OCR
Oxford Cambridge and RSA

Wednesday 13 May 2015 – Morning

AS GCE MATHEMATICS (MEI)

4751/01 Introduction to Advanced Mathematics (C1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4751/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

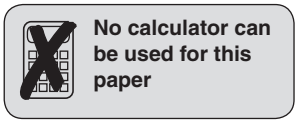
INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



Section A (36 marks)

1 Make r the subject of the formula $A = \pi r^2(x+y)$, where $r > 0$. **[2]**

2 A line L is parallel to $y = 4x+5$ and passes through the point $(-1,6)$. Find the equation of the line L in the form $y = ax+b$. Find also the coordinates of its intersections with the axes. **[5]**

3 Evaluate the following.

(i) 200^0 **[1]**

(ii) $\left(\frac{25}{9}\right)^{-\frac{1}{2}}$ **[3]**

4 Solve the inequality $\frac{4x-5}{7} > 2x+1$. **[3]**

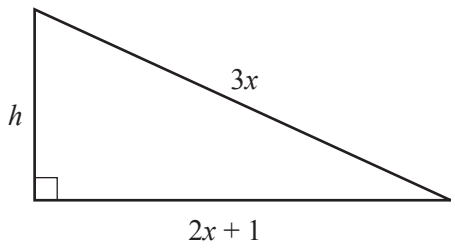
5 Find the coordinates of the point of intersection of the lines $y = 5x-2$ and $x+3y = 8$. **[4]**

6 **(i)** Expand and simplify $(3+4\sqrt{5})(3-2\sqrt{5})$. **[3]**

(ii) Express $\sqrt{72}+\frac{32}{\sqrt{2}}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. **[2]**

7 Find and simplify the binomial expansion of $(3x-2)^4$. **[4]**

8 Fig. 8 shows a right-angled triangle with base $2x+1$, height h and hypotenuse $3x$.



Not to scale

Fig. 8

(i) Show that $h^2 = 5x^2-4x-1$. **[2]**

(ii) Given that $h = \sqrt{7}$, find the value of x, giving your answer in surd form. **[3]**

9 Explain why each of the following statements is false. State in each case which of the symbols $\Rightarrow$, $\Leftarrow$ or $\Leftrightarrow$ would make the statement true.

(i) ABCD is a square $\Leftrightarrow$ the diagonals of quadrilateral ABCD intersect at 90° **[2]**

(ii) x^2 is an integer $\Rightarrow$ x is an integer **[2]**

Section B (36 marks)

10 You are given that $f(x) = (x+3)(x-2)(x-5)$.

(i) Sketch the curve $y = f(x)$. **[3]**

(ii) Show that $f(x)$ may be written as $x^3 - 4x^2 - 11x + 30$. **[2]**

(iii) Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, where $g(x) = x^3 - 4x^2 - 11x - 6$. **[2]**

(iv) Show that $g(-1) = 0$. Hence factorise $g(x)$ completely. **[5]**

11

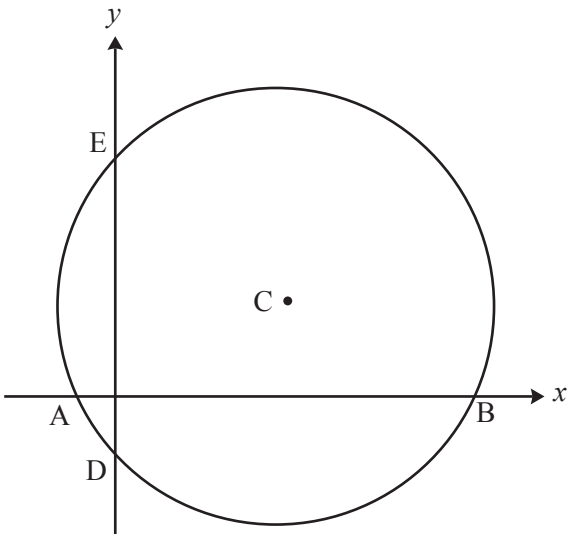


Fig. 11

Fig. 11 shows a sketch of the circle with equation $(x-10)^2 + (y-2)^2 = 125$ and centre C. The points A, B, D and E are the intersections of the circle with the axes.

