Game-based Multi-hop Broadcast Including Power Control and MRC in Wireless Networks

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Abstract—A wireless Ad Hoc network consisting of a source and multiple receiving nodes is considered. The source wants to transmit a common message throughout the whole network. The message has to be spread in a multi-hop fashion, as the transmit powers at the source and the nodes are limited. The goal of this paper is to find the multi-hop broadcast tree with a minimum energy consumption in the network. To reach this goal, a new decentralized game theoretic approach is proposed which considers the following two aspects jointly for the first time: Firstly, it optimizes the transmit powers at the source and at the individual intermediate nodes. Secondly, it employs maximum ratio combining at the receiving nodes following the fact that a node can receive several copies of the message from different sources in different time slots. The game is modeled such that the nodes are incentivized to forward the message to their neighbors. In terms of the total transmit energy, the results show that the proposed algorithm outperforms other conventional algorithms.

I. INTRODUCTION

Recently, Ad Hoc networks receive huge attention due to their numerous applications in several fields. An Ad Hoc network consists of multiple wireless nodes. It has no central control unit and a dynamically changing topology [1]. The nodes are usually small devices, e.g., tablets or smart phones with limited transmit powers. Nowadays, there are many emerging applications, for instance live video streaming for a social event, where many of the network participants are interested in the same service. The present work focuses on broadcast scenarios where the nodes are demanding the same data from a particular source. Due to the transmit power limitation of the nodes, the data has to be transmitted in a multi-hop fashion to cover the whole network. If multiple unicast transmissions are established to distribute the message throughout the network, a large amount of data and overhead signaling are required to be distributed over the network which will cause a degradation of the quality of service (QoS).

An important goal in this paper is to minimize the total transmit energy in multi-hop broadcast networks [2] in which two challenges are involved. Firstly, the optimum topology has to be found which in this case is a rooted tree where the source is the root of the tree. Secondly, the transmit power of the individual nodes has to be optimized. These challenges have been previously addressed in different ways. In [3], a centralized algorithm aiming at minimizing the total transmit power is proposed. Basically, the algorithm constructs the tree on a node by node basis. In each time slot, it selects the node demanding the minimum power from the previously existing nodes in the tree. A decentralized implementation of this algorithm based on incremental power at each node is proposed in [4] in which the nodes exchange their decisions with each other and use signal strength measurements to find the required power for each link. Unfortunately, this algorithm requires higher total transmit energy as compared to the central solution and it experiences long delays required for avoiding decision conflicts among the nodes. Furthermore, the authors of [5] extend the algorithm of [3] by exploiting the overheard signals at the receiving nodes and they use the maximum ratio combining (MRC) technique at the receiving nodes.

Game theory modeling is a promising approach in minimizing the total energy in multi-hop broadcast networks in a decentralized way. Different algorithms based on cooperative games are proposed in [6] and [7]. In cooperative games, nodes form coalitions. A drawback is that these algorithms require a large amount of signaling among the nodes in each coalition for making a decision. Therefore, non-cooperative games are more suitable for solving this problem. In non-cooperative games, nodes can decide independently on their own and, thus, decentralized algorithms with a minimal signaling overhead can be developed. The authors of [8] propose a decentralized game-based algorithm constructing the broadcast tree with a minimum number of hops. However, they do not consider power control, i.e., the nodes always transmit with their maximum power. In [9], a game-based approach is proposed which considers power control, but does not exploit MRC.

In this paper, the goal is to find a multi-hop broadcast tree with minimum energy consumption in the network. A new decentralized algorithm is proposed based on game theory which considers two aspects jointly for the first time. The first aspect is the optimization of the transmit powers at the source and the individual intermediate nodes. The second aspect is the use of MRC at the receiving nodes following the fact that a node can receive several copies of a message from different transmitting nodes. More precisely, if a node receives multiple copies of the message from different transmitting nodes, it combines the received signals constructively to decode the message successfully. In general, this MRC capability allows to reduce the transmit powers. The optimum powers of the transmitting nodes depend on which signals are being combined and vice versa, so the two aspects need to be

treated jointly. The multi-hop broadcast transmission problem is modeled as a non-cooperative game where the nodes are the players. By employing power control at the transmitting nodes and MRC at the receiving nodes, the game model can find the best broadcast tree and the best transmit powers at the individual nodes. Each node minimizes its transmit power rationally and selfishly. To incentivize the nodes to forward the message to their neighbors, cost sharing rules, namely, marginal contribution [10] and Shapley value [11] are used.

