INDR 371 Fall 2022
HOMEWORK 3

1. Semiconductor wafer fabrication is very much error-prone. Assume that a wafer production facility received an order for a specially designed prototype wafer. The cost of producing each wafer is estimated to be \$20,000. The customer agrees to pay \$150,000 for 3 good wafers, \$200,000 for 4 good wafers, and \$250,000 for 5 good wafers. Other than 3, 4, or 5 good wafers, all other wafers must be destroyed (i.e. have no value). To obtain the contract, the wafer fab offers to pay the customer a penalty of \$100,000 if at least three good wafers are not produced. Assume that each wafer is produced independently and the probability that a wafer is acceptable is 0.65.

Build a function in your favorite coding environment (Python, R, Java, etc.), to answer the following questions. It is important to note that your calculations should be general enough to hold when different inputs (revenues, probabilities, etc.) are changed. For this purpose, always define all inputs as variables whose values can be changed. Writing your function keep in mind that the number of successfully produced items would have a Binomial distribution. Submit your code along with the answers for the following questions.

a. (10 pts) Construct an expected profit table for *Q*=1,2,…,10 (report the expected profit obtained when *Q* wafers are manufactured). What is the optimal quantity to manufacture to maximize the expected profit?

b. (10 pts) Plot how does the optimal quantity change when the probability of a wafer being acceptable changes in the range of 0.5 to 0.95?

c. (10 pts) Plot how does the optimal order quantity change if the production cost per wafer changes in the range from \$10,000 to \$40,000?

d. (10 pts) The wafer company is risk-averse and would also like to avoid the risk of losing money. Using the binary loss/gain probabilities. Report the probability of losing money for *Q*=1,2,…,10.

e. (20 pts) Now assume that consecutive wafer productions are not necessarily independent. Consider the following model: *P(wafer i+1 is good | wafer i is good)=0.9* and *P(wafer i+1 is bad| wafer i is good)=0.1*. In addition, *P(wafer i+1 is good | wafer i is bad)=0.6, P(wafer i+1 is bad | wafer i is bad)=0.4.* Assume that the first wafer is good with probability 0.75 and find the expected profit when 4 wafers are scheduled.

2. A firm has received an order for 25 die cast parts made from precious metals. The parts will sell for \$5000 each and it will cost \$2500 to produce an individual part. The probability of a part meeting final inspection equals 0.75. Parts that meet the inspection but are not sold can be recycled at a value of \$900. Parts that do not meet the inspection have a unit salvage value of \$100. A penalty clause in the contract results in the firm having to pay the customer \$500 <u>per unit short</u> if the number of parts that can be delivered is less than 25. In addition, if the number that can be delivered is less than 23, there is an additional fixed penalty of \$10,000 (i.e. if 20 parts can be delivered, the total penalty is (25-20)×500 + 10,000=\$12,500).

a. (10 pts) Determine the expected profit for a given production quantity.

b. (10 pts) Find the production quantity that maximizes the expected profit.

c. (20 pts) Assume that there is a process improvement that can improve production quality. If this improvement takes place, the probability of a given part meeting the final inspection will raise to 95%. What is the maximum amount that should be paid for such an improvement?