. In this network node j is a parent node for node i and a child node for S.

than centralized ones, their performance is poor compared to the well-known centralized algorithms such as BIPSW.

In this paper, a non-cooperative dynamic game is designed based on the potential game model [8]. In this game, unlike in [3], the action set of a node varies at each iteration and adapts itself with respect to the actions of other nodes. In other words, in [3], the nodes' decision are restricted, while in this work, a node could connect to any of the nodes around itself. Moreover, in this paper using the weakly dominant strategy at the nodes is proposed, i.e., a node updates its actions if two transmitters offer the same cost. The game is designed such that although the nodes rationally decide to minimize their own costs on each iteration, the total transmit power in the network will be minimized accordingly. The results show that our proposed decentralized algorithm not only performs better than conventional decentralized algorithms, but also when the network is not dense, it outperforms the BIPSW.

The rest of this paper is organized as follows. Section II describes the system model. The game elements, properties and convergence are explained in Section III. Simulation results are provided in Section IV and finally, Section V concludes the paper.

II. System Model

A network composed of $N+1$ wireless nodes with random locations is considered; a source node S and a set of N other nodes denoted by $\mathcal{P} = \{1, \dots, N\}$. The nodes are equipped with a single antenna and have transmit power constraint $p^{\max}$. The source node has a message for all other nodes and due to the transmit power constraint at the nodes, the data should be disseminated in a multi-hop manner. The message dissemination from the source node to the nodes in $\mathcal{P}$ can be modeled as a graph with a tree structure. In this tree, the source is the root, the vertices represent the nodes and the edges are the links between the transmitting nodes in the network and their respective receivers. This tree is called *broadcast tree*. For a single hop point to point transmission in this tree, node j as a transmitter and node i as a receiver are called the *parent node* and the *child node*, respectively. In a broadcast tree, a child node has one parent node, but in order to benefit from the broadcast nature of wireless channel, a parent node may serve multiple child nodes. The set of child nodes served by a parent node j is denoted by $\mathcal{M}_j$, see Fig. 1.

The transmission scheme in this network is composed of two phases. The first phase is for broadcast tree construction and the second phase is for transmission using the constructed broadcast tree. This paper focuses on the broadcast tree construction in the first phase. It is assumed that the transmission in this network takes seconds or minutes. Therefore, an averaged value for the channel gains is used. The channel gain between the child node i and parent node j is given by $|h_{i,j}|^2$. In order to decode the data sent from its parent node successfully, a child node requires a minimum signal to noise ratio (SNR) denoted by γ^{th}. The SNR of the transmitted signal from parent node j and received by child node i is given by

$$\gamma_i = \frac{p_j^{\mathrm{Tx}} |h_{i,j}|^2}{\sigma^2} \tag{1}$$

in which p_j^{Tx} is the transmit power of the node j and σ^2 is the noise power. Therefore, for a unicast transmission, the transmit power of node j, in order to guarantee at least SNR of γ^{th} at child node i, is calculated as

$$p_{i,j}^{\mathrm{uni}} = \frac{\gamma^{\mathrm{th}} \sigma^2}{|h_{i,j}|^2}. \tag{2}$$

In the broadcast tree shown in Fig. 1, the weight of the edge between a parent node j and child node i is equal to $p_{i,j}^{\mathrm{uni}}$. A node can be a parent of another node if they are in each other's neighborhood. Let $\mathcal{N}_i$ be the set of neighboring nodes for node i, then, node j is in $\mathcal{N}_i$ if the required unicast power between them is less than $p^{\max}$, i.e.,

$$\mathcal{N}_i = \left\{ j \,\middle|\, j \in \mathcal{P} \cup \{S\}, p_{i,j}^{\mathrm{uni}} \leq p^{\max} \right\}. \tag{3}$$

The transmit power of a parent node is adjustable based on the required unicast powers of its child nodes. The multicast transmit power of a parent node is dominated by the highest required unicast power of its child nodes and is given by

$$p_j^{\mathrm{Tx}}(\mathcal{M}_j) = \max_{i \in \mathcal{M}_j} \quad \left\{ p_{i,j}^{\mathrm{uni}} \right\}. \tag{4}$$

The goal of this paper is to minimize the total transmit power in the network

$$p^{\mathrm{net}} = \sum_{j=1}^{N+1} p_j^{\mathrm{Tx}} \tag{5}$$

where p_{N+1}^{Tx} represents the transmit power of the source.