This paper is organized as follows. Section II describes the system model. Sections III and IV introduce the broadcast tree and the game model, respectively. In Section V, the best response and convergence are discussed. Section VI explains some implementation issues. The performance assessment of the proposed algorithm is discussed in Section VII. Finally, Section VIII concludes the paper.

II. SYSTEM MODEL

A scenario consisting of a source S and $N \in \mathcal{N}$ nodes randomly deployed over a specified area is considered. Each of the nodes and the source are equipped with a single omni-antenna and have a maximum transmit power $p^{\max}$. The source has a common message for all nodes in the scenario. Since the transmit power of the source is limited, the source cannot directly transmit to all nodes. Therefore, the message has to be forwarded by the intermediate nodes such that it can reach all nodes in the scenario.

Based on the transmit power of an intermediate node, the multicast set of nodes which can be served by this intermediate node can be determined. The higher the transmit power of an intermediate node, the larger the distance this node can cover. This implies that the size of the multicast set of nodes in general increases if the transmit power of an intermediate node is increased. Serving more nodes may lead to a higher transmit power at the transmitting node. Since the nodes are randomly deployed over the scenario, a node can be usually served by different intermediate nodes. Therefore, an allocation of the transmit powers at the source and the intermediate nodes determines the association of the multicast set to a node. The present paper focuses on finding the optimum association of nodes and their multicast sets of nodes as well as the optimum transmit power per node such that the total transmit energy is minimized.

The association of nodes and their multicast sets of nodes can be modeled as a spanning tree. The root of the tree is the source and the leaves are the farthest reached nodes who do not need to forward the message. This tree is also known as broadcast tree. In a broadcast tree, the nodes which receive the message from other transmitting nodes are called child nodes and the transmitting nodes which serve the child nodes are called parent nodes, see Fig. 1. A rank attribute used for classifying the neighboring nodes will be described in the next section.

The proposed transmission scheme is composed of two phases. The first phase is the construction of the broadcast tree where the best association of the nodes and their multicast
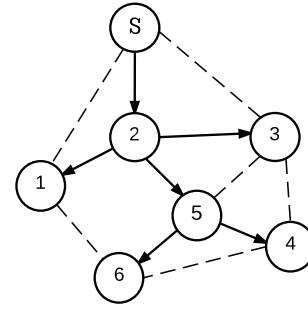

Fig. 1: A broadcast scenario consisting of a source S and 6 nodes.

sets is determined. Then, the transmission of the message throughout the scenario is taking place in the second phase.

The channel gain between the transmitting node j and the receiving node i is denoted by $|h_{i,j}|^2$. It is assumed that the transmission takes long time scales, e.g., seconds or minutes. Hence, the small scale variations of the channel such as fast fading will not be considered and $|h_{i,j}|^2$ represents an averaged value. Video streaming can be an example of such a scenario in which the transmission may take seconds or minutes long. Assuming that a node correctly decodes the message if the received power is not below a certain threshold p^{th}, the required transmit power for unicast and multicast transmissions can be calculated as follows.

For a unicast transmission, the transmit power required by the j-th node such that the received message can be correctly decoded at the i-th node is given by

$$p_{i,j}^{\text{uni}} = \frac{p^{\text{th}}}{|h_{i,j}|^2}, \tag{1}$$

with $p_{i,j}^{\text{uni}} \leq p^{\max}$. Because of the broadcast nature of the wireless channel, the transmit power of the j-th node for multicasting to the set $\mathcal{M}_j$ of its child nodes, is given by

$$p_j^{\text{Tx}}(\mathcal{M}_j) = \max \left\{ p_{1,j}^{\text{req}}, \dots, p_{|\mathcal{M}_j|,j}^{\text{req}} \right\}, \tag{2}$$

in which $|\mathcal{M}_j|$ denotes the number of child nodes for the j-th parent node and $p_{i,j}^{\text{req}}$ with $p_{i,j}^{\text{req}} \leq p_{i,j}^{\text{uni}}$ is the power which the i-th node requires from node j. The equality holds when node i has only one parent, i.e., node j. Since by using MRC, a fraction of the required power can be received by other parents in the network, in general $p_{i,j}^{\text{req}}$ can be less than $p_{i,j}^{\text{uni}}$.