(i) Write down the radius of the circle and the coordinates of C. **[2]**

(ii) Verify that B is the point (21, 0) and find the coordinates of A, D and E. **[4]**

(iii) Find the equation of the perpendicular bisector of BE and verify that this line passes through C. **[6]**

12 **(i)** Find the set of values of k for which the line $y = 2x + k$ intersects the curve $y = 3x^2 + 12x + 13$ at two distinct points. **[5]**

(ii) Express $3x^2 + 12x + 13$ in the form $a(x+b)^2 + c$. Hence show that the curve $y = 3x^2 + 12x + 13$ lies completely above the x-axis. **[5]**

(iii) Find the value of k for which the line $y = 2x + k$ passes through the minimum point of the curve $y = 3x^2 + 12x + 13$. **[2]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$[r =]\sqrt{\frac{A}{\pi(x+y)}}$ or $[r =]\sqrt{\frac{A}{\pi x+\pi y}}$ as final answer	2 **[2]**	square root symbol must extend below fraction line; accept to power ½ with appropriate brackets **M1** for a triple decker fraction or for $r^2 = \frac{A}{\pi(x+y)}$ or for $[r =]\pm\sqrt{\frac{A}{\pi(x+y)}}$ or for their final answer for r ft their r^2	condone missing end bracket in denominator eg M1 for $[r =]\sqrt{\frac{A}{\frac{\pi}{(x+y)}}}$
2			$y = 4x + 10$ (0, 10) or ft (–10/4, 0) oe or ft	B3 B1 B1 **[5]**	**M1** for $y = 4x + b$ oe and **M1** for $y – 6 =$ their a $(x + 1)$ oe or for (–1, 6) subst in $y =$ (their a)$x + b$ oe or **M1** for $y = ax + 10$ condone $y = 10$ isw condone $x = –10/4$ isw	condone lack of brackets and eg $y = 10$, $x = –2.5$ or ft isw but B0, SC1 for poor notation such as (–2.5, 10) with no better answers seen Throughout the scheme, note that for evaluated rational answers, unless specified otherwise, fractional or decimal equivalents are acceptable, but not triple-decker fractions etc; integer answers must be simplified to an integer

Question			Answer	Marks	Guidance	Question
3	(i)		1	1 **[1]**		
3	(ii)		$\frac{3}{5}$ or 0.6	3 **[3]**	allow **B3** for ±0.6 oe; **M1** for $\left(\frac{25}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{\frac{1}{2}}$ soi or $\frac{1}{\left(\frac{25}{9}\right)^{\frac{1}{2}}}$ and **M1** for at least one of 3 and 5 found	M1 for inversion even if they have done something else first, eg may be earned after 2nd M1 for inversion of their $\frac{5}{3}$
4			$4x - 5 > 14x + 7$ $-12 > 10x$ or $-10x > 12$ or ft $x < -\frac{12}{10}$ or $-\frac{12}{10} > x$ oe isw or ft	M1 M1 M1 **[3]**	for correctly multiplying by 7 to eliminate the fraction, including expanding bracket if this step done first for correctly collecting x terms on one side and number terms on the other and simplifying ft their ax [inequality] b, where $b \neq 0$ and $a \neq 0$ or ±1	may be earned later; the first two Ms may be earned with an equation or wrong inequality ft wrong first step award 3 marks only if correct answer obtained after equations or inequalities are used with no errors
5			$x + 3(5x - 2) = 8$ or $y = 5(8 - 3y) - 2$ $16x = 14$ or $16y = 38$ (7/8, 19/8) oe	M1 M1 A2 **[4]**	for subst to eliminate one variable; condone one error; for collecting terms and simplifying; condoning one error ft or $x = 14/16$, $y = 38/16$ oe isw allow **A1** for each coordinate	or multn or divn of one or both eqns to get a pair of coeffts the same, condoning one error appropriate addn or subtn to eliminate a variable, condoning an error in one term; if subtracting, condone eg y instead of 0 if no other errors

