Robotics Tutorial 1

Forward kinematics of SCARA robot

Let consider the robot given in Fig 1. Composed of four axes, the robot has a RRPR structure (known as SCARA structure, Selective Compliant Assembly Robot Arm). According to the posture (configuration) illustrated in Fig. 1, all the joint variables are null (zero).

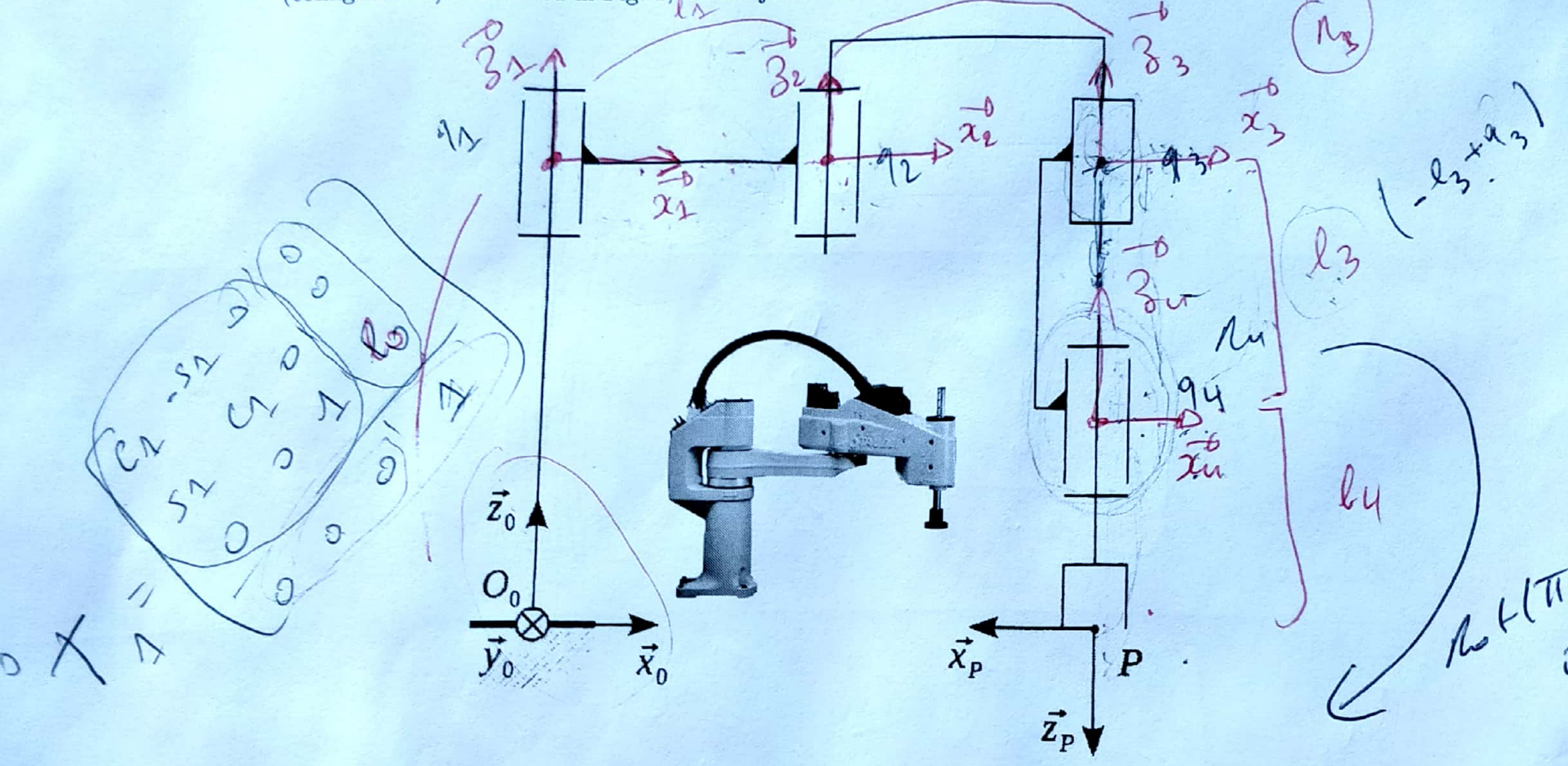
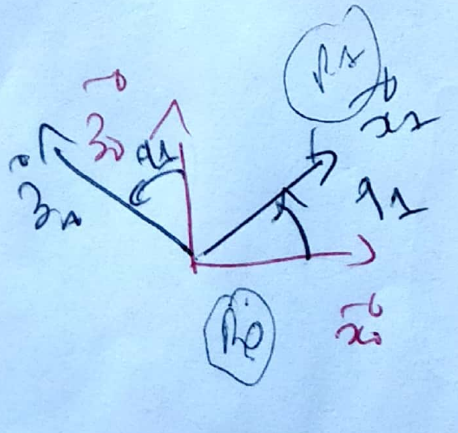
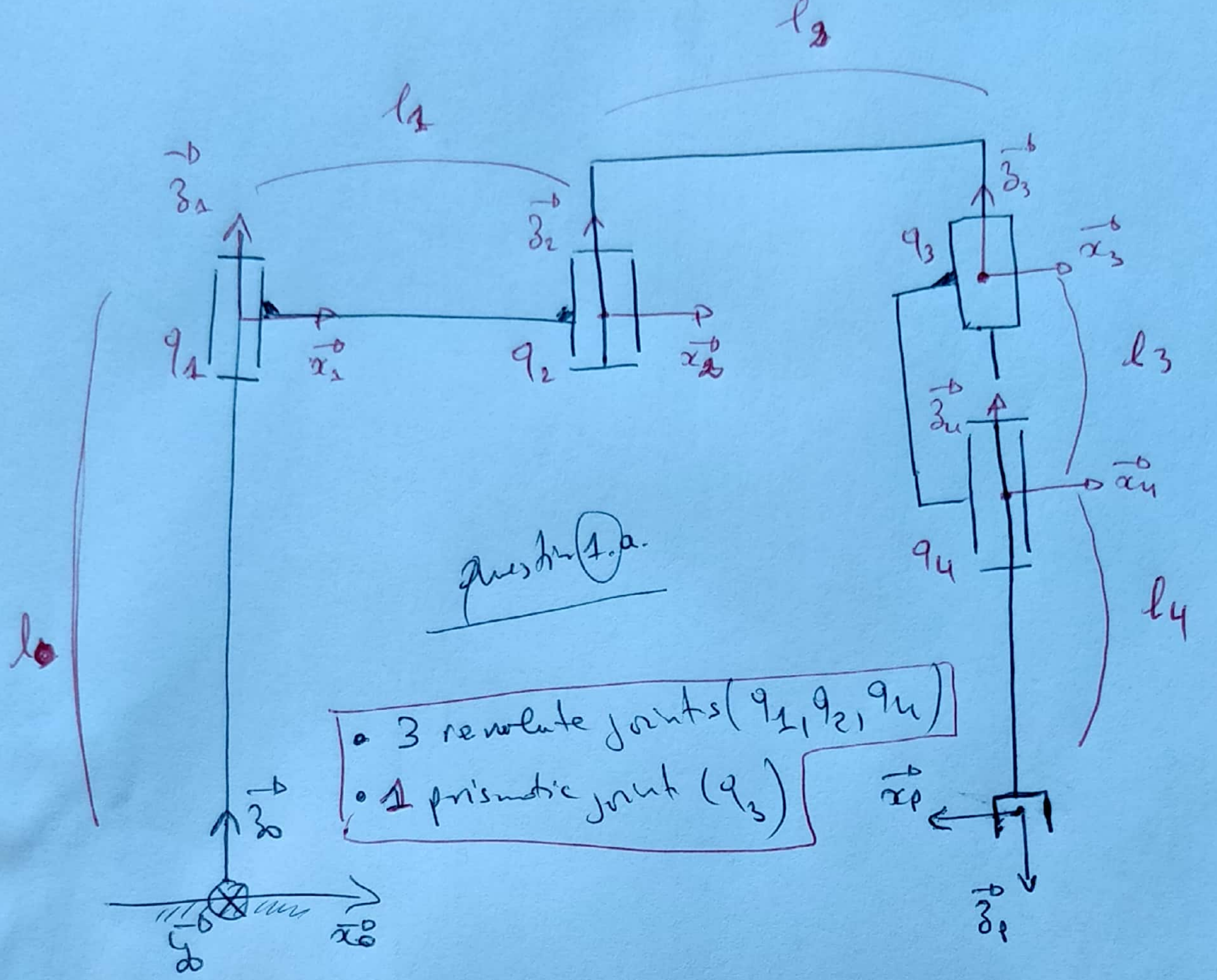