As a node i can receive multiple signals from several parent nodes in its neighborhood, it can apply MRC by assuming that the parent nodes transmit at different time slots. Let $\mathcal{K}_i$ be the set of the parent nodes for the node i, then, the aggregate received power at node i is calculated as

$$p_{i,\mathcal{K}_i}^{\text{mrc}} = \sum_{j=1}^{|\mathcal{K}_i|} p_j^{\text{Tx}} |h_{i,j}|^2, \tag{3}$$

assuming that the received noise at all nodes is Gaussian distributed with the same variance. If $p_{i,\mathcal{K}_i}^{\text{mrc}} \geq p^{\text{th}}$ holds, the i-th node can decode the message.

III. BROADCAST TREE

The problem of constructing broadcast tree can be modeled as assigning every node to a subset of intermediate nodes such that the source can be reached by every node through the edges of the tree. A child node can be assigned only to the parent nodes in its neighborhood. The neighboring nodes of a node i are all nodes which can communicate directly with the i-th node. For instance, node i is a neighbor of node j if

$$p^{\max}|h_{i,j}|^2 \geq p^{\text{th}} \qquad (4)$$

holds. To guarantee that the proposed algorithm will end up with a tree, we classify a neighboring node of the i-th node as either a child node or a parent node using a power based rank attribute proposed in [9]. The power rank of a child node is the power rank of its parent node plus the required power for unicast transmission from the parent node to the child node. The power rank of the source is set to 0. Accordingly, node j for which (4) holds and its power rank is lower than the power rank of node i, is called a candidate parent for node i. A set of candidate parents for node i is denoted by $\mathcal{K}_i$. Note that, due to the use of MRC, a child node will in general be assigned to more than one parent node.

IV. GAME-BASED APPROACH

To find the broadcast tree with the minimum total energy consumption, a game theoretic approach is proposed. The proposed game is non-cooperative in the sense that the nodes are competing rationally and selfishly in selecting parents which offer the minimum individual total cost. To incentivize the parent nodes to transmit the requested powers to their children, the children have to pay virtual costs to their parents. Basically, the game is performed among the nodes in a sequential manner without a particular order. In each round, every node i calculates its required power from its candidate parents i.e., $p_{i,j}^{\text{req}}, j \in \mathcal{K}_i$, such that its total cost is minimized. Because a parent node can serve multiple child nodes, the cost of the transmitted power can be shared among the child nodes. Furthermore, parent nodes have to transmit in different time slots if they are selected by the same child node so that MRC can be employed.

A. Game properties

The game can be described as

$$G = (\mathcal{N}, \{\mathcal{A}_i\}_{i \in \mathcal{N}}, \{\mathcal{C}_i\}_{i \in \mathcal{N}}), \qquad (5)$$

where $\mathcal{N}$ represents the set of players, and for player $i \in \mathcal{N}$, the total cost and the action space are shown by $\mathcal{C}_i$ and $\mathcal{A}_i$, respectively. In this game, $\mathcal{A}_i \in \mathbb{R}^{|\mathcal{K}_i|}$ is the set of all possible requested powers from all candidate parents by node i, i.e.,

$$\mathcal{A}_i = \left\{ \mathcal{A}_{i,j} \in [0, p^{\max}] \,\middle|\, j \in \mathcal{K}_i \right\}. \qquad (6)$$

The action of node i is $a_i \in \mathcal{A}_i$ which is a vector of requested powers by the i-th node from its parents, i.e.,

$$a_i = \left\{ \mathbf{p}_i^{\text{req}} = \left(p_{i,1}^{\text{req}}, \dots, p_{i,|\mathcal{K}_i|}^{\text{req}} \right) \,\middle|\, \sum_{j \in \mathcal{K}_i} p_j^{\text{Tx}} |h_{i,j}|^2 \geq p^{\text{th}} \right\}. \qquad (7)$$

