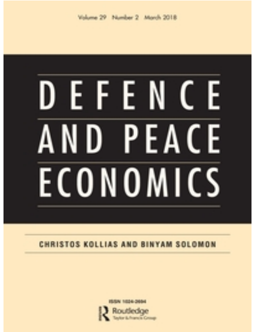

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THE TIMING OF THIRD-PARTY INTERVENTION IN SOCIAL CONFLICT

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This paper analyzes how the equilibrium outcome of social conflict between factions is strategically altered by third-party intervention. We consider an intervening third party that commits financial support to one of two contending factions for reducing its cost in conflict. Within the framework of three-player sequential-move games, we investigate the questions as follows. What is the optimal intervention intensity in terms of the third party's financial support? Is there a first-mover advantage in conflict when there is third-party intervention? Fighting against all odds, will the unsupported faction have a chance to prevail when its opponent receives third-party support? What is the optimal timing of third-party intervention? The analysis in the paper has implications for the conditions under which the strategic intervention of a third party may or may not break a conflict between factions.

Keywords: Conflict; Sequential game; Third-party intervention; First-mover advantage

JEL Codes: D72; D74

1. INTRODUCTION

For better or for worse, third-party interventions are typical responses to persistent social conflicts. Despite that their forms and contexts may differ considerably, interventions by third parties commonly take place in such non-violent conflicts as litigation by interested parties over property rights (Robson and Skaperdas 2008) and water disputes across jurisdictional boundaries (Ansink and Weikard 2009). Not surprisingly, third-party interventions have frequently been observed in the events of violent conflicts or terrorist attacks.[1] Interventions by third parties from time to time occupy the center stage in international politics when conflicts show no signs of ending. Given the persistence of many intrastate conflicts in particular, it is important to understand the role that an intervening third party might play in influencing the outcome of a conflict between factions. Although the goal of certain

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[1]See, e.g. Regan (1996, 1998, 2002), Werner (2000), Regan and Aydin (2006), Rowlands and Carment (2006), Azam and Thelen (2010), and Bandyopadhyay, Sandler, and Younas (2011).

third-party intervention is unbiased mediation for reducing or eliminating conflict (e.g. United Nations peacekeeping missions), such an ideal goal does not, by any means, drive all third-party actions. Many studies contend that third parties may choose to intervene, directly or indirectly, when their own interests are at stake.[2] For example, in his classic study on third-party involvements in armed conflicts, Morgenthau (1967, 430) states that 'All nations will continue to be guided in their decisions to intervene … by what they regard as their respective national interests.' Betts (1994, 21) indicates that interventions can end a conflict efficiently when 'the intervener takes sides, tilts the local balance of power, and helps one of the rivals to win.' Regan (1996, 1998) argues that the paradigm of realism is the dominant philosophy in third-party interventions.

This paper is concerned with social conflict and the political economy of third-party intervention. We extend the standard rent-seeking game with asymmetric valuations to analyze various issues on the intervention of a third party into social conflict between factions. Social conflicts arise when different factions or interest groups compete for valuable resources, the property rights of which are either imperfectly specified or imperfectly enforced. The faction that gains control over resources will be able to pursuit its own interests. For gaining the control, each faction allocates a fraction of its resources to contest for political power or dominance.[3] In our analysis, we wish to examine how the equilibrium outcome of social conflict between two factions is affected by third-party intervention. Specifically, we consider an intervening third party that commits financial support to one of the two factions for lowering its fighting cost. Based on the analytical framework of three-player sequential-move games, we investigate a set of questions related to conflicts with biased interventions. Will a supported faction be able to increase its winning probability and expected payoff when receiving financial subsidies from an intervening third party? Is there a first-mover advantage for the two contending factions, given the intervention decision of the third party? In terms of intervention subsidies to the supported faction, is there an advantage for both the third party and its supported 'ally' when the former is the overall first mover in a sequential game? Fighting against all odds, under what circumstances will the unsupported faction (i.e. the underdog) have a chance to prevail when its opponent receives third-party support? What is the optimal timing of an effective third-party intervention in the two-faction conflict? In other words, is the sequence of move in an intervention game crucial to the third party's effectiveness in increasing its own expected payoff and the winning probability of the supported faction?