III. Proposed Mechanism Using Potential Game

In this section, the game theoretic algorithm is presented. The game is designed in a way that minimizing the cost at each individual node minimizes the total transmit power of the network introduced in (5).

A. Game Properties

The considered game is characterized by a set $\mathcal{P}$ of rational players which are the nodes in the network except the source. The game is played iteratively such that at each iteration, a node makes decision, given the decision of the other nodes. Different iterations of the game are shown by parameter t. The action of player $i \in \mathcal{P}$ shown by $a_i \in \mathcal{A}_i^{(t)}$, is to choose a node in its neighborhood as its parent node, where $\mathcal{A}_i^{(t)}$ represents the action set of the i-th player at iteration t. The set of joint actions of the players is denoted by $\mathcal{A}^{(t)} = \prod_{i=1}^{N} \mathcal{A}_i^{(t)}$.

The action profile of the game $\mathbf{a} = (a_1, \dots, a_N) \in \mathcal{A}^{(t)}$ is a vector which contains the actions of all players and $\mathbf{a}_{-i} \in \mathcal{A}^{(t)}$ represents the actions of all players except the i-th one. Based on the action profile of the game, a non-negative cost will be assigned to each player, i.e., $C_i(a_i, \mathbf{a}_{-i}) : \mathcal{A}^{(t)} \rightarrow \mathbb{R}^+ \cup \{0\}$ in which $\mathbb{R}^+$ represents positive real numbers. A non-cooperative dynamic game is designed for this network as $\mathcal{G} = (\mathcal{P}, \{\mathcal{A}_i^{(t)}\}_{i\in\mathcal{P}}, \{C_i\}_{i\in\mathcal{P}})$.

Game $\mathcal{G}$ is child-driven, that is, a node as child selects a parent with minimum cost. In this game, defining a proper action set for the nodes is of high importance. The action set must be defined in a way to ensure that the resulting graph is a tree, rooted at the source. A cycle must not occur during the iterations of this non-cooperative game where the nodes individually decide to minimize their own cost, regardless of what the others do. When a cycle occurs in a tree, a part of the network loses the connections to S. Based on the definition in [9], a cycle occurs in a rooted tree when a node connects to one of its ancestors. The ancestors of a node in a tree are the nodes which are on the route from S to the node. Therefore, node j could be a parent for node i at iteration t, if node i is a neighbor and it is not one of the node j's ancestors at iteration $t-1$. This means that the action set of a node depends on the state of the broadcast tree [10] resulting from the actions of other nodes taken in previous iterations.

Let $\mathcal{R}_j^{(t)}$ be the set of nodes which are on the route from S to node j at iteration t. For instance, in Fig. 1, node j just has node S on its route to source, i.e., $\mathcal{R}_j^{(t)} = \{S\}$. Then, the action set of node i at iteration t is defined as

$$\mathcal{A}_i^{(t)}) = \left\{ j \,\middle|\, j \in \mathcal{N}_i, \mathcal{R}_j^{(t-1)} \neq \{\}, i \notin \mathcal{R}_j^{(t-1)} \right\} \tag{6}$$

in which $\mathcal{R}_j^{(t-1)} \neq \{\}$ denotes a non-empty set, that is, node j as a parent node for node i must be connected to the broadcast tree.

