OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

AS GCE MATHEMATICS (MEI)

4761/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4761/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows four forces acting at a point. The forces are in equilibrium.

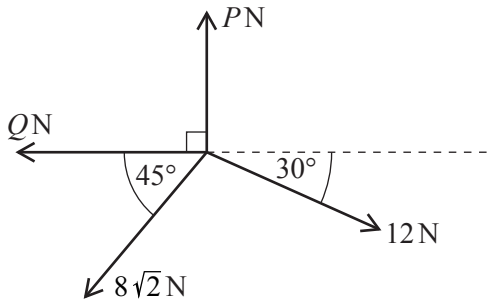


Fig. 1

Show that $P = 14$.

Find Q, giving your answer correct to 3 significant figures. **[5]**

2 Fig. 2 shows a 6 kg block on a smooth horizontal table. It is connected to blocks of mass 2 kg and 9 kg by two light strings which pass over smooth pulleys at the edges of the table. The parts of the strings attached to the 6 kg block are horizontal.

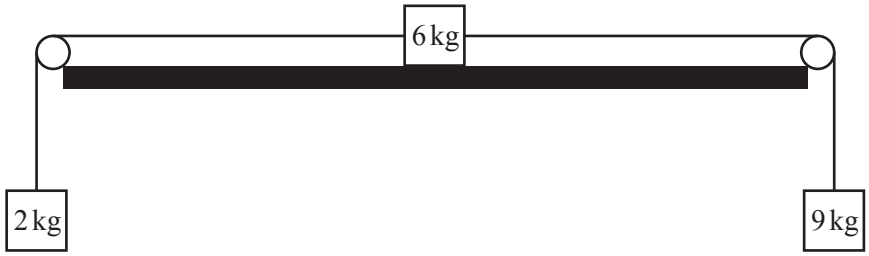


Fig. 2

(i) Draw three separate diagrams showing all the forces acting on each of the blocks. **[3]**

(ii) Calculate the acceleration of the system and the tension in each string. **[5]**

3 The map of a large area of open land is marked in 1 km squares and a point near the middle of the area is defined to be the origin. The vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are in the directions east and north.

At time t hours the position vectors of two hikers, Ashok and Kumar, are given by:

Ashok $\quad \mathbf{r}_A = \begin{pmatrix}-2\\0\end{pmatrix} + \begin{pmatrix}8\\1\end{pmatrix}t,$

Kumar $\quad \mathbf{r}_K = \begin{pmatrix}7t\\10-4t\end{pmatrix}.$

(i) Prove that the two hikers meet and give the coordinates of the point where this happens. **[4]**

(ii) Compare the speeds of the two hikers. **[3]**

4 Fig. 4 illustrates a straight horizontal road. A and B are points on the road which are 215 metres apart and M is the mid-point of AB.

When a car passes A its speed is $12\,\text{m s}^{-1}$ in the direction AB. It then accelerates uniformly and when it reaches B its speed is $31\,\text{m s}^{-1}$.

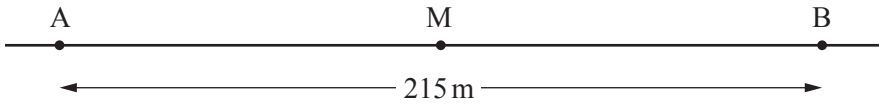


Fig. 4

(i) Find the car's acceleration. **[2]**

(ii) Find how long it takes the car to travel from A to B. **[2]**

(iii) Find how long it takes the car to travel from A to M. **[3]**

(iv) Explain briefly, in terms of the speed of the car, why the time taken to travel from A to M is more than half the time taken to travel from A to B. **[1]**

5 A golf ball is hit at an angle of 60° to the horizontal from a point, O, on level horizontal ground. Its initial speed is $20\,\text{m s}^{-1}$. The standard projectile model, in which air resistance is neglected, is used to describe the subsequent motion of the golf ball. At time t s the horizontal and vertical components of its displacement from O are denoted by x m and y m.

(i) Write down equations for x and y in terms of t. **[2]**

(ii) Hence show that the equation of the trajectory is

$$y = \sqrt{3}x - 0.049x^2.$$ **[2]**

(iii) Find the range of the golf ball. **[2]**

(iv) A bird is hovering at position $(20, 16)$.

Find whether the golf ball passes above it, passes below it or hits it. **[2]**

Section B (36 marks)

6 The battery on Carol and Martin's car is flat so the car will not start. They hope to be able to "bump start" the car by letting it run down a hill and engaging the engine when the car is going fast enough. Fig. 6.1 shows the road leading away from their house, which is at A. The road is straight, and at all times the car is steered directly along it.

