Quiz 3/12 Answers

COGS 526

1. Translate to first-order logic:

   - **[2pts*]** "Not everything that is faced can be changed, but nothing can be changed until it is faced." (James Baldwin)

     Predicates:
     - $F(x)$: x is faced.
     - $C(x)$: x can be changed.

     "Not everything that is faced can be changed" can be formulated as:

     $$\neg(\forall x.F(x) \rightarrow C(x))$$

     "Nothing can be changed until it is faced" can be formulated as:

     $$\neg(\exists x.\neg F(x) \wedge C(x))$$

     The whole formula is the conjunction of two:

     $$\neg(\forall x.F(x) \rightarrow C(x)) \wedge \neg(\exists x.\neg F(x) \wedge C(x))$$

     $$\equiv \exists x(F(x) \wedge \neg C(x)) \wedge \forall x(C(x) \rightarrow F(x))$$

   - **[5pts]** "Some cause happiness wherever they go; others whenever they go." Key: $R(x)$: 'x is a person', $P(x)$: 'x is a place', $T(x)$: 'x is a time', $C(x, y)$: 'x causes happiness at the place y', $G(x, y, z)$: 'x goes to place y at time z', $L(x, y, z)$: 'x leaves the place y at time z'.

"Some cause happiness wherever they go" can be reformulated as: There exists a person x such that for all places y and all times z, if x goes to y at time z, then x causes happiness at y.

$$\exists x.(R(x) \wedge \forall y \forall z.((P(y) \wedge T(z) \wedge G(x, y, z)) \rightarrow C(x, y)))$$

"Others whenever they go" can be reformulated as: There exists a person a such that for all places y and all times z, if a leaves place y at time z, then a causes happiness at y.

$$\exists a.(R(a) \wedge \forall y \forall z.((P(y) \wedge T(z) \wedge L(a, y, z)) \rightarrow C(a, y)))$$

The whole expression can be formulated as:

$$\exists x \exists a \Bigg(R(x) \wedge R(a) \wedge x \neq a$$
$$\wedge \Bigg[\forall y \forall z \left((P(y) \wedge T(z) \wedge G(x, y, z)) \rightarrow C(x, y) \right) \Bigg]$$
$$\wedge \Bigg[\forall y \forall z \left((P(y) \wedge T(z) \wedge L(a, y, z)) \rightarrow C(a, y) \right) \Bigg] \Bigg)$$