Quiz 17/12 Answers

COGS 526

1. **[5pts]** Translate to first-order logic:

   - "Either John found something, or John lost something." (Symbolize "John", "x found y" and "x lost y".)
     - j': John
     - $F(x, y)$: x found y
     - $L(x, y)$: x lost y

     (a) John found something: $\exists x.F(j', x)$

     (b) John lost something: $\exists x.L(j', x)$

     The final logical formula is the disjunction of (a) and (b):

     $$\exists x.F(j', x) \vee \exists y.L(j', y)$$

   - "Either John found something, or John found nothing."

     (a) John found something: $\exists x.F(j', x)$

     (b) John found nothing: $\forall x.\neg F(j', x)$

     The final logical formula is the disjunction of (a) and (b):

     $$\exists x.F(j', x) \vee \forall y.\neg F(j', y)$$

     Interpretation:"Either there exists something that John found, or for everything, John did not find it."

     NOTE: $\forall x.\neg F(j', x)$ is logically equivalent to $\neg\exists x.F(j', x)$ (There does not exist a thing that John found.)

   - "Every man gave some kid every toy." (Symbolize "x is a man", "x is a kid", "x is a toy", and "x gave y to z".)
     - $M(x)$: x is a man
     - $K(x)$: x is a kid
     - $T(x)$: x is a toy

- $G(x, y, z)$: x gave y to z

$$\forall x.(M(x) \rightarrow \exists y.(K(y) \wedge \forall z.(T(z) \rightarrow G(x, z, y))))$$

Interpretation: "For every x, if x is a man, then there exists a y such that y is a kid, and for every z, if z is a toy, then x gave z to y."

- "At least one tree isn't climbed up by every cat in a garden." (Symbolize "x is a tree", "x is a cat", "x is a garden", and "x climbed up y in z".)
  - $T(x)$: x is a tree
  - $C(x)$: x is a cat
  - $G(x)$: x is a garden
  - $Climb(x, y, z)$: x climbed up y in z

  $$\exists x.(T(x) \wedge \exists y.(G(y) \wedge \neg\forall z.(C(z) \rightarrow Climb(z, x, y))))$$

  Interpretation: "There exists a tree (x) and there exists a garden (y) such that it is not the case that for every z, if z is a cat, then z climbed up x in y."

  NOTE: $\neg\forall z.(C(z) \rightarrow Climb(z, x, y))$ is logically equivalent to $\exists z.(C(z) \wedge \neg Climb(z, x, y))$

- "For every cat there is a garden in which there is a tree that she climbs up."

  $$\forall x.(C(x) \rightarrow \exists y.(G(y) \wedge \exists z.(T(z) \wedge Climb(x, z, y))))$$

Interpretation: "For every x, if x is a cat, then there exists a garden y and there exists a tree z such that x climbed z in y."