MIRROR SYMMETRY FOR LOOIJENGA INTERIORS AND BEYOND

For the purposes of this document, define a smooth Looijenga interior to be a smooth manifold U diffeomorphic to $X \setminus D$, for some smooth complex projective surface X and a simple normal crossings anti canonical divisor D. Fixing a diffeomorphism $\Phi : U \to X_\Phi \setminus D_\Phi$, U can be equipped with a complex structure and a holomorphic volume form Ω_Φ, which extends to X_Φ as a meromorphic form with simple poles along D_Φ. Note that Ω_Φ is well defined up to multiplication by a scalar $c \in \mathbb{C}^*$. An important special case is when D_Φ supports an ample Weil divisor.

One loose goal of this workshop/conference is to understand the relationship between various mirror symmetry constructions and results where (U, ω), ω being some symplectic structure on a smooth Looijenga interior U, is the A-side. In the literature, symplectic structures of the following forms are considered

- Restrictions of Kahler forms from X_Φ.
- Phases of Ω_Φ.
- Weinstein structures built using handle attachments or descriptions of Lagrangian skeleton.
- Total spaces of nodal Lagrangian torus fibrations described by base diagrams [Sym02].

These are of course not mutually exclusive classes.

Here is a partial list of such works: Kontsevich-Soibelman [KS06], Auroux-Katzarkov-Orlov [AKO06], Gross-Hacking-Keel [GHK15], Shende-Treumann-Williams [STW16], Tu [Tu14], Yuan [Yua20], Cheung-Lin [CL21], Collins-Jacob-Lin [CJL21], Hacking-Keating [HK20], Cheung-Evans-Hong-Lin [BECHL21], Groman-Varolgunes [GV21]. We also note Mandel [Man19] as a helpful article.

The second goal then would be to learn how the different approaches generalize by considering Looijenga interiors (with their various symplectic structures) as dimension/rank 2 cases of

- interiors of complex log Calabi-Yau's
- cluster varieties
- Weinstein manifolds with handle decompositions dictated by seed data
- Total spaces of complete Lagrangian torus fibrations with a special class of slideable singularities (e.g. [BM09], [AS21], [Gam21])

Again these are not mutually exclusive.

Here is a partial list of such works: Gross-Hacking-Keel-Kontsevich [GHKK18], Hacking-Keel [HK18], Keel-Yu [KY19], Gammage-Le [GL21], Gross-Siebert [GS19], Arguz-Gross [AG20], Groman [Gro18], Groman-Varolgunes [GV21], Yuan [Yua20].

References

[AG20] Hülya Argüz and Mark Gross. The higher dimensional tropical vertex. *arXiv preprint arXiv:2007.08347*, 2020.

[AKO06] Denis Auroux, Ludmil Katzarkov, and Dmitri Orlov. Mirror symmetry for del pezzo surfaces: vanishing cycles and coherent sheaves. *Inventiones mathematicae*, 166(3):537–582, 2006.

[AS21] Mohammed Abouzaid and Zachary Sylvan. Homological Mirror Symmetry for local SYZ singularities. *arXiv preprint arXiv:2107.05068*, 2021.

[BECHL21] Sam Bardwell-Evans, Man-Wai Mandy Cheung, Hansol Hong, and Yu-Shen Lin. Scattering Diagrams from Holomorphic Discs in Log Calabi-Yau Surfaces. *arXiv preprint arXiv:2110.15234*, 2021.

[BM09] Ricardo Castano Bernard and Diego Matessi. Lagrangian 3-torus fibrations. *Journal of Differential Geometry*, 81(3):483–573, 2009.

[CJL21] Tristan C Collins, Adam Jacob, and Yu-Shen Lin. Special lagrangian submanifolds of log calabi–yau manifolds. *Duke Mathematical Journal*, 170(7):1291–1375, 2021.

[CL21] Man-Wai Mandy Cheung and Yu-Shen Lin. Some examples of family floer mirror. *arXiv preprint arXiv:2101.07079*, 2021.

[Gam21] Benjamin Gammage. Local mirror symmetry via SYZ. *arXiv preprint arXiv:2105.12863*, 2021.

[GHK15] Mark Gross, Paul Hacking, and Sean Keel. Mirror symmetry for log Calabi-Yau surfaces I. *Publications Mathématiques de l'IHES*, 122(1):65–168, 2015.

[GHKK18] Mark Gross, Paul Hacking, Sean Keel, and Maxim Kontsevich. Canonical bases for cluster algebras. *Journal of the American Mathematical Society*, 31(2):497–608, 2018.

[GL21] Benjamin Gammage and Ian Le. Mirror symmetry for truncated cluster varieties. *arXiv preprint arXiv:2103.12232*, 2021.

[Gro18] Yoel Groman. The wrapped Fukaya category for semi-toric SYZ fibrations. *arXiv preprint arXiv:1805.03635*, 2018.

[GS19] Mark Gross and Bernd Siebert. Intrinsic mirror symmetry. *arXiv preprint arXiv:1909.07649*, 2019.

[GV21] Yoel Groman and Umut Varolgunes. Locality of relative symplectic cohomology for complete embeddings. *arXiv preprint arXiv:2110.08891*, 2021.

[HK18] Paul Hacking and Sean Keel. Mirror symmetry and cluster algebras. In *Proceedings of the International Congress of Mathematicians: Rio de Janeiro 2018*, pages 671–697. World Scientific, 2018.

[HK20] Paul Hacking and Ailsa Keating. Homological mirror symmetry for log Calabi-Yau surfaces. *arXiv preprint arXiv:2005.05010*, 2020.

[KS06] Maxim Kontsevich and Yan Soibelman. Affine structures and non-archimedean analytic spaces. In *The unity of mathematics*, pages 321–385. Springer, 2006.

[KY19] Sean Keel and Tony Yue Yu. The frobenius structure theorem for affine log calabi-yau varieties containing a torus. *arXiv preprint arXiv:1908.09861*, 2019.

[Man19] Travis Mandel. Classification of rank 2 cluster varieties. *SIGMA. Symmetry, Integrability and Geometry: Methods and Applications*, 15:042, 2019.

[STW16] Vivek Shende, David Treumann, and Harold Williams. On the combinatorics of exact lagrangian surfaces. *arXiv preprint arXiv:1603.07449*, 2016.

[Sym02] Margaret Symington. Four dimensions from two in symplectic topology. *arXiv preprint math/0210033*, 2002.

[Tu14] Junwu Tu. On the reconstruction problem in mirror symmetry. *Advances in Mathematics*, 256:449–478, 2014.

[Yua20] Hang Yuan. Family floer program and non-archimedean syz mirror construction. *arXiv preprint arXiv:2003.06106*, 2020.