The total cost assigned to node i is calculated as

$$C_i(a_i, a_{-i}) = \sum_{j \in \mathcal{K}_i} C_{i,j}^{\text{f}}(p_{i,j}^{\text{req}}) \qquad (8)$$

where a_{-i} denotes the action of other nodes except the node i and $C_{i,j}^{\text{f}}$ is the cost assigned by node j to node i based on the cost sharing rule f. In the next subsection, we will discuss two cost sharing rules in details.

B. Cost sharing rules

In this section, the cost of providing the requested power from a parent is determined using cost sharing rules. For a requested power vector $\mathbf{p}_i^{\text{req}} \in \mathcal{A}_i$, the cost assigned to a node i by requesting a power $p_{i,j}^{\text{req}}$ from the j-th parent is calculated using either marginal contribution or Shapley value rules as follows.

1) Marginal contribution (MC) rule: For the j-th parent node with the set $\mathcal{M}_j$ of children, the cost based on marginal contribution [11] assigned to the node $i \in \mathcal{M}_j$ is calculated as

$$C_{i,j}^{\text{MC}}(p_{i,j}^{\text{req}}) = p_j^{\text{Tx}}(\mathcal{M}_j) - p_j^{\text{Tx}}(\mathcal{M}_j \setminus \{i\}) \qquad (9)$$

From (9), one can infer that all nodes in $\mathcal{M}_j$ will pay nothing to their parent j except the one with highest power. If we sort the requested powers of each node in $\mathcal{M}_j$ in an increasing order, i.e., $p_{1,j}^{\text{req}} \leq p_{2,j}^{\text{req}} \leq \cdots \leq p_{|\mathcal{M}_j|,j}^{\text{req}}$, then

$$C_{i,j}^{\text{MC}}(p_{i,j}^{\text{req}}) = \begin{cases} 0 & \text{if } 1 \leq i < |\mathcal{M}_j| \\ p_{|\mathcal{M}_j|,j}^{\text{req}} - p_{|\mathcal{M}_j|-1,j}^{\text{req}} & \text{if } i = |\mathcal{M}_j| \end{cases}. \qquad (10)$$

2) Shapley value (SV) rule: When the transmit power of a parent node is dominated by the highest requested power from the child nodes, the SV formulation can be obtained according to [12]. By sorting the powers requested by the nodes in an increasing order, i.e., $p_{1,j}^{\text{req}} \leq p_{2,j}^{\text{req}} \leq \cdots \leq p_{|\mathcal{M}_j|,j}^{\text{req}}$, the SV can be calculated as

$$C_{i,j}^{\text{SV}}(p_{i,j}^{\text{req}}) = \sum_{m=1}^{i} \frac{p_{m,j}^{\text{req}} - p_{m-1,j}^{\text{req}}}{|\mathcal{M}_j| + 1 - m} \qquad (11)$$

where $p_{0,j}^{\text{req}}$ equals zero.

V. BEST RESPONSE AND CONVERGENCE

A. Best response

The considered game is repeated over all nodes iteratively. In each iteration, one of the nodes takes an action based on (7) considering the status of the game in the previous iteration. In other words, a node i selects a new vector $\mathbf{p}_i^{\text{req}}$ of requested powers from its candidate parents based on the previous requests of other nodes in the network so that its total cost is minimized. For the best response, a node i has to solve the optimization problem

$$\mathbf{p}_i^{\text{opt}} = \underset{\mathbf{p}_i^{\text{req}}}{\arg\min} \left\{ C_i(a_i, a_{-i}) = \sum_{j \in \mathcal{K}_i} C_{i,j}^{\text{f}}(p_{i,j}^{\text{req}}) \right\}, \qquad (12)$$

subject to $\qquad \sum_{j \in \mathcal{K}_i} p_j^{\text{Tx}} |h_{i,j}|^2 \geq p^{\text{th}}, \qquad (13)$

where $C_{i,j}^{\mathrm{f}}$ is defined based on MC or SV rules.

