OCR
Oxford Cambridge and RSA

Thursday 14 May 2015 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

This paper has been pre modified for carrier language

Section A (36 marks)

1 Given that $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 8 & 21 \end{pmatrix}$, find x and y. **[6]**

2 Find the roots of the quadratic equation $z^2 - 4z + 13 = 0$.

Find the modulus and argument of each root. **[5]**

3 The equation $2x^3 + px^2 + qx + r = 0$ has a root at $x = 4$. The sum of the roots is 6 and the product of the roots is -10. Find p, q and r. **[6]**

4 Indicate, on a single Argand diagram

(i) the set of points for which $\arg(z - (-1 - \mathrm{j})) = \frac{\pi}{4}$, **[2]**

(ii) the set of points for which $|z - (1 + 2\mathrm{j})| = 2$, **[2]**

(iii) the set of points for which $|z - (1 + 2\mathrm{j})| \geqslant 2$ and $0 \leqslant \arg(z - (-1 - \mathrm{j})) \leqslant \frac{\pi}{4}$. **[2]**

5 **(i)** Show that $\sum_{r=1}^{n} (2r - 1) = n^2$. **[3]**

(ii) Show that $\dfrac{\sum_{r=1}^{n} (2r-1)}{\sum_{r=n+1}^{2n} (2r-1)} = k$, where k is a constant to be determined. **[4]**

6 A sequence is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 5$. Prove by induction that $u_n = \dfrac{3^{n-1} + 5}{2}$. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(3x+2)(x-3)}{(x-2)(x+1)}$.

(i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. **[4]**

(ii) Sketch the curve, justifying how it approaches the horizontal asymptote. **[5]**

(iii) Find the set of values of x for which $y \geqslant 3$. **[3]**

8 The complex number 5 + 4j is denoted by α.

(i) Find α^2 and α^3, showing your working. **[3]**

(ii) The real numbers q and r are such that $\alpha^3 + q\alpha^2 + 11\alpha + r = 0$. Find q and r. **[4]**

Let $\mathrm{f}(z) = z^3 + qz^2 + 11z + r$, where q and r are as in part **(ii)**.

(iii) Solve the equation $\mathrm{f}(z) = 0$. **[3]**

(iv) Solve the equation $z^4 + qz^3 + 11z^2 + rz = z^3 + qz^2 + 11z + r$. **[2]**

9 The triangle ABC has vertices at $\mathrm{A}(0,0)$, $\mathrm{B}(0,2)$ and $\mathrm{C}(4,1)$. The matrix $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ represents a transformation T.

(i) The transformation T maps triangle ABC onto triangle $\mathrm{A'B'C'}$. Find the coordinates of A′, B′ and C′. **[3]**

Triangle $\mathrm{A'B'C'}$ is now mapped onto triangle $\mathrm{A''B''C''}$ using the matrix $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$.

(ii) Describe fully the transformation represented by **M**. **[3]**

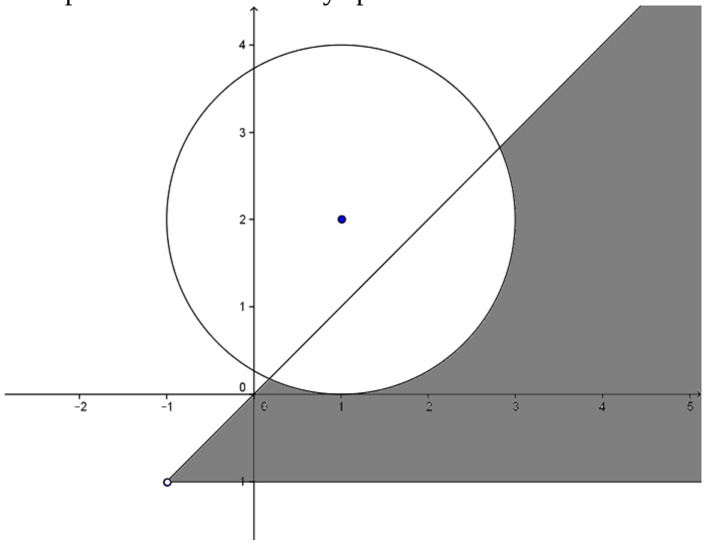
(iii) Triangle $\mathrm{A''B''C''}$ is now mapped back onto ABC by a single transformation. Find the matrix representing this transformation. **[3]**

