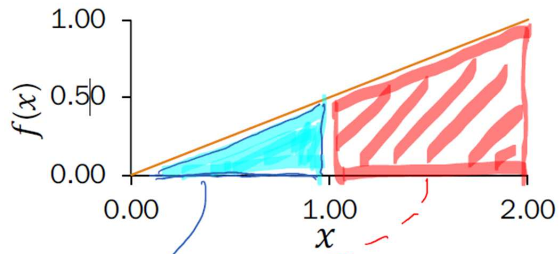
Computing probability

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

Let X be a continuous RV with PDF:

confirm: $\int_0^2 \frac{x}{2} dx = 1$, so valid pdf

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$



What is $P(X \ge 1)$?

Strategy 1: Integrate

$$P(1 \le X < \infty) = \int_1^\infty f(x)dx = \int_1^2 \frac{1}{2} x dx$$

$$= \frac{1}{2}\left(\frac{1}{2}x^2\right)\Big|_1^2 = \frac{1}{2}\left[2 - \frac{1}{2}\right] = \frac{3}{4}$$

Strategy 2: Know triangles

$$1 - \frac{1}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$$

Wait! Is this even legal?

$$P(0 \le X < 1) = \int_0^1 f(x)dx\,??$$

Earthquakes

$X \sim \text{Exp}(\lambda)$ $\quad E[X] = 1/\lambda$ $\quad f(x) = \lambda e^{-\lambda x}$ if $x \ge 0$

Major earthquakes (magnitude 8.0+) occur on average once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?

We know on average:

$$500 \ \frac{\text{years}}{\text{earthquake}}$$

$$0.002 \ \frac{\text{earthquakes}}{\text{year}}$$

$$1 \ \frac{\text{earthquakes}}{500 \text{ years}} = 1 \ \frac{\text{"success"}}{500 \text{ years}}$$

if earthquakes are "successes", then $\frac{1}{500} = 0.002$ is our lambda value, since λ is the average number of successes per time unit (in this case, a year).

*In California, according to historical data form USGS, 2015

Earthquakes

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Major earthquakes (magnitude 8.0+) occur once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?

Define events/ RVs & state goal

X: when next earthquake happens

$X \sim \text{Exp}(\lambda = 0.002)$

λ: year^{-1} $= 1/500$

Want: $P(X < 30)$

Solve

$$P(X<30) = \int_0^{30} 0.002\, e^{-0.002x}\, dx$$

$$= 0.002 \frac{-1}{0.002} e^{-0.002x} \Big|_0^{30}$$

$$= -\left(e^{-0.06} - e^{0.00}\right)$$

$$= 1 - e^{-0.06} \approx \boxed{0.058}$$

Recall $\int e^{cx} dx = \frac{1}{c} e^{cx}$

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Earthquakes

$X \sim \text{Exp}(\lambda)$ $E[X] = 1/\lambda$ $f(x) = \lambda e^{-\lambda x}$ if $x \geq 0$

Major earthquakes (magnitude 8.0+) occur once every 500 years.*

1. What is the probability of a major earthquake in the next 30 years?
2. What is the standard deviation of years until the next earthquake?

Define events/ RVs & state goal

X: when next earthquake happens

$X \sim \text{Exp}(\lambda = 0.002)$

λ: year^{-1}

Want: $P(X < 30)$

Solve

$$\text{Var}(X) = \frac{1}{\lambda^2} = \frac{1}{(0.002 \text{ year}^{-1})^2} = 250{,}000 \text{ years}^2$$

$$SD(X) = \sqrt{\text{Var}(X)} = \boxed{500 \text{ years}}$$

In general, $SD(X) = E[X] = \frac{1}{\lambda}$

whenever $X \sim \text{Exp}(\lambda)$

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Earthquakes

Major earthquakes (magnitude 8.0+) occur independently on average once every 500 years.*

What is the probability of zero major earthquakes next year?

Strategy 1: Exponential RV

Define events/RVs & state goal

T: when first earthquake happens
$T \sim \text{Exp}(\lambda = 0.002)$

Want: $P(T > 1) = 1 - F(1)$

Solve

$$P(T > 1) = 1 - \left(1 - e^{-\lambda \cdot 1}\right) = e^{-\lambda}$$

$= e^{-0.002} \approx 0.998$

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Earthquakes

$Y \sim \text{Poi}(\lambda) \quad p(k) = e^{-\lambda}\frac{\lambda^k}{k!}$

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What is the probability of zero major earthquakes next year?

Strategy 1: Exponential RV

Define events/RVs & state goal

T: when first earthquake happens
$T \sim \text{Exp}(\lambda = 0.002)$

Want: $P(T > 1) = 1 - F(1)$

Solve

$$P(T > 1) = 1 - \left(1 - e^{-\lambda \cdot 1}\right) = e^{-\lambda}$$

Strategy 2: Poisson RV

Define events/RVs & state goal

N: # earthquakes next year
$N \sim \text{Poi}(\lambda = 0.002)$

$\lambda: \frac{\text{earthquakes}}{\text{year}}$

Want: $P(N = 0)$

Solve

$$P(N = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} \approx 0.998$$

*In California, according to historical data form USGS, 2015

Replacing your laptop

$X \sim \text{Exp}(\lambda)$ $\quad E[X] = 1/\lambda$ $\quad F(x) = 1 - e^{-\lambda x}$

Let X = # hours of use until your laptop dies.

- X is distributed as an Exponential RV, where
- On average, laptops die after 5000 hours of use.
- You use your laptop 5 hours a day.

What is P(your laptop lasts 4 years)?

Define

X: # hours until laptop death

$X \sim \text{Exp}(\lambda = 1/5000)$

Want: $P(X > 5 \cdot 365 \cdot 4)$

Solve

$$P(X > 7300) = 1 - F(7300)$$
$$= 1 - \left(1 - e^{-7300/5000}\right) = e^{-1.46} \approx 0.2322$$

Better plan ahead if you're co-terming!

- 5-year plan: $P(X > 9125) = e^{-1.825} \approx 0.1612$
- 6-year plan: $P(X > 10950) = e^{-2.19} \approx 0.1119$