In order to benefit from the broadcast nature of the wireless channel, the nodes will be incentivized to choose a common parent. To this end, the Marginal Contribution (MC) principle [11] is used to assign the cost to the child nodes. Using MC, the cost of a node depends on the node's effect on its parent node's final transmit power. More precisely, the cost of a child node i when $a_i = j$ is defined as

$$C_i(j, \mathbf{a}_{-i}) = p_j^{\mathrm{Tx}}(\mathcal{M}_j) - p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}). \tag{7}$$

Based on (7), a positive cost will be assigned from parent node j to the child in $\mathcal{M}_j$ which has the highest required unicast power and the cost of the rest of the children in $\mathcal{M}_j$ will be zero. The assigned positive cost is equal to the difference between the highest and second highest required unicast powers in $\mathcal{M}_j$.

The best response of a player to the action of other players is to choose a parent with minimum cost, i.e.,

$$a_i = \underset{a_i \in \mathcal{A}_i^{(t)}}{\operatorname{argmin}} \quad C_i(a_i, \mathbf{a}_{-i}), \quad \forall i \in \mathcal{N}. \tag{8}$$

The Nash Equilibrium (NE) is considered as the solution concept of this game. An action profile $\mathbf{a}^* \in \mathcal{A}$ is a NE if

$$C_i(a_i^*, \mathbf{a}_{-i}^*) \leq C_i(a_i, \mathbf{a}_{-i}^*), \quad \forall i \in \mathcal{N}, a_i \in \mathcal{A}_i. \tag{9}$$

The NE in (9) is defined based on a weakly dominant strategy [11]. With this strategy, a player updates its action if a new action with either lower or equal cost exists. Clearly, by using a weakly dominant strategy, a player may not gain by updating its action. Nevertheless, as system designer we proposed this strategy at the nodes because updating the action to a weakly dominant action by a node changes the state of the broadcast tree. Since the action set of the nodes varies based on the state of the broadcast tree, this may help the game to reach to a better result.

B. Convergence and Discussions

In this subsection it is shown how using weakly dominant strategy guarantees reaching a better result compared to strictly dominant strategy.

Definition: A game $\mathcal{G}$ is an exact potential game [8] if there exists a function $\Phi : \mathcal{A} \rightarrow \mathbb{R}$, called potential function, such that for every $i \in N$, $a_i \in \mathcal{A}_i$ and $a_i' \in \mathcal{A}_i$,

$$C(a_i', \mathbf{a}_{-i}) - C(a_i, \mathbf{a}_{-i}) = \Phi(a_i', \mathbf{a}_{-i}) - \Phi(a_i, \mathbf{a}_{-i}). \tag{10}$$

Theorem 1: The proposed game is an exact potential game with potential function $\Phi = \sum_{j=1}^{N+1} p_j^{Tx}$ where p_j^{Tx} is defined in (4).

Proof: Based on theorem 1, the potential function of the game is equal to the total transmit power in the network presented in (5) , i.e., $\Phi = p^{\mathrm{net}}$. The function Φ can be written as

$$\Phi = \sum_{j=1}^{N+1} p_j^{Tx} = p_j^{Tx} + p_k^{Tx} + \sum_{m=1, m\neq\{j,k\}}^{N+1} p_m^{Tx}. \tag{11}$$

Suppose that node i in a transition changes its action from $a_i = j$ to $a_i' = k$, see Fig. 1. Based on the concept introduced in (9), in the following, (10) is verified for different cases that a node changes its action.

Case 1 $(0 < C_i(k, \mathbf{a}_{-i}) < C_i(j, \mathbf{a}_{-i}))$*:* In this case, the costs of node i when it is connected to either parent node j or parent node k are positive. It means that in both cases, i.e., when node $i \in \mathcal{M}_j$ and when node $i \in \mathcal{M}_k$, the highest required unicast power among the children of parent node j and parent node k belongs to child node i. The transmit power of parent nodes j and k are dominated by node i and are equal to $p_{i,j}^{\mathrm{uni}}$ and $p_{i,k}^{\mathrm{uni}}$, respectively. The costs of node i when it is connected to parent node j or parent node k are equal to

