Question 1

A. The plot of the coffee price index can be seen in Figure 1 in the appendix. There is an obvious upward trend, but seasonality cannot be observed from the time series plot.

B. Naive Forecast MAE from 1991 to 2020: 2.7447940745555557
Naive Forecast MAPE from 1991 to 2020: 0.03563979274636223
Naive Forecast RMSE from 1991 to 2020: 4.13839665488288

Time series plot of the "naive" forecast can be seen in Figure 2.

C. Five MA Forecast MAE from 1991 to 2020: 4.961328926305556
Five MA Forecast MAPE from 1991 to 2020: 0.06356983921661971
Five MA Forecast RMSE from 1991 to 2020: 7.363141715664006

Time series plot of the "five MA" forecast can be seen in Figure 3.

From Figure 8, it can be observed that prediction intervals do not perform accurately after 6 months as the real price exceeds the upper bound of the prediction interval.

D. ES Forecast MAE from 1991 to 2020: 2.7447940745555557
ES Forecast MAPE from 1991 to 2020: 0.03563979274636223
ES Forecast RMSE from 1991 to 2020: 4.13839665488288

Time series plot of the "ES" forecast can be seen in Figure 4.

Optimal alpha value is 1.0, which means ES algorithm puts all weight to the latest sale value. Therefore, "ES" forecast is equivalent to "naive" forecast. Both of them perform better than the "five MA" forecast in terms of errors. "ES" forecast has a slightly tighter prediction interval than the "five MA" forecast for 2021 and the real price is inside the prediction interval.

E. Naive Forecast with trend MAE from 1991 to 2020: 3.9266112891944456
Naive Forecast with trend MAPE from 1991 to 2020: 0.05124078373307825
Naive Forecast with trend RMSE from 1991 to 2020: 5.410511999371555

Time series plot of the "naive forecast with trend" can be seen in Figure 5.

From Figure 8, it can be observed that prediction intervals perform good as the real prices are inside the interval. The performance of "naive forecast with trend" is better than "five MA" forecast but worse than "naive" and "ES" forecasts. The real price is between the prediction interval of "naive forecast with trend" but the interval is slightly larger than the prediction interval of "ES" forecast which means the prediction interval of "ES" forecast is better.

F. Smoothed Trend Forecast MAE from 1991 to 2020: 6.157771417175611
Smoothed Trend Forecast MAPE from 1991 to 2020: 0.078055016335063
Smoothed Trend Forecast RMSE from 1991 to 2020: 8.306143226665506

Time series plot of the "smoothed trend" forecast can be seen in Figure 6.

From Figure 8, it can be observed that prediction intervals perform good as the real prices are inside the interval. The performance of "smoothed trend" forecast is better than "five MA" forecast but it's hard to compare the prediction interval of "smoothed trend" forecast with the prediction interval of other forecasts because they all perform good and they have more or less similar interval widths.

G. 6 month ahead Forecast MAE from 1991 to 2020: 18.019234087901516
6 month ahead Forecast MAPE from 1991 to 2020: 0.23371194755157096
6 month ahead Forecast RMSE from 1991 to 2020: 26.506122531366092

Time series plot of the "6-month ahead" forecast can be seen in Figure 7. Predicted values of sales from Jan 21 to Jun 22 can be found in the code.

Question 2

A. Time series plot of 500 AR-1 generalizations can be observed in Figure 9.
B. ACF plot of 500 AR-1 generalizations after period 100 can be observed in Figure 10.
C. RMSE starting from period 100: 21.844471227159115
Implementation of "naive" forecast can be observed in the code.

Question 3

A. ACF and PACF plots of coffee price index can be observed in Figure 11 and 12. From the ACF plot, we can infer that there is definitely an upward trend, but we cannot infer an AR or MA model. From the PACF plot, we can infer an AR(2) model with a positive coefficient at lag-1 and a negative coefficient at lag-2 but further examination with some transformations would result in a more definite solution.

B. ACF and PACF plots of first order difference of coffee price index can be observed in Figure 13 and 14. In the ACF plot, there is a positive spike at lag-1 and a smaller but still significant spike at lag-2. In the PACF plot, there is also one significant (ignoring the significant values close to the bounds) value which is at lag-1. From these 2 plots, I infer that this should be an AR(1) model with a positive coefficient (for the first order difference)

C. From the PACF plot of original data together with the ACF and PACF plots of first order difference of the original data, I came up with an AR(2) model. The PACF plot of original data is clear and I stated my reasoning above. From the plots of first order differenced data, I came up with an AR(1) model as such: $Z_t = \phi_1 Z_{t-1} + \varepsilon_t$ where $Z_t = Y_t - Y_{t-1}$ and the original equation becomes $Y_t = Y_{t-1} + \phi_1(Y_{t-1} - Y_{t-2}) + \varepsilon_t$ which is an AR(2) model.