In case of applying the MC rule, the power requested by a child node i from parent node j is set to the minimum power requested by other children of j except the node i, i.e., $p_{1,j}^{\mathrm{req}}$. This way, based on the definition of the MC rule in (10), the cost that has to be paid by node i is zero. If the received power from the candidate parents of node i is still not enough to reach p^{th}, then node i demands the remaining power from the parent node k, which has the highest channel gain to node i. In this case, the new requested power by node i from the parent node k is calculated as

$$p_{i,k}^{\mathrm{req,new}} = p_k^{\mathrm{Tx}} + \frac{\left(p^{\mathrm{th}} - \sum\limits_{j \in \mathcal{K}_i} p_j^{\mathrm{Tx}} |h_{i,j}|^2 \right)}{|h_{i,k}|^2} \qquad (14)$$

with $\sum\limits_{j \in \mathcal{K}_i} p_j^{\mathrm{Tx}} |h_{i,j}|^2 < p^{\mathrm{th}}$.

In case of applying SV rule, according to (11), the minimum cost from a parent node j will be assigned to the child node which has the lowest power request. This cost, by setting $i = 1$ in (11), is equal to $\frac{p_{1,j}^{\mathrm{req}}}{|\mathcal{M}_j|}$. In order to minimize its cost, a node t sorts its candidate parents in $\mathcal{K}_t$ based on the minimum requested power from a parent, over the number of children that the parent has as

$$\frac{p_{1,1}^{\mathrm{req}}}{|\mathcal{M}_1|} < \cdots < \frac{p_{1,j}^{\mathrm{req}}}{|\mathcal{M}_j|} < \cdots \frac{p_{1,|\mathcal{K}_t|}^{\mathrm{req}}}{|\mathcal{M}_{|\mathcal{K}_t|}|}, \qquad (15)$$

by assuming $\forall j \in \mathcal{K}_t, t \in \mathcal{M}_j$.

Based on (15), node t requests $p_{1,j}^{\mathrm{req}}$ from the parent node $j \in \mathcal{K}_t$ until (13) holds. If the received power from all candidate parents of node i is still less than p^{th}, then node t searches over $\mathcal{K}_t$ and demands the remaining power from a parent k which minimizes its final cost according to (12). The new demand can be calculated using (14) with $i = t$.

B. Convergence

Accoring to [10], a game $G = (\mathcal{N}, \mathcal{A}_{i \in \mathcal{N}}, \mathcal{C}_{i \in \mathcal{N}})$ is an exact potential game for a joint action space $\mathcal{A} = \prod_{i=1}^{N} \mathcal{A}_i$, if a function $\Phi : \mathcal{A} \to \mathbb{R}$ called potential function exists such that for all $i \in \mathcal{N}$, $a_i \in \mathcal{A}_i$, $a_i' \in \mathcal{A}_i$ and $a_i \neq a_i'$ we have [10]

$$C_i(a_i, a_{-i}) - C_i(a_i', a_{-i}) = \Phi(a_i, a_{-i}) - \Phi(a_i', a_{-i}). \quad (16)$$

Theorem 1: The proposed game with MC rule is an exact potential game with potential function $\Phi = \sum_{j=1}^{N+1} p_j^{Tx}$.

Proof: Node i changes its action from a_i to a_i', i.e., it requests a different set of powers from its candidate parents, if $0 \leq C_i(a_i', a_{-i}) < C_i(a_i, a_{-i})$ holds. Therefore, two cases can be distinguished, i.e., $C_i(a_i', a_{-i}) > 0$ and $C_i(a_i', a_{-i}) = 0$. First we consider the case when $C_i(a_i', a_{-i}) > 0$. Based on the best response strategy explained in Section V-A, a node i pays to at most one of its selected parents in $\mathcal{K}_i$. For instance let a node i changes its action such that it pays to the parent node k instead of the parent node j. Since node i requests the highest power from j and k, the difference of the cost for node i in this transition from parent node j to parent node k

can be calculated according to (10) as

$$C_i(a_i', a_{-i}) - C_i(a_i, a_{-i}) = \\ \left(p_{i,k}^{\mathrm{req}} - p_k^{\mathrm{Tx}} \right) - \left(p_{i,j}^{\mathrm{req}} - p_{|\mathcal{M}_j|-1,j}^{\mathrm{req}} \right). \quad (17)$$