Nash equilibrium models of contests and conflicts adopt simultaneous-move games. But one of the contending parties may commit to use the first-mover strategy (e.g. Dixit 1987; Baik and Shogren 1992; Leininger 1993; Gershenson and Grossman 2000; Morgan 2003; Aanesen 2011). A player who moves first may be able to influence the outcome of a game. This suggests the importance of timing in choosing an optimal effort in a Stackelberg-type sequential-move game other than the Nash equilibrium effort (Congleton, Hillman, and Konrad 2008).[4] It has been argued in the literature that in a sequential game with complete information, the moving order is critical in determining the equilibrium outcome of a two-party contest or competition. For example, Morgan

[2]See, e.g. Morgenthau (1967), Bull (1984), Betts (1994), Blechman (1995), Carment and James (1995), Dowty and Loescher (1996), Amegashie and Kutsoati (2007), Amegashie (2010), and Chang, Sanders, and Walia (2010).
[3]See, e.g. Gershenson and Grossman (2000), Azam and Mesnard (2003), Collier and Hoeffler (2004), Garfinkel and Skaperdas (2007), Collier, Hoeffler, and Rohner (2009), and Vahabi (2010).
[4]Congleton, Hillman, and Konrad (2008) present a systematic review of contributions on rent seeking in the past several decades. The authors discuss different formats of contests including those with an optimal choice of timing in a sequential-move game.

(2003) analyzes and compares the expected payoffs of two contenders from a sequential-move game to a simultaneous-move game. The author shows that contenders prefer a sequential-move game as it generates relatively higher expected payoffs for both sides. In the present paper, we extend the Morgan (2003) model of two-party competition to allow for the possibilities of third-party intervention (in the form of subsidies). In analyzing the strategic intervention of a third party into conflicts between factions, we show how the three players moving sequentially affect the optimal allocations of combative inputs by the factions, their equilibrium winning probabilities, the incentives of the third party in providing financial support for its ally, as well as the expected payoffs of all the three players involved.

In recent years there has been growing interest in investigating the effectiveness of third-party interventions into conflicts from a game-theoretic perspective. Gershenson (2002) systematically examines the effect of a third party who imposes sanctions on a faction in a civil conflict. Siqueira (2003) further uses a conflict model to analyze the strategies of third-party intervention. In his analysis, the third party acts strictly as peacemaker to reduce the level of conflict, regardless of the stakes involved in a specific conflict. Siqueira's work helps pave the foundation for extensions in that he suggests cost-reducing arms subsidies as a mode of third-party conflict intervention. Amegashie and Kutsoati (2007) model intervention by a third party in a two-faction conflict as a simultaneous-move game. The authors find that, unless the game is indefinitely repeated, a third party tends to intervene on behalf of a relatively strong faction when winning probability is directly related to combative efforts or when two parties are similar in abilities. Chang, Potter, and Sanders (2007a) and Chang and Sanders (2009) present a sequential-move game to model intervention into conflict where a third party chooses to intervene by providing cost-reducing military assistance to its ally. Chang and Sanders (2009) show that an intervening third party takes into account an ally's relative strength in fighting as complementary to intervention subsidies. That is, the third party finds it beneficial to provide more subsides to a relatively capable ally. Amegashie (2010) presents a recent review on issues in third-party intervention. The author points out possible difficulties in analyzing intervention from a purely economic perspective. In the present paper, we further look at some important issues that seem not to have been adequately examined in the theoretical conflict literature. These issues include the potential benefits of intervention subsidies to a supported faction in a sequential game, the optimal timing of third-party intervention, and the first-mover advantage on the part of an intervening third party. This paper also extends the conflict models of Gershenson and Grossman (2000) and Chang, Potter, and Sanders (2007b). These two studies show that if two factions value political dominance so differently, the faction with a higher valuation exerts more effort and is able to gain the control over the other. But if the parties' valuations of political dominance are considerably close, conflict is never-ending.

We attempt to move beyond the never-ending outcome by investigating conditions under which the strategic intervention of a third party is able to break the persistent conflict. It is often important for the third party to commit its support to its favorite faction before the supported makes a decision, for three complementary reasons: (i) the favorite faction may not be solely powerful enough to defeat its rival; (ii) the favorite faction may not overcome the collective action dilemma (Olson 1965, 1982), which appears as 'the paradox of revolution (Tullock 1974, 1987)' when the favorite faction is a repressed group in a dictatorship regime; and (3) the third-party may expect cost-benefit efficiency out of its intervention. That is to say, with its initiative move before the favorite faction acts, the third party may effectively provoke the supported[5] to make the possibility out of the seeming impossibility,[6] and equally importantly, maximizes its benefits from the dominance of the supported[7] before the supported has already suffered heavy losses in the battles.

