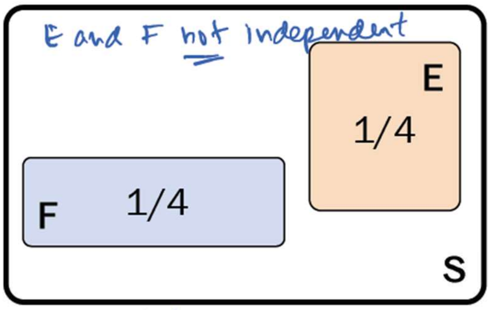
Independence?

Independent events E and F	⟺	$P(EF) = P(E)P(F)$ $P(E\|F) = P(E)$

assume $P(E), P(F) > 0$

1. True or False? Two events E and F are independent if:
   A. no Knowing that F happens means that E can't happen. $P(E|F) = 0 \neq P(E)$
   B. yes Knowing that F happens doesn't change probability that E happened.
   $P(E|F) = P(E)$ definition of independence
2. Are E and F independent in the following pictures?

A.

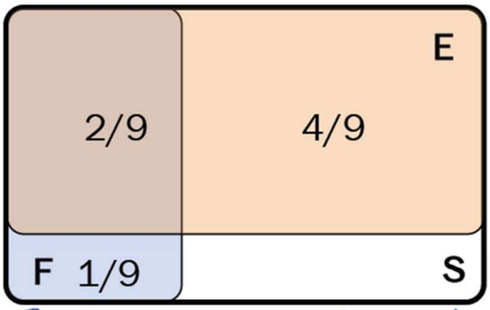


$EF = \emptyset$ $\quad P(E) = 1/4$

$P(EF) = 0$ $\quad P(F) = 1/4$

product of the two $= 1/16 \neq 0$

B.



E and F are independent!

$P(E) = \frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$ $\quad P(EF) = 2/9$

$P(F) = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$ $\quad P(E) \cdot P(F) = \frac{2}{3} \cdot \frac{1}{3} = 2/9$

(biased) Coin Flips

Suppose we flip a coin n times. Each coin flip is an **independent trial** with probability p of coming up heads. Write an expression for the following:

1. $P(n$ heads on n coin flips$)$ → n consecutive heads $\underbrace{H}_{p}\underbrace{H}_{p}\underbrace{H}_{p}\underbrace{H}_{p} \cdots \underbrace{H}_{p} \Rightarrow \boxed{p^n}$
2. $P(n$ tails on n coin flips$)$ → n consecutive tails $\underbrace{T}_{1-p}\underbrace{T}_{1-p}\underbrace{T}_{1-p} \cdots \underbrace{T}_{1-p} \Rightarrow (1-p)^n = \boxed{q^n}$ where $q = 1-p$
3. $P($first k heads, then $n-k$ tails$)$ → $\underbrace{HH \cdots H}_{k}\ \underbrace{TTT \cdots T}_{n-k} \Rightarrow p^k (1-p)^{n-k} \boxed{= p^k q^{n-k}}$
4. $P($exactly k heads on n coin flips$)$

→ any single sequence of n flips with k heads somewhere is $p^k (1-p)^{n-k}$

there are $\binom{n}{k}$ such sequences

total probability is $\boxed{\binom{n}{k} p^k (1-p)^{n-k}}$

$$\binom{n}{k} \; p^k (1-p)^{n-k}$$

# of mutually exclusive outcomes — $P($a particular outcome's k heads on n coin flips$)$

Make sure you understand #4! It will come up again.