Solutions

For MEI students

12 Let $f(x) = \frac{x+2}{x^3-8}$ where $-3 < x < 3$.

(a) Write down the equation of the vertical asymptote of $f(x)$.

(b) Sketch $f(x)$. Clearly mark all asymptotes and points of interest.

13 Let $f(x) = \frac{a-x}{ax+1}$ where a is an integer and $-2 < x < 2$.

(a) Find the points where $f(x)$ crosses the x and y axes in terms of a.

(b) Find the equation of the vertical asymptote of $f(x)$ in terms of a.

(c) Hence sketch $f(x)$ for $a = 4$.

(d) If $a \geq 1$, find the smallest value of x such that $f(x) \geq 1$.

14 The quadratic $ax^2 + bx + c$ has roots α and β.

(a) Form a new quadratic with the repeated root $\frac{1}{\alpha+\beta}$.

(b) Form a new quadratic with roots $\alpha - 1$ and $\beta - 1$.

15 You are given that $\frac{5}{r(r+5)} = \frac{1}{r} - \frac{1}{r+5}$.

(a) Evaluate the sum

$$\sum_{r=0}^{n} \frac{5}{r(r+5)}$$

(b) Hence find

$$\lim_{n\to\infty} \sum_{r=0}^{n} \frac{5}{r(r+5)}$$

12. $y = \dfrac{x+2}{x^3-8}$

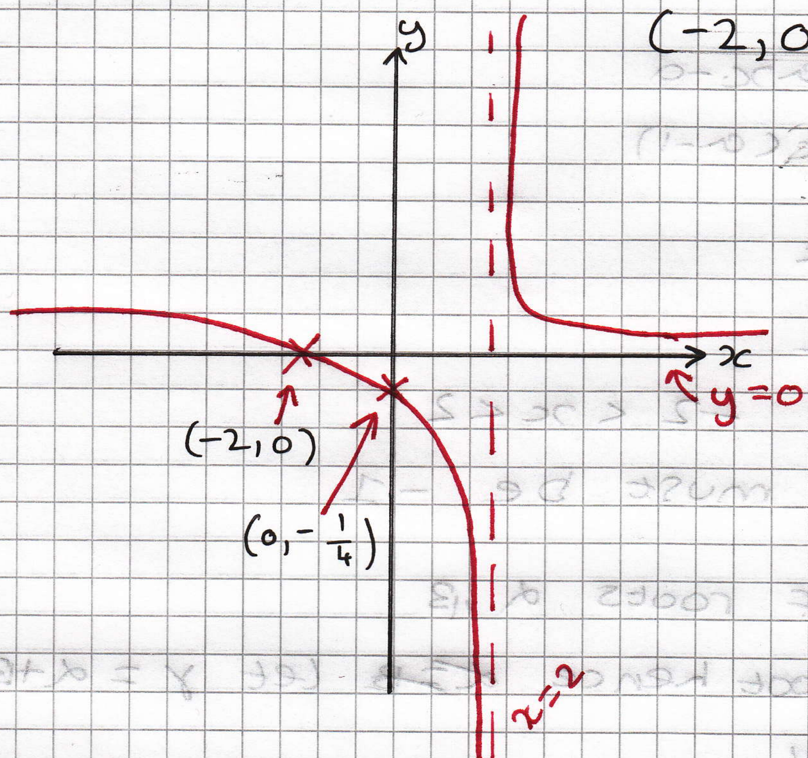
a) $x^3 - 8 = 0$

$x^3 = 8 \rightarrow \underline{x = 2}$

b) Horizontal Asymptote $\Rightarrow y = 0$

* For large positive value of X (ie, 100), curve is above horizontal asymptote
* For large negative value of X (ie, −100) curve is above asymptote

Points of Intersection: $(0, -1/4)$
$(-2, 0)$



13. $y = \dfrac{a-x}{ax+1}$

a) When $x = 0$, $y = \dfrac{a}{1}$

$\therefore (0, a)$

When $y = 0$, $a = x$

$(a, 0)$

b) Vertical $\rightarrow ax + 1 = 0$

hence $x = -\dfrac{1}{a}$

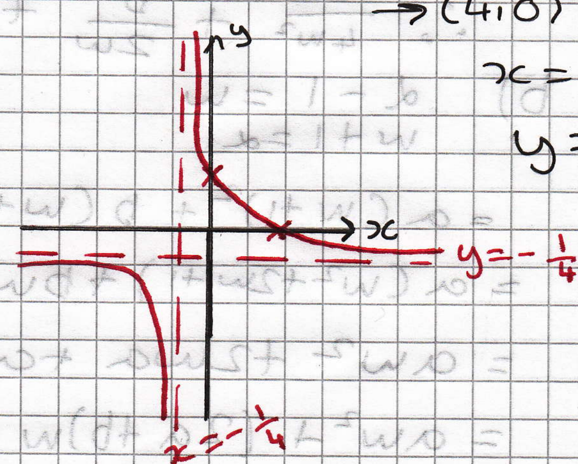
c) When $(a = 4)$

$y = \dfrac{4-x}{4x+1}$

Intersections $\rightarrow (0, 4)$
$\rightarrow (4, 0)$

$x = -\dfrac{1}{4}$

$y = -\dfrac{1}{4}$



13 d) Where $a \geq 1$, find x when $f(x) \geq 1$ (least)

$f(x) = \cancel{\frac{ax-1}{}}$

$\hookrightarrow \dfrac{a-x}{ax+1}$

- $\dfrac{a-x}{ax+1} \geq 1$
- $a - x \geq ax + 1$
- $a - 1 \geq ax - a$
- $a - 1 \geq x(a-1)$
- $\dfrac{a-1}{a-1} \geq x$
- $x \leq 1$

Condition $\rightarrow$ $-2 < x < 2$

$\therefore$ Least value must be -1

$\underline{x = -1}$

14. $ax^2 + bx + c$ of roots α, β

a) Repeated Root hence $\cancel{\alpha = \beta}$ let $\gamma = \alpha + \beta$

$\dfrac{1}{2\alpha} = w$

$1 = 2\alpha w \rightarrow \dfrac{1}{2w} = \alpha$

$= a\left(\dfrac{1}{2w}\right)^2 + b\left(\dfrac{1}{2w}\right) + c$

$\therefore \dfrac{a}{4w^2} + \dfrac{b}{2w} + c$

b) $\alpha - 1 = w$

$w + 1 = \alpha$

$= a(w+1)^2 + b(w+1) + c$

$= a(w^2 + 2w + 1) + bw + b + c$

$= aw^2 + 2wa + a + bw + b + c$

$= aw^2 + (2a+b)w + (a+b+c)$

15. a) $$\frac{5}{r(r+5)} = \frac{1}{r} - \frac{1}{r+5}$$

$$= \frac{1}{1} - \frac{1}{n+5}$$

$$= \frac{(n+5) - 1}{n+5} = \cancel{\frac{n-6}{n+5}} = \underline{\frac{n+4}{n+5}}$$

b) when $n \to \infty$, $\left(\frac{1}{r+5}\right) \to 0$

hence when $n \to \infty$

$$\to \frac{1}{1} - \frac{1}{n+5} = \frac{1}{1} - \frac{1}{\infty}$$

$$= \frac{1}{1} - 0$$

$$= \underline{1}$$

Limit = 1