- From A to B the road is horizontal.
- Between B and C, it goes up a hill with a uniform slope of 1.5° to the horizontal.
- Between C and D the road goes down a hill with a uniform slope of 3° to the horizontal. CD is 100 m. (This is the part of the road where they hope to get the car started.)
- From D to E the road is again horizontal.

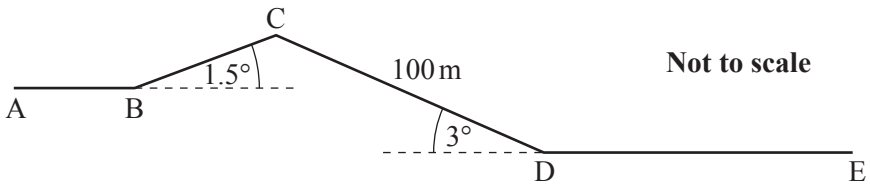


Fig. 6.1

The mass of the car is 750 kg, Carol's mass is 50 kg and Martin's mass is 80 kg.

Throughout the rest of this question, whenever Martin pushes the car, he exerts a force of 300 N along the line of the car.

(i) Between A and B, Martin pushes the car and Carol sits inside to steer it. The car has an acceleration of $0.25\,\text{m}\,\text{s}^{-2}$.

Show that the resistance to the car's motion is 100 N. **[3]**

Throughout the rest of this question you should assume that the resistance to motion is constant at 100 N.

(ii) They stop at B and then Martin tries to push the car up the hill BC.

Show that Martin cannot push the car up the hill with Carol inside it but can if she gets out.

Find the acceleration of the car when Martin is pushing it and Carol is standing outside. **[6]**

(iii) While between B and C, Carol opens the window of the car and pushes it from outside while steering with one hand. Carol is able to exert a force of 150 N parallel to the surface of the road but at an angle of 30° to the line of the car. This is illustrated in Fig. 6.2.

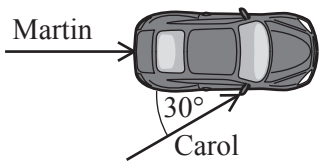


Fig. 6.2

Find the acceleration of the car. **[4]**

(iv) At C, both Martin and Carol get in the car and, starting from rest, let it run down the hill under gravity. If the car reaches a speed of $8\,\text{m}\,\text{s}^{-1}$ they can get the engine to start.

Does the car reach this speed before it reaches D? **[5]**

7 A box of emergency supplies is dropped to victims of a natural disaster from a stationary helicopter at a height of 1000 metres. The initial velocity of the box is zero.

At time t s after being dropped, the acceleration, $a\,\text{m s}^{-2}$, of the box in the vertically downwards direction is modelled by

$$a = 10 - t \quad \text{for} \quad 0 \leqslant t \leqslant 10,$$
$$a = 0 \quad \text{for} \quad t > 10.$$

(i) Find an expression for the velocity, $v\,\text{m s}^{-1}$, of the box in the vertically downwards direction in terms of t for $0 \leqslant t \leqslant 10$.

Show that for $t > 10$, $v = 50$. **[4]**

(ii) Draw a sketch graph of v against t for $0 \leqslant t \leqslant 20$. **[3]**

(iii) Show that the height, h m, of the box above the ground at time t s is given, for $0 \leqslant t \leqslant 10$, by

$$h = 1000 - 5t^2 + \frac{1}{6}t^3.$$

Find the height of the box when $t = 10$. **[4]**

(iv) Find the value of t when the box hits the ground. **[2]**

(v) Some of the supplies in the box are damaged when the box hits the ground. So measures are considered to reduce the speed with which the box hits the ground the next time one is dropped. Two different proposals are made. Carry out suitable calculations and then comment on each of them.

(*A*) The box should be dropped from a height of 500 m instead of 1000 m. **[2]**

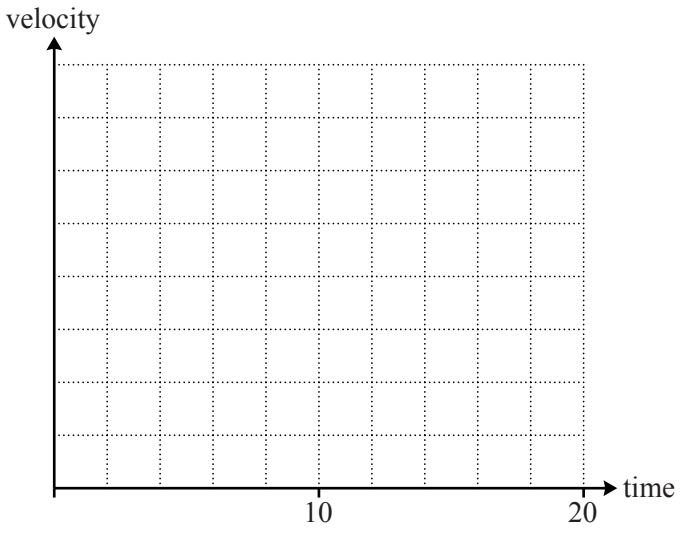
(*B*) The box should be fitted with a parachute so that its acceleration is given by