Appendix

Figure 1: Time series plot of coffee price

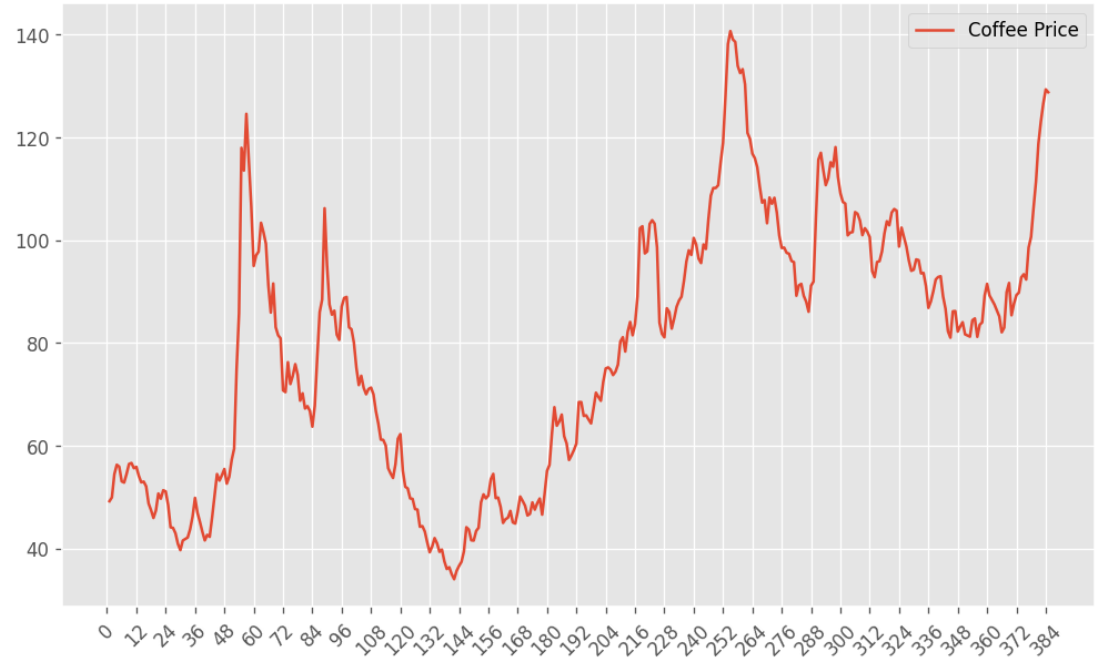

Figure 2: Time series plot of real sales vs naive forecast

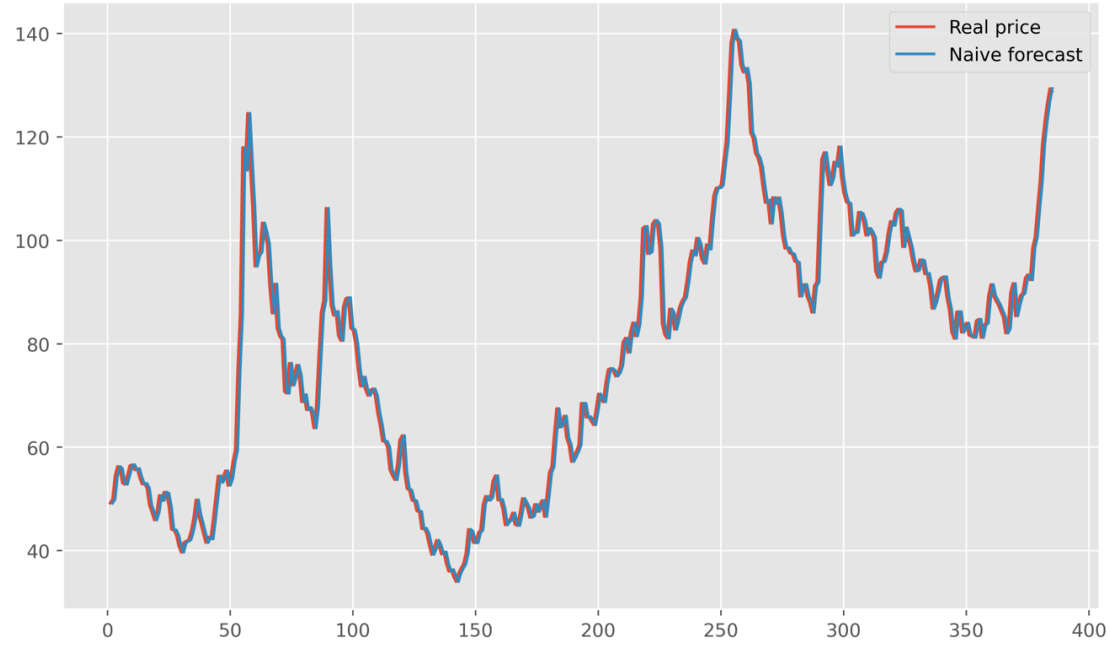

Figure 3: Time series plot of real sales vs five MA forecast