(iv) Calculate the area of $\mathrm{A''B''C''}$. **[3]**

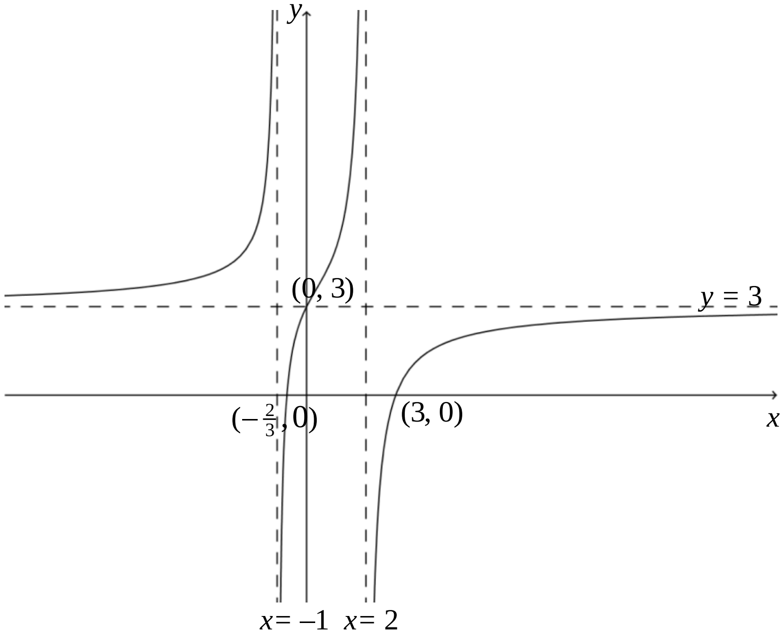
END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1			$\mathbf{M}^{-1}=\frac{1}{108}\begin{pmatrix}21 & 3\\ -8 & 4\end{pmatrix}$	M1* M1* A1	Attempt to find $\mathbf{M}^{-1}$ or $108\mathbf{M}^{-1}$ Divide by their determinant, Δ, at some stage Correct determinant, (A0 for det **M**= $\frac{1}{108}$ stated, all other marks are available)
			$\frac{1}{108}\begin{pmatrix}21 & 3\\ -8 & 4\end{pmatrix}\begin{pmatrix}1\\ 3\end{pmatrix}=\begin{pmatrix}\frac{5}{18}\\ \frac{1}{27}\end{pmatrix}$	M1 A1	Attempt to **pre**-multiply by inverse or by Δ $\mathbf{M}^{-1}$ Correct matrix multiplication (allow one slip)
			$x=\frac{5}{18},\ y=\frac{1}{27}$,oe	A1dep* **[6]**	For both, cao x and y must be specified, may be in column vectors **SC** answers only B1
		OR			
			$4x-3y=1$ $8x+21y=3$	M1 A1	Using **M** to create two equations Correct equations
			Eliminating x or y	M1	Any valid method
			Finding second unknown	M1	Valid method
			$x=\frac{5}{18}, y=\frac{1}{27}$ Allow 3 dp or better.	A1A1	For each cao. **SC** Answers only B1
				[6]	
2			$2+3\text{j}$ and $2-3\text{j}$	B1	For both, accept $2\pm3\text{j}$
			Modulus $=\sqrt{(2^2+3^2)}=\sqrt{13}$	M1	Attempt at modulus of their complex roots
			Argument $=\pm\arctan\left(\frac{3}{2}\right)=\pm0.983$	M1	Attempt at $\arctan\left(\pm\frac{3}{2}\right)$ ft their complex roots
			$2+3\text{j}$ has modulus $\sqrt{13}$ and argument 0.983	A1ft	Moduli specified, ft their roots. Accept √13 only
			$2-3\text{j}$ has modulus $\sqrt{13}$ and argument -0.983	A1ft **[5]**	ft their roots - must be in $(-\pi,\pi]$ Accept ±0.983, ±56.3° If 2 sf given accuracy MUST be stated.