$$C_i(j, \mathbf{a}_{-i}) = p_{i,j}^{\mathrm{uni}} - p_j^{\mathrm{Tx}}\left(\mathcal{M}_j \setminus \{i\}\right) \tag{12}$$

and

$$C_i(k, \mathbf{a}_{-i}) = p_{i,k}^{\mathrm{uni}} - p_k^{\mathrm{Tx}}\left(\mathcal{M}_k\right), \tag{13}$$

respectively. When node i leaves set $\mathcal{M}_j$, the transmit power of node j decreases from $p_{i,j}^{\mathrm{uni}}$ to $p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\})$ and by joining set $\mathcal{M}_k$, the transmit power of node k increases from $p_k^{\mathrm{Tx}}(\mathcal{M}_k)$ to $p_{i,k}^{\mathrm{uni}}$. Therefore, for these two actions, Φ in (11) can be rewritten as

$$\Phi(j, \mathbf{a}_{-i}) = p_{i,j}^{\mathrm{uni}} + p_k^{\mathrm{Tx}}(\mathcal{M}_k) + \sum_{m=1, m\neq\{j,k\}}^{N+1} p_m^{Tx} \tag{14}$$

and

$$\Phi(k,\mathbf{a}_{-i}) = p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}) + p_{i,k}^{\mathrm{uni}} + \sum_{m=1, m\neq\{j,k\}}^{N+1} p_m^{Tx}. \quad (15)$$

Using (12) – (15), the difference in potential function is

$$\begin{aligned}\Phi(k,\mathbf{a}_{-i}) - \Phi(j,\mathbf{a}_{-i}) &= p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\})) + p_{i,k}^{\mathrm{uni}} - p_{i,j}^{\mathrm{uni}} - p_k^{\mathrm{Tx}}(\mathcal{M}_k)\\ &= C_i(k,\mathbf{a}_{-i}) - C_i(j,\mathbf{a}_{-i}),\end{aligned}$$

which shows that the condition in (10) holds.

Case 2 ($0 = C_i(k,\mathbf{a}_{-i}) < C_i(j,\mathbf{a}_{-i})$)*:* In this case, (13) equals zero but (12) and (14) hold. Moreover, since $C_i(k,\mathbf{a}_{-i}) = 0$, then $p_{i,k}^{\mathrm{uni}} \leq p_k^{\mathrm{Tx}}(\mathcal{M}_k)$, i.e., even if node i joins set $\mathcal{M}_k$, the power of k remains the same and when $i \in \mathcal{M}_k$ the potential function is equal to

$$\Phi(k,\mathbf{a}_{-i}) = p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}) + p_k^{\mathrm{Tx}}(\mathcal{M}_k) + \sum_{m=1, m\neq\{j,k\}}^{N+1} p_m^{Tx}. \quad (16)$$

Using (16), (14) and (12) we find

$$\begin{aligned}\Phi(k,\mathbf{a}_{-i}) - \Phi(j,\mathbf{a}_{-i}) &= p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}) - p_{i,j}^{\mathrm{uni}}\\ &= C_i(k,\mathbf{a}_{-i}) - C_i(j,\mathbf{a}_{-i}),\end{aligned}$$

which confirms that (10) is valid for this case.

Case 3 ($0 < C_i(k,\mathbf{a}_{-i}) = C_i(j,\mathbf{a}_{-i})$)*:* In this case, since the costs of node i when either $i \in \mathcal{M}_j$ or $i \in \mathcal{M}_k$ are the same, the difference in the cost of node i is zero. Therefore (12) and (13) are equal and

$$p_{i,j}^{\mathrm{uni}} - p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}) = p_{i,k}^{\mathrm{uni}} - p_k^{\mathrm{Tx}}(\mathcal{M}_k). \quad (17)$$

Considering (17), by using (14) and (15), yields

$$\begin{aligned}&\Phi(k,\mathbf{a}_{-i}) - \Phi(j,\mathbf{a}_{-i})\\ &\quad = p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\})) + p_{i,k}^{\mathrm{uni}} - p_{i,j}^{\mathrm{uni}} - p_k^{\mathrm{Tx}}(\mathcal{M}_k) = 0,\end{aligned}$$

which shows that the difference in the potential function of the game, as well as the difference of the node i's cost, is zero. Therefore (10) holds.

