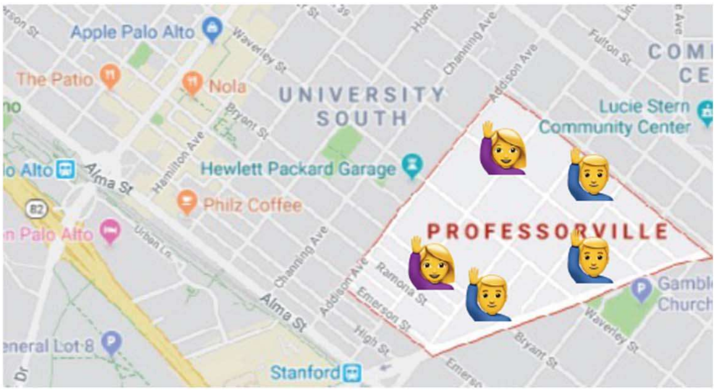
Algorithmic ride sharing



Probability of k requests from this area in the next 1 minute?

On average, $\lambda = 5$ requests per minute

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Poisson distribution

Earthquakes

$X \sim \text{Poi}(\lambda)$ $\quad p(k) = e^{-\lambda} \frac{\lambda^k}{k!}$

$E[X] = \lambda$

There are an average of 2.79 major earthquakes in the world each year, and major earthquakes occur independently.

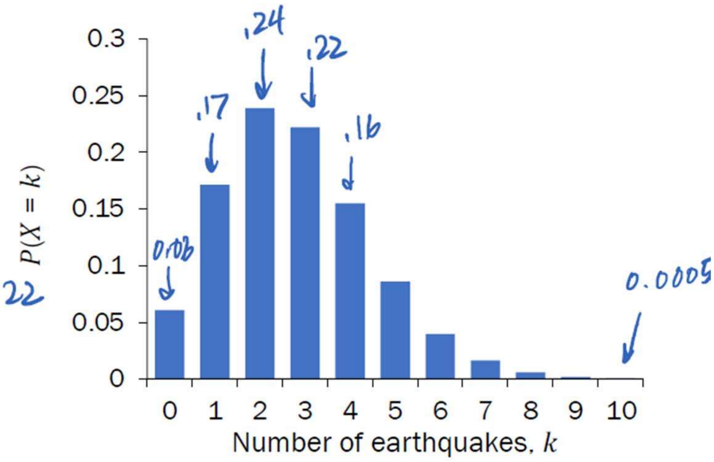
What is the probability of 3 major earthquakes happening next year?

1. Define RVs

$X \sim \text{Poi}(\lambda = 2{,}79)$

2. Solve

$P(X=3) = e^{-2{,}79} \frac{(2{,}79)^3}{3!} \approx 0{,}22$



1. Web server load

$X \sim \text{Poi}(\lambda)$ $\quad p(k) = e^{-\lambda}\frac{\lambda^k}{k!}$

$E[X] = \lambda$

Consider requests to a web server in 1 second.

- In the past, server load averages 2 hits/second, where requests arrive independently.
- Let X = # requests the server receives in a second.

What is $P(X < 5)$?

Define RVs

$X \sim Poi(\lambda = 2)$

unit of time is the second

Solve

$$P(X<5) = P(X \le 4) = \sum_{k=0}^{4} e^{-2}\frac{2^k}{k!} = e^{-2}\sum_{k=0}^{4}\frac{2^k}{k!} = 0.9473$$

Alternatively, you could compute $P(X \geq 5)$, but no good reason to do that!!

2. DNA

base n

What is the probability that DNA storage stays uncorrupted?

- In DNA (and real networks), we store large strings.
- Let string length be long, e.g., $n \approx 10^4$
- Probability of corruption of each base pair is very small, e.g., $p = 10^{-6}$
- Let X = # of corruptions.

What is P(DNA storage is uncorrupted) = $P(X = 0)$?

1. Approach 1:

$$X \sim \text{Bin}(n = 10^4, p = 10^{-6})$$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

unwieldy! ⚠️ $= \binom{10^4}{0} 10^{-6\cdot 0}(1 - 10^{-6})^{10^4 - 0}$

≈ 0.990049829

2. Approach 2:

$\lambda = np = E[X]$

$$X \sim \text{Poi}(\lambda = 10^4 \cdot 10^{-6} = 0.01)$$

$$P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!} = e^{-0.01}\frac{0.01^0}{0!}$$

$k = 0$

$$= e^{-0.01}$$

≈ 0.990049834 a good approximation! ✅