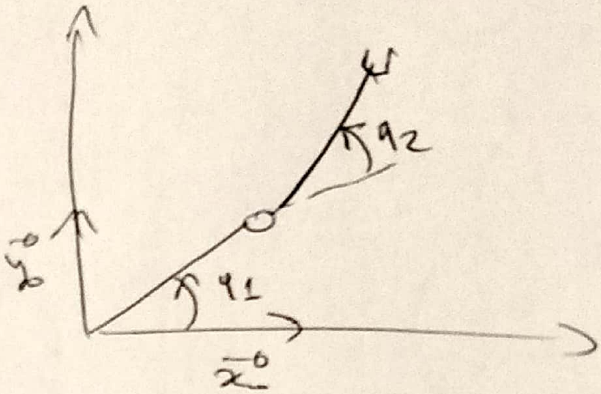
4.3.1 Jacobian matrix.

$$J^{a}(q) = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \vdots & \ddots & & \vdots \\ \frac{\partial f_d}{\partial q_1} & \cdots & & \frac{\partial f_d}{\partial q_n} \end{bmatrix}$$

Example:



FKM

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{pmatrix}.$$

$$\dot{x} = \begin{bmatrix} -\dot{q}_1 (l_1 s_1 + l_2 s_{12}) & \dot{q}_2 l_2 s_{12} \\ \dot{q}_1 (l_1 c_1 + l_2 c_{12}) & \dot{q}_2 l_2 c_{12} \end{bmatrix}$$

$$\dot{x} = \boxed{\begin{bmatrix} -l_1 s_1 - l_2 s_{12} & l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}.$$

Jacobian matrix

4.3.2 Kinematic Jacobian

The movement of the end effector can be expressed or defined by its instantaneous velocity and rotation velocity using a kinematic tensor.

$$ {}^{0}V_{n/0}(P) = {}^{0}J^{c}(q)\,\dot{q} $$

${}^{0}V_{n/0}(P)$: kinematic tensor

4.4 : Achievable speeds and manipulability

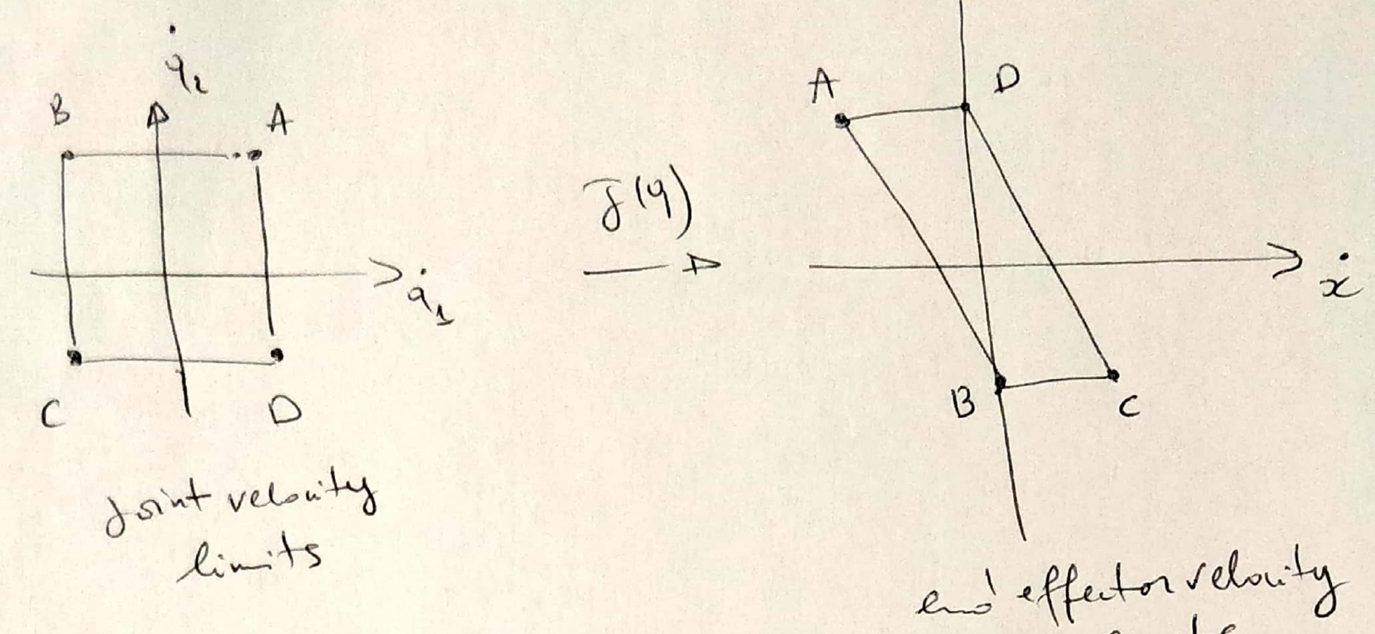
The Jacobian matrix allows to characterize the achievable velocities of the robot given a configuration. In fact, if we know the maximal joint velocities, we have:

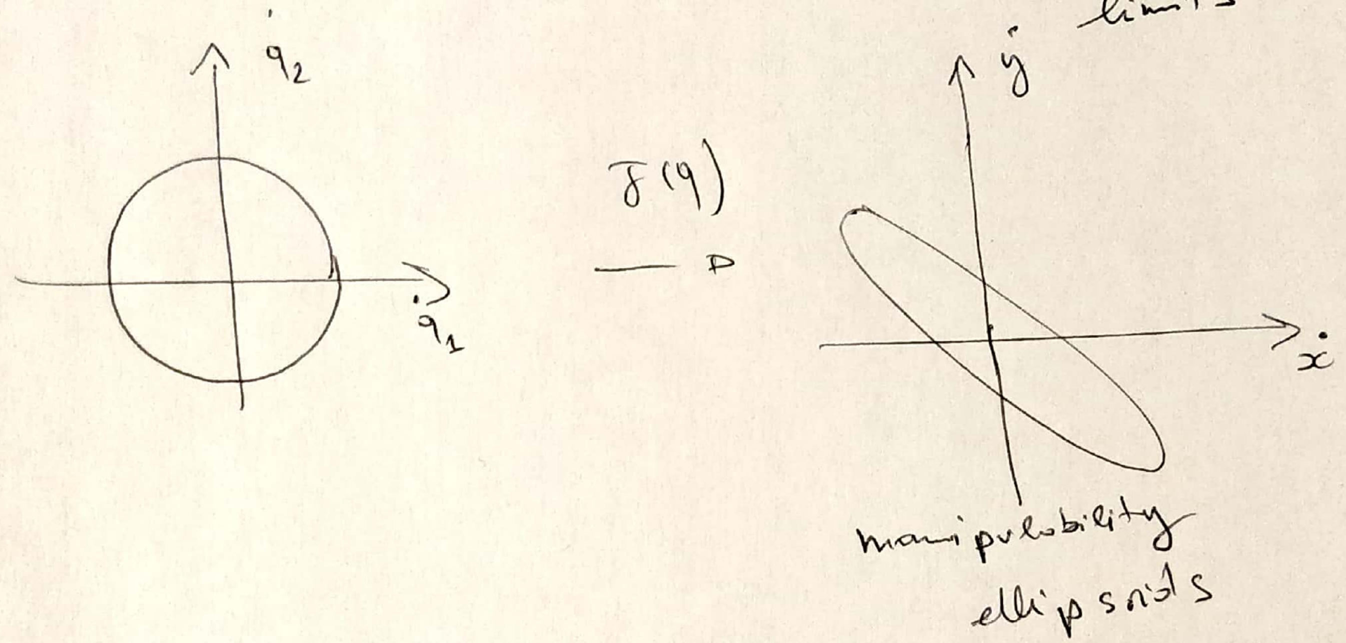
$$ -\dot{q}_{max} < \dot{q} < \dot{q}_{max} $$

Therefore

$$ \min J(q)\,\dot{q} \leq \dot{x} \leq \max J(q)\,\dot{q} $$

example:





The volume of the ellipsoid (surface) is given by

$$\boxed{m(q) = \sqrt{\det\left(J(q)\, J(q)^T\right)}}$$

This metric is called the velocity manipulability of the robot. It allows to mesure the capacity of the robot to generate a velocity.

4.5 Singular Configurations:

According to the previous chapter, we remind that the calculation of the IKM could lead to infinity of solutions for two cases:

1 _ The robot is redundant according to the task.

2 _ The robot configuration is a specific configuration that generates undeterminate situation or a local redundancy at the expence of one or several DoF. In this case, the configuration is called a $\boxed{\text{singular configuration.}}$

- From mathematical point of view, the analysis of the singularities is based on the calculation of the IKM or the resolution of a set of equations with n equation and n variables. For simplicity, we can use the VKM instead of IKM for this purpose.
- The VKM is not inversible if $\boxed{\det(J(q)) = 0}$. From this equation, we can extract all the singular configurations.

example: planar Robot

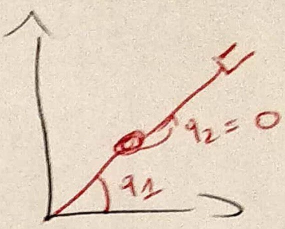
$$J(q) = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix}$$

$$\boxed{\det J(q) = l_1 l_2 s_2}$$ after simplification.