While peacebuilding and peacekeeping, in the context of interstate or civil wars, necessarily involves outside parties, the third party in our analytical framework is an expected utility maximizer driven by self-interests.[8] In other words, in our model, third-party intervention may induce escalation of the conflict rather than pacification. Our framework also rules out any post-conflict involvement by the third party (where it moves last). Nonetheless, our model is connected to two aspects commonly considered in the literature of economics of peacekeeping. First, the third party attaches private values on the two fighting factions (Berkok and Solomon 2011). Second, the subsidy provided by the third party to its favorite faction is similar in the spirit of a side payment (Bove and Smith 2011).

The key findings of the paper are as follows. First, biased interventions into social conflict between factions through financial subsidies to one faction may increase its winning probability and expected payoff. Second, there are scenarios that the strategic intervention of a third party may fail to improve its ally's position, depending on the asymmetric valuations of the contested 'prize' (e.g. political dominance) as held by the two contending factions, the stakes the third party holds with each of the factions, and the timing or sequence of the moves. Third, the intervening third party plays a vital role on whether the first mover of the two factions can effectively deter the follower in the Stackelberg sense. Fourth, when the third party is the overall first mover in committing financial subsidies in a three-player sequential game, its supported faction has an incentive to increase its effort or combative input of fighting. This shows the conditions under which a third party's intervention subsidies and its supported faction's fighting effort are 'strategic complements.' In this case, whether the third party is able to reap a higher expected payoff depends crucially on the marginal effect of intervention subsidies on reducing the fighting cost of its ally.

The remainder of the paper is organized as follows. Section 2 presents the analytical framework of conflict between two factions when there is an intervening third party. In this section, we lay out basic assumptions of conflict and intervention technologies, and discuss the expected payoffs of the three players. In Section 3, we first examine the equilibrium outcomes of conflict between factions with and without third-party intervention, and then compare their differences in terms of winning probabilities and equilibrium payoffs. Section 4 contains concluding remarks. To focus on economic implications of the model results, we present all the mathematical proofs for our propositions in Appendix 1.

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[5]A classic example is the Communism movements provoked by the former Soviet Union. For instances, (i) in 1921 the former Soviet Union coined the Chinese Communist Party which later rebelled and defeated the then-central government of China; (ii) after the World War II, the former Soviet Union trained Kim II-Sung, sent him to rule North Korea, and then supported him to initiate the Korea War.

[6]The third-party intervention is not effective if it fails to support its favorite faction. One failed scenario is that the third party cannot successfully provoke the collective action of the disorganized favorite faction. Another failure occurs when the favorite faction is already smitten before the third party gives a hand.

[7]For instance, Gaddafi-ruled Libya supported Mahdi to rebel against Sudan government in 1976, backgrounded by historical territory disputes between the two countries. Similarly, during the 1st Congo war against Congolian dictator Mobutu in 1996, Rwanda and Uganda supported the repellants for the sake of ethnic disputes.

[8]For recent surveys on the economics of peacekeeping, see the special issue of *Defence and Peace Economics* (2006, Issue 5), Solomon (2007), and Bove and Smith (2011).

2. THE ANALYTICAL FRAMEWORK

We extend the standard rent-seeking game to analyze the effects of third-party intervention on the equilibrium outcome of social conflict between factions. For analytical simplicity, we assume that there are two factions, denoted as 1 and 2, competing for political power in a winner-take-all game. We further assume that the intrinsic value of political dominance to faction $i(=1, 2)$ is exogenously given as $V_i(>0)$, where V_1 and V_2 differ. This is consistent with the notion of asymmetric valuations in rent-seeking activities (Hillman and Riley 1989; Nti 1999; Gershenson and Grossman 2000; Morgan 2003).

