

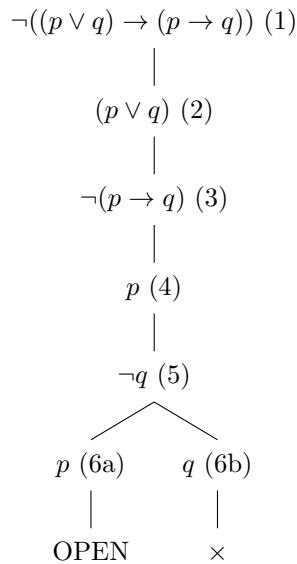
Quiz 19/11 Answers

COGS 526

1. [2pts] Is the following formula valid (tautology, true under every interpretation)? Justify your answer using a truth tree.

$$(p \vee q) \rightarrow (p \rightarrow q)$$

To test for tautology, we assume the negation of the formula and construct a truth tree. If all branches close, the original formula is a tautology.



- In (1) we assume the negation of the formula is true.
- (2) and (3) are derived from (1) using the rule of negation of a conditional.
- (4) and (5) are derived from (3) using the rule of negation of a conditional.
- (6a) and (6b) are derived from (2) using the rule of disjunction.

The right branch closes since it involves a contradiction; (5) and (6b). However, the left branch remains open. Thus, the original formula is not a tautology

2. [2pts] Determine which of the four imply q and which imply $\neg p$. Any method is fine.

$$p \wedge (p \rightarrow q), \quad \neg q \wedge (p \rightarrow q), \quad p \wedge (\neg p \rightarrow q), \quad p \rightarrow (\neg q \wedge q)$$

(a) For $p \wedge (p \rightarrow q)$

$$\begin{aligned} p \wedge (p \rightarrow q) &\equiv p \wedge (\neg p \vee q) \quad (\text{Implication Definition}) \\ &\equiv (p \wedge \neg p) \vee (p \wedge q) \quad (\text{Distribution}) \\ &\equiv \text{F} \vee (p \wedge q) \quad (\text{Contradiction}) \\ &\equiv p \wedge q \end{aligned}$$

- If $p \wedge q$ is true, then q must be true. It implies q .
- $p \wedge q$ does not imply $\neg p$.

(b) For $\neg q \wedge (p \rightarrow q)$

$$\begin{aligned} \neg q \wedge (p \rightarrow q) &\equiv \neg q \wedge (\neg p \vee q) \quad (\text{Implication Definition}) \\ &\equiv (\neg q \wedge \neg p) \vee (\neg q \wedge q) \quad (\text{Distribution}) \\ &\equiv (\neg p \wedge \neg q) \vee \text{F} \quad (\text{Contradiction}) \\ &\equiv \neg p \wedge \neg q \end{aligned}$$

- $\neg p \wedge \neg q$ does not imply q .
- $\neg p \wedge \neg q$ implies $\neg p$.

(c) For $p \wedge (\neg p \rightarrow q)$

$$\begin{aligned} p \wedge (\neg p \rightarrow q) &\equiv p \wedge (\neg(\neg p) \vee q) \quad (\text{Implication Definition}) \\ &\equiv p \wedge (p \vee q) \quad (\text{Double Negation}) \end{aligned}$$

$p \wedge (p \vee q)$ can be reduced to p , since the formula is only true if p is true, and false if p is false.

$$\equiv p$$

- It does not imply q .
- It does not imply $\neg p$.

(d) For $p \rightarrow (\neg q \wedge q)$

$$\begin{aligned} p \rightarrow (\neg q \wedge q) &\equiv \neg p \vee (\neg q \wedge q) \quad (\text{Implication Definition}) \\ &\equiv \neg p \vee \text{F} \quad (\text{Contradiction}) \\ &\equiv \neg p \end{aligned}$$

- It does not imply q .
- The formula is equivalent to $\neg p$.