$\det J(q) = 0$ if $S_2 = 0 \Rightarrow q_2 = 0$

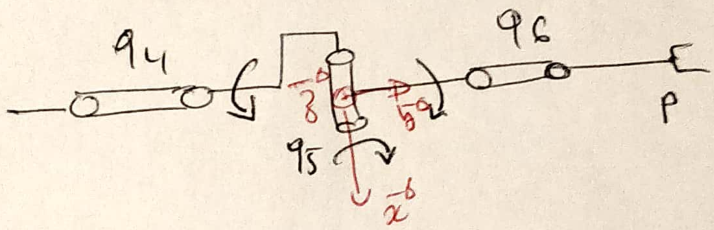
$q_2 = 0$: singular configuration (~~type~~).

type 1



example 2: Singular configuration of an anthropomorphic robot (6DoF) with a spherical wrist. (3 revolute joints).

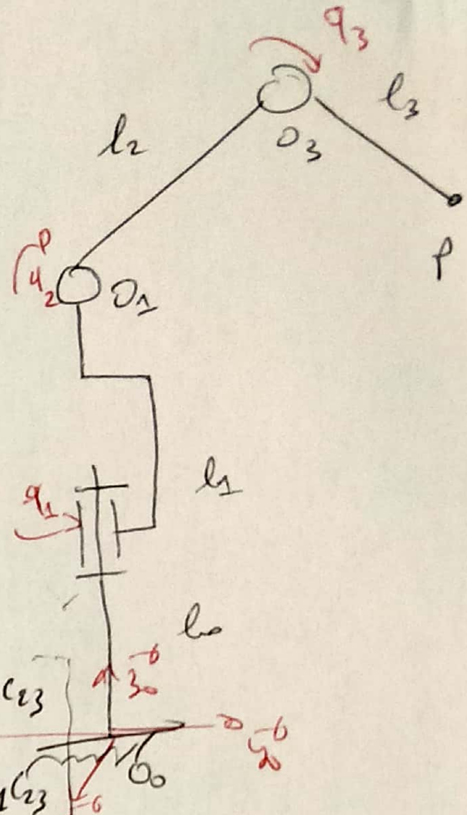
⇒ Singularity of the wrist:



In this configuration, it seems clearly that q_4 and q_6 are redundant to generate a rotation about $\vec{y}$. However the rotation about $\vec{z}$ cannot be done. We can find this result by calculating the jacobian of the wrist.

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{matrix with a rank of } 2 \Rightarrow \det(J) = 0} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}$$

⇒ Singularity of the manipulator:



FKM

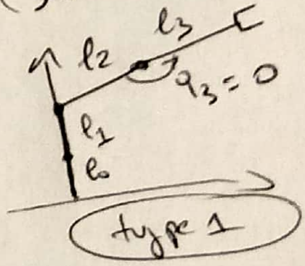
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s_1(l_2 s_2 + l_3 s_{23}) \\ -c_1(l_2 s_2 + l_3 s_{23}) \\ l_1 + l_2 c_2 + l_3 c_{23} \end{pmatrix}$$

$$J = \begin{pmatrix} c_{q_1}(l_2 c_2 + l_3 c_{23}) & s_1(l_2 c_2 + l_3 c_{23}) & l_3 s_1 c_{23} \\ s_1(l_2 s_2 + l_3 s_{23}) & -c_1(l_2 c_2 + l_3 c_{23}) & -l_3 c_1 c_{23} \\ 0 & -(l_2 c_2 + l_3 s_{23}) & -l_3 s_{23} \end{pmatrix}$$

$$\Rightarrow \det(J(q)) = l_2 l_3 s_3 (l_2 s_{q_2} + l_3 s_{23}).$$

- Singular configurations:

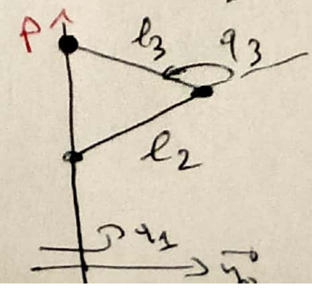
type 1: $q_3 = 0$ or π ⇒ rank$(J) = 2$ (same singularity as the planar robot).



type 2: $l_2 c_2 + l_3 c_{23} = 0$

⇒ rank$(J) = 2$, lost of 1 Dof

In this case P belongs to $\vec{z}_0$ axis and q_1 cannot modify the position of point P.



Conclusions:

- → The VKM gives the robot velocity performances, singular configuration and the robot postures.

- → The IKM allows to calculate the joint velocities references according to the operational velocity references (generates standard movements).

- → The VKM allows also to calculates the joint efforts corresponding to a static effort applied on the robot and the compliance matrices of the robot. This model is mandatory for effort control and vision control of the robot.

Some aspects of robot that need to be investigated:

- Robot dynamics
- trajectory generation and planification
- Linear control of robot and non-linear control.
- force control.
- Robot programming
- parallel robots..
- Robot identification and calibration.
- Robot vision serving.