Course: Digital Control

A report
of
TP2: Modeling, Simulation and Numerical Control of a Positioning Axis Feedback System

Supervisor: Benjamin MAUZE

Done by:

UMUHOZA Jean d'Amour

YANGOUA YANGOUA Franklin Rostan

Département Automatique et Robotique

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1.introduction

Rapidly evolving production processes have driven the need for motion control systems that provide higher accuracy, speed, resolution and repeatability. In response, motion control manufacturers have unleashed a number of mechanisms, position and force feedback technologies, and electromechanical actuation technologies.

The dynamic behavior of a servomechanism must be studied as a whole without distinction between its parts, in order to assess correctly the overall performance of the system. For this reason, in many cases of practical interest, it may be useful to analyze the dynamics of a mechatronic device using a mathematical model and a computer simulation software, in order to verify the consequences arising from the modification of a particular parameter of the system. That is the reason we have conducted this lab experiment [1].

The purpose of this lab was to model, simulate and perform the numerical control of an axis of feedback positioning representing a gantry crane (used in ports) or a bridge rolling (used in factories).
This lab focused on the study of the engine and the entire chain of mechanical transmission in order to be able to model the complete dynamic behavior of the mock-up of practical work First, a theoretical study of the system will be carried out and this will be completed by numerical modeling using Matlab Simulink. In a second time, measurements were made on the experimental bench in order to extract the parameters of the model. Finally, an experimental implementation of different digital controls was performed and the experimental results will be compared to the theoretical results.

2. Experimental setup and material used

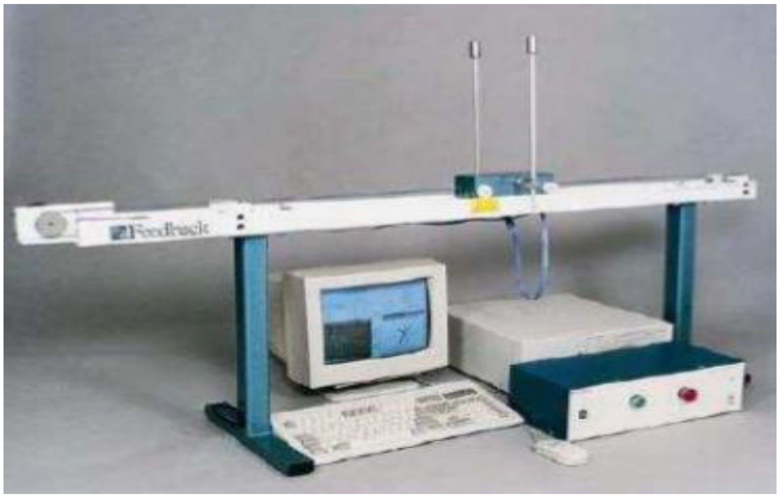

Figure 1: The experimental step Up

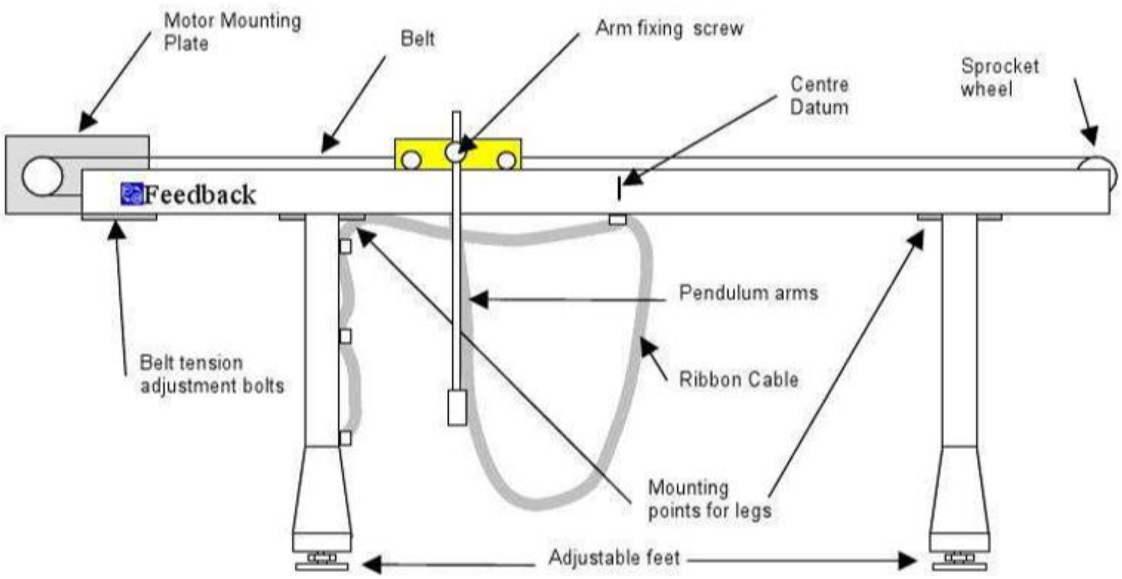

Figure 2:Description of the feedback model

The operative part consists of the following elements:

1. a DC motor of the Crouzet brand 82850002 with the following specifications

Table 1: The DC motor specifications

Parameters	values
Current rating	2.15 A
DC motor type	Brushed
Maximum output torque	100 mNm
Output speed	3200 rpm
Shaft diameter	6mm
Core construction	Iron core
Power rating	33.5 w
Supply voltage	24 v dc
length	112.6 mm

2.A transmission and conversion system of the pinion / belt type notched driving a carriage in translation.
3.An incremental encoder on the motor and another on the crane's pendulum;
4.Various all-or-nothing sensors (limit switches);
5.A pendulum with adjustable masses allowing to simulate the cables of a crane to gantry (not used in this lab).

Part A

A.1 Modeling of the motor

A DC electric motor is an electromechanical energy conversion system that can be modeled by the block diagram of *figure 3*. The quantities $U_m(s), I(s), \Omega_m(s), C_m(s), and C_c(s)$ are respectively the Laplace transform of the motor supply voltage $U_m(t)$, its current $i(t)$, its induced inductance $e(t)$, its rotation speed $\omega_m(t)$, its motor torque $C_m(t)$ and finally the resistive torque applied by the load $C_c(t)$.