To examine the role of strategic biases in third-party intervention into social conflict, we consider the scenario that an intervening party (denoted as Party 3) chooses to support Faction 1. There may have different forms of intervention, but for ease of illustration, we follow Siqueira (2003) and Chang, Potter, and Sanders (2007a) by considering an 'intervention technology' under which a subsidy transfer (I) provided by Party 3 helps lower the effort cost of Faction 1 in conflict.[9] We use the value of I to capture the *intensity* of intervention indirectly exerted by the third party. Despite the biased commitment of intervention subsidies to Faction 1, Party 3 also attaches an intrinsic value to each of the two factions. It is plausible to assume that, economically and/or politically, there are potential benefits to the third party should either faction successfully retains its power. When faction i is able to obtain the control, the value that Party 3 attaches to the political regime is assumed to be given as B_i. All else being equal, the assumption that $B_1 > 0$ and $B_2 = 0$ imply that Party 3 chooses to support Faction 1.[10]

As in the literature on rent seeking and conflict, we use a canonical contest success function (Tullock 1980; Skaperdas 1996) to determine each faction's winning probability. Denoting G_i as effort or combative input invested by faction i, its winning probability is then given as[11]

$$p_i = \frac{G_i}{G_1 + G_2} \text{ for } i = 1, 2. \tag{1}$$

It is easy to verify that the marginal effect of G_i on p_i, $p_i' = \partial p_i(G_1, G_2)/\partial G_i = G_j/(G_1 + G_2)^2$, is positive but is subject to diminishing returns for $i, j = 1, 2$ and $i \neq j$.

For the two factions competing for political dominance in the presence of intervention, we assume that their expected payoffs are given, respectively, as

$$Y_1 = p_1 V_1 - \frac{1}{(1+I)^\theta} G_1; \tag{2a}$$

$$Y_2 = p_2 V_2 - G_2; \tag{2b}$$

[9] See detailed discussions in Chang, Potter, and Sanders (2007a) and Chang and Sanders (2009).

[10] For a detailed proof of this result, see Chang, Potter, and Sanders (2007a). The analytical framework of two-party conflict in the present analysis is fundamentally identical to the standard models of rent-seeking contests with independent private valuations of the contest prize. See, e.g. Nti (1999) and Morgan (2003). Under this simplification, one could consider B_1 as the difference of the values of stakes the third party put onto the two contending groups. This is also consistent with the assumption that the third party is a biased intervener.

[11] Effort or combative input by each of the contending parties can broadly be defined as gun, weapon, or armament in military conflict, rent-seeking effort in contest, or expenditure in litigation.

where I, as mentioned earlier, represents intervention intensity in terms of financial subsidies committed by Party 3 to Faction 1. Faction 1's expected payoff function in (2a) is its expected value of political dominance minus fighting cost, which is $C_1 = G_1/(1+I)^{\theta}$. This cost function reflects the assumption that Party 3's intervention is of a cost-reducing technology, with parameter θ measuring the marginal effectiveness of financial subsidies.[12] For $I > 0$ and $G_1 > 0$, we have $\partial C_1/\partial\theta < 0$, which implies that, other things being equal, the larger the value of θ the lower the fighting cost for Faction 1 and hence the more effective the intervention subsidies. Faction 2's expected payoff function in (2b) is its expected value of political dominance minus combative cost, which is $C_2 = G_2$. Faction 1 and Faction 2 independently choose their optimal levels of combative inputs, $\{G_1, G_2\}$, which maximize their respective payoffs in (2a) and (2b).

To characterize the political economy of third-party intervention, we assume that Party 3's expected payoff function (denoted as Y_3) is the weighted sum of the intrinsic values associated with the political dominance of the two factions, $\{B_1, B_2\}$, minus its own intervention cost, with the weights being the winning probability of each faction. That is,

$$Y_3 = p_1 B_1 + p_2 B_2 - I. \tag{3}$$

The intervening Party 3 independently commits to its favored ally (i.e. Faction 1) an amount of financial subsidies, I, which maximizes its own expected payoff in (3).

For our notations and conventions, we use *i–j–k* to represent the moving sequence of a game in which player i makes the first move and player k makes the last, where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. There are six possible combinations of moving orders in the sequential-game framework. The moving orders of interest to our study are 2–3–1, 3–2–1, and 3–1–2. In these sequences, the third party moves before Faction 1. From a theoretical standpoint, since the third party values Faction 1's expected payoff but not vice versa, it is necessary for the third party's expected payoff to be a function of Faction 1's effort level. With backward induction, this happens only when the third party moves before Faction 1. From an empirical standpoint, since our analysis rules out post-conflict peacekeeping, it is necessary to exclude the moving orders 1–2–3 and 2–1–3.