Question			Answer	Marks	Guidance	
3			$\frac{-p}{2}=6 \Rightarrow p=-12$	M1,M1	M1 use of $\sum\alpha$ for p and M1 use of $\alpha\beta\gamma$ for r - allow one sign error; 2 sign errors is M1 M0	
			$\frac{-r}{2}=-10 \Rightarrow r=20$	A1 A1	for p, cao for r, cao	
			OR $\alpha+\beta+4=6$, $4\alpha\beta = -10$	**OR**		
			Implies α,β satisfy $2x^2-4x-5=0$	M1	Valid method to create a quadratic equation	
			Roots $1\pm\frac{\sqrt{14}}{2}$	M1	Attempt to solve a 3-term quadratic	
			$-\frac{p}{2}=1+\frac{\sqrt{14}}{2}+1-\frac{\sqrt{14}}{2}+4=6 \Rightarrow p=-12$	A1	for p, cao	
			Product of roots $=-10=-\frac{r}{2}\Rightarrow r=20$	A1	for r, cao	
			THEN	**THEN**		
			EITHER $x=4$ is a root, so $2\times64+16p+4q+r=0$	M1	Substitution and attempt to solve for coefficient of x^2,(or for the remaining unknown.) Allow making q the subject if p and r not found.	
			OR $\alpha+\beta+4=6\Rightarrow\alpha+\beta=2$ $4\alpha\beta=-10\Rightarrow\alpha\beta=-\frac{10}{4}$			
			$\frac{q}{2}=4\alpha+4\beta+\alpha\beta=4\times2-\frac{5}{2}$		OR M1 using $\sum\alpha\beta$ OR use of remainder after division	
			$\Rightarrow q=11$	A1 **[6]**	for q, cao	

Question			Answer	Marks	Guidance
4	(i)		Accept un-numbered evenly spaced marks on axes to show scale 	B1 B1 **[2]**	Line at acute angle, all or part in Im z>0 Half line from -1- j through 0 [don't penalise if point -1- j is included] Allow near miss to 0 if π/4 marked SC correct diagram, no annotations seen B1 B0
4	(ii)			B1 B1 **[2]**	Circle centre 1 + 2j Radius 2 Must touch real axis SC correct diagram, no annotations seen B1 B0
4	(iii)			B1 B1 **[2]**	The shaded region must be outside their circle and have a border with the circumference Fully correct SC correct diagram, no annotations seen allow B1 B1
5	(i)		$\sum_{r=1}^{n}(2r-1)=2\sum_{r=1}^{n}r-n$ $=n(n+1)-n \ = \ n^2$	M1 M1 A1 **[3]**	Attempt to split into two sums (May be implied) Use of standard result for Σr cao (must be in terms of n) SC Induction: B1 case n = 1 : E1 sum to k + 1 terms correctly found : E1 argument completely correct
5	(ii)		$\dfrac{\sum_{r=1}^{n}(2r-1)}{\sum_{r=n+1}^{2n}(2r-1)}=\dfrac{n^2}{(2n)^2-n^2}$ $=\dfrac{n^2}{3n^2}=\dfrac{1}{3}=k$	M1 M1 A1 A1 **[4]**	Use of result from (i) in numerator of a fraction Expressing denominator as $\sum_{r=1}^{2n}....-\sum_{r=1}^{n}....$ need not be explicit, or other valid method. Correct sums $k=\dfrac{1}{3}$