Case 4 ($C_i(k,\mathbf{a}_{-i}) = C_i(j,\mathbf{a}_{-i}) = 0$)*:* In this case, the cost of node i is zero in both cases of staying in the set $\mathcal{M}_j$ or joining the set $\mathcal{M}_k$. It means the node i has no effect on the transmit powers of parent nodes j and k. Therefore by leaving the set $\mathcal{M}_j$ and joining the set $\mathcal{M}_k$, not only the cost of node i does not change, but also no change occurs in Φ.

In the first two studied cases, the transition from parent node j to parent node k is a strictly dominant strategy for node i. The latter two cases are based on weakly dominant strategy. In all cases, (10) holds. Therefore, game $\mathcal{G}$ with the cost function defined in (7) is an exact potential game. ■

Theorem 2: Starting with an initial tree, after some iterations, the Game $\mathcal{G}$ converges to a NE and the total transmit power in the network monotonically decreases.

Proof: Based on Theorem 1, since $\mathcal{G}$ is an exact potential game, the existence of a NE is guaranteed [11]. Moreover, based on (9), when a node updates its action, the cost of the node either reduces or remains the same. Since the game $\mathcal{G}$ is an exact potential game, the potential function of the game, i.e., Φ, has the same behavior as the cost function. Therefore, Φ is a monotonically decreasing function. Since the total transmit power in the network is bounded from below, by starting from an initial broadcast tree, the game converges after some iterations to a state which is a NE of the game. ■

Based on Theorems 1 and 2, not only the total transmit power in the network in (5), i.e., Φ, is exactly aligned with the best response strategy of a node defined in (8), but also Φ is a monotonically decreasing function with guaranteed convergence. In other words, with the proposed mechanism, regardless of the decision made by a node at an iteration, Φ never increases. Hence, by defining a proper strategy for the players, which here is exploiting the weakly dominant strategy, along with a proper definition of the action set, which here is defining a variable action set, it would be possible to further reduce the total transmit power of the network by means of more iterations. More precisely, although in a weakly dominant transition, the cost of the node remains the same, by defining a variable action set, new action sets will be available for other nodes in the network. Since theorem 2 guarantees that the broadcast tree will never go to a state with higher transmit power than its current state, therefore, if a node reduces its cost by its new action set, then, the total transmit power in the network decreases accordingly.

In practice, when there are multiple weakly dominant actions available for a node, the node randomly chooses an action which is different from its current action. To terminate the game, since the number of neighboring nodes is limited, a predefined number which is less than N could be considered for each node as the maximum number of iterations. When this number is reached and there is no strictly dominant action, the nodes stop updating their actions.

IV. Simulation Results

For the simulations, randomly deployed nodes in a square region of 500m×500m are considered. The number of nodes varies between 8 and 24 and the maximum transmit power of a node is set to $p^{\max} = 20$ dBm. The simulation is based on the Monte Carlo method and at each network realization, the source node will be chosen randomly. The realizations of the network is considered that based on (3), the nodes have at least one neighbor in their neighborhood. The channel is based on a path-loss model such that $|h_{i,j}|^2 = 1/d^{\alpha}$ in which d represents the distance between nodes i and j and α shows the attenuation exponent considered as $\alpha = 3$. The Benchmarks to our proposed algorithm are the BIP and BIPSW algorithms [2] as well as the game theoretic algorithm introduced in [3] which have been described in Sec. I. The minimum required SNR at the receiving nodes is considered as $\gamma^{\mathrm{th}} = 10$ dB and the noise power is set to -90 dBm. The broadcast tree is initialized based on a distributed implementation of the Dijkstra algorithm explained in [3]. The total transmit power in the network is considered as the performance measure.