Question			Answer	Marks	Guidance	Question
6	(i)		$-31 + 6\sqrt{5}$	3 **[3]**	**B2** for –31 or **B1** for 9 – 40 or **SC1** for 49 and **B1** for $6\sqrt{5}$ if 0, allow **M1** for three terms correct in $9-6\sqrt{5}+12\sqrt{5}-40$	
6	(ii)		$22\sqrt{2}$	2 **[2]**	**M1** for $\sqrt{72}=6\sqrt{2}$ soi or for $\frac{32}{\sqrt{2}}=16\sqrt{2}$ soi or for $\frac{12+32}{\sqrt{2}}$ oe	
7			$81x^4 - 216x^3 + 216x^2 - 96x + 16$	4 **[4]**	**M3** for 4 terms correct or for all coefficients correct except for sign errors or for correct answer seen then further 'simplified' or for all terms correct eg seen in table but not combined or **M2** for 3 terms correct or for correct expansion seen without correct evaluation of coefficients [if brackets missing in elements such as $(3x)^2$ there must be evidence from calculation that $9x^2$ has been used] or **M1** for 1 4 6 4 1 row of Pascal's triangle seen	condone eg +(–96x) or +–96x instead of –96x any who multiply out instead of using binomial coeffts: look at their final answer and mark as per main scheme if 3 or more terms are correct, otherwise M0 binomial coefficients such as ${}^4C_2$ or $\binom{4}{2}$ are not sufficient – must show understanding of these symbols by at least partial evaluation;

Question			Answer	Marks	Guidance	Question
8	(i)		$(3x)^2 = h^2 + (2x + 1)^2$ oe	B1	for a correct Pythagoras statement for this triangle, in terms of x, with correct brackets	condone another letter instead of h for one mark but not both unless recovered at some point
			$9x^2 = h^2 + 4x^2 + 4x + 1$ and completion to given answer , $h^2 = 5x^2 - 4x - 1$	B1 **[2]**	for correct expansion, with brackets or correct signs; must complete to the given answer with no errors in any interim working may follow $3x^2 = h^2 + (2x + 1)^2$ oe for **B0 B1**	eg B1 for $h^2 = 9x^2 - (4x^2 + 4x + 1)$ and completion to correct answer but B0 for $h^2 = 9x^2 - 4x^2 + 4x + 1$
8	(ii)		$[0 =] 5x^2 - 4x - 8$	B1	for subst and correctly rearranging to zero	
			$\frac{4 \pm \sqrt{(-4)^2 - 4 \times 5 \times -8}}{2 \times 5}$ or ft	M1	for use of formula in their eqn rearranged to zero, condoning one error; ft only if their rearranged eqn is a 3-term quadratic; no ft from $5x^2 - 4x - 1$ [=0]	or M1 for $\left(x - \frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^2 + \frac{8}{5}$ oe, (condoning one error), which also implies first M1 if not previously earned M0 for factorising ft
			$\frac{4 + \sqrt{176}}{10}$ or $\frac{2}{5} + \frac{\sqrt{44}}{5}$ oe	A1 **[3]**	isw wrong simplification; **A0** if negative root also included	
9	(i)		the diagonals of a rhombus also intersect at 90°	B1	oe for kite or other valid statement/sketch **B0** if eg rectangle or parallelogram etc also included as having diagonals intersecting at 90°	accept 'diamond' etc reference merely to 'other shapes' having diagonals intersecting at 90° is not sufficient; sketches must have diagonals drawn, intersecting approx. at right angles but need not be ruled
			ABCD is a square ⇒ the diagonals of quadrilateral ABCD intersect at 90°	B1 **[2]**	oe; **B0** if no attempt at explanation (explanation does not need to gain a mark)	Do not accept → oe

