

# MIRROR SYMMETRY FOR LOOIJENGA INTERIORS AND BEYOND

For the purposes of this document, define a smooth Looijenga interior to be a smooth manifold  $U$  diffeomorphic to  $X \setminus D$ , for some smooth complex projective surface  $X$  and a normal crossings anti canonical divisor  $D$ . Fixing a diffeomorphism  $\Phi : U \rightarrow X_\Phi \setminus D_\Phi$ ,  $U$  can be equipped with a complex structure and a holomorphic volume form  $\Omega_\Phi$ , which extends to  $X_\Phi$  as a meromorphic form with simple poles along  $D_\Phi$ . Note that  $\Omega_\Phi$  is well defined up to multiplication by a scalar  $c \in \mathbb{C}^*$ . An important special case is when  $D_\Phi$  supports an ample Weil divisor.

One loose goal of this workshop/conference is to understand the relationship between various mirror symmetry constructions and results where  $(U, \omega)$ ,  $\omega$  being some symplectic structure on a smooth Looijenga interior  $U$ , is the  $A$ -side. In the literature, symplectic structures of the following forms are considered

- Restrictions of Kahler forms from  $X_\Phi$ .
- Phases of  $\Omega_\Phi$ .
- Weinstein structures built using handle attachments or descriptions of Lagrangian skeleton.
- Total spaces of nodal Lagrangian torus fibrations described by base diagrams [Sym02].

These are of course not mutually exclusive classes.

Here is a partial list of such works: Kontsevich-Soibelman [KS06], Auroux-Katzarkov-Orlov [AKO06], Gross-Hacking-Keel [GHK15], Shende-Treumann-Williams [STW16], Tu [Tu14], Yuan [Yua20], Cheung-Lin [CL21], Collins-Jacob-Lin [CJL21], Hacking-Keating [HK20], Cheung-Evans-Hong-Lin [BECHL21], Groman-Varolgunes [GV21]. We also note Mandel [Man19] as a helpful article.

The second goal then would be to learn how the different approaches generalize by considering Looijenga interiors (with their various symplectic structures) as dimension/rank 2 cases of

- interiors of complex log Calabi-Yau's
- cluster varieties
- Weinstein manifolds with handle decompositions dictated by seed data
- Total spaces of complete Lagrangian torus fibrations with a special class of slideable singularities (e.g. [BM09], [AS21], [Gam21])

Again these are not mutually exclusive.

Here is a partial list of such works: Gross-Hacking-Keel-Kontsevich [GHKK18], Hacking-Keel [HK18], Keel-Yu [KY19], Gammage-Le [GL21], Gross-Siebert [GS19], Arguz-Gross [AG20], Groman [Gro18], Groman-Varolgunes [GV21], Yuan [Yua20].

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