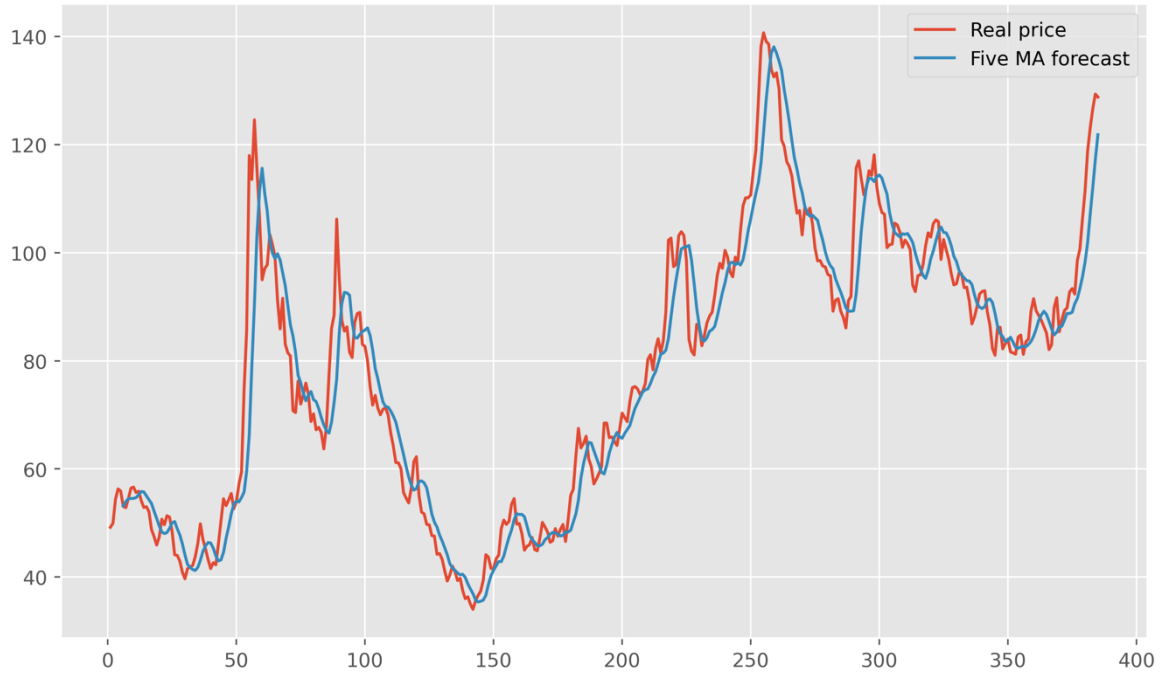

Figure 4: Time series plot of real sales vs ES forecast

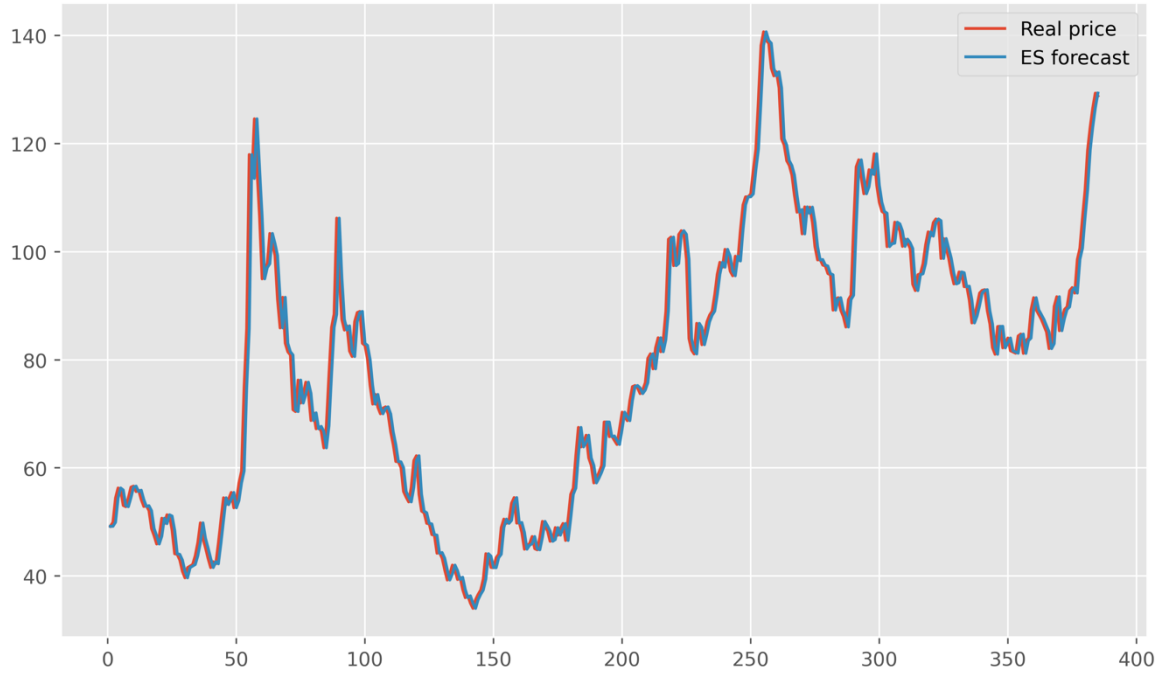

Figure 5: Time series plot of real sales vs naive forecast with trend

