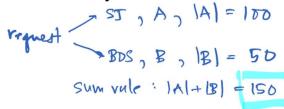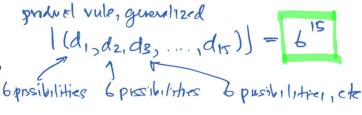# Let's try it out

Sum Rule, Product Rule, or something else? How many outcomes?

1. **Video streaming application**
   - Your application has distributed servers in 2 locations (SJ: 100, Boston: 50).
   - If a web request is routed to a server, how large is the set of servers it can get routed to?

   request $\to$ SJ, A, $|A| = 100$
   $\to$ BDS, B, $|B| = 50$
   Sum rule: $|A| + |B| = 150$

2. **Dice**
   - How many possible outcomes are there from rolling fifteen six-sided dice?

   product rule, generalized
   $|(d_1, d_2, d_3, \ldots, d_{15})| = 6^{15}$
   6 possibilities 6 possibilities 6 possibilities, etc

3. **Strings**
   - How many **different** orderings of letters are possible for the string PEPPER?

   $\# = \dfrac{6!}{3!\, 2!\, 1!} = 60 \text{ orderings}$

Transmitting bytes over a network

An 8-bit string is sent over a network.
- The receiver only accepts strings that either start with 01 or end with 10.

How many 8-bit strings will the receiver accept?

01001100
byte (8 bits)

Define

A : 8-bit strings starting with 01

B : 8-bit strings ending with 10

A: $\underline{0}\ \underline{1}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}$ $|A| = 2^6$

B: $\underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{0}\ \underline{1}$ $|B| = 2^6$

A∩B: $\underline{0}\ \underline{1}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{0}\ \underline{1}$ $|A \cap B| = 2^4$

$|A \cup B| = |A| + |B| - |A \cap B| = 2^6 + 2^6 - 2^4$
$= 2^7 - 2^4 = 112$

License plates

How many CA license plates are possible if...

(pre-1982)

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$$
$$\underbrace{}_{A-2} \quad \underbrace{}_{A-2} \quad \underbrace{}_{A-2} \quad \underbrace{}_{0-9} \quad \underbrace{}_{0-9} \quad \underbrace{}_{0-9}$$

(present day)

allows for leading $1,2,3,4,\ldots,9$ (no zero)

approach 1: $9 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$
$+ \; 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 175,760,000$

original count, pre-1982

approach 2: $(9+1) \cdot 17,576,000 = 175,760,000$

leading digit 1-9   no leading digit   pre-1982

Arrange *n* distinct objects

Ayesha   Tim   Irina   Joey   Waddie

1st   2nd   3rd   4th   5th

Steps:

1. Choose 1st can   5 options
2. Choose 2nd can   4 options

...

5. Choose 5th can   1 option

$$\text{Total} = 5 \times 4 \times 3 \times 2 \times 1$$
$$= 120$$

Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of **six** distinct numbers?

Total $= 6!$
$= 720$ passcodes

```
>>> import math
>>> math.factorial(6)
720
```