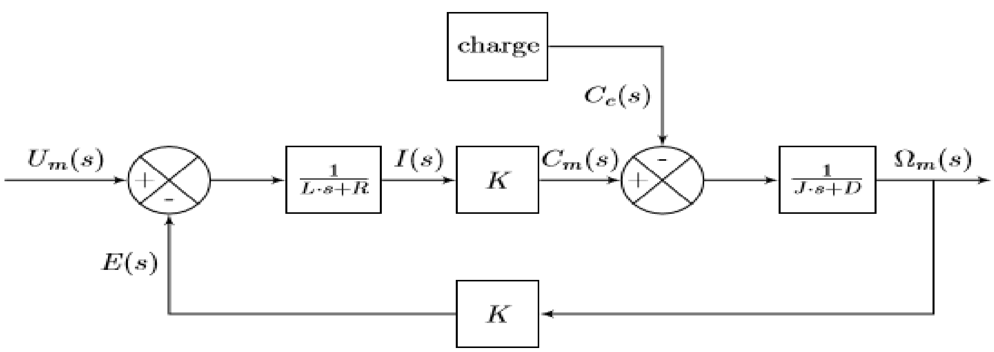

Figure 3:Block diagram of DC motor

The motor parameters are supplied by the manufacturer and are shown in the *table 2*. For the single motor (without transmission system), and we had $J = J_m$ and $D = D_m$.This table shows the limits of the operation for the motor which includes the electrical characteristics and mechanical characteristics.

Table 2: Crouzet engine manufacturer data

Limits of operations	$U_{\max}$ $24V$	$i_{\max}$ $9.6A$	$\omega_{\max}$ $4050\, {}^{tr}\!/_{\min}$
Electrical characteristics	K $52.10^{-3}\, {}^{N.m}\!/_{A}$	R 2.5Ω	L $2.5mH$

Mechanical characteristics	$j_{\max}$ $14.10^{-6}\,kg.m^2$	$D_{\max}$ $1{,}07.10^{-3}\ {}^{Nm}\!/_{rad.s^{-1}}$	-

A.1.1 Determination of sampling frequency

The first thing was to estimate the dynamics involved in this engine (time constants) and then we chose a sampling period h to numerically simulate its function. The dynamic behavior involved in this engine can be calculated by using the following two equations from the following feedback block diagram.

$$\frac{1}{Ls+R} \text{ and } \frac{1}{js+D} \tag{1}$$

The aim is to find the equation with this form $\frac{1}{1+\tau s}$ where τ represent the time constant of the system. From the above two equations (1), they can help to find the values of time constant as below:

$\frac{1}{1+\frac{L}{R}s} = \frac{1}{1+\tau_1 s}$, then from this we have found that

$$\tau_1 = \frac{2.5\times10^{-3}}{2.5} = 10^{-3}\sec = 1ms$$

$\frac{1}{1+\frac{j}{D}s} = \frac{1}{1+\tau_2 s}$, and also from this we have found that the value of

$$\tau_2 = \frac{14\times10^{-6}}{1.07\times10^{-3}} = 13.084\times10^{-3}\sec = 13.083ms$$

By having the two time constant we Have found the sampling frequency for each dynamics

$h_1 = \frac{1}{\tau_1} = \frac{1}{0.001} = 1000Hz$ and

$$h_2 = \frac{1}{\tau_2} = \frac{1}{13.083\times10^{-3}} = 76.435Hz$$

And according to the Shannon Nyquist criteria in order to avoid the aliasing phenomena, $f_s \geq 2 f_{\max}$ and for us to avoid this aliasing phenomena we chose the smallest sampling frequency which is $h_1 = 1000Hz$

A.1.2 Modeling of the mechanical transmission

The mechanical transmission of this model is a system of transformation of the rotation / translation movement using a pinion and a toothed belt. This makes it possible to adapt the speed of the load $V_c(t) \xrightarrow{L} V_c(s)$ to the speed of the motor $\omega_m(t)$ by the choice of a transmission ratio T which depends on the dimensions of the gears and the gear wheels according to the following given equation (2).

$$T = \frac{V_c(t)}{\omega_m(t)} \Rightarrow V_c(t) = T.\omega_m(t) \qquad (2)$$

The rotation energy can be calculated by the following equation with the aim to find the expression of the transmission ration in relation to diameter *d*, the number of teeth *N* and inertia of the pinion J_p.

$E_{rot} = \frac{1}{2} j\omega_m{}^2$ = the rotation energy

$E_{trans} = \frac{1}{2} mV_C{}^2$=the translation energy

And $E_{trans} = E_{rot}$

$$\frac{1}{2} J_p \omega_m{}^2 = \frac{1}{2} mV_C^2$$

$$\frac{J_p}{m} = \left(\frac{V_C}{\omega_m}\right)^2 = T^2$$

$$T^2 = \frac{J_p}{\rho V} \qquad (3)$$

And the volume of the gear can be calculated as follow:

$$V = \pi \left(\frac{r}{N}\right)^2 \Delta x = \pi \left(\frac{d}{2N}\right) \Delta x$$

Where Δx is the thickness of the gear is the gear used in this experiment has Δx of 1cm and N is the number of teeth and the used gear has 25 teeth.

Now replacing the value of V in equation (3), transmission ratio become:

$$T^2 = \frac{J_p}{\pi\rho\left(\frac{d}{2N}\right)^2 \Delta x} \Leftrightarrow T = \sqrt{\frac{J_p}{0.01\pi\rho\left(\frac{d}{2N}\right)}} = \sqrt{\frac{J_p}{27\pi\left(\frac{d}{2N}\right)}}$$

Finally, $T = \frac{1}{6.512}\sqrt{\frac{NJ_p}{d}}$ (4)

From the specifications of the motor in the *table 1,* we can find the J_p

$J_p = \frac{T_p}{\omega_p}$, where ω_p and T_p are the output speed and maximum output torque of the motor

$$J_p = \frac{100\times10^{-3}\,Nm}{335.103\,rad/s} = 298.415\times10^{-6}\,kg.m^2$$

Now by using the equation (4) it is easier to find the value of this transmission ratio and was found to be as follow:

Now $T_1 = \frac{1}{6.512}\sqrt{\frac{25\times298.415\times10^{-6}}{5\times10^{-2}}} = 0.0593\,m/rad$

And from the given 1 tour → 0,156 m and this is the perimeter of the gear

$$Perimeter(c) = \pi d = 2\pi r$$

$$d = \frac{c}{\pi} = \frac{0.156}{\pi} = 0.04965m$$

The new transmission ratio now is

$$T_2 = \frac{1}{6.512}\sqrt{\frac{25\times298.415\times10^{-6}}{0.04965}} = 0.05952\,m/rad$$

So, According to the calculation, we found that the theoretical transmission ratio is almost equal to the practical value of Transmission ratio $(T_1 \simeq T_2)$.

