INDR371 - HW1

Umur Berkay Karakaş

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Question 1

- $\bar{F}$: set of open plants
- F: set of new candidate locations
- q_f: capacity of facility f
- d_k: demand of customer k
- c: unit transportation cost per unit distance
- t_{fk} distance between facility f and customer k
- K(f): set of customer that facility f can serve
- x_f: binary for whether facility f is open or not
- p_{fk}: percentage of the demand of customer k satistied by facility f
- M: the number of customers

$$
\begin{aligned}
\min \quad & \sum_{k \in K} \sum_{f \in F \cup \bar{F}} c t_{fk} p_{fk} d_k \\
\text{s.t.} \quad & \sum_{f \in F \cup \bar{F}} p_{fk} = 1 \quad , \quad \forall k \in K, \\
& \sum_{k \in K} p_{fk} d_k = q_f \quad , \quad \forall f \in F \cup \bar{F}, \\
& \sum_{f \in F} x_f = 2, \\
& \sum_{k \in K} p_{fk} \leq M x_f \quad , \quad \forall f \in F \cup \bar{F}, \\
& p_{fk} = 0 \quad , \quad \forall f \in F \cup \bar{F}, \forall k \notin K(f), \\
& x_f \in \{0, 1\} \quad , \quad \forall f \in F \cup \bar{F}, \\
& p_{fk} \geq 0 \quad , \quad \forall f \in F \cup \bar{F}, \forall k \in K(f)
\end{aligned}
$$

Question 2

a)

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Gurobi Optimizer version 9.5.2 build v9.5.2rc0 (mac64[rosetta2])
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 1 rows, 81 columns and 81 nonzeros
Model fingerprint: 0xa9e17c44
Variable types: 0 continuous, 81 integer (81 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+06, 5e+06]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 1e+00]
Found heuristic solution: objective 1666660.7651
Presolve removed 1 rows and 81 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.01 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 1.25501e+06 1.66666e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.255005784623e+06, best bound 1.255005784623e+06, gap 0.0000%
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Figure 1: Results of the Gurobi model for Question 2, part A

b)

I have adapted the solution in this article using "Problem UMApHMP-N". The only difference in the model is instead of limiting the number of hubs with p using

$$\sum_{k \in N} H_k = p$$

I have used

$$\sum_{k \in N} C_k H_k \leq 2400$$

where C_k corresponds to the fixed hub cost of hub k and H_k corresponds to the binary that represents whether hub k is open or not.

Other than that, I have solved the problem in 2 ways, one way is using (distance * fixed link cost) between cities directly. The other one is calculating the shortest (distance * fixed link cost) between each node and then using that matrix in my optimization model. In order to calculate shortest paths, I have used Floyd-Warshall algorithm.

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Barrier performed 99 iterations in 100.64 seconds (51.76 work units)
Barrier solve interrupted - model solved by another algorithm

Concurrent spin time: 65.17s (can be avoided by choosing Method=3)

Solved with dual simplex

Root relaxation: objective 6.799391e+06, 38774 iterations, 94.15 seconds (44.94 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 6799391.23    0    1          - 6799391.23      -     -  100s
H    0     0                    6986410.7891 6799391.23  2.68%     -  102s
     0     0 6800626.28    0    3 6986410.79 6800626.28  2.66%     -  113s
     0     0 6800626.28    0    1 6986410.79 6800626.28  2.66%     -  121s
     0     0 6801466.85    0    3 6986410.79 6801466.85  2.65%     -  122s
     0     0 6801466.85    0    3 6986410.79 6801466.85  2.65%     -  125s
     0     2 6801466.85    0    3 6986410.79 6801466.85  2.65%     -  128s
    10     0 infeasible    4      6986410.79 6936773.12  0.71%   669  130s

Cutting planes:
  Flow cover: 6
  Flow path: 1
  RLT: 1

Explored 14 nodes (58269 simplex iterations) in 130.11 seconds (81.33 work units)
Thread count was 8 (of 8 available processors)

Solution count 2: 6.98641e+06 6.98641e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 6.986410789128e+06, best bound 6.986410789128e+06, gap 0.0000%
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Figure 2: Results of the Gurobi model for Question 2, part B, without shortest paths

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Root relaxation: objective 7.655747e+06, 31369 iterations, 79.56 seconds (39.90 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 7655746.87    0    3          - 7655746.87      -     -   85s
H    0     0                    1.065475e+07 7655746.87  28.1%     -   86s
H    0     0                    9974369.5122 7655746.87  23.2%     -   87s
     0     0 7796970.98    0    5 9974369.51 7796970.98  21.8%     -  102s
     0     0 7797848.46    0    5 9974369.51 7797848.46  21.8%     -  106s
     0     0 7801281.41    0    2 9974369.51 7801281.41  21.8%     -  109s
     0     0 7801314.81    0    2 9974369.51 7801314.81  21.8%     -  112s
     0     0 7803841.96    0    2 9974369.51 7803841.96  21.8%     -  118s
     0     0 7804803.02    0    2 9974369.51 7804803.02  21.8%     -  122s
     0     0 7804843.89    0    2 9974369.51 7804843.89  21.8%     -  122s
     0     0 7804886.41    0    2 9974369.51 7804886.41  21.8%     -  123s
     0     0 7804888.52    0    2 9974369.51 7804888.52  21.8%     -  124s
     0     0 7804991.25    0    2 9974369.51 7804991.25  21.7%     -  130s
     0     0 7804992.71    0    2 9974369.51 7804992.71  21.7%     -  133s
     0     0 7805954.27    0    2 9974369.51 7805954.27  21.7%     -  136s
     0     0 7805954.36    0    2 9974369.51 7805954.36  21.7%     -  139s
     0     0 7805955.17    0    2 9974369.51 7805955.17  21.7%     -  141s
     0     0 7805955.72    0    2 9974369.51 7805955.72  21.7%     -  144s
     0     0 7805955.72    0    2 9974369.51 7805955.72  21.7%     -  149s
H    0     0                    7807322.0084 7805955.72  0.02%     -  151s

Cutting planes:
  Cover: 1
  MIR: 9
  StrongCG: 1
  Flow cover: 32
  Flow path: 11

Explored 1 nodes (36564 simplex iterations) in 153.71 seconds (130.69 work units)
Thread count was 8 (of 8 available processors)

Solution count 4: 7.80732e+06 7.80732e+06 9.97437e+06 1.06548e+07

Optimal solution found (tolerance 1.00e-04)
Best objective 7.807322008449e+06, best bound 7.807322008449e+06, gap 0.0000%
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Figure 3: Results of the Gurobi model for Question 2, part B, with shortest paths