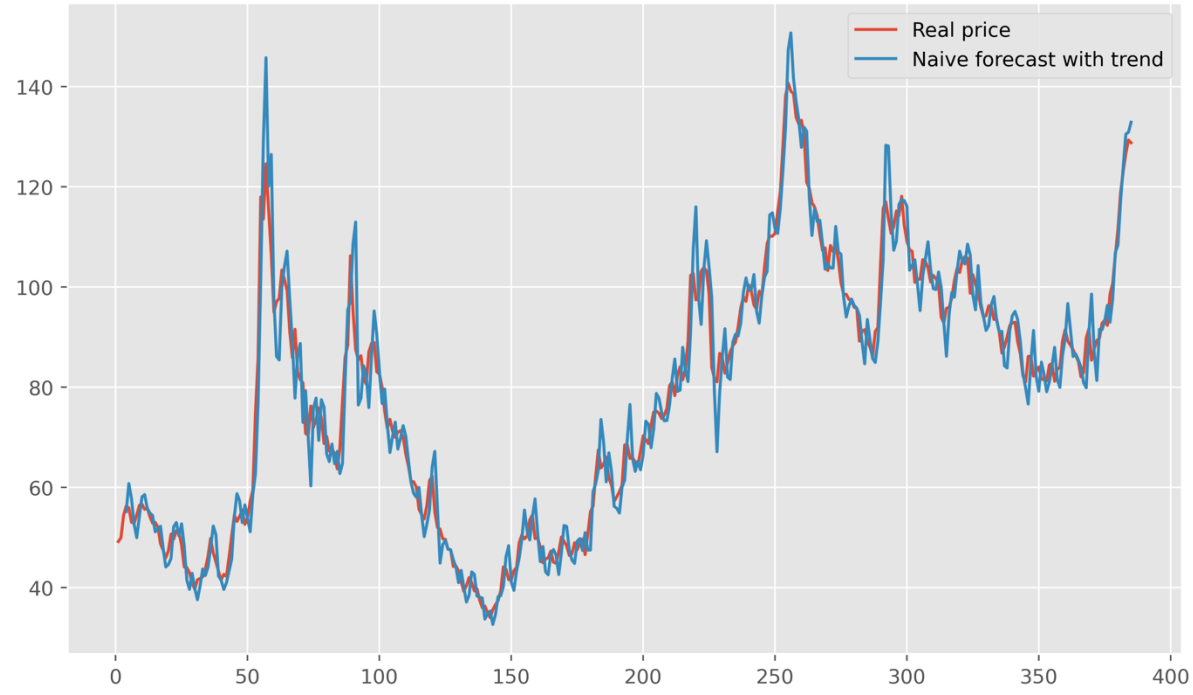

Figure 6: Time series plot of real sales vs smoothed trend forecast (alpha = 0.7, beta = 0.2)

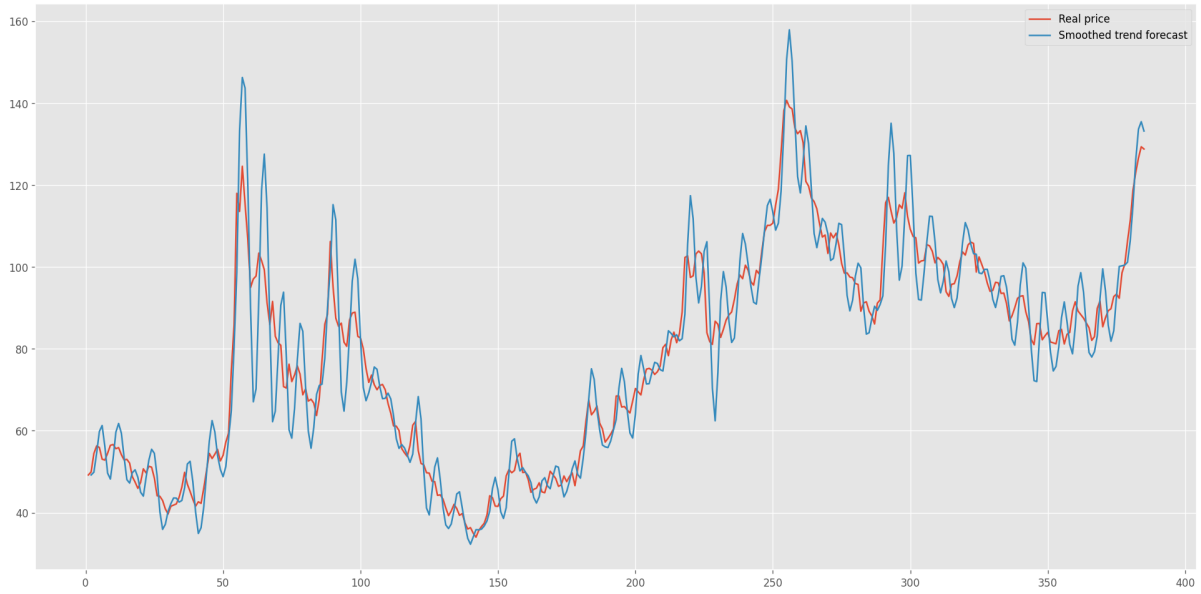

Figure 7: Time series plot of real sales vs 6-month ahead smoothed trend forecast