A.1.3 Modeling of the complete system

The complete block diagram of the positioning system can be represented by the following block diagram where the source is the unit step $u(t)$ which is also the input source for the electrical component of the motor but with additional feedback component $E(s)$. For the mechanical component, the source is due to the load effort $f_c(t)$ and is shown on the diagram.

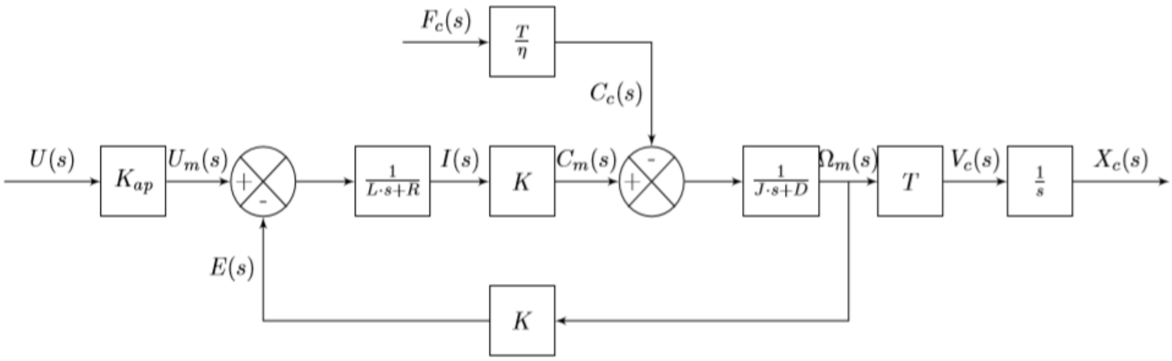

Figure 4: Complete block diagram of positioning system

Now, if the influence of load effort and electrical dynamics are neglected, it means ($f_c(t) \approx 0$ and $L \approx 0$), the block diagram in *figure 4* above will be the following:

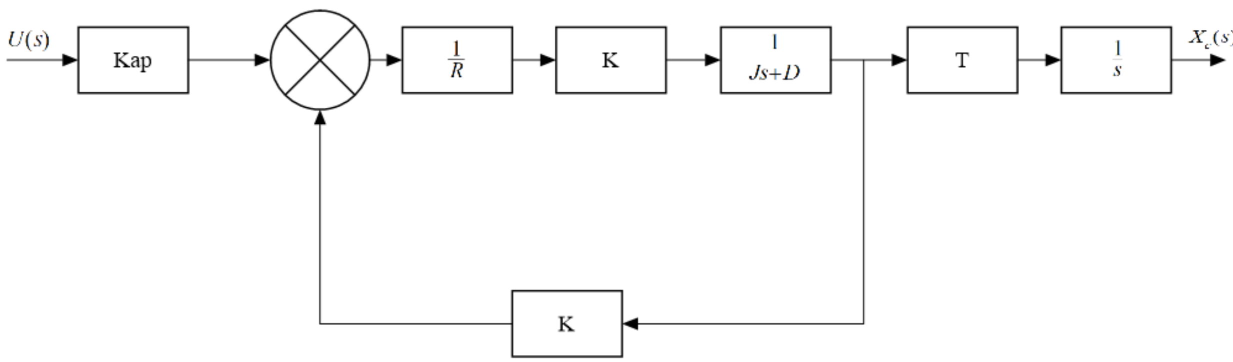

Figure 5: Block diagram when load effort and electrical dynamics are neglected

Now from this block diagram in *figure 5*, we can calculate the position transfer function of the complete positioning system as follow.

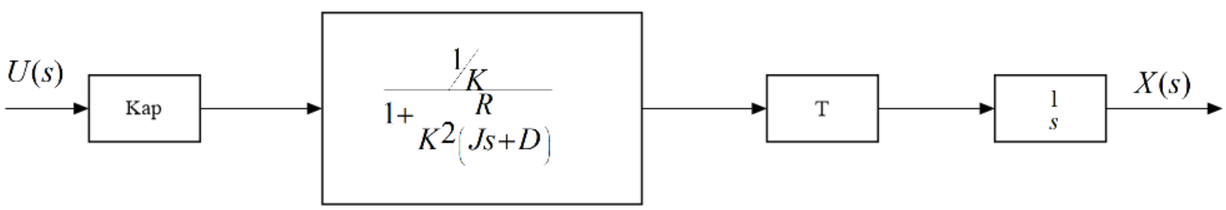

Figure 6: The reduce block diagram

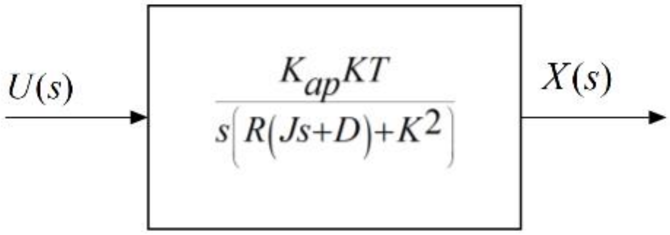

Figure 7: Final reduced block diagram of positional system

$$G_{\infty}(s)=\frac{X_c(s)}{U(s)}=\frac{K_{ap}KT}{s\left(R\left(Js+D\right)+K^2\right)} \tag{5}$$

And by replacing the given parameters with $K_{ap}=10$, this transfer function is:

$$G_{\infty}(s)=\frac{52\times10^{-6}K_{ap}}{35\times10^{-6}s^2+2.675\times10^{-3}s+2.704\times10^{-3}}$$

$$G_{\infty}(s)=\frac{52\times10^{-5}}{35\times10^{-6}s^2+2.675\times10^{-3}s+2.704\times10^{-3}}$$

A.2 Analog PID controller

Analogue position feedback The correctors used by the T.P. model are digital correctors, however, if the sampling period h is sufficiently small, these correctors behave substantially like analogical correctors. They are then qualified as pseudo continuous corrector. The digital realization of the correctors makes it possible to develop more complex correctors than simple PIDs, however, the digital PIDs are still very used because they are robust and quite easy to adjust.

A proportional-integral-derivative controller (PID) is a generic control loop feedback mechanism widely used in industrial control systems. A PID controller will correct the error between the output and the desired input or set point by calculating and give an output of correction that will adjust the process accordingly [3].

$$u(t)=k_p e(t)+k_i\int_0^t e(\tau)d\tau+k_d\frac{de}{dt} \tag{6}$$

Where k_p is proportional gain, k_i is the integral gain, and k_d is the derivative gain. The Proportional value determines the reaction to the current error, the Integral determines the reaction based on the sum of recent errors and the Derivative determines the reaction to the rate at which the error has been changing.

