Slicing up the spam

$P(E|F) = \frac{|EF|}{|F|}$ Equally likely outcomes

24 emails are sent, 6 each to 4 users.

- 10 of the 24 emails are spam.
- All possible outcomes are equally likely.

Let E = user 1 receives 3 spam emails.

What is $P(E)$?

$$P(E) = \frac{\binom{10}{3}\binom{14}{3}}{\binom{24}{6}} \approx 0.3245$$

Let F = user 2 receives 6 spam emails.

What is $P(E|F)$?

$$P(E|F) = \frac{\binom{4}{3}\binom{14}{3}}{\binom{18}{6}} \approx 0.0784$$

Let G = user 3 receives 5 spam emails.

What is $P(G|F)$?

$$P(G|F) = \frac{\binom{4}{5}\binom{14}{1}}{\binom{18}{6}} = 0$$

No way to choose 5 spam from 4 remaining spam emails!

Finding $P(E)$ from $P(E|F)$

$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$ Law of Total Probability

- Flip a fair coin.
- If heads: roll a fair 6-sided die.
- Else: roll a fair 3-sided die.

You win if you roll a 6. What is P(winning)?

1. Define events & state goal

Let: E: win, F: flip heads

Want: $P(\text{win}) = P(E)$

2. Identify known probabilities

$P(\text{win}\,|\,\text{H}) = P(E|F) = 1/6$

$P(\text{H}) = P(F) = 1/2$

$P(\text{win}\,|\,\text{T}) = P(E|F^C) = 0$

$P(\text{T}) = P(F^C) = 1 - 1/2$

3. Solve

$$P(E) = (1/6)(1/2) + (0)(1/2) = \frac{1}{12} \approx 0.083$$

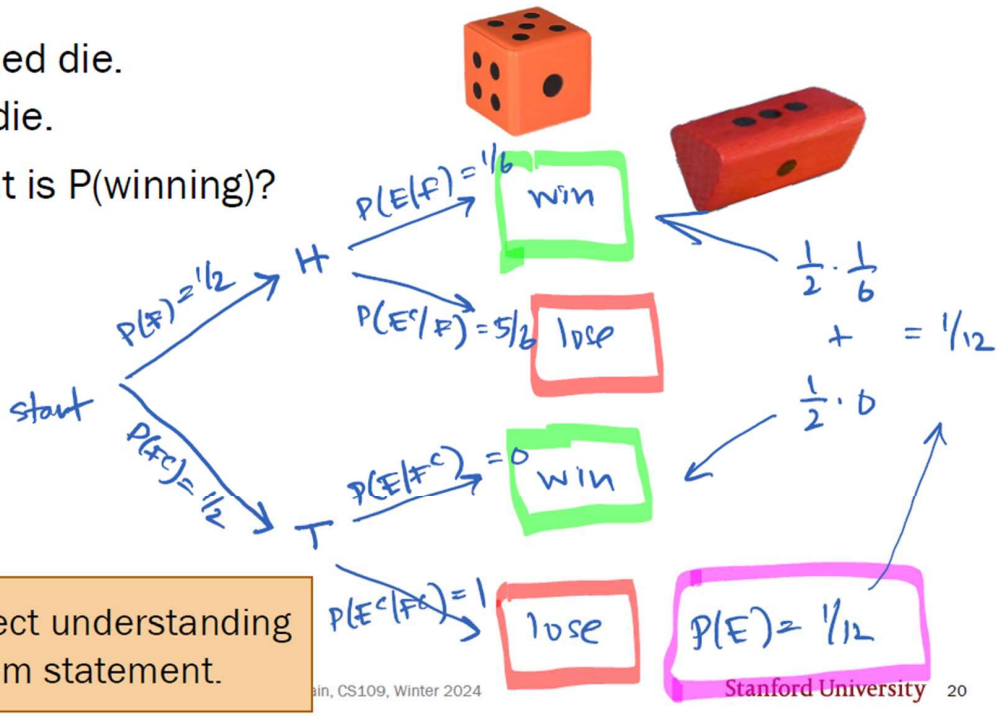
Finding $P(E)$ from $P(E|F)$, an understanding

- Flip a fair coin.
- If heads: roll a fair 6-sided die.
- Else: roll a fair 3-sided die.

You win if you roll a 6. What is P(winning)?

1. Define events & state goal

Let: E: win, F: flip heads
Want: $P(\text{win}) = P(E)$



"Probability trees" can help connect understanding of the experiment with the problem statement.