Fig. 2 shows the total transmit power as a function of the number of nodes in the network. Our proposed algorithm performs relatively better than all benchmarks. The Dijkstra algorithm does not consider the broadcast nature of wireless

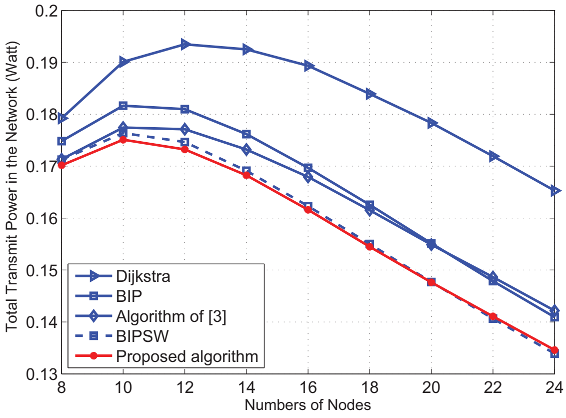

Fig. 2: Total transmit power in the network for different algorithms.

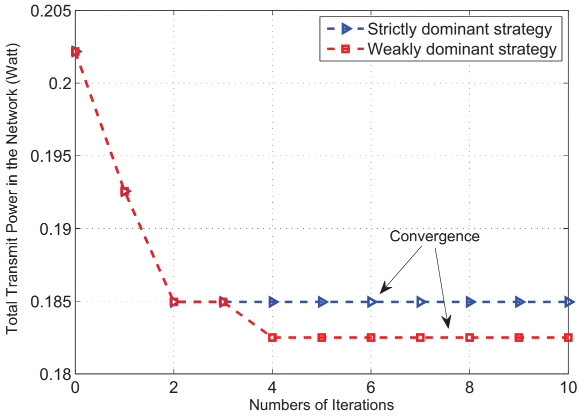

Fig. 3: Benefit of exploiting weakly dominant strategy for the proposed algorithm. There are 10 nodes in the network.

channel and it cannot perform well in such networks. Moreover, in the BIP algorithm there are unnecessary transmissions which results in a poorer performance than of our algorithm. Comparing with BIPSW algorithm, the performance of our proposed algorithm is better when the network is sparse. When the network becomes denser, the performance of the BIPSW becomes better. This is due to the fact that in a dense network, more information about the links and connections are available for BIPSW as a centralized algorithm compared to our proposed algorithm which naturally considers one hop information at a node for decision making. Hence, BIPSW reaches to a better result in dense networks. Moreover, the proposed algorithm outperforms the algorithm in [3]. The reason, on one hand, is because of variable action sets of the nodes which adapt to the state of the game. On the other hand, as discussed in Sec. III, having a weakly dominant strategy along with a proper design of a potential game helps to benefit from the definition of variable action set.

Fig. 3 shows the convergence of the proposed algorithm when there are 10 nodes in the network. As also proved in Sec. III, the total transmit power in the network monotonically decreases and converges after some iterations, depending on the number of nodes operating in the network. The benefit of considering the weakly dominant strategy, i.e., updating the action to an action with the same cost, can be clearly observed in this figure. Without considering this strategy, the game stops after two iterations. By considering the weakly dominant strategy, a node updates its action to a weakly dominant state at iteration three. Although this action has no effect on the network's total transmit power, i.e., from iteration two to iteration three, it changes the action set of other nodes and gives them the opportunity to reduce their cost at the next iterations. This leads to a lower total transmit power for the network, here at iteration four.

V. Conclusion

In this paper, a game theoretical mechanism for the minimum power broadcast tree problem is proposed based on potential game model. Variable action sets are defined for the nodes in the network and using the weakly dominant strategy at the nodes is proposed. It is proved that by this mechanism, when the nodes in the network minimize their own costs, the total transmit power in the network is minimized, accordingly. The proposed decentralized algorithm outperforms both the BIP and the BIPSW centralized algorithms, especially when the network is not dense.

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