The transfer function of the analog PID controller is found by applying the Laplace transform to the equation above and is found to as follow:

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

A.4 Digital Positional Commutations

The discrete command u_k and the discrete position error ε_k for k ∈ N corresponds to the digitization of a PID corrector are related according to the following equations,

$$\begin{cases} U_k = k_p \varepsilon_k + k_d(\varepsilon_k - \varepsilon_{k-1}) + I_k \\ I_k = I_{k-1} + k_i \varepsilon_k \end{cases} \tag{7}$$

Now, from the recurrence equation (7) we calculated the transfer function of the digital PID corrector. From this recurrence equation we can find the past of this equation when there a delay of one unit then subtract from equation (7) above to get:

$$U_{k-1} = k_p \varepsilon_{k-1} + k_d(\varepsilon_{k-1} - \varepsilon_{k-2}) + I_{k-1}$$

$$U_k - U_{k-1} = k_p(\varepsilon_k - \varepsilon_{k-1}) + k_d(\varepsilon_k - 2\varepsilon_{k-1} + \varepsilon_{k-2}) + I_k - I_{k-1} \tag{8}$$

By apply the Z-transform to the above equation (8) the result will be:

$$\begin{cases} U(z)\left(1 - z^{-1}\right) = k_p(1 - z^{-1})E(z) + k_d(1 - 2z^{-1} + z^{-2}) + I(z)\left(1 - z^{-1}\right) \\ I(z) = \dfrac{k_i E(z)}{(1 - z^{-1})} \end{cases}$$

$$U(z) = \frac{k_p(1 - z^{-1})E(z) + k_d(1 - 2z^{-1} + z^{-2}) + k_i E(z)}{1 - z^{-1}} \tag{9}$$

So, the transfer function of PID is given as

$$H(z) = \frac{U(z)}{E(z)} = \frac{(k_p + k_i + k_d) - z^{-1}(k_p + 2k_d) + k_d z^{-2}}{1 - z^{-1}}$$

$$H(z) = \frac{U(z)}{E(z)} = \frac{(k_p + k_i + k_d)z^2 - z(k_p + 2k_d) + k_d}{z(1 - z)} \tag{10}$$

The new open transfer function is given by

$H_n(z) = H(z)Z\left(G_\infty(s)\right)$

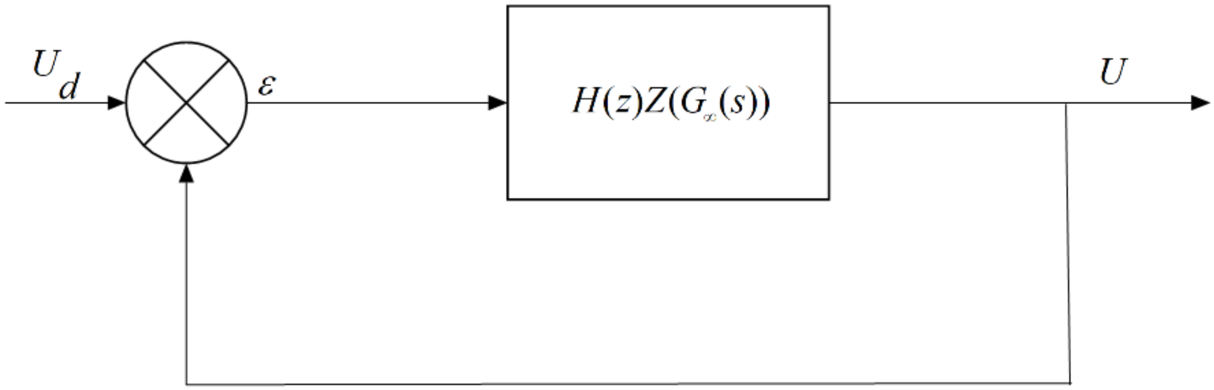

Figure 8: The new feedback closed loop

So the new feedback transfer function will be equal to $\dfrac{U_d(z)}{U(z)} = \dfrac{H(z)Z(G_\infty(z))}{1 + H(z)Z(G_\infty(z))}$

2.2 Simulations and digital controls of the model

2.2.1 Numerical simulations

The simulations will be carried out using Matlab/Simulink, which makes it possible to dynamically describe dynamic systems by block diagram and to simulate their temporal evolutions using finite difference algorithms. The figure below shows the Simulink model for the block diagram of DC motor.

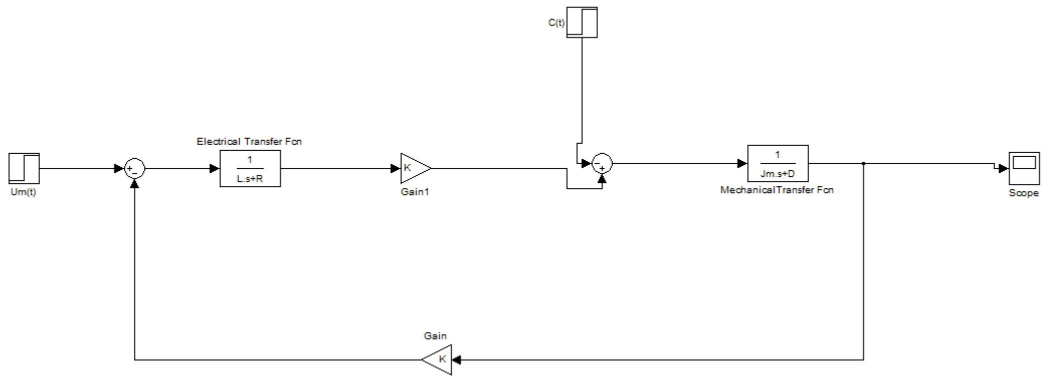

Figure 9: The Simulink model for Block diagram in figure1

By simulating the Simulink block diagram designed according to the block diagram in figure1 when subjected to a voltage step $u_m(t)$ of 24 V applied at t = 0 seconds and at a load torque step $c(t)$ of $5\times10^{-3}\,N\cdot$ m applied to t = 0.05 seconds for a chosen sampling step of 0.001 second, we get the following result

