Cats and sharks (ordered solution)

$P(E) = \frac{|E|}{|S|}$ Equally likely outcomes

4 cats and 3 sharks in a bag. 3 drawn.

What is P(1 cat and 2 sharks drawn)?

pretend all stuffed animal are unique

Make indistinct items distinct to get equally likely outcomes.

Define

- S = Pick 3 distinct items, and retain order
- E = 1 distinct cat, 2 distinct sharks

$|S| = 7 \cdot 6 \cdot 5$

C stands for cat

S stands for shark

$|E| =$ sum of three distinct cases

$$\left.\begin{aligned} |E_{CSS}| &= 4 \cdot 3 \cdot 2 = 24 \\ |E_{SCS}| &= 3 \cdot 4 \cdot 2 = 24 \\ |E_{SSC}| &= 3 \cdot 2 \cdot 4 = 24 \end{aligned}\right\} 72$$

$\therefore$ probability is $P(E) = \frac{|E|}{|S|} = \frac{72}{210} = \frac{12}{35}$

Cats and sharks (unordered solution)

$P(E) = \frac{|E|}{|S|}$ Equally likely outcomes

4 cats and 3 sharks in a bag. 3 drawn.

What is P(1 cat and 2 sharks drawn)?

Make indistinct items distinct to get equally likely outcomes.

Define

- S = Pick 3 distinct items, ignore order
- E = 1 distinct cat, 2 distinct sharks

$|S| = \binom{7}{3} = 35$

$|E| = \binom{4}{1} \cdot \binom{3}{2} = 4 \cdot 3 = 12$

number of ways to choose one cat from four

number of ways to choose any two of three sharks

because we're ignoring order with this approach, we rely on combinations and choose terms instead of multiplication

$P(E) = \frac{|E|}{|S|} = \frac{12}{35}$

Counting? Probability? Distinctness?

We choose 3 books from a set of ↙ but distinguishable from the first four
4 distinct (distinguishable) and 2 indistinct (indistinguishable) books.
Each set of 3 books is equally likely.

Let event E = our choice excludes one or both indistinct books.

1. How many distinct outcomes are in E? } restated, how many visibly different subsets

   $\binom{4}{2}$ ways to include one of the two copies

   $\binom{4}{3}$ ways to exclude both identical copies

   $\Rightarrow \binom{4}{2} + \binom{4}{3} = 6 + 4 = 10$

2. What is $P(E)$?

make identical copies distinguishable from each other, else some subsets are more likely than others, and we want equally likely outcomes.

$|E| = 2\binom{4}{2} + \binom{4}{3} = 16$

$|S| = \binom{6}{3} = 20$

$P(E) = \frac{16}{20} = \frac{4}{5}$

distinct, equally likely outcomes → make indistinct → report count

distinct, equally likely outcomes → keep distinct → compute probability

1. Any Poker Straight

assume Ace can be either low or high

Consider equally likely 5-card poker hands.

- "straight" is 5 consecutive rank cards of any suit

What is P(Poker straight)?

possibilities: A2345
23456
34567
⋮
10JQKA

Define

- S (unordered)

$|S| = \binom{52}{5}$

- E (unordered, consistent with S)

$|E| = \boxed{10} \left[\underbrace{\binom{4}{1}}\right]^5$

10, because the lowest rank card in the straight can be one of ten different ranks (rest are constrained)

$\binom{4}{1}$ is the number of ways to choose a suit for each of the 5 cards

Compute $P(\text{Poker straight}) = \frac{|E|}{|S|} = \frac{10 \cdot 4^5}{\binom{52}{5}} = 0.00394$

2. Chip defect detection

$\binom{1}{1}$ is the number of ways we can choose the one defective chip!

$\binom{n-1}{k-1}$ is the number of ways to choose an additional $k-1$ chips from the $n-1$ good ones.

n chips are manufactured, 1 of which is defective.
k chips are randomly selected from n for testing.

What is P(defective chip is in k selected chips?)

Define

- S (unordered)
- E (unordered, consistent with S)

$|S| = \binom{n}{k}$ → all possible subsets of size k

$|E| = \binom{1}{1}\binom{n-1}{k-1} = \binom{n-1}{k-1}$

Compute

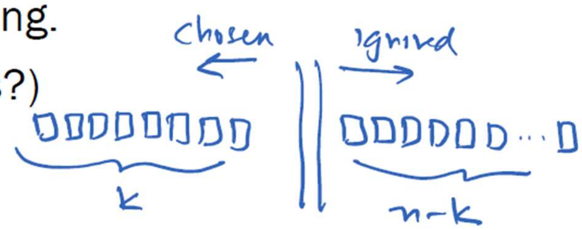
$$P(E) = \frac{|E|}{|S|} = \frac{\binom{n-1}{k-1}}{\binom{n}{k}}$$

$$= \frac{\frac{(n-1)!}{(k-1)!\,\cancel{(n-k)!}}}{\frac{n!}{k!\,\cancel{(n-k)!}}} = \frac{(n-1)!}{n!}\cdot\frac{k!}{(k-1)!} = \frac{k}{n} \Rightarrow \boxed{P(E) = \frac{k}{n}}$$

2. Chip defect detection, solution #2

n chips are manufactured, 1 of which is defective.
k chips are randomly selected from n for testing.

What is P(defective chip is in k selected chips?)



Redefine experiment

1. Choose k indistinct chips (1 way)
2. Throw a dart and make one defective

probability of hitting one to the left of the divider is $\frac{k}{n}$

$\therefore \boxed{P(E) = \frac{k}{n}}$

Define

- S (unordered)
- E (unordered, consistent with S)

Serendipity

you are friends with 100
you are not friends with 16,900

- The population of Stanford is $n = 17{,}000$ people.
- You are friends with $r = 100$ people.
- Walk into a room, see $k = 223$ random people.
- Assume each group of k Stanford people is equally likely to be in the room.

What is the probability that you see at least one friend in the room?

$|S| = \binom{17000}{223}$

$|E^c| = \binom{100}{0}\binom{16900}{223} = \binom{16900}{223}$

Define

- S (unordered)
- E: ≥ 1 friend in the room

$$P(E) = 1 - P(E^c) = 1 - \frac{\binom{16900}{223}}{\binom{17000}{223}} = 0.7340$$

It is often much easier to compute $P(E^c)$.