Since node i leaves set $\mathcal{M}_j$ and joins set $\mathcal{M}_k$, the transmit power of node j decreases from $p_{i,j}^{\mathrm{req}}$ to $p_{|\mathcal{M}_j|-1,j}^{\mathrm{req}}$ and the transmit power of node k increases from p_k^{Tx} to $p_{i,k}^{\mathrm{req}}$. Hence, the difference in the total transmit power of the network is given by

$$\Phi(a_i', a_{-i}) - \Phi(a_i, a_{-i}) = \\ \left(p_{i,k}^{\mathrm{req}} + p_{|\mathcal{M}_j|-1,j}^{\mathrm{req}} \right) - \left(p_k^{\mathrm{Tx}} + p_{i,j}^{\mathrm{req}} \right). \quad (18)$$

By inserting (17) and (18) it can be seen that (16) is fulfilled for $C_i(a_i', a_{-i}) > 0$.

For the case when $C_i(a_i', a_{-i}) = 0$, the difference in the cost of node i is calculated as

$$C_i(a_i', a_{-i}) - C_i(a_i, a_{-i}) = 0 - \left(p_{i,j}^{\mathrm{req}} - p_{|\mathcal{M}_j|-1,j}^{\mathrm{req}} \right). (19)$$

The transmit power of the parent node k will not change when node i leaves node j and joins node k. Therefore, for the difference in potential function by this transition we have

$$\Phi(a_i', a_{-i}) - \Phi(a_i, a_{-i}) = \\ \left(p_k^{\mathrm{Tx}} + p_{|\mathcal{M}_j|-1,j}^{\mathrm{req}} \right) - \left(p_k^{\mathrm{Tx}} + p_{i,j}^{\mathrm{req}} \right). \quad (20)$$

Eq. (16) is also fulfilled for $C_i(a_i', a_{-i}) = 0$, by inserting (19) and (20). Therefore, game G with MC rule is an exact potential game. $\blacksquare$

Theorem 2: The proposed game with SV rule is an exact potential game with potential function $\Phi = \sum_{j=1}^{N+1} p_j^{Tx} - \sum_{k=1, k \neq i}^{N} C_k$.

Proof: The SV cost sharing rule is budget balanced [13], i.e, $\sum_{i \in \mathcal{M}_j} C_{i,j}^{\mathrm{SV}} = p_j^{Tx}$ which implies

$$\sum_{j=1}^{N+1} \sum_{i \in \mathcal{M}_j} C_{i,j}^{\mathrm{SV}} = \sum_{j=1}^{N+1} p_j^{Tx} \qquad (21)$$

in which $N + 1$ represents the number of all nodes including the source. The left side of (21) is the total cost received by the parent nodes in the network. The total cost received by the parent nodes is equal to the to total the cost paid by the child nodes of the network. Therefor, (21) can be rewritten as

$$\sum_{i=1}^{N} \sum_{j \in \mathcal{K}_i} C_{i,j}^{\mathrm{SV}} = \sum_{j=1}^{N+1} p_j^{Tx}. \qquad (22)$$

Eq. (22) can be written as $\sum_{i=1}^{N} C_i = \sum_{j=1}^{N+1} p_j^{Tx}$, based on (8). Hence, the cost of node i can be obtained by

$$C_i = \sum_{j=1}^{N+1} p_j^{Tx} - \sum_{k=1, k \neq i}^{N} C_k. \qquad (23)$$

Eq. (23) shows that the potential function of the game with the SV rule can be expressed as $\Phi = \sum_{j=1}^{N+1} p_j^{Tx} - \sum_{k=1, k \neq i}^{N} C_k$. Based on (23), when a node updates its action, the change that

occurs in the cost of the node, effects the total transmit power of the network and cost of other nodes in the network, jointly. ∎

Based on theorems 1 and 2, the proposed game with either the MC or the SV rule is an exact potential game for which the convergence of the game to the Nash Equilibrium point is guaranteed [14].

