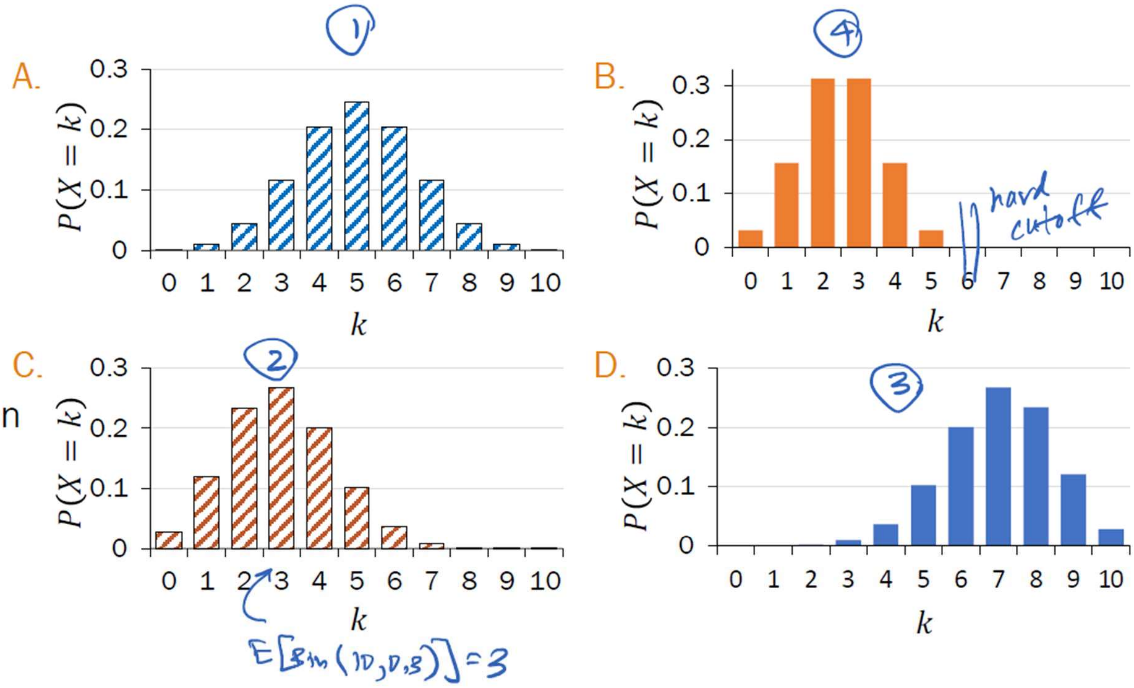
Visualizing Binomial PMFs

$E[X] = np$

$X \sim \text{Bin}(n,p) \quad p(i) = \binom{n}{k} p^k (1-p)^{n-k}$

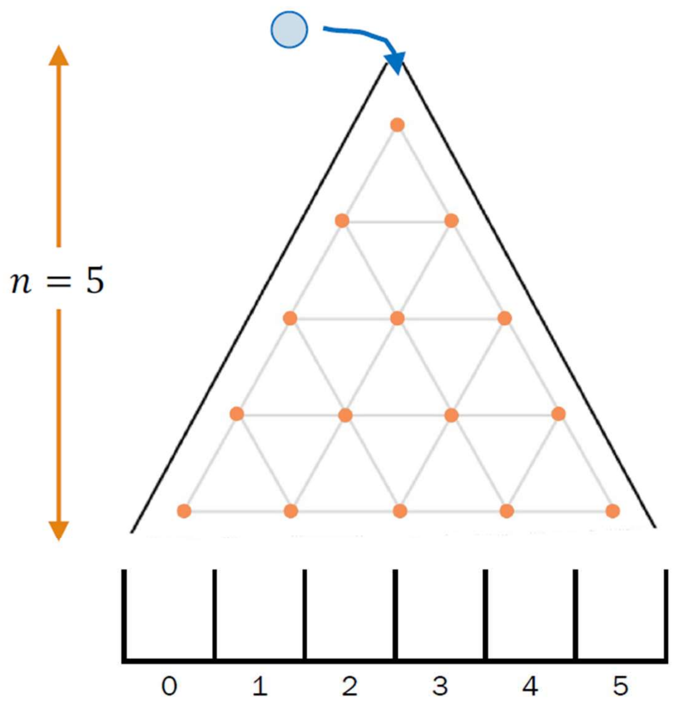
Match the distribution of X to the graph:

1. $\text{Bin}(10, 0.5)$
2. $\text{Bin}(10, 0.3)$
3. $\text{Bin}(10, 0.7)$
4. $\text{Bin}(5, 0.5)$



Galton Board

$X \sim \text{Bin}(n,p) \quad p(k) = \binom{n}{k} p^k (1-p)^{n-k}$



When a marble hits a pin, it has an equal chance of going left or right.
Let B = the bucket index a ball drops into.
What is the **distribution** of B?