Question			Answer	Marks	Guidance	Question
9	(ii)		eg 8 is an integer but $\sqrt{8}$ is not an integer	B1	oe with another valid number, or equivalent explanation **B1** for the square root of some integers is a surd / irrational number / decimal	0 for 'the square root of some integers is a fraction'
			x^2 is an integer $\Leftarrow$ x is an integer	B1 **[2]**	**B0** if no attempt at explanation	Do not accept ← oe
10	(i)		graph of cubic correct way up	B1	**B0** if stops at x-axis	must not have any ruled sections; no curving back; condone slight 'flicking out' at ends but not approaching a turning point; allow max on y-axis or in 1st or 2nd quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x-axis at -3, 2 and 5	B1	on graph or nearby; may be in coordinate form mark intent for intersections with both axes	allow if no graph, but marked on x-axis condone intercepts for x and / or y given as reversed coordinates
			crossing y-axis at 30	B1 **[3]**	or $x = 0$, $y = 30$ seen if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on y-axis nowhere near their indicated 30
10	(ii)		correct expansion of two of the linear factors	M1	may be 3 or 4 terms	condone lack of brackets if correct expansions as if they were there
			correct expansion and completion to given answer, $x^3 - 4x^2 - 11x + 30$	A1 **[2]**	must be working for this step before given answer	or for direct expansion of all three factors, allow M1 for $x^3 + 3x^2 - 2x^2 - 5x^2 - 6x - 15x + 10x + 30$, condoning an error in one term , and A1 if no error for completion by stating given answer

Question			Answer	Marks	Guidance	Question
10	(iii)		translation $\begin{pmatrix} 0 \\ -36 \end{pmatrix}$	B1 B1 **[2]**	0 for shift or move etc without stating translation or 36 down, or –36 in *y* direction oe	0 if eg stretch also mentioned if conflict, eg between '–36 in *y* direction' and wrong vector, award B0 0 for '–36 down'
10	(iv)		$-1 - 4 + 11 - 6 = 0$ attempt at division by $(x + 1)$ as far as $x^3 + x^2$ in working correctly obtaining $x^2 - 5x - 6$ factorising the correct quadratic factor $x^2 - 5x - 6$, that has been correctly obtained $(x - 6)(x + 1)^2$ oe isw	B1 M1 A1 M1 A1 **[5]**	or **B1** for correct division by $(x + 1)$ or for the quadratic factor found by inspection, and the conclusion that no remainder means that $g(-1) = 0$ or inspection with at least two terms of three–term quadratic factor correct; or finding f(6) = 0 or $(x - 6)$ found as factor for factors giving two terms of quadratic correct or for factors ft one error in quadratic formula or completing square; **M0** for formula etc without factors found for those who have used the factor theorem to find $(x - 6)$, **M1** for working with cubic to find that $(x + 1)$ is repeated condone inclusion of '= 0'	NB examiners must use annotation in this part; a tick where each mark is earned is sufficient M0 for trials of factors to give cubic unless correct answer found with clear correct working, in which case award the M1A1M1A1 allow for $(x - 6)$ and $(x + 1)$ given as factors eg after quadratic formula etc isw roots found, even if stated as factors just the answer $(x - 6)(x + 1)^2$ oe gets last 4 marks

Question			Answer	Marks	Guidance	Question
11	(i)		[radius =] $\sqrt{125}$ isw or $5\sqrt{5}$	B1		
			[C =] (10, 2)	B1 **[2]**	condone $x = 10$, $y = 2$	
11	(ii)		verifying / deriving that (21, 0) is one of the intersections with the axes	B1	using circle equation or Pythagoras; or putting $y = 0$ in circle equation and solving to get 21 and -1; condone omission of brackets	equation may be expanded first
			(−1, 0)	B1		condone not written as coordinates
			(0, −3) and (0, 7)	B2 **[4]**	**B1** each; if B0 for D and E, then **M1** for substitution of $x = 0$ into circle equation or use of Pythagoras showing $125 - 10^2$ or $h^2 + 10^2 = 125$ ft their centre and/or radius	condone not written as coordinates; condone not identified as D and E; condone D = (0, 7), E = (0, −3) – will penalise themselves in (iii)