$$a = 10 - 2t \quad \text{for} \quad 0 \leqslant t \leqslant 5,$$
$$a = 0 \quad \text{for} \quad t > 5.$$ **[3]**

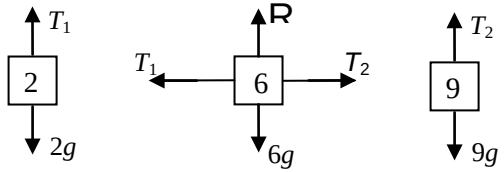
END OF QUESTION PAPER

7 (i)

7 (ii) **A spare copy of this diagram can be found on page 2.**



Question			Answer	Marks	Guidance	
1			$P = 8\sqrt{2}\sin 45° + 12\sin 30°$	**M1**	Considering equilibrium in the vertical direction	
				M1	Resolution of forces of 12 N and $8\sqrt{2}$ N in the vertical direction. Do not allow sin-cos interchange for the 30^o angle.	
			$P = 14$	**A1**	Dependent on both M marks	
			$Q + 8\sqrt{2}\cos 45° = 12\cos 30°$	**B1**		
			$Q = 2.39$	**B1**		
				[5]		

2	(i)			**B1** **B1** **B1** **[3]**	Diagrams for both 2 and 9 kg blocks. The tensions must be different from each other. No extra forces. Tensions on 6 kg block. The tensions must be different from each other. No extra forces. 6g and R on 6 kg block. No extra forces. **Special Case** When the tensions are given as T_1, T_2, T_3, T_4 (or equivalent) award up to SC1 SC0 for the first two marks.	
2	(ii)		$9g - T_2 = 9a$ $T_2 - T_1 = 6a$ $T_1 - 2g = 2a$ $a = \frac{7}{17}g = 4.04$ (m s^{-2}) $T_1 = 27.7$ (N) $T_2 = 51.9$ (N)	**M1** **M1** **A1** **A1** **A1** **[5]**	First equation correct Both the remaining two equations correct. Do not give this mark if both tensions are shown as the same. The final three marks are dependent on both M marks a, T_1 and T_2 may be found in any order and FT should be allowed from the first of these found	
	(ii)		**Alternative: Whole system** $9g - 2g = 17a$ $a = \frac{7g}{17} = 4.04$ $T_1 - 2g = 2a$ and $9g - T_2 = 9a$ $T_1 = 27.7$ (N) $T_2 = 51.9$ (N)	 **M1** **A1** **M1** **A1** **A1**	 Both equations correct. Oe. The final two marks are dependent on both M marks. T_1 and T_2 may be found in either order and FT should be allowed from their value for a.	

3	(i)		**Either** $-2+8t=7t$ **Or** $t=10-4t$ $\Rightarrow t=2$ Substituting $t=2$ in **both** expressions They meet at (14, 2)	**M1** **A1** **B1** **B1** **[4]**	Forming an equation for *t*. Accept vector equation for this mark. May be implied by a statement that *t* = 2. oe, eg showing $t=2$ satisfies both equations or a vector equation. Accept $\begin{pmatrix}14\\2\end{pmatrix}$	
3	(ii)		Ashok's speed is $\sqrt{8^2+1^2}=\sqrt{65}$ Kumar's speed is $\sqrt{7^2+(-4)^2}=\sqrt{65}$ km h^{-1} They both walk at the same speed	**B1** **B1** **B1** **[3]**	CAO from correct speeds SC1 for finding both velocities correctly but neither speed	

Follow through between parts of Question 4 should be allowed for the value of *a* found in part (i) into parts (ii) and (iii).						
4	**(i)**		$v^2 - u^2 = 2as$ $31^2 - 12^2 = 2 \times 215 \times a$ $a = 1.9$ so $1.9\ \text{m s}^{-2}$	**M1** **A1** **[2]**	Selection and use of appropriate equation(s)	
4	**(ii)**		$v = u + at$ $31 = 12 + 1.9t$ $t = 10$ so 10 s	**M1** **A1** **[2]**	Selection and use of appropriate equation(s) FT from their value of *a* from part (i).	

