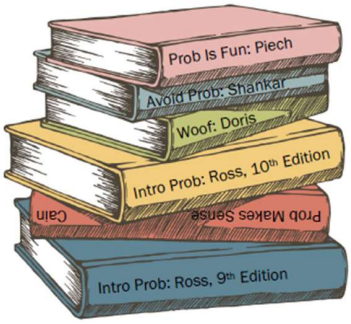
Probability textbooks

Choose k of n distinct objects $\binom{n}{k}$

How many ways are there to choose a set of 3 from a set of 6 distinct books? Assume order doesn't matter.



$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

we select three books but don't care about order of selection

we don't care about order of books we ignore either.

Probability textbooks

Choose k of n distinct objects $\binom{n}{k}$

1. How many ways are there to choose 3 books from a set of 6 distinct books?

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

2. Two are by the same author. What if we don't want to choose both?

Strategy 1: Sum Rule

answer is $2\binom{4}{2} + \binom{4}{3} = 16$

two books are by author with last name Steinbeck, others by authors A, B, C, and D

partition into three cases

Steinbeck	Steinbeck	A	B	C	D	
✓	✗					choose any two — 1 → $\binom{4}{2}$
✗	✓					choose any two — 1 → $\binom{4}{2}$
✗	✗					choose any three — 1 → $\binom{4}{3}$

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1. How many ways are there to choose 3 books from a set of 6 distinct books?

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

2. Two are by the same author. What if we don't want to choose both?

answer is $\binom{6}{3} - \binom{4}{1} = 16$

Strategy 2: "Forbidden method" (unofficial name)

count number of illegal / forbidden subsets

Steinbeck Steinbeck A B C D

✓ ✓ ← select one →

$= \binom{4}{1}$

Forbidden method: It is sometimes easier to exclude invalid cases than to include cases.

Venture capitalists. #1

Divider method (n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

You have \$10 million to invest in 4 companies (in units of \$1 million).

1. How many ways can you fully allocate your \$10 million?

Set up

$$x_1 + x_2 + x_3 + x_4 = 10$$

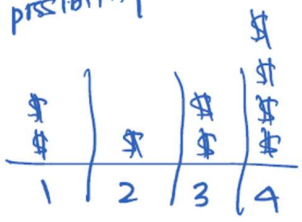
x_i: amount invested in company i

$$x_i \geq 0$$

x_i are integers

Solve

one such possibility



$\binom{10+4-1}{4-1} = 286$

Venture capitalists. #2

Divider method (n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

You have \$10 million to invest in 4 companies (in units of \$1 million).

1. How many ways can you fully allocate your \$10 million?
2. What if you want to invest at least \$3 million in company 1?

Set up

$$x_1 + x_2 + x_3 + x_4 = 10$$

x_i: amount invested in company i

constraint that \$3 million goes to company 1, but remaining \$7 million can be freely allocated.. We're really solving $x_1 + x_2 + x_3 + x_4 = 7$

Solve

$$\text{answer} = \binom{7+4-1}{4-1} = 120$$

Venture capitalists. #3

Divider method (n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

You have \$10 million to invest in 4 companies (in units of \$1 million).

1. How many ways can you fully allocate your \$10 million?
2. What if you want to invest at least \$3 million in company 1?
3. What if you don't have to invest all your money?

Set up

$$x_1 + x_2 + x_3 + x_4 \leq 10$$

x_i: amount invested in company i

$$x_i \geq 0$$

you are really solving this

$x_1 + x_2 + x_3 + x_4 + x_5 = 10$, where x_5 counts the money you don't invest

Solve

$$\text{answer is } \binom{10+5-1}{5-1} = 1001$$