SCARA robot (inverse kinematics and singular configurations).

① SCARA inverse kinematics: Here the idea is to express q_1, q_2, q_3 and q_4 according to f, l_1, l_2, l_3, l_4, P_x and P_y and P_z (P_x and P_y and P_z are the coordinates of the end effector). ~~the end~~ P_z

1.1 q_4?

we know from the FKM that $\alpha = q_1 + q_2 + q_4$

$$\Rightarrow \boxed{q_4 = \alpha - q_1 - q_2}$$

1.2 q_3? we know from the FKM that

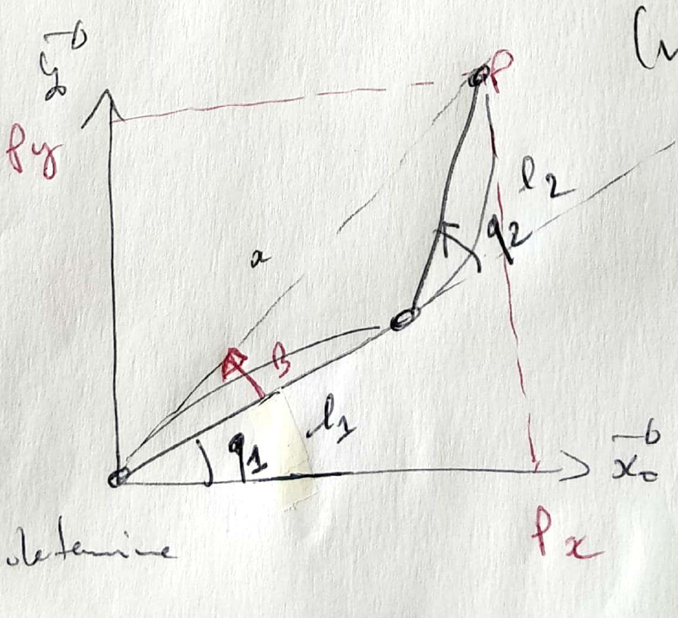
$$P_z = l_0 - l_3 - l_4 + q_3$$

$$\Rightarrow \boxed{q_3 = P_z - l_0 + l_3 + l_4}$$

1.3 q_1, q_2?

top view of the robot

(here $q_2 > 0$)



Here the geometric method can be used to determine q_1 and q_2.

Let start with q_2:

$$a^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\pi - q_2)$$

$$\cos q_2 = \frac{a^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

therefore $$q_2 = a\cos\left(\frac{a^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

or $$q_2 = -a\cos\left(\frac{a^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

with $$\boxed{a^2 = P_x^2 + P_y^2}$$

Let calculate q_1:

Here we have

$$l_2^2 = a^2 + l_1^2 - 2 a l_1 \cos B.$$

$$B = a\cos\left(\frac{a^2 + l_1^2 - l_2^2}{2 a l_1}\right)$$

$\cos(B)$ is positive $B \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

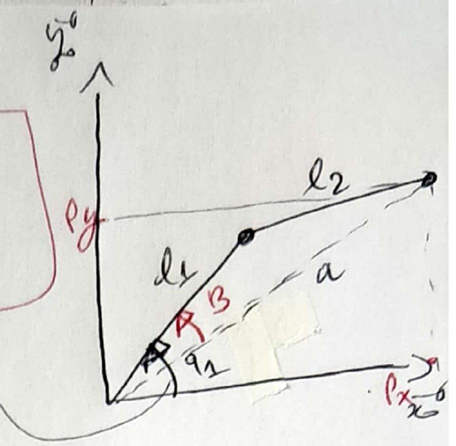
here: if $q_2 > 0$ (arm right).

$$tg(q_1 + B) = \frac{P_y}{P_x}$$

$$\Rightarrow \boxed{q_1 = atan2(P_y, P_x) - B}$$

if $q_2 < 0$ (arm left).

$$\boxed{q_1 = \operatorname{atan2}(P_y . P_x) + B}$$



B. SCARA Robot singular configuration:

The FKM of the SCARA model is given by the following model:

$$\begin{pmatrix} x \\ y \\ z \\ \alpha \end{pmatrix} = \begin{pmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \\ d_0 - l_3 - l_4 + q_3 \\ q_1 + q_2 + q_4 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

To determine the singular configuration, we need first to derive the Jacobian matrix. Then all the configurations that verify $\det(\text{jacobian matrix}) = 0$ ~~are to~~ are considered as singular configurations.

B.1 Jacobian matrix.

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} & \frac{\partial f_1}{\partial q_4} \\ \vdots & & & \vdots \\ & & & \frac{\partial f_4}{\partial q_4} \\ \frac{\partial f_4}{\partial q_1} & \frac{\partial f_4}{\partial q_2} & \cdots & \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\det(J) = \det \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

$$\det|J| = (-l_1 s_1 - l_2 s_{12})(l_2 c_{12}) - (+l_1 c_1 + l_2 c_{12})(-l_2 s_{12})$$

$$= -l_1 l_2 q_1 \cdot c_{12} - \cancel{l_2^2 \cdot s_{12} c_{12}} + l_1 l_2 c_1 s_{12} + \cancel{l_2^2 c_{12} s_{12}}$$

$$= l_1 l_2 (s_{12} c_1 - s_1 c_{12})$$

$$\boxed{\det(J) = l_1 l_2 s q_2}$$

$$\det(J) = 0 \Rightarrow s q_2 = 0 \Rightarrow \boxed{q_2 = 0}$$

$q_2 = 0 \Rightarrow$ singular configuration.

<u>trigonometric formulas</u> $s_{12} c_1 - s_1 c_{12} = s(q_1 + q_2 - q_1) = s q_2 = s_2$