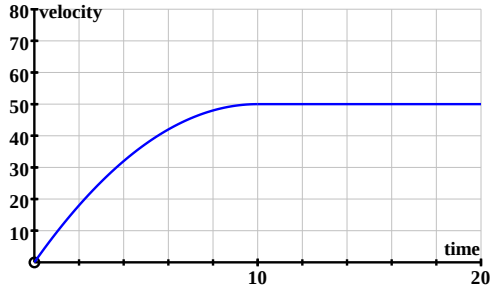
4	(iii)		$s = ut + \frac{1}{2}at^2$ $\frac{215}{2} = 12t + \frac{1}{2} \times 1.9 \times t^2$ $\left(t = \frac{-12 \pm \sqrt{12^2 + 4 \times 0.95 \times 107.5}}{1.9} \right)$ $t = 6.055$ (or -18.69)	 **M1** **M1** **A1** **[3]**	 Selection and use of $s = ut + \frac{1}{2}at^2$, oe. Correct elements but condone minor arithmetic errors. Use of quadratic formula (may be implied by answer), oe. FT their *a* only.	
			Alternative: Finding a 2-stage method $v^2 - u^2 = 2as$ and $s = \frac{(u+v)}{2}t$ $v = \pm\sqrt{12^2 + 2 \times 1.9 \times 107.5} = (\pm)23.505...$ $s = \frac{(u+v)}{2}t \Rightarrow t = \frac{2 \times 107.5}{(12 + 23.505...)}$ $\left(\text{or } t = \frac{2 \times 107.5}{(12 - 23.505...)} \right)$ $t = 6.055$ (or 18.69)	 **M1** **M1** **A1**	 Selection and use of a complete valid 2-stage method Using the output from the first stage to find *t* FT their *a* only.	

4	**(iv)**		Because it is accelerating, it travels less fast in the first half of the distance and so takes more time.	**B1** **[1]**	The answer must refer to the two parts of the distance (or "the same distance") so no credit is given to answers like "Because it is accelerating" and "Because its speed is not uniform". Most successful answers will refer to the times to cover AM and MB but this may be implicit. So B1 should be given for an answer like "It is travelling faster between M and B than it is between A and M" Notice that the fact that the acceleration is uniform is irrelevant.	

5	(i)		$x = 10t$ $y = 10\sqrt{3}t - 4.9t^2$	B1 B1 [2]	Allow $x = 20\cos 60^\circ t$ Allow $y = 20\sin 60° \, t - \frac{g}{2}t^2$ or $y = 17.3\, t - \frac{9.8}{2}t^2$	
5	(ii)		Substitute $t = \frac{x}{10}$ in equation for y $\Rightarrow y = \sqrt{3}x - 0.049x^2$	M1 A1 [2]	Substitution of a correct expression for t. Notice that this is a given result	
5	(iii)		When $y = 0$, $x = \frac{1.732}{0.049}$ (or 0) The range is 35.3 m	M1 A1 [2]	Use of $y = 0$, or 2 × Time to maximum height	
5	(iv)		When $x = 20, y = 1.732 \times 20 - 0.049 \times 20^2$ Height is 15.04 m so passes below the bird whose height is 16 m	M1 A1 [2]	Use of equation of trajectory **Special Case** Allow **SC2** for substituting $y = 16$ in the trajectory, showing the equation for x has no real roots and concluding the height of the ball is always less than16 m. This can also be done with the equation for vertical motion.	
	(iv)		**Alternative: Using time** When $x = 20$, $t = 2$ $y = 10\sqrt{3} \times 2 - 4.9 \times 2^2$ Height is 15.04 m so passes below the bird whose height is 16 m	 M1 A1	 Use of equation for the height	
	(iv)		**Alternative: Maximum height** The maximum height of the ball (is 15.3 m) Since15.3 < 16, it is always below the bird	 M1 A1	 A valid method for finding the maximum height	