Question			Answer	Marks	Guidance
6			$u_1 = 3$ and $\frac{3^{1-1}+5}{2} = 3$, so true for $n = 1$	B1	Must show working on given result with $n = 1$
			Assume true for $n = k$ $\Rightarrow u_k = \frac{3^{k-1}+5}{2}$	E1	Assuming true for k Allow "Let $n = k$ and (result)" "If $n = k$ and (result)" Do not allow " $n = k$" or "Let $n = k$", without the result quoted, followed by working
			$\Rightarrow u_{k+1} = 3\left(\frac{3^{k-1}+5}{2}\right) - 5$	M1	u_{k+1} with substitution of result for u_z and some working to follow
			$= \frac{3^k+15}{2} - 5$ $= \frac{3^k+15-10}{2}$		
			$= \frac{3^k+5}{2}$	A1	Correctly obtained
			$= \frac{3^{n-1}+5}{2}$ when $n = k+1$		Or target seen
			Therefore **if** true for $n = k$ it is **also true** for $n = k+1$.	E1	Both points explicit Dependent on A1 and previous E1
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[6]**	Dependent on B1 and previous E1
7	(i)		Asymptotes: $y = 3$, $x = 2$, $x = -1$	B1 B1	 (both) Allow $x = 2, -1$
			Crosses axes at $(0,\ 3)$	B1	Must see values for x and y if not written as co-ordinates
			$\left(\frac{-2}{3},\ 0\right)$, $(3,\ 0)$	B1	(both) Must see values for x and y if not written as co-ordinates.
				[4]	

Question			Answer	Marks	Guidance	
7	(ii)			B1	Intercepts labelled (single figures on axes suffice)	
				B1	Asymptotes correct and labelled. Allow $y = 3$ shown by intercept labelled at (0,3) and $x = 2$ and $x = -1$ likewise	
				B2	Three correct branches (-1 each error) Any poorly illustrated asymptotic approaches penalised once only.	
			When x is large and positive, graph approaches $y = 3$ from below, e.g. for $x = 100$, $\frac{302\times97}{98\times101}=2.9...$ When x is large and negative, graph approaches $y = 3$ from above, e.g. for $x = -100$, $\frac{-298\times-103}{-102\times-99}=3.03...$	B1 **[5]**	Approaches to $y = 3$ justified There must be a result for y	
7	(iii)		$y\geq3\Rightarrow0\leq x<2$ or $x<-1$	B1 B1B1 **[3]**	$x<-1$ $0\leq x<2$ (B1 for $0 < x < 2$ or $0 \leq x \leq 2$) isw any more shown	

Question			Answer	Marks	Guidance
8	(i)		$(5+4\mathrm{j})^2=(5+4\mathrm{j})(5+4\mathrm{j})=25+40\mathrm{j}-16=9+40\mathrm{j}$ $(5+4\mathrm{j})^3=-115+236\mathrm{j}$	M1 A1 A1 **[3]**	Use of $\mathrm{j}^2=-1$ at least once
8	(ii)		$\alpha^3+q\alpha^2+11\alpha+r=0$ $\Rightarrow -115+236\mathrm{j}+9q+40q\mathrm{j}+55+44\mathrm{j}+r=0$ $\Rightarrow (236+40q+44)\mathrm{j}=0$, $-115+9q+55+r=0$ $\Rightarrow q=-7$ $\Rightarrow r=123$	 M1 M1 A1ft A1ft **[4]**	 Substitute for α Compare either real or imaginary parts $q=-7$ ft their α^2 and α^3 $r=123$ ft their α^2 and α^3
8	(iii)		$\mathrm{f}(z)=z^3-7z^2+11z+123$ Sum of roots $=7$ $\Rightarrow (5+4\mathrm{j})+(5-4\mathrm{j})+w=7$ $\Rightarrow w=-3$ Roots are $5+4\mathrm{j}$ and $5-4\mathrm{j}$ and -3	 M1 B1 A1 **[3]**	 Valid method for the third root. (division , factor theorem, attempt at linear x quadratic with complex roots correctly used) quoted cao real root identified, A0 if extra roots found
8	(iv)		$z\mathrm{f}(z)=\mathrm{f}(z)\Rightarrow(z-1)f(z)=0$ $\Rightarrow z=1$ or $\mathrm{f}(z)=0$ $\Rightarrow z=1,\ z=-3, z=5+4\mathrm{j}, z=5-4\mathrm{j}$	 M1 A1ft **[2]**	 solving $z-1=0$, **and** $f(z)=0$ (may be implied) For all four solutions [ft (iii)] NB incomplete method giving z = 1 only is M0 A0