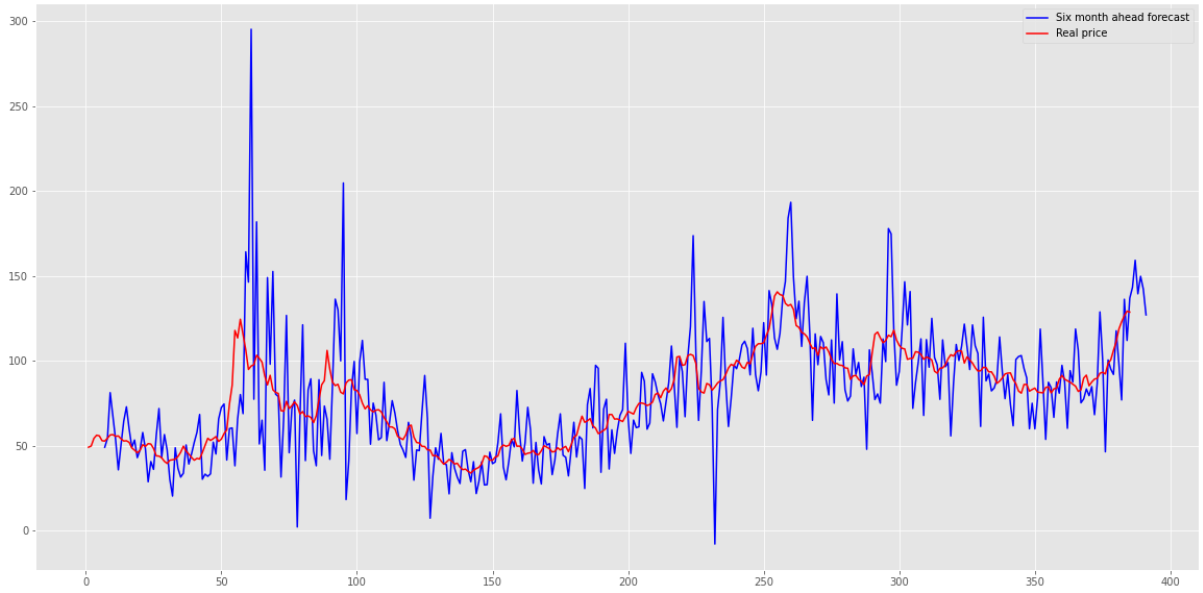

Figure 8: All forecasts prediction intervals in 2021

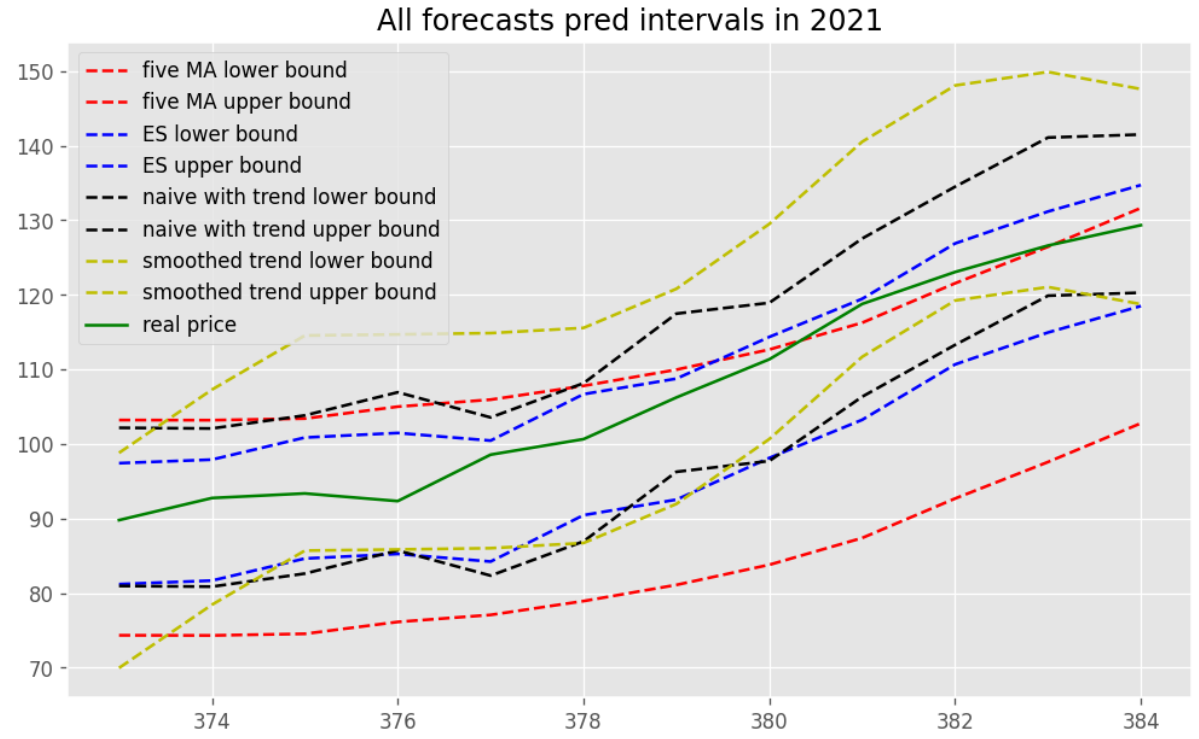

Figure 9: Time series plot of 500 AR-1 generalizations

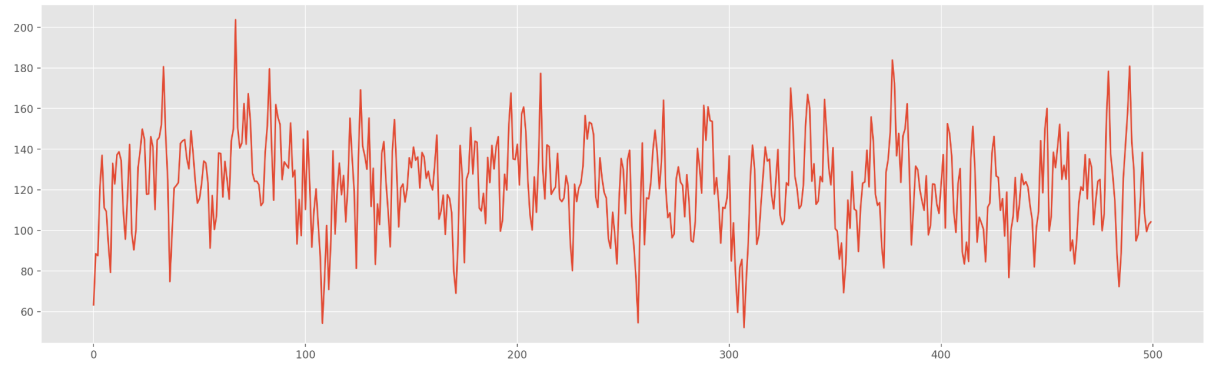

