General approach to counting permutations

When there are n objects such that

n_1 are the same (indistinguishable or indistinct), and

n_2 are the same, and

...

n_r are the same,

The number of unique orderings (permutations) is

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}.$$

Simple example:
how many strings can be formed from the letters TENSE

answer is $\frac{5!}{1!\,2!\,1!\,1!} = 60$

For each group of indistinct objects,
Divide by the overcounted permutations.

Sort semi-distinct objects

Order n semi-distinct objects $\frac{n!}{n_1!\, n_2! \cdots n_r!}$

How many permutations?

number of distinct orderings is $\frac{5!}{2!\,3!}$

Coke Coke0 Coke Coke0 Coke0

Strings

Order n semi-distinct objects $\frac{n!}{n_1! n_2! \cdots n_r!}$

How many letter orderings are possible for the following strings?

1. SUBCOMMITTEE — 12 letters; 2 M's, 2 T's, 2 E's $\Rightarrow \frac{12!}{2!2!2!}$

2. MISSISSIPPI — 11 letters; 4 I's, 4 S's, 2 P's $\Rightarrow \frac{11!}{4!4!2!}$

Unique 6-digit passcodes with five smudges

Order n semi-distinct objects $\frac{n!}{n_1! n_2! \cdots n_r!}$

How many unique 6-digit passcodes are possible if a phone password uses each of five distinct numbers?

Steps:

1. Choose digit to repeat — 5 outcomes
2. Create passcode — (sort 6 digits: 4 distinct, 2 indistinct)

$$\text{Total} = 5 \times \frac{6!}{2!} = 1{,}800 \text{ passcodes}$$