Follow through between parts of Question 6 should be allowed for values found in parts (ii) and (iii) providing the questions are not simplified.						
6	**(i)**		$F - R = ma$ $300 - R = (750 + 50) \times 0.25$ $R = 100$	**M1** **A1** **A1** **[3]**	Use of Newton's 2nd Law Correct elements present This is a given result	
6	**(ii)**		**Carol in** Component of weight down slope $= 800g \sin 1.5° \ (= 205.2 \text{ N})$ Martin has to overcome 305.2 N $300 < 305.2$ Martin cannot manage	**M1** **A1** **A1**	Resolving down the slope. Accept use of 750 instead of 800. For this mark only condone no g and allow sin-cos interchange. Give M1 A1 for $800g\sin 15^o$ seen This mark may be awarded for an argument based on Newton's 2nd law leading towards a = -0.006	
			Carol out Martin has to overcome $750g \sin 1.5° + 100 = 292.4\text{N}$ $300 > 292.4$ so Martin manages $300 - 292.4 = 7.6 = 750a$ The acceleration is 0.010 m s^{-2}	**B1** **M1** **A1** **[6]**	Explanation, based on correct working, that Martin can manage. This can be given retrospectively with a comment on a positive value for a. Use of Newton's 2nd Law Cao. Accept 0.01 or an answer that rounds to 0.01.	
6	**(iii)**		Component of Carol's force parallel to the line of the car $= 150 \cos 30° \ (= 129.9)$ Resultant forward force $= 7.6 + 129.9 = 137.5$ $750a = 137.5$ The acceleration is 0.183 m s^{-2}	**M1** **A1** **M1** **A1** **[4]**	For attempt at resolution in the correct direction. For this mark only, condone sin-cos interchange. Give M1 A1 for $150\cos 30^o$ seen All forces parallel to the slope present and correct. Sign errors condoned. FT their force parallel to the slope from part (ii) (correct value 7.6 N)	

6	(iv)		Component of weight down the slope $=(750+50+80)\times 9.8\times \sin 3°$			
			$880a = 451.3 - 100$	**M1**	Newton's 2nd law with correct elements present. No sin-cos interchange. The same mass must be used in both places.	
			$a = 0.399$	**A1**		
			$v^2 - u^2 = 2as$			
			When $v = 8$, $s = 8^2 \div (2\times 0.399)$	**M1**	Selection and use of an appropriate formula (unless with $a = g$)	
			$s = 80.1$	**A1**	FT their value of a	
			$80.1 < 100$ so Yes they get the car started	**A1**	FT their value of a	
				[5]		
	(iv)		**Alternative: Finding the speed after 100 m**			
			Component of weight down the slope $=(750+50+80)\times 9.8\times \sin 3°$	**M1**	Newton's 2nd law with correct elements present. No sin-cos interchange	
			$880a = 451.3 - 100$			
			$a = 0.399$	**A1**		
			$v^2 - u^2 = 2as$			
			$v^2 = (0^2) + 2\times 0.399\times 100$	**M1**	Selection and use of an appropriate formula (unless with $a = g$)	
			$v = (\sqrt{79.8}) = 8.93...$	**A1**	FT their value of a	
			($v > 8$) so they get the car started	**A1**	FT their value of a	

Follow through between parts of Question 7 should be allowed for the value of *h* (when *t* = 10) found in part (iii) if it is used in part (iv) or in part (v)(*A*).						
7	**(i)**		Integrate a to obtain v $v = 10t - \frac{1}{2}t^2 \quad (+c)$ $t = 10 \Rightarrow v = 100 - 50 = 50$ Since $a = 0$ for $t > 10$, $v = 50$ for $t > 10$	**M1** **A1** **M1** **A1** **[4]**	Attempt to integrate Substitution of $t = 10$ to find v Sound argument required for given answer. It must in some way refer to $a = 0$.	
7	**(ii)**		Continuous two part v-t graph  Curve for $0 \le t \le 10$ Horizontal straight line for $10 \le t \le 20$	**B1** **B1** **B1** **[3]**	The graph must cover $t = 0$ to $t = 20$ B0 if no vertical scale is given	

7	(iii)		Distance fallen = $\int\left(10t-\frac{1}{2}t^2\right)\mathrm{d}t$ $d = 5t^2-\frac{1}{6}t^3+c \quad (c=0)$ Height = $1000 - d$ Height = $1000-5t^2+\frac{1}{6}t^3$ When $t=10$, $h=667$	**M1** **A1** **A1** **B1** **[4]**	Attempt to integrate This mark should only be given if the signs are correctly obtained. oe	
7	(iv)		Time at constant vel $=667\div 50=13.3$ Total time $t=10+13.3=23.3$	**B1** **B1** **[2]**	FT for *h* from part (iii) FT	
7	(v)	***A***	Since $500>333$ The box will have reached terminal speed. So there is no improvement	**M1** **A1** **[2]**	For finding the height at which the crate reaches terminal velocity, eg $h = 167$, or equivalent relevant calculation. FT for *h* from part (iii) if used. Allow either one (or both) of these two statements.	
7	(v)	***B***	$v=10t-t^2$ (for $t\le 5$) Terminal velocity is 25 $\mathrm{m\,s^{-1}}$ So better	**M1** **A1** **A1** **[3]**	Integration to find *v*	