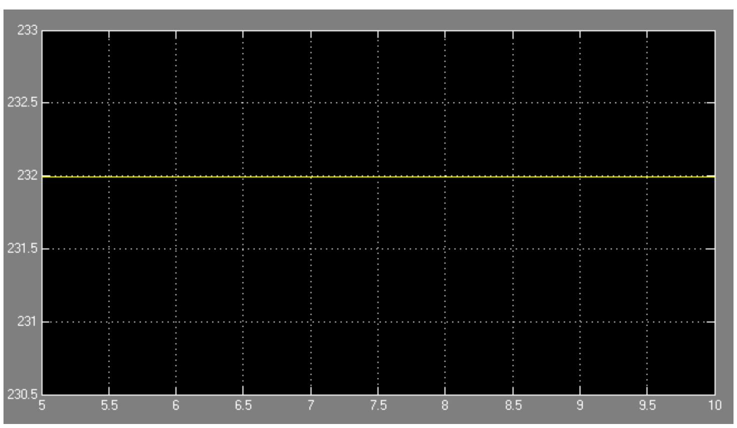

Figure 10: Result obtained with sampling step of 0.001s

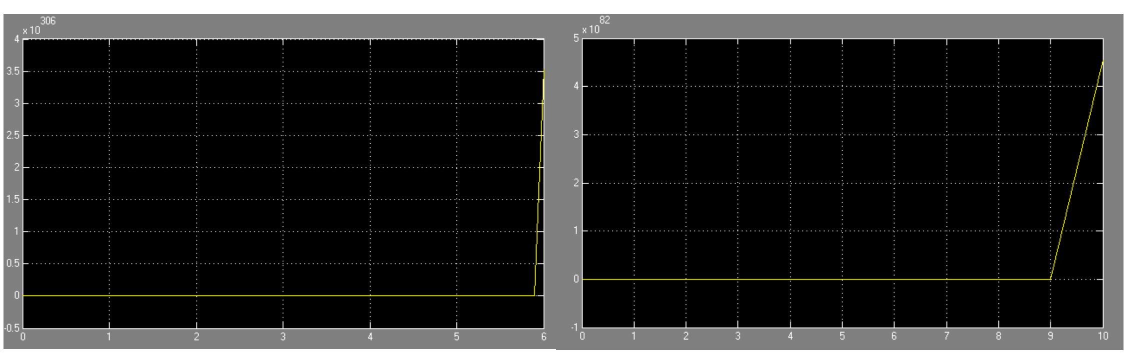

Figure 11: Result for sampling step of 0.1s left and for sampling step of 1s right

Sampling step time is a critical parameter in Matlab Simulink simulation control algorithms, because the controller is required to complete one set of control computations within one sampling interval. With those three graphs, it clearly that when the sampling step is very small there is no enough information about the result but by increasing this value the result is much clearly.

2.1.2 Dynamic behavior simulation for the complete system

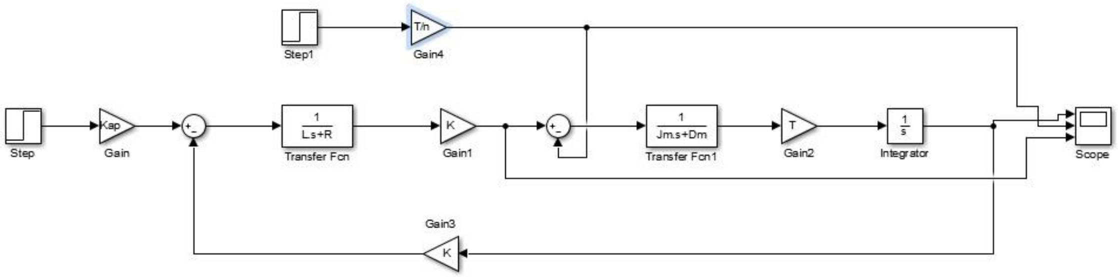

Figure 12: Simulink model for the complete system

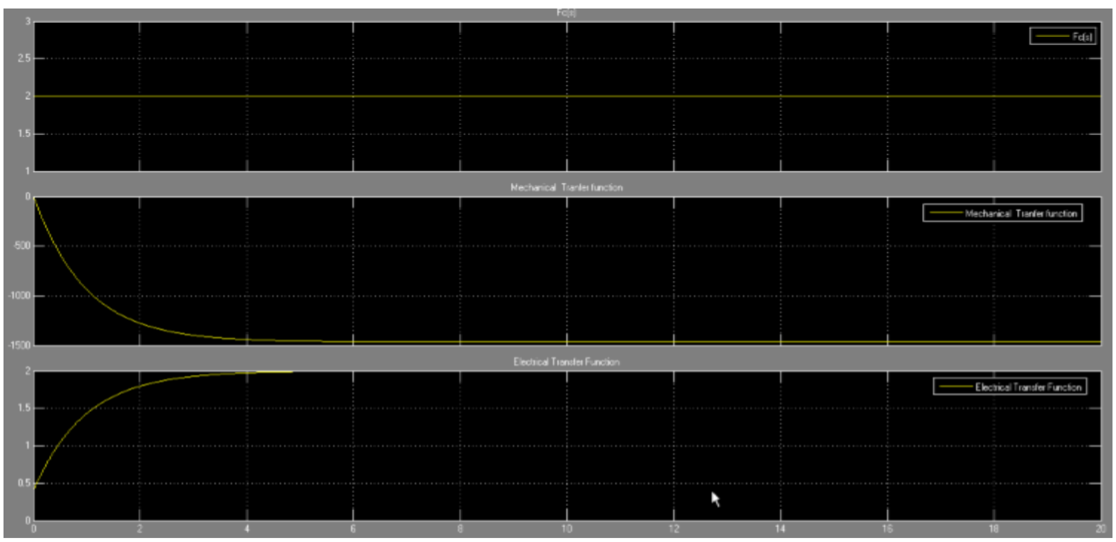

Figure 13: The result after simulation of the complete system

Part B

2.3 Experimental identification of the operative part

Because in practice, some parameters of the previous model are not always available. Now, it is possible to resort to an experimental identification to obtain the transfer functions of the system. In this model, the gain of the power amplifier and the values of certain mechanical parameters such as viscous friction coefficients or certain inertias are not known. And during this lab the carriage which is placed on the wheels, the effect $f_c(t)$ which corresponds substantially to dry friction, are neglected. By raising the position of the carriage according to the set point applied to the input of the power amplifier, it is then possible to determine the transfer functions in position and speed of the operative part. So, in this Part B the velocity information has been reconstructed/estimated by digital derivation and filtering of the position information measured by the incremental encoder.

Below, there is the Simulink model for speed transfer function used in order to study and control the position of the system.