Figure 10: ACF plot of 500 AR-1 generalizations after period 100

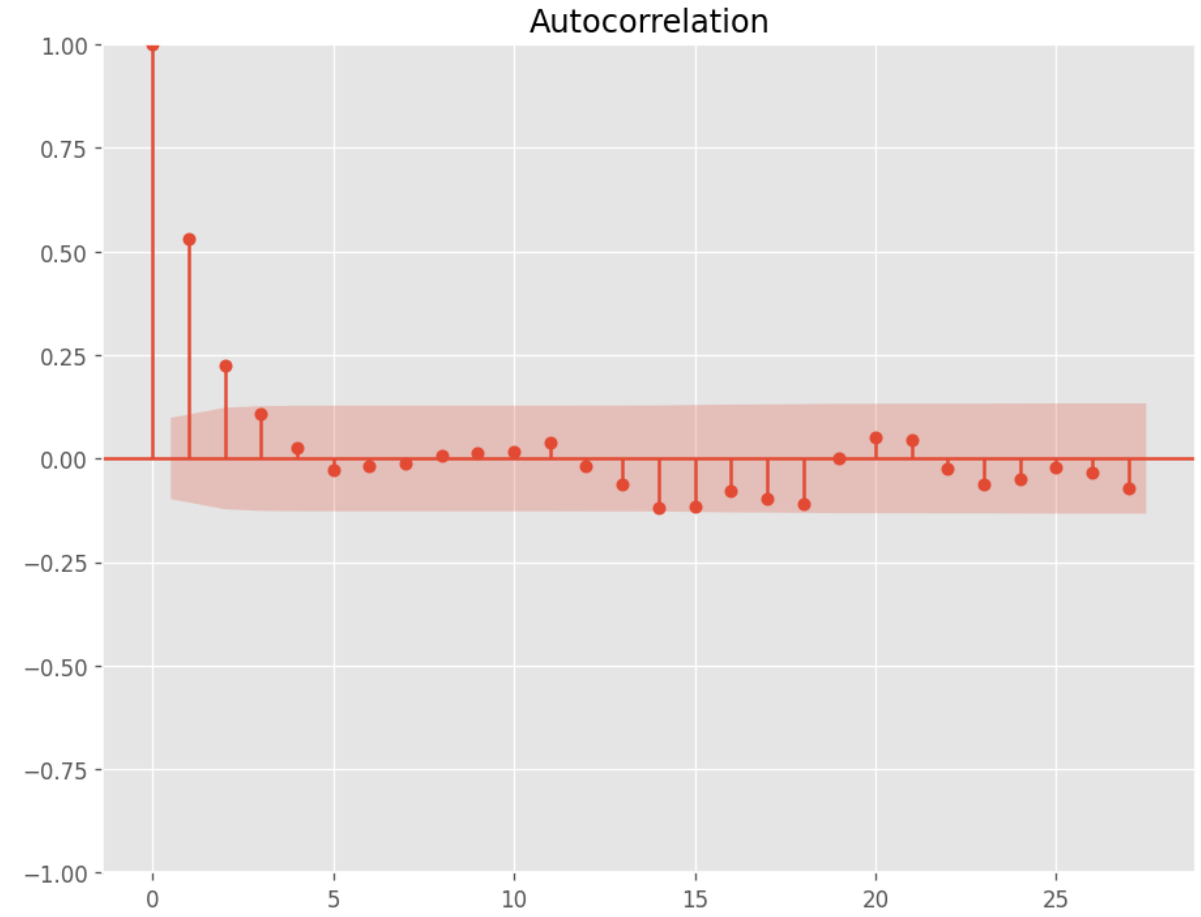

Figure 11: ACF plot of coffee price index

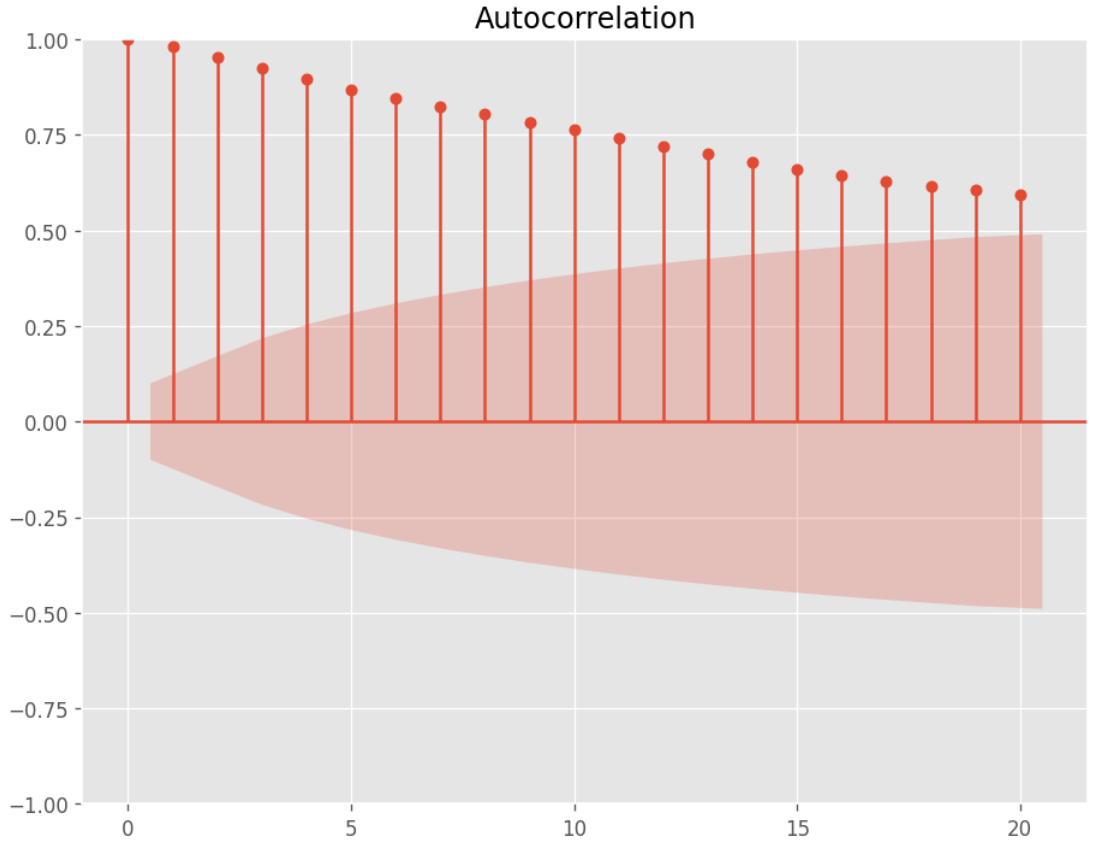

Figure 12: PACF plot of coffee price index

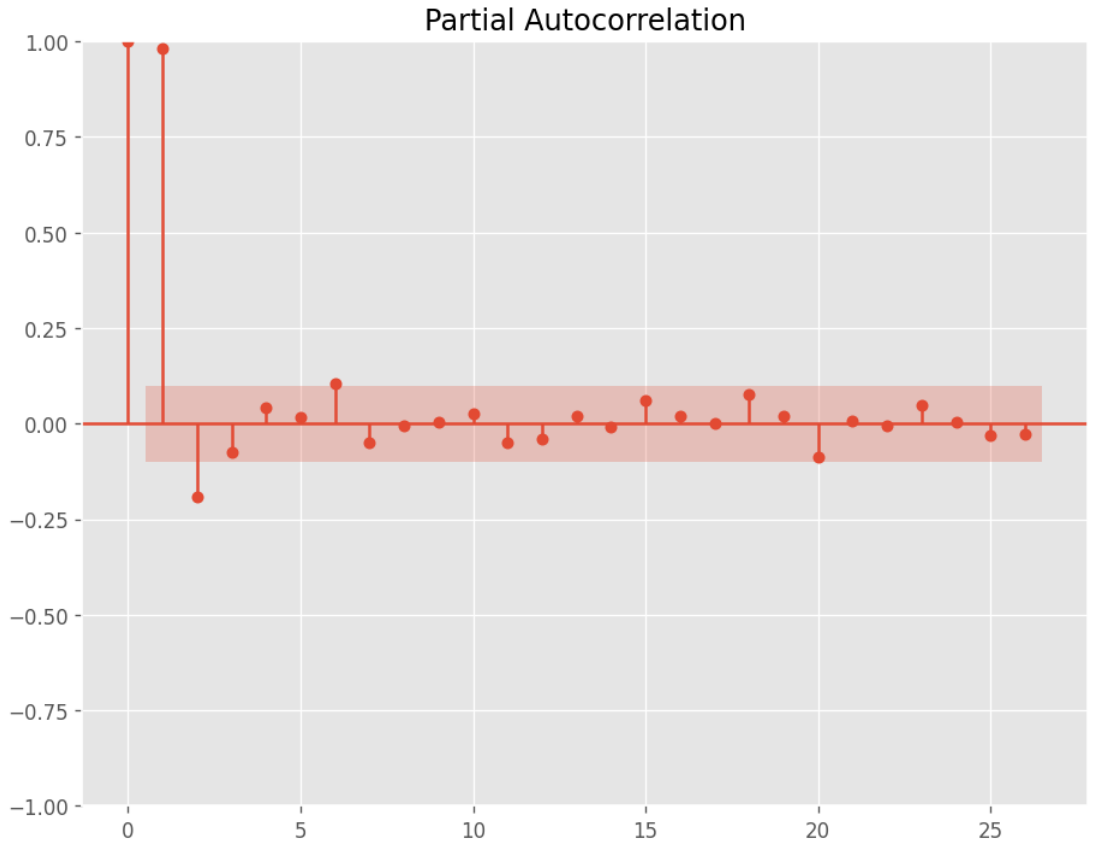

