(s,S) Inventory

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This example is adapted (almost verbatim) from the article Kleijnen, J.P.C. et al. *Constrained Optimization in Simulation: A Novel Approach*, Discussion Paper 2008-95, Tilburg University, Center for Economic Research.

Consider a (*s*,*S*) inventory model with full backlogging. Demand during each period, D_t is distributed exponential with mean μ. At the end of each period, the inventory position (IP_t = Stock on hand - Backorders + Outstanding Orders) is calculated and, if it is below s, an order to get back up to S is placed ($O_t = max\{I(IP_t < s)(S - IP_t), 0\}$). Lead times have a Poisson distribution with mean θ days and all replenishment orders are received at the beginning of the period. Note that, since orders are placed at the end of the day, an order with lead time l placed in period n will arrive at the beginning of period $n + l + 1$.

A per unit holding cost h is charged for inventory on-hand; furthermore, there is a fixed order cost f and a variable, per unit, cost c. Our goal is to find s and S in order to minimize the E[Total cost per period] such that the stockout rate δ – the fraction of demand not supplied from stock on-hand – is at most 10%. To further clarify the order of events and the calculation of costs, a 5-day example in which $s = 1000$ and $S = 1500$, the initial inventory on hand is 1000 and there are no outstanding orders is provided in Table 1.

Recommended Parameter Settings: Take $\mu = 100$, $\theta = 6$, $h = 1$, $f = 36$ and $c = 2$.

Starting Solutions: $s = 1000$, $S = 2000$. If multiple solutions are needed, use s ~Uniform(700,1000), S ~Uniform(1500,2000).

Measurement of Time: Days simulated

Optimal Solution: Unknown

Table 1: 5-day Example

Day	**1**	**2**	**3**	**4**	**5**
Starting inventory (backorders)	1000	900	500	200	(50)
Orders received	0	0	0	0	1000
Demand	100	400	300	250	250
Ending inventory (backorders)	900	500	200	(50)	700
Inventory position	900	500	1200	950	700
Orders placed	0	1000	0	0	800
Order lead time	-	2	-	-	3
Period cost	$900	$2536	$200	$0	$2336
Cumulative on-time delivery (1-δ)	100%	100%	100%	95.23%	96.15%