- Each pin is an independent trial
- One decision made for level $i = 1, 2, \ldots, 5$
- Consider a Bernoulli RV with success R_i if ball went right on level i
- Bucket index B = # times ball went right

$$B \sim \text{Bin}(n = 5, p = 0.5)$$

Galton Board

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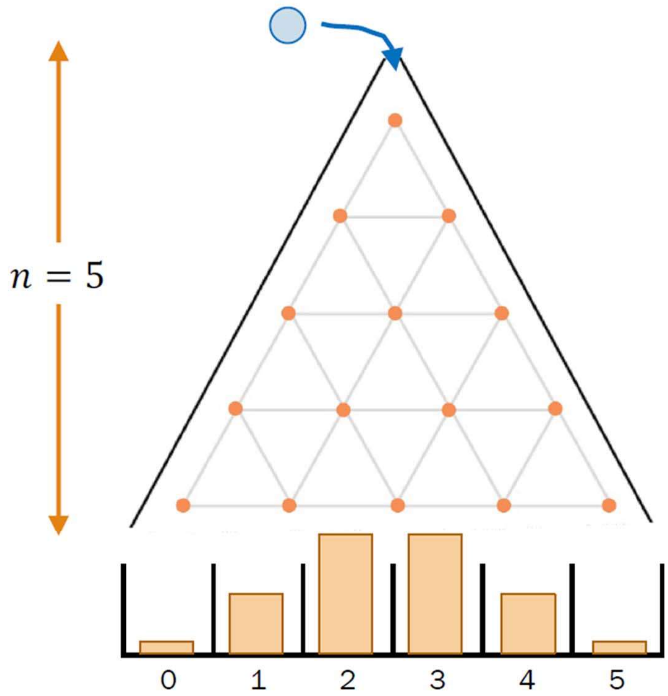

When a marble hits a pin, it has an equal chance of going left or right.
Let B = the bucket index a ball drops into.
B is distributed as a Binomial RV,

$$B \sim \text{Bin}(n = 5, p = 0.5)$$

Calculate the probability of a ball landing in bucket k.

$$P(B = 0) = \binom{5}{0} 0.5^5 \approx 0.03$$

$$P(B = 1) = \binom{5}{1} 0.5^5 \approx 0.16$$

$$P(B = 2) = \binom{5}{2} 0.5^5 \approx 0.31$$

PMF of Binomial RV!

Genetic inheritance

1. Each parent has 2 genes per trait (e.g., eye color).

- Child inherits 1 gene from each parent with equal likelihood.
- Brown eyes are "dominant", blue eyes are "recessive":
  - Child has brown eyes if either or both genes for brown eyes are inherited.
  - Child has blue eyes otherwise (i.e., child inherits two genes for blue eyes)
- Assume parents each have 1 gene for blue eyes and 1 gene for brown eyes.

Two parents have 4 children. What is P(exactly 3 children have brown eyes)?

Big Q: Fixed parameter or random variable?

Parameters What is common among all outcomes of our experiment?

$n=4$, $p_{Brown} = 0.75$

Random variable What differentiates our event from the rest of the sample space?

$X = \{0, 1, 2, 3, 4\}$

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1. Define events/ RVs & state goal
2. Identify known probabilities
3. Solve

X: # brown-eyed children,

$X \sim \text{Bin}(4, p)$, with $p = 0.75$

p: $P(\text{brown}-\text{eyed child})$

Want: $P(X = 3)$

$p = 0.75$

	R	L
R	.25	.25
L	.25	.25

brown = R

blue = L

$P(X=3) = \binom{4}{3}(0.75)^3(0.25)$

$= 0.4219$

RRRL RLRR

RRLR LRRR