Quiz 19/11 Answers

COGS 526

1. **[2pts]** Is the following formula valid (tautology, true under every interpretation)? Justify your answer using a truth tree.

$$(p \lor q) \to (p \to q)$$

To test for tautology, we assume the negation of the formula and construct a truth tree. If all branches close, the original formula is a tautology.

$$\neg((p \lor q) \to (p \to q)) \ (1)$$
$$|$$
$$(p \lor q) \ (2)$$
$$|$$
$$\neg(p \to q) \ (3)$$
$$|$$
$$p \ (4)$$
$$|$$
$$\neg q \ (5)$$
$$\swarrow \qquad \searrow$$
$$p \ (6a) \qquad q \ (6b)$$
$$| \qquad\qquad |$$
$$\text{OPEN} \qquad \times$$

- In (1) we assume the negation of the formula is true.
- (2) and (3) are derived from (1) using the rule of negation of a conditional.
- (4) and (5) are derived from (3) using the rule of negation of a conditional.
- (6a) and (6b) are derived from (2) using the rule of disjunction.

The right branch closes since it involves a contradiction; (5) and (6b). However, the left branch remains open. Thus, the original formula is not a tautology

2. **[2pts]** Determine which of the four imply q and which imply $\neg p$. Any method is fine.

$$p \land (p \to q), \quad \neg q \land (p \to q), \quad p \land (\neg p \to q), \quad p \to (\neg q \land q)$$

(a) For $p \land (p \to q)$

$$\begin{aligned} p \land (p \to q) &\equiv p \land (\neg p \lor q) \quad \text{(Implication Definition)} \\ &\equiv (p \land \neg p) \lor (p \land q) \quad \text{(Distribution)} \\ &\equiv \mathrm{F} \lor (p \land q) \quad \text{(Contradiction)} \\ &\equiv p \land q \end{aligned}$$

- If $p \land q$ is true, then q must be true. It implies q.
- $p \land q$ does not imply $\neg p$.

(b) For $\neg q \land (p \to q)$

$$\begin{aligned} \neg q \land (p \to q) &\equiv \neg q \land (\neg p \lor q) \quad \text{(Implication Definition)} \\ &\equiv (\neg q \land \neg p) \lor (\neg q \land q) \quad \text{(Distribution)} \\ &\equiv (\neg p \land \neg q) \lor \mathrm{F} \quad \text{(Contradiction)} \\ &\equiv \neg p \land \neg q \end{aligned}$$

- $\neg p \land \neg q$ does not imply q.
- $\neg p \land \neg q$ implies $\neg p$.

(c) For $p \land (\neg p \to q)$

$$\begin{aligned} p \land (\neg p \to q) &\equiv p \land (\neg(\neg p) \lor q) \quad \text{(Implication Definition)} \\ &\equiv p \land (p \lor q) \quad \text{(Double Negation)} \end{aligned}$$

$p \land (p \lor q)$ can be reduced to p, since the formula is only true if p is true, and false if p is false.

$$\equiv p$$

- It does not imply q.
- It does not imply $\neg p$.

(d) For $p \to (\neg q \land q)$

$$\begin{aligned} p \to (\neg q \land q) &\equiv \neg p \lor (\neg q \land q) \quad \text{(Implication Definition)} \\ &\equiv \neg p \lor \mathrm{F} \quad \text{(Contradiction)} \\ &\equiv \neg p \end{aligned}$$

- It does not imply q.
- The formula is equivalent to $\neg p$.