As standard in game theory, we use backward induction to solve for the subgame perfect Nash equilibrium of a sequential-move game. Hereafter, we use a superscript for the entry order of a game and subscripts for variables associated with the three players. For example, $G_1^{2\text{–}3\text{–}1}$ denotes Faction 1's effort in the 2–3–1 sequential game and $Y_3^{3\text{–}1\text{–}2}$ denotes the third party's payoff in the 3–1–2 sequential game.

It should be mentioned at the outset that we use the term 'peace' to indicate an absence of fighting. That is, if a first mover is able to effectively deter the follower from attacking, this means that the deterred faction exerts zero effort. This definition is consistent with the notion of 'acquiescence' or 'deterrence' in sequential-move games of conflict as discussed in Grossman (1999), Gershenson and Grossman (2000), and Gershenson (2002), a 'nonaggressive equilibrium' in Grossman and Kim (1995), and 'peace' in Chang, Potter, and Sanders (2007a, 2007b). These papers also provide conditions when peace is expected to occur. In our analysis, conditions of deterrence are discussed in Section 3.3.

[12]The value of θ is assumed to be positive but is less than one, which implies that the third party's intervention investment I is subject to diminishing returns (Chang, Potter, and Sanders 2007b).

3. EQUILIBRIUM OUTCOMES AND THEIR IMPLICATIONS

3.1. Effectiveness of Third-party Intervention

Based on the framework of the three-player sequential-move games as outlined in the last section, we first investigate the effectiveness of intervention by the third party in the conflict. It is a conventionally held thinking that the presence of third-party intervention shall increase the winning probability and the expected payoff of a supported faction at the expense of an unsupported faction. We present a formal analysis to this thinking by calculating and comparing the equilibrium winning probabilities and the expected payoffs for the two contending factions, with and without intervention. It will be shown that our findings are consistent with the conventional thinking on third-party intervention. We assume throughout the analysis that $B_2 = 0$ and $B_1 > 0$, which ensures that the third party always supports Faction 1.

Denote 1–2 as the game in which Faction 1 moves first without third-party intervention. We show in Appendix A-1 that the equilibrium effort allocations, winning probabilities, and expected payoffs have the following results:

$$G_1^{3-1-2} > G_1^{1-2} \text{ and } G_2^{3-1-2} < G_2^{1-2}; \tag{4a}$$

$$p_1^{3-1-2} > p_1^{1-2} \text{ and } p_2^{3-1-2} < p_2^{1-2}; \tag{4b}$$

$$Y_1^{3-1-2} > Y_1^{1-2}. \tag{4c}$$

Next, denote 2–1 as the other two-player game when Faction 2 moves first in the absence of third-party intervention. We show also in Appendix A-1 the following results:

$$G_1^{3-2-1} > G_1^{2-1} \text{ and } G_2^{3-2-1} < G_2^{2-1}; \tag{5a}$$

$$p_1^{3-2-1} > p_1^{2-1} \text{ and } p_2^{3-2-1} < p_2^{2-1}; \tag{5b}$$

$$Y_1^{3-2-1} > Y_1^{2-1}. \tag{5c}$$

Relative to the 2–1 game without intervention, we further examine the possibility that the third party commits its support to Faction 1 after observing the action of Faction 2 in the 2–3–1 sequential game. We find that

$$p_1^{2\text{–}3\text{–}1} > p_1^{2\text{–}1};\, Y_1^{2\text{–}3\text{–}1} > Y_1^{2\text{–}1}. \tag{6}$$

The results outlined in Equations (4), (5), and (6) lead to the first proposition as follows:

Proposition 1. *Regardless of the moving order of the sequential games, the supply of a third party's intervention subsidies to its supported faction unambiguously raises the winning probability of the faction, ceteris paribus. In equilibrium, the supported faction's expected payoff is strictly higher with the intervention than without it.*

The implication of Proposition 1 is straightforward. Factions directly involved in fighting have a strong incentive to seek support from a third party because, other things being equal, such intervention enhances one's winning probability and expected payoff. Put alternatively, Proposition 1 suggests that eliminating external supports to a rival is an effective strategy in weakening its ability to engage in fighting.