VI. IMPLEMENTATION ISSUES

To implement this distributed algorithm, we use an initial message at the nodes called Hello message. A Hello message contains the ID of the node and its power rank. By using rank, we can prevent occurring loops in the tree. In different iterations, a node joins new parents by sending JOIN messages and leaves its old parents by sending LEAVE messages.

In the case of using the MC rule, a Hello message of node j contains the highest and second highest of the requested powers in $\mathcal{M}_j$. For the SV rule, the Hello message at node j contains all requested powers for all children in $\mathcal{M}_j$. This way, after receiving a Hello message, a node can calculate its new cost based on (10) and (11) for the case of using the MC rule and the SV rule, respectively, and adjust its action in choosing new parents. The explained procedure is shown by the following pseudo-code:

1. **Start with an initial tree**
2. **For every node until convergence, do**
3. Sort the parents as explained in Section V-A
4. Choose the parents to reach p^{th}
5. **If** receiving power is less than p^{th}, **then**
6. Update requested power based on (14)

VII. SIMULATION RESULTS

Throughout the simulations, we consider a square region of 1000 m × 1000 m in which the nodes are randomly deployed. The number of nodes in this network varies between 20 and 80. Let λ and d_0 be the signal wavelength and the reference distance, respectively. Then, by considering α as the path loss exponent, the power p^{Rx} of the received signal at an antenna with distance d from the transmitter with power p^{Tx} is obtained by

$$\frac{p^{\text{Rx}}}{p^{\text{Tx}}} = \left(\frac{\lambda}{4\pi d_0}\right)^2 \left(\frac{d_0}{d}\right)^{\alpha}. \tag{24}$$

For the simulations, we set $\lambda = 0.125$ m, $d_0 = 10$ m and the path loss exponent is considered to be $\alpha = 3$. The maximum transmit power at parent nodes and the threshold power at child nodes are set to $p^{\text{max}} = -20$ dBm and $p^{\text{th}} = -80$ dBm, respectively. The results are based on Monte Carlo simulations.

We compare our results with the proposed algorithms in [3], [4] and [9] where the authors considered related problems. All mentioned papers are restricted to the case that the nodes just can have one parent, i.e., they do not include MRC. The algorithm provided in [3] is centralized in which all the network information is available at a central entity. The algorithm, called broadcast incremental power (BIP), constructs the tree on a node by node basis. In each time slot, it selects
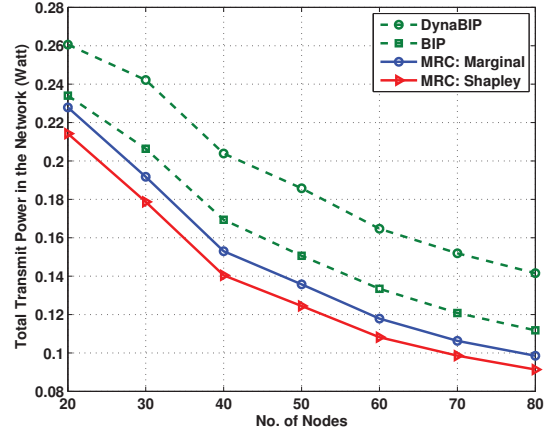

Fig. 2: Total transmit power in the network with the proposed algorithm is less than BIP and DynaBIP algorithms for different numbers of nodes.

the node which requires minimum transmit power from the available nodes in the tree. The second reference scheme [4] is a decentralized implementation of [3] which is called dynamic BIP (DynaBIP). In DynaBIP, the link costs are determined based on the signal strength which is measured distributedly at each node. DynaBIP performs worse than the BIP because the nodes do not have sufficient information to make the best decision. In [9] a game-based approach is proposed for finding the minimum power broadcast tree and the authors use cost sharing rules in their algorithm. For the simulation of our new proposed algorithm for both the MC and the SV cases, we initialize the broadcast tree using the power rank attribute, explained in Section III such that a node selects an initial parent which results in minimum power rank.