Question			Answer	Marks	Guidance
9	(i)		$\begin{pmatrix}1&-2\\3&0\end{pmatrix}\begin{pmatrix}0&0&4\\0&2&1\end{pmatrix}$	M1	Any valid method – may be implied
			$=\begin{pmatrix}0&-4&2\\0&0&12\end{pmatrix}$	A1	Correct position vectors found (need not be identified)
			$A'=(0,0),\ B'=(-4,0),\ C'=(2,12)$	A1ft	co-ordinates, ft their position vectors A', B', C' identifiable. Coordinates only, M1A0A1
				[3]	
9	(ii)		**M** represents a two-way stretch	B1	Stretch. (enlargement B0)
			factor 4 parallel to the *x* axis factor 2 parallel to the *y* axis	B1 B1	Directions indicated
				[3]	
9	(iii)		$\begin{pmatrix}4&0\\0&2\end{pmatrix}\begin{pmatrix}1&-2\\3&0\end{pmatrix}$	M1	Attempt at MT in correct sequence
			$=\begin{pmatrix}4&-8\\6&0\end{pmatrix}$	A1	cao
			Represents the composite transformation T followed by M		
			$\begin{pmatrix}4&-8\\6&0\end{pmatrix}^{-1}=\frac{1}{48}\begin{pmatrix}0&8\\-6&4\end{pmatrix}$ represents the single transformation	A1	cao
				[3]	
		OR	$\frac{1}{6}\begin{pmatrix}0&2\\-3&1\end{pmatrix}\quad\frac{1}{8}\begin{pmatrix}2&0\\0&4\end{pmatrix}=\frac{1}{48}\begin{pmatrix}0&8\\-6&4\end{pmatrix}$	B1 M1 A1 **[3]**	for T^{-1} and M^{-1} correct for attempt at $T^{-1}M^{-1}$ cao
		OR	$\begin{pmatrix}0&-16&8\\0&0&24\end{pmatrix}$ whence $\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}0&-16&8\\0&0&24\end{pmatrix}=\begin{pmatrix}0&0&4\\0&2&1\end{pmatrix}$	M1	Finding A'', B'' and C'' coordinates or position vectors
			$\Rightarrow\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=\frac{1}{48}\begin{pmatrix}0&8\\-6&4\end{pmatrix}$	A1 A1	For correct position vectors Inverse matrix correctly found
				[3]	

Question			Answer	Marks	Guidance
9	(iv)		Area scale factor = 48 Area of triangle ABC = 4 square units Area of triangle $A''B''C'' = 48\times$ area of triangle ABC $= 192$ (square units)	B1 M1 A1 **[3]**	Using their "48" and their area of triangle ABC, correct triangle Or other valid method cao
		OR	Finding $A''B''C''$ (0,0) (-16, 0) (8, 24) and using them Finding the area of $A''B''C''$ Area of triangle $=192$ (square units)	B1 M1 A1	$A''B''C''$ may be in (iii) Any valid method attempted cao (possibly after rounding to 3 sf)