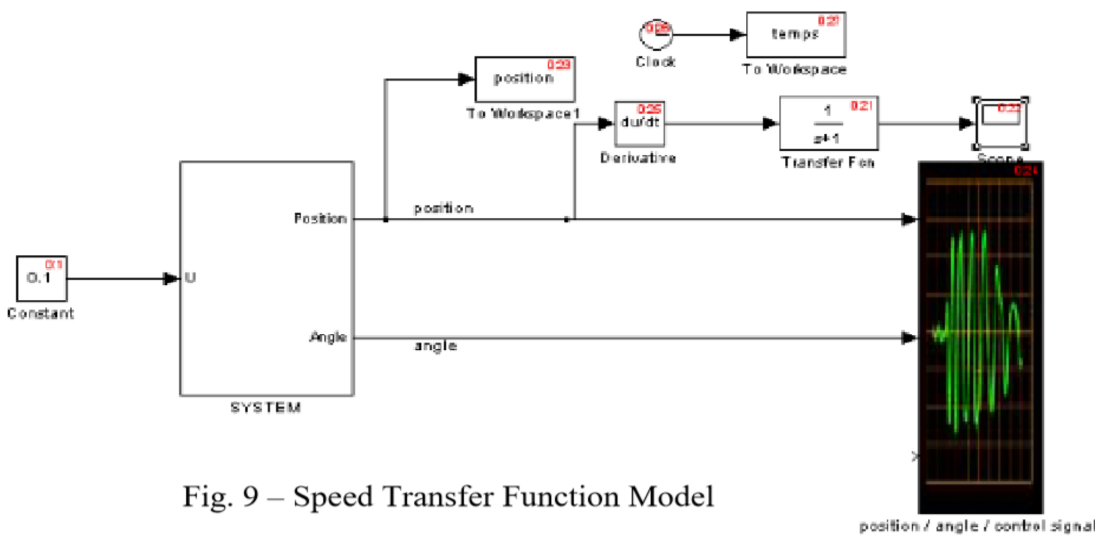

Fig. 9 – Speed Transfer Function Model

Figure 14: The speed transfer function

2.3.a Estimation of the speed of truck

By Performing an index response test by applying an input voltage step of the power amplifier $u(t) = U_0.U(t)$ with $U_0 = 0.1V$, and after simulation the following is the graph which represent the position of the motor with respect to time. From this graph we have found the speed by calculating the slope.

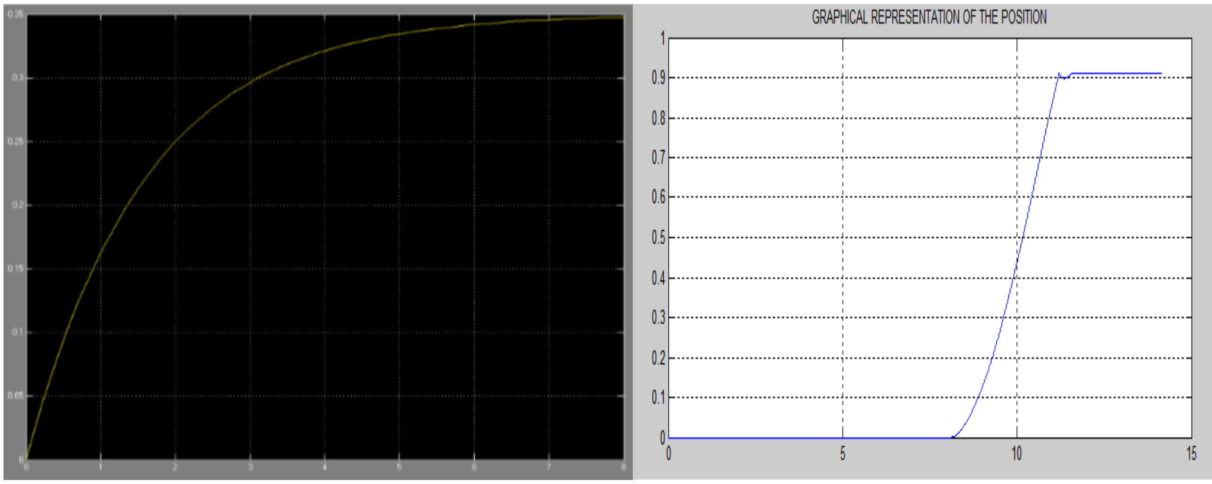

Figure 15: The result after simulation of the speed in left and the position in right side

Table 3: The parameters for finding the speed of truck

parameters	Final value	Initial value
Position (y) in m	$y_2 = 0.8978$	$y_1 = 0.4675$
Time (t) in s	$t_2 = 3.366$	$t_1 = 3.123$

$$V = \frac{dy}{dt} = \frac{0.8978 - 0.4675}{3.366 - 2.123} = \frac{0.4303}{1.243} = 0.346\,{}^{m}\!/_{s}$$

FOR small input voltage the response of the motor is very slowly but by increasing the input voltage the response of the motor increase.

2.3.b The velocity transfer function

The transfer function of the velocity is written as follow:

$$G_v(s) = \frac{V_c(s)}{U(s)} = \frac{K}{1+\tau s}.e^{-Ts}$$

In order to find those parameters in above transfer function we have used the following graph of the speed

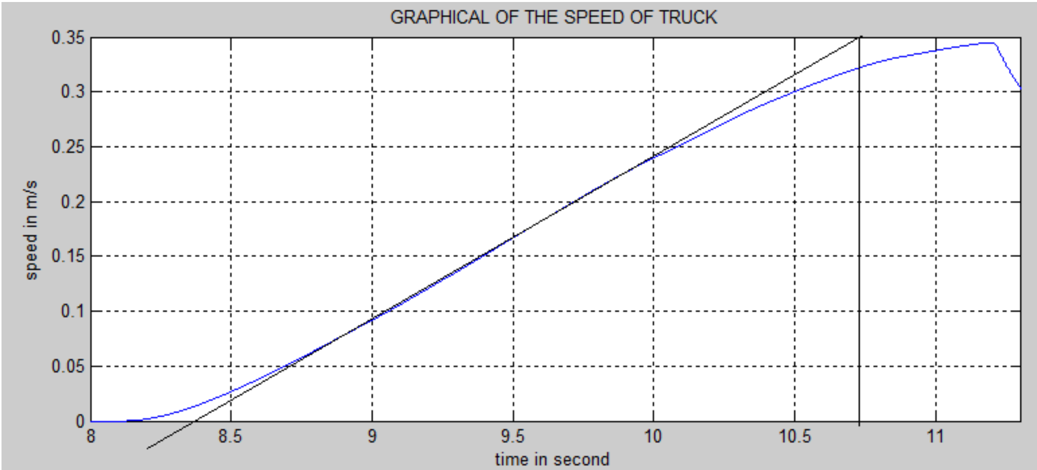

Figure 16: The speed of truck used to find the missing parameters of the complete system

And according to the above graph we have found that the values of the parameters are:

The time constant τ =2.323 second

K=3.45

And T=0.258 s

So now, $G_v(s) = \dfrac{3.45}{1+2.322s} e^{-0.258s}$

2.3.1 position control by PID

Due to its simplicity, robustness and successful practical application, PID (Proportional-Integral-Derivative) controllers have become most widely used controller in the industry. There are several different methods through which the PID controller can generate automatic control efficiently [2,3].