Fig. 2 shows the performance of the SV and the MC rules compared to the BIP and DynaBIP algorithms for different numbers of nodes. As it can be seen, since our proposed solution exploits MRC for both the MC and the SV cases, it outperforms both benchmark algorithms. For instance, when there are 50 nodes in the network, by applying the SV rule, the total transmit power in the network is 35% less than for the DynaBIP algorithm. This improvement is due to using MRC in our algorithms which allows the receiving nodes to combine different copies of the messages coming from different parents. Moreover, by increasing the number of nodes in the network, the total transmit power in the network decreases. This is because when the number of nodes increases, the distance between parents and child nodes on average decreases and, consequently, the transmit powers decrease. In fact, increasing the number of nodes in the network may increase the number of parents, and thus, the number of transmissions may increase, however, the impact of the reduction of transmit powers due to the reduction of the coverage area of each transmitting node is more significant.

The numbers of iterations required for convergence of the algorithm is shown in Fig. 3. As it can be seen, the proposed
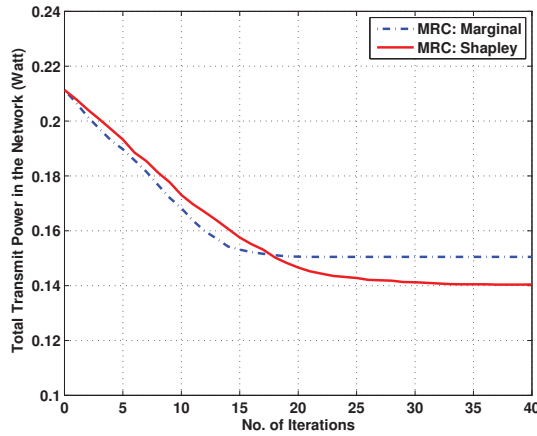
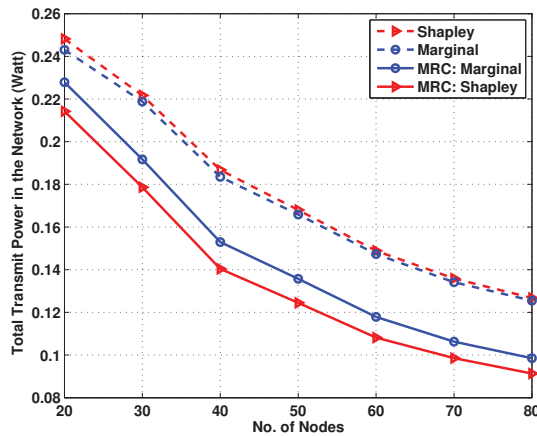Fig. 3: Number of iterations required for 40 nodes in the network.

Fig. 4: Total transmit power in the network with and without using MRC technique for different numbers of nodes.

algorithm with SV rule needs a higher number of iterations to converge than MC rule. This is due to the cost assigned to the children in SV rule which is always greater than zero and the nodes trying to minimize their cost by updating their actions. In case of applying MC rule, since the cost assigned to some nodes are zero, the nodes have no incentive to update their actions and the game converges faster.

Fig. 4 compares the result of our proposed algorithm with and without applying MRC in the network [9]. As it can be seen, when we exploit MRC, the total transmit power in the network decreases significantly. Comparing the MC and the SV rules, the reduction of the transmit power is higher when the SV rule is applied. As explained earlier, this is due to the cost assigned to children by applying SV rule which is always positive and the nodes try to minimize their cost in more iterations which results in a higher gain. When there are 50 nodes in the network, exploiting MRC with the MC rule results in 20% of improvement while the improvement in case of using the SV rule is about 25%.

VIII. CONCLUSION

In this paper, minimizing the total transmit energy in multi-hop broadcast networks is studied. We proposed a decentralized algorithm based on game theory, which employs MRC technique at receiving nodes, to combine different copies of the received signal constructively. Cost sharing rules are applied in order to minimize the total energy in the network. Two cost sharing rules, marginal contribution and Shapley value, are studied. Simulation results show that the proposed game theoretic algorithm significantly decreases the total energy consumption in the network.

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