Variance of a 6-sided die

$\text{Var}(X) = E[(X - E[X])^2]$ Variance
$= E[X^2] - (E[X])^2$ of X

Let Y = outcome of a single die roll. Recall $E[Y] = 7/2$.

Calculate the variance of Y.

1. Approach #1: Definition

$$\text{Var}(Y) = \frac{1}{6}\left(1-\frac{7}{2}\right)^2 + \frac{1}{6}\left(2-\frac{7}{2}\right)^2 + \frac{1}{6}\left(3-\frac{7}{2}\right)^2 + \frac{1}{6}\left(4-\frac{7}{2}\right)^2 + \frac{1}{6}\left(5-\frac{7}{2}\right)^2 + \frac{1}{6}\left(6-\frac{7}{2}\right)^2$$

$$= 35/12$$

2. Approach #2: A property

2nd moment

$$E[Y^2] = \frac{1}{6}[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2] = 91/6$$

$$\text{Var}(Y) = 91/6 - (7/2)^2$$

$$= 35/12$$

Statistics: Expectation and variance

If you can identify common RVs, just look up statistics instead of rederiving from scratch.

1. a. Let X = the outcome of a fair 24-sided die roll. What is $E[X]$?
   b. Let Y = the sum of seven rolls of a fair 24-sided die. What is $E[Y]$?

support = $\{1,2,3,4,5,\ldots,23,24\}$
$E[X] = 12{,}5$, by symmetry
$E[Y] = E[X_1 + X_2 + X_3 + \cdots + X_7]$
$= E[X_1] + E[X_2] + E[X_3] + \cdots$
$= 7E[X_1] = 87{,}5$

2. Let Z = # of ***tails*** on 10 flips of a biased coin, with p = 0.71. What is $E[Z]$?

$p = 0{,}71$
$E[Z] = 10p = 7{,}1$

3. Compare the variances of $B_0 \sim \text{Ber}(0.0)$, $B_1 \sim \text{Ber}(0.1)$, $B_2 \sim \text{Ber}(0.5)$, and $B_3 \sim \text{Ber}(0.9)$.

$\text{Var}(B_0) = 0 \Rightarrow$ no spread, no variation
$\text{Var}(B_1) = 0{,}1(1-0{,}1) = 0{,}09 \leftarrow$ nonzero, but small
$\text{Var}(B_2) = 0{,}5^2 = 0{,}25 \leftarrow$ relatively substantial
$\text{Var}(B_3) = \text{Var}(B_1) = 0{,}09$
$p = 0{,}5$ maximizes variance