This is a complex question in two parts— process PID tuning for motion control PID tuning but as good engineer we have follow some general guidelines. The one we have chosen to guide are:

1. Always perform sizing to take in account the load to motor inertial mismatch, which is the most common problem and the source of loop instability

2. Remove system backlash as much as possible in the driven components

3. Use the step response method (command vs. response) to see how the system is reacting. This is commonly performed using an oscilloscope or is sometimes provided with the controller manufacturer's software. You need to see what you are doing; otherwise, you are just shooting in the dark

4. Start tuning with integral gain at zero; increase proportional gain to get a bit of overshoot response; and then adjust the derivative gain a bit at the same time to damp the oscillations. Add integral gain at the end to remove the static error. Think of it as a pyramid. The base of the pyramid is K_D; the middle of the pyramid is K_P; and the top of the pyramid is K_I. The K_D (derivative gain) would have the highest value, followed by K_P (proportional gain) with a lower value, followed by K_I (integral gain) with the lowest value. Manual tuning is a trial-and-error process.
5. At all time, make sure the current output of the amplifier is not saturated; otherwise, this procedure is invalid, and tuning is impossible. Saturated current can mean you are asking too much torque from the motor and amplifier or you are asking for a speed higher than what your system can achieve. A deeper analysis of the complete system might still be needed to help diagnose flaws or weaknesses.

For this lab we have used the Ziegler-Nichols method as a common used in tuning of PID controllers.

The proposed feedback which include the PID is as follow:

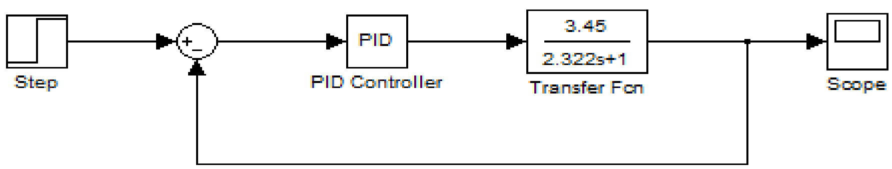

Figure 17: The feedback block diagram of the modeled system after implementing PID

In this model, with Ziegler-Nichols tuning method was used, the response is shown in the figure (18). And here the proportional gain K_{ap} is equal to 1, the integral gain K_i is equal to 1 and made the derivative gain K_D to be 0 and the response found is as follow:

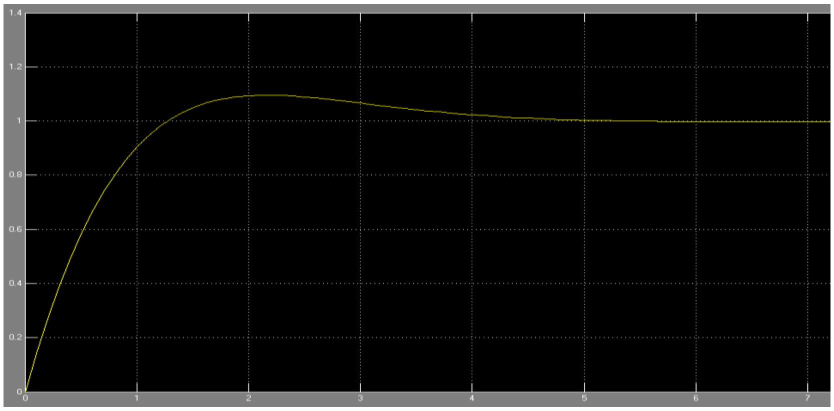

Figure 18: The response of the proposed PID

2.3.2 PID limits and order improvement

2.3.2.1 Derivative proportional

The D-term in PID means the Differentiation Damping increases with increasing derivative time, but decreases again when derivative time becomes too large. Recall that derivative action can be interpreted as providing prediction by linear extrapolation over the time Td. Using this interpretation, it is easy to understand that derivative action does not help if the prediction time Td is too large [2].

2.3.2.2 Integral proportional

As state in part A, the term I represent the Integral and determines the reaction based on the sum of recent errors. The k_i as the integral gain decreases the rise time, increases both the overshoot and the settling time, and eliminates the steady-state error. The integral gain responds proportionally to the accumulation of speed error over a time period. It is provided to prevent speed regulation as described above. Raising this gain will decrease the amount of time taken to reach the speed set point, thereby increasing the system stiffness. Integral gain also reduces system dampening, which leads to overshoot after a transient. For a given integral gain the dampening can be improved by increasing proportional gain. In general, a compromise between system response, stiffness and dampening must be reached for a given application [2].