3.2. The Unsupported Faction Fights against All Odds

Although a supported faction is able to improve its winning probability and expected payoff by the means of third-party intervention, this does not imply that it will always have relatively higher winning probability and expected payoff than the unsupported one. Fighting against all odds, the unsupported faction may prevail. We show in Appendix A-2 that

$$G_2^r > G_1^r \text{ and } p_2^r > p_1^r \text{ for } r = 3\text{–}1\text{–}2, 3\text{–}2\text{–}1, \text{ and } 2\text{–}3\text{–}1,$$

if the following sufficient conditions are satisfied:

$$\frac{V_2}{V_1} > 1 + \frac{\theta}{2} > 1 \text{ and } B_1 < \frac{2}{\theta}\left(\frac{V_2}{V_1}\right)^{\frac{1}{\theta}}. \tag{7}$$

The conditions in (7) permit us to establish

Proposition 2. *In the presence of a biased intervention into a conflict between factions, the unsupported faction has a relatively higher probability of winning in contest provided that (i) the faction has a critically higher intrinsic value of its political dominance than its opponent and (ii) the stakes the third party holds with each of the two factions are not significantly different.*

The intuition behind Proposition 2 is interesting. For the case in which Faction 2's intrinsic value of political dominance (V_2) is relatively higher than that of Faction 1 (V_1), as indicated by the first inequality in (7), Faction 2 has a stronger incentive to allocate more resources than its opponent. On the other hand, when the values that the third party attaches to the political dominance of the two conflicting factions are not significantly different, Faction 1 is unlikely to receive a large amount of aids. Despite the presence of support from the third party, the winning probability of Faction 1 may end up being relatively lower than that of Faction 2's. This finding is consistent with the non-intervention model of a two-faction contest as analyzed by Nti (1999). The author shows that a 'player who values the prize more expends more effort in equilibrium (419),' with the consequence that the player's winning probability is relatively higher. Proposition 2 suggests that a faction's lack of outside support (Faction 2 in our analysis) is not the decisive factor to conclude that it is doomed to fail.

3.3. First-mover Advantage in the Presence of Third-party Intervention

Another issue of interest concerns the first-mover advantage commonly observed in sequential games. Morgan (2003) shows that in a two-player sequential game first mover and second mover may have the same expected payoff. In theory, second-mover advantage is often observed in two settings of games: (i) when coordination/matching is required to win the game, such as in the matching pennies game or rock–paper–scissors, (ii) when there is positive externality and free riders cannot be prevented. In our setting with two contending factions and an intervening third party, there are two possible types of first-mover advantage. First, whether Faction 1 or Faction 2 is the first mover, given the intervention decision of the third party. The first mover among the two contending factions, which is defined as the 'provoker' in our analysis, can potentially deter the 'follower.' In this section, we wish to examine how the addition of a third party might strategically change such result. Specifically, we want to identify conditions under which receiving a financial support from an intervening outside party guarantees that Faction 1 will not be deterred by Faction 2 when the later moves first as the 'provoker.'

Second, given the sequence of the two contending factions, the third party may act as the overall first mover in the sequential game. We shall analyze the second type of the first-mover advantage in details in Section 3.4 as it is related to the timing of intervention in increasing the equilibrium winning probability and the expected payoff for the supported faction.

We first focus our attention on scenarios where the supported faction, as a 'follower,' may not be deterred by the unsupported faction, the 'provoker.' For the 3–2–1 sequential game where the third party acts as the overall first mover, we show in Appendix A-3 the sufficient condition that Faction 2 does not have any opportunity to deter Faction 1 is

$$\theta B_1 > 1. \tag{8}$$

For the 2–3–1 sequential game where Faction 2 is the overall first mover, a similar condition is given by

$$\theta B_1 > 2. \tag{9}$$

These conditions show that the presence of a third party significantly alters the first mover advantage commonly observed in a two-player sequential rent-seeking game. Based on the results in (8) and (9), we establish the following proposition:

Proposition 3. *In the three-player sequential-move game where Faction 2 acts as the provoker (first mover), Faction 1 may escape being deterred if the conditions in (8) and (9) are satisfied. The sufficient