ME 3210 Mechatronics Modelling
Linear Graphs

Linear Graphs

The other method that will be used in this class is called linear graphs. It is useful primarily because it enables us to solve all systems, mechanical, electrical, hydraulic in the same manner.

The idea is to draw a circuit of the system. Then Kirchhoff's voltage and current laws can be used to derive the equations of motion. First, the linear graph representations of the translational mechanical passive elements are shown in figures 1 through 3. Note that each element representation is simply a line. At each end of the line there are two dotes. These represent the across variable of the element. For translational mechanical elements the across variable is velocity (or position, or acceleration). The arrow on the line represents the direction for the through variable (force for a translational mechanical system). For passive elements, the direction of the arrow is arbitrary.

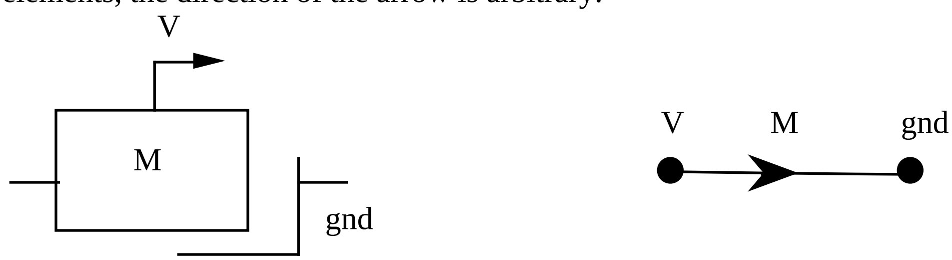

Figure 1 Linear graph representation of a mass. Note that one across variable is always ground for a mass.

Figure 2 Linear graph representation of a spring

Figure 3 Linear graph representation of a damper

To make a linear graph of a system comprised of many elements, the individual element graphs are connected at the dots. Common dots represent common across variables (same position, velocity, etc).

To see how a compete system is represented in a linear graph, a linear graph will be made for the same example that was used for the section on free body diagrams. A set of rules will be developed which will be used to create the linear graph. Recall the example system shown in figure 4.

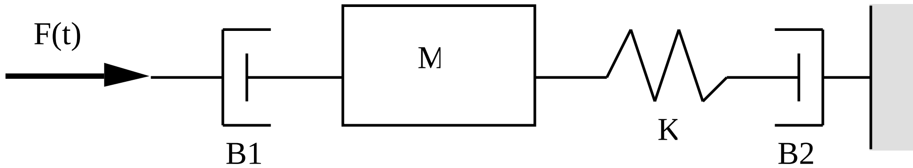

Figure 4 Example mechanical system

The first step is to identify all of the displacements (and velocities and accelerations). There are four displacements needed for this example — always include ground. This is shown in figure 5. Note that this step is exactly the same as the free body diagram technique.

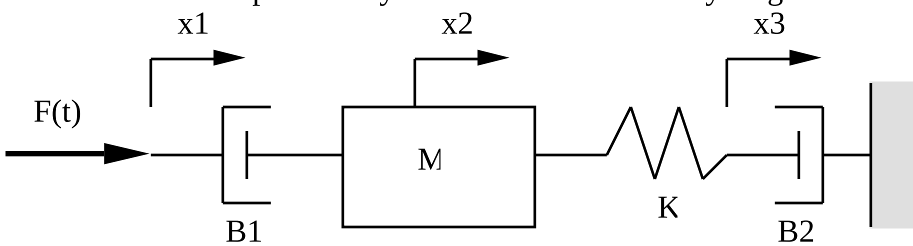

Figure 5 System with displacements assigned

To start the linear graph, draw dots (called nodes or terminals) to represent each across variable. For a translational mechanical system this is velocity. Each element has two velocities associated with it. Next draw a line for each element between each of the nodes that that element connects. This is shown in figure 6 for this example. Three displacements are indicated by x1, x2, and x3. The fourth is ground.

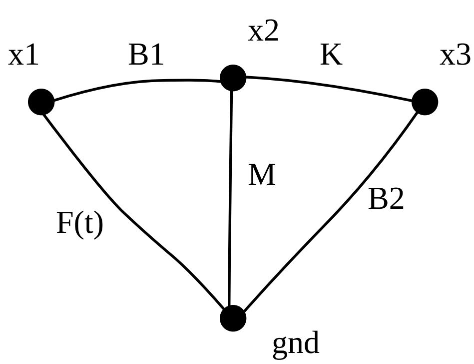

Figure 6 Start of the linear graph with the displacements and elements shown.

The next step is to draw arrows on each of the element lines representing the direction of the through variable – force for a translational mechanical system. For passive elements, the direction is arbitrary. It is not for the sources. In this example, there is a force source, F(t). If F(t) were positive, there would be a positive velocity of the damper, B1. Therefore, the arrow for F(t) must point towards x1. If x1 were defined in the opposite direction or the force F(t) were defined in the opposite direction, the arrow for F(t) would point away from x1, towards ground. Figure 7 shows the directions assigned for this system.

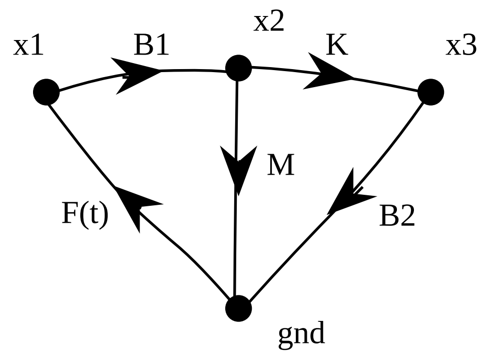

Figure 7 Linear graph for the example system.

This is the complete linear graph for the example system.

In the next section, the derivation of electrical systems will be shown, followed by hydraulic, and finally, 1-D thermal systems. Once the linear graphs are developed for each system, the techniques for deriving the differential equations will be presented. The derivation of the equations is exactly the same for all systems.