2.3.2.3 The simulation using the PID controller

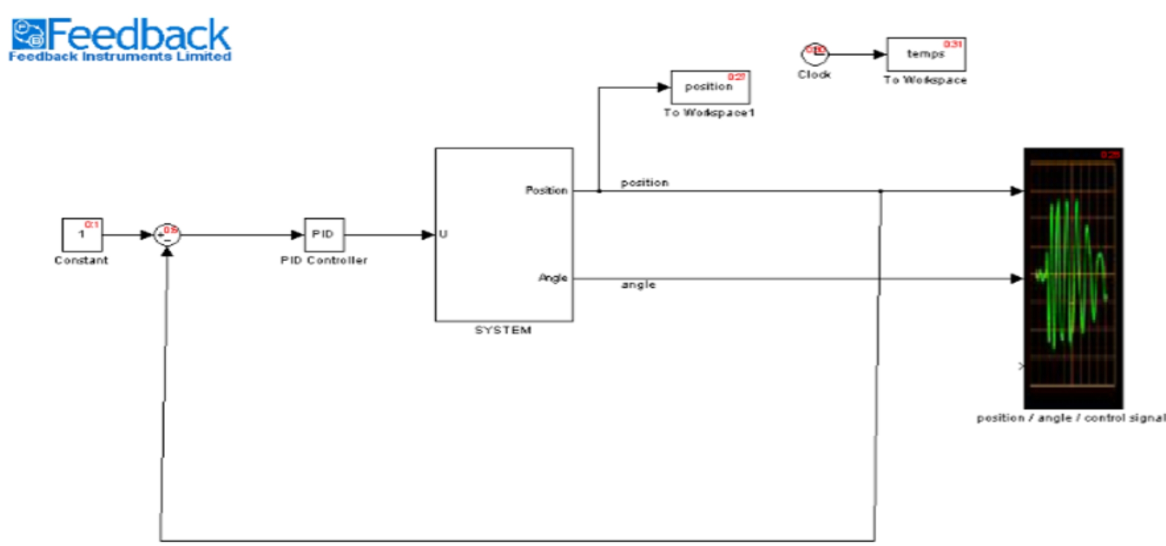

Figure 19: Simulink block diagram of the complete system with PID controller

And after simulation the following graphical result has been found

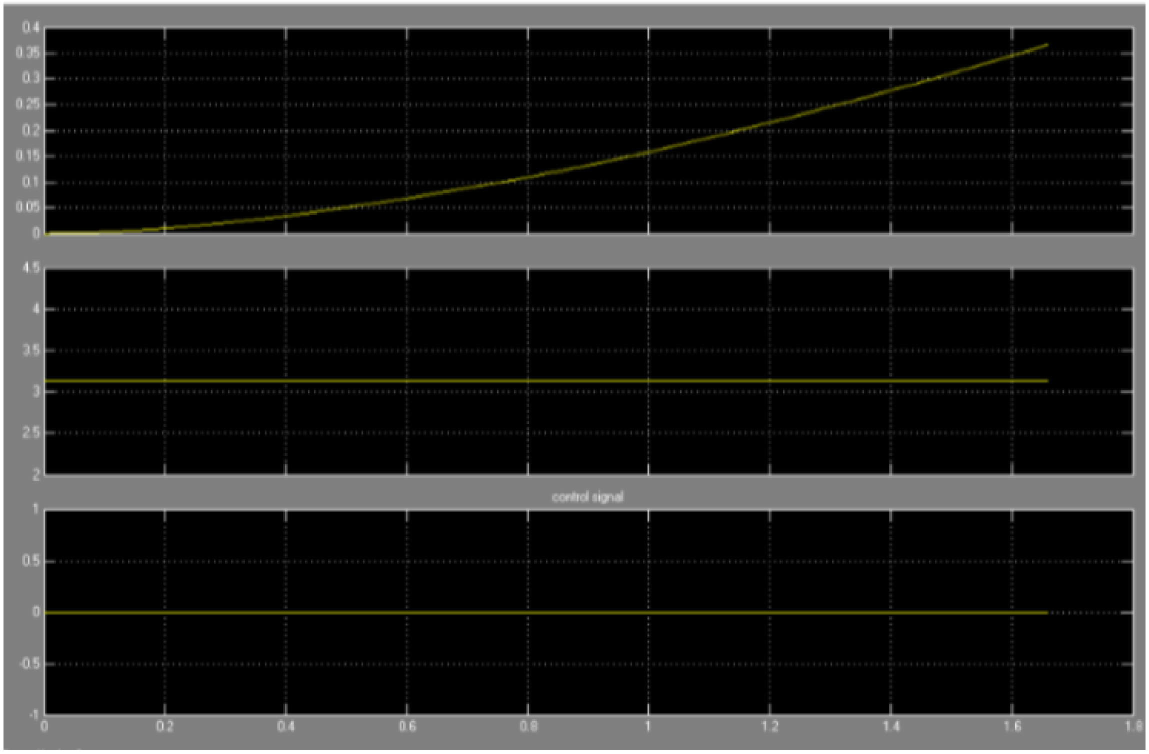

Figure 20: The result after simulation of complete diagram with PID

Test by Using open-loop is shown below, which has helped us to find the maximum acceleration of the motor and adjust accordingly the trajectory generation of the position servo-control. We found this by developing an experimentally test this device and then we took a look at the curves obtained experimentally and from this we were able to find the maximum acceleration as follow and the proposed open loop with its simulation are shown below also.

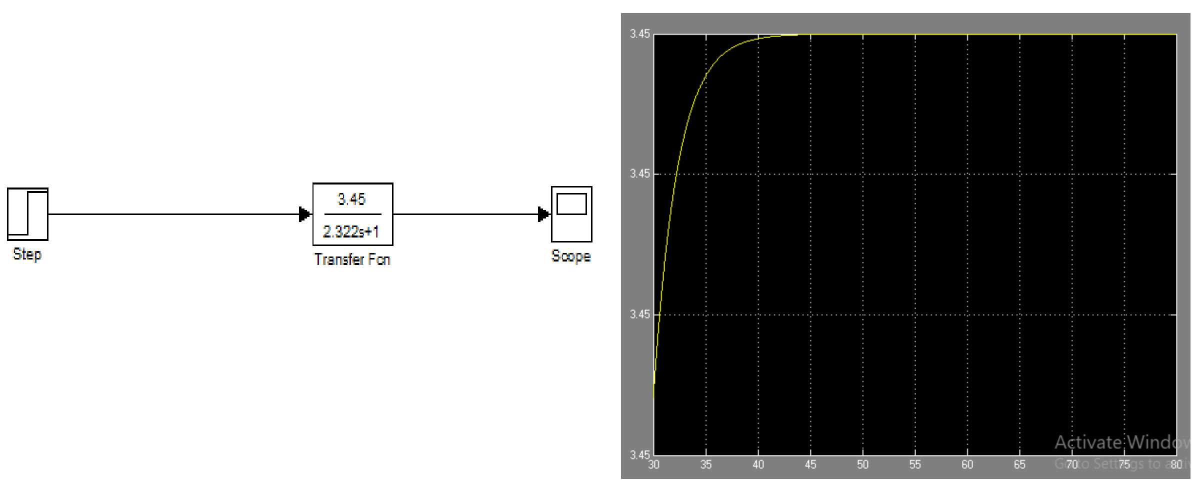

Figure 21: The proposed open loop and its simulation graph

The acceleration of the truck can be found by finding the slope using the above graph and as from the previous equation we have found that the speed of the truck is equal to $V = 0.346\,{}^{m}\!/_{s}$ and by considering the same time the acceleration come to be equal to:

$$a = \frac{dv}{dt} = \frac{0.346}{1.243} = 0.278\,{}^{m}\!/_{s^2}$$

So, once there is no PID controller, the position of the system was not easier to be regulated but once we used the PID controller, it is easier to regulate the position of the pendulum.

3.Conclusion

During this lab, as its aim was to focus on the study of the engine and the entire chain of electromechanical transmission in order to be able to model the complete dynamic behavior of the given system. The theoretical study of the system has been carried out and the numerical modeling using Matlab Simulink of the positioning axis systems. The position of the pendulum has been controlled by the used of PID corrector. We have made as much as possible tuning in order to find the best Proportional gain, Integral gain and Derivative gain which are best fit to our system to control its position. Therefore, with this study the position of the truck can be controlled which is mostly used now days in many industries.

4.References

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