FIGURE 1 – SCARA robot

1.a Attach frames R_1, R_2, R_3 and R_4 to each axis.
1.b Compute ${}^0T_1$, ${}^1T_2$, ${}^2T_3$, ${}^3T_4$, ${}^4T_P$ and ${}^0T_P$.
1.c Express the forward kinematic model of the robot.
1.d Check the validity of the model by using some trivial postures.
1.e Express the rotation of the robot tool using Euler (yaw, pitch, roll) angles.
1.f Plot the projection of the robot working domain (area) relative to $O_0\vec{x_0}\vec{y_0}$ plan. Robot mechanical stops are as follows : $q_1 \in [-\frac{2\pi}{3}, \frac{2\pi}{3}]$; $q_2 \in [-\frac{2\pi}{3}, \frac{2\pi}{3}]$.



SCARA robot



→ This robot has: 4 axis, 4 DOF, 3 rotations 1 translation, serial robot.

1b Homogeneous matrices. (

$$
{}^0T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ +S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_1 \\ +S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^3T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & \cancel{-l_3+q_3} \\ +S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & -l_3+q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

NB important! here I consider frame l_3 fix (the prismatic joint is translating according to this frame).

$$\underbrace{\left({}^0T_1\,{}^1T_2\right)}_{{}^0T_2}\,{}^2T_3 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 \\ +S_{12} & C_{12} & 0 & +l_1 S_1 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ +S_{12} & C_{12} & 0 & +l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\left({}^0T_1\,{}^1T_2\,{}^2T_3\right)}_{{}^0T_3}\,{}^3T_4 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ +S_{12} & C_{12} & 0 & -l_1 S_1 - l_2 S_{12} \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ +S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & -l_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{12} C_4 - S_{12} S_4 & -(C_{12} S_4 + S_{12} C_4) & 0 & l_1 C_1 + l_2 C_{12} \\ +S_{12} C_4 + C_{12} S_4 & -S_{12} S_4 + C_{12} C_4 & 0 & +l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & l_0 - l_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = \begin{bmatrix} C_{124} & -S_{124} & 0 & l_1 C_1 + l_2 C_{12} \\ S_{124} & C_{124} & 0 & l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & l_0 - l_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_{124} = \cos(q_1 + q_2 + q_3)$

$$ {}^{4}T_p = \begin{bmatrix} C\pi & 0 & S\pi & 0 \\ 0 & 1 & 0 & 0 \\ -S\pi & 0 & C\pi & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$ (fix rotation of π around $\vec{y}^{\,0}$) ($R_4 \to R_p$)

$$ {}^{4}T_p = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_p = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 \, {}^{3}T_4 \, {}^{4}T_p $$

$$ {}^{0}T_1 \, {}^{1}T_2 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ +S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & l_1 \\ +S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} C_1C_2 - S_1S_2 & -(C_1S_2 + S_1C_2) & 0 & l_1 C_1 \\ +S_1C_2 + C_1S_2 & -S_1S_2 + C_1C_2 & 0 & +l_1 S_1 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 \\ +S_{12} & C_{12} & 0 & +l_1 S_1 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Remark: $C_1C_2 - S_1S_2 = C_{12}$, $C_1S_2 + S_1C_2 = S_{12}$) trigonometric formulas.

$${}^0T_p = {}^0T_4\,{}^4T_p = \begin{bmatrix} C_{124} & -S_{124} & 0 & l_1C_1+l_2C_{12} \\ S_{124} & C_{124} & 0 & l_1S_1+l_2S_{12} \\ 0 & 0 & 1 & l_0-l_3+q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_p = \begin{bmatrix} -C_{124} & -S_{124} & 0 & l_1C_1+l_2C_{12} \\ -S_{124} & C_{124} & 0 & l_1S_1+l_2S_{12} \\ 0 & 0 & -1 & l_0-l_3-l_4+q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

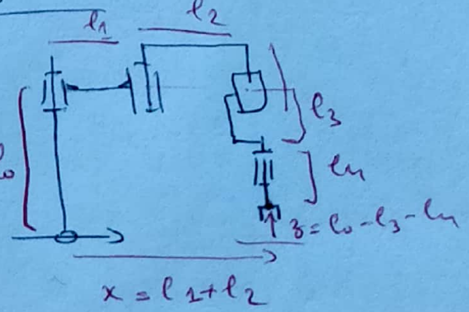
1.c Forward kinematics of the robot: (FKM).

$$X = {}^0T_p \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} l_1C_1+l_2C_{12} \\ l_1S_1+l_2S_{12} \\ l_0-l_3-l_4+q_3 \end{pmatrix}$$

1.d Validity of the FKM for specific configurations:

Configuration ① $q_1=q_2=q_3=q_4=0$.

$$X = \begin{pmatrix} l_1+l_2 \\ 0 \\ l_0-l_3-l_4 \end{pmatrix}$$



1.e: The structure of the robot is simple, therefore it is easy to express the orientation of the wrist (you can also use identification methods)

* ~~All the rotation~~ All the rotations are taking place around z axis, No other rotation, Thus $\boxed{\alpha = q_1+q_2+q_4}$ $\boxed{\beta=0}$ $\boxed{\gamma=0}$

1.e.a

with the orientation, the new expression of FKM can be written as:

$$\boxed{\begin{pmatrix} x \\ y \\ z \\ \alpha \end{pmatrix} = \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 S_1 + l_2 S_{12} \\ l_0 - l_3 - l_4 + q_3 \\ q_1 + q_2 + q_4 \end{pmatrix}}$$

FKM of SCARA robot.

1.f Robot working space: (projection relative to plan $O_0 \vec{x}_0 \vec{y}_0$.

top view of the robot)