Question			Answer	Marks	Guidance	Question
11	(iii)		midpt BE = (21/2 , 7/2 ft) oe	B1	ft their E	NB examiners must use annotation in this part; a tick where each mark is earned is sufficient
					or stating that the perp bisector of a chord always passes through the centre of the circle	must be explicit generalised statement; need more than just that C is on this perp bisector
			grad BE = $\frac{7-0}{0-21}$ oe isw	M1	ft their E; M0 for using grad BC (= –2/11)	condone $-1/3x$ oe
			grad perp bisector = 3 oe	M1	for use of $m_1m_2 = -1$ oe soi; ft their grad BE; no ft from grad BC used	condone $3x$ oe; allow M1 for eg $-1/3 \times 3 = -1$
			$y - 7/2 = 3(x - 21/2)$ oe	M1	ft; M0 for using grad BE or perp to BC allow this M1 for C used instead of midpoint	or use of $y = 3x + c$ and subst of (21/2 , 7/2) oe ft
			$y = 3x - 28$ oe	A1	must be a simplified equation	no ft; those who assume that C is on the line and use it to find $y = 3x - 28$ can earn B0M1M1M1A1A0 those who argue that the perp bisector of a chord always passes through the centre of the circle and then uses C rather than midpt of BE are eligible for all 6 marks
			verifying that (10, 2) is on this line	A1	no ft; A0 if C used to find equation of line, unless B1 earned for correct argument	
				[6]		

Question			Answer	Marks	Guidance	Question
12	(i)		$3x^2 + 12x + 13 = 2x + k$	M1	oe eg M1 for $3x^2 + 10x + 13 = k$	condone $3x^2 + 10x + 13 - k = y$ for this M1
			$3x^2 + 10x + 13 - k$ [= 0]	M1	for rearranging to 0; condone one error in adding/subtracting; but M0 for $3x^2 + 10x + 13 = k$ or $3x^2 + 10x + 13 - k = y$	$3x^2 + 10x + 13 - k$ [= 0] will also earn the first M1 if a separate statement has not already done so
			$b^2 - 4ac > 0$ oe soi	M1	may be earned near end with correct inequality sign used there	allow '$b^2 - 4ac$ is positive' oe; 0 for just 'discriminant > 0' unless implied by later work
			$100 - 4 \times 3 \times (13 - k)$ (> 0) oe	M1	for correct substitution ft into $b^2 - 4ac$, dep on second M1 earned; brackets / signs must be correct	can be earned with equality or wrong inequality, or in formula M0 for trials of values of k in $b^2 - 4ac$
			$k > 14/3$ oe	A1	accept $k >$ 56/12 or better, isw incorrect conversion of fraction but not wrong use of inequalities if A0, allow **B1** for 56/12 oe obtained with equality or wrong inequality (ie 3rd M1 has not been earned)	
				[5]		

Question			Answer	Marks	Guidance	Question
12	(ii)		$3(x + 2)^2 + 1$ www as final answer	B4	**B1** for $a = 3$ and **B1** for $b = 2$ and **B2** for $c = 1$ or **M1** for $13 - 3 \times$ their b^2 or for $13/3 -$ their b^2 or **B3** for $3\left[(x+2)^2 + \frac{1}{3}\right]$	condone omission of square symbol; ignore equating to zero in working or answer
			y-minimum = 1 [hence curve is above x-axis]	B1 **[5]**	Stating min pt is (–2, 1) is sufft allow ft if their $c > 0$ B0 for only showing that discriminant is negative oe; need also to justify that it is all above not all below x-axis B0 for stating min point = 1 or ft	must be done in this part; ignore wrong x-coordinate
12	(iii)		5 cao	B2 **[2]**	**M1** for substitution of their (–2, 1) in $y = 2x + k$	allow M1 ft their $3(x + 2)^2 + 1$; or use of (–2,1) found using calculus; M0 if they use an incorrect minimum point inconsistent with their completed square form