Figure 13: ACF plot of first order difference of coffee price index

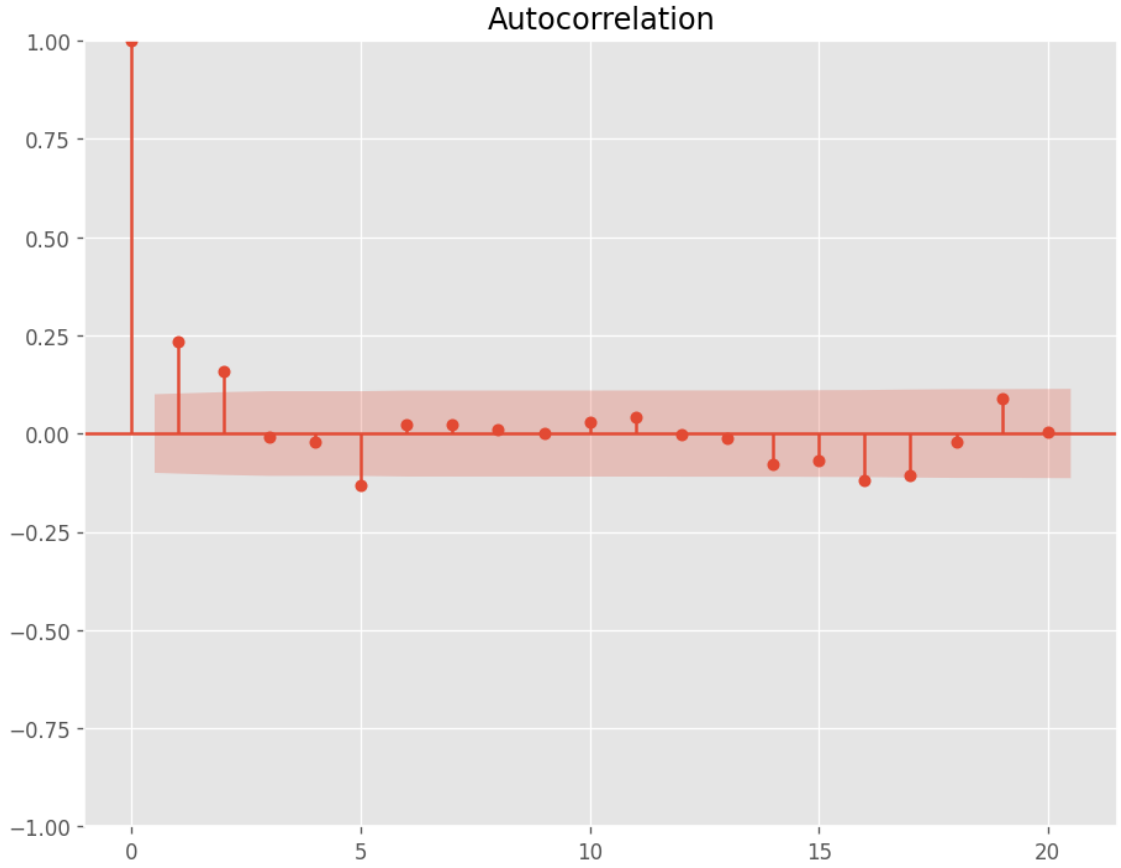

Figure 14: PACF plot of first order difference of coffee price index

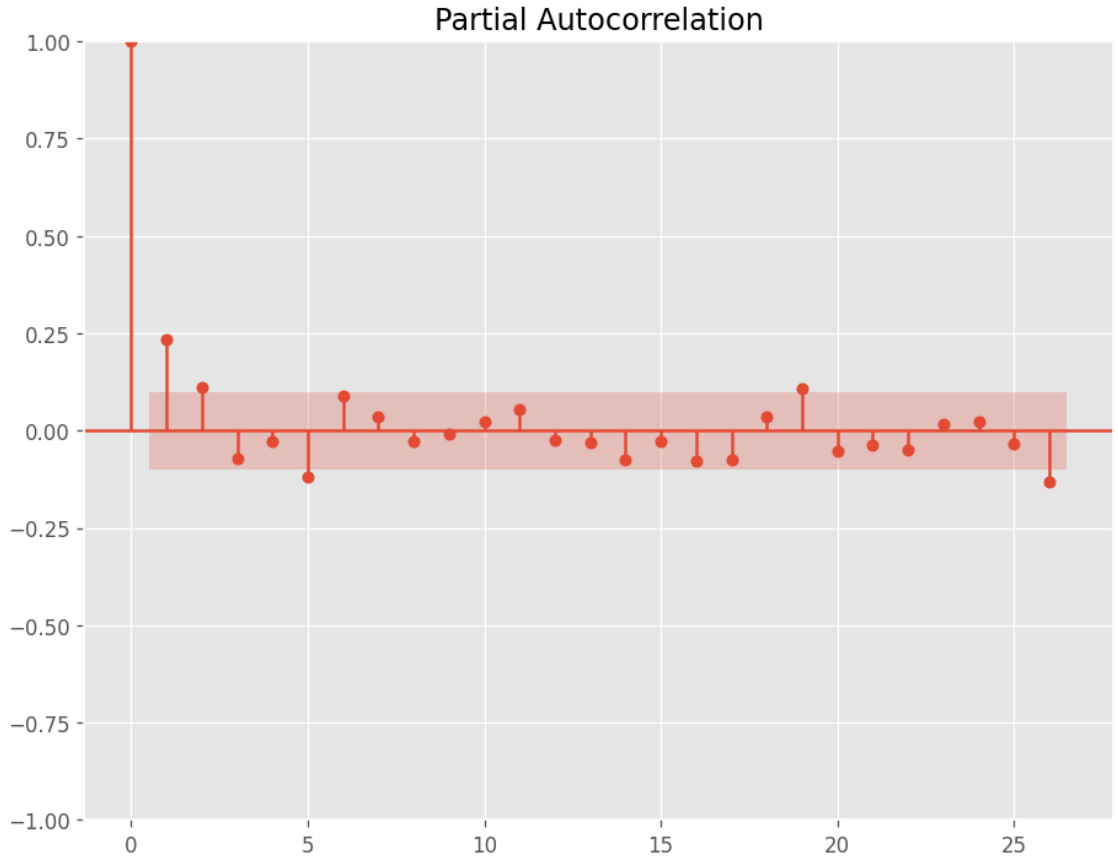

Table 1: Model specifications and corresponding MAPE & RMSE values for forecasts

Method	**Spec.**	**MAPE**	**RMSE**
Benchmark 1	-	0.036	4.14
MA-5	-	0.064	7.36
ES	$\alpha = 1.0$	0.036	4.14
Trend	-	0.051	5.41
Smoothed Trend	$\alpha = 0.7$ $\beta = 0.2$	0.078	8.31
6-month ahead	$\alpha = 0.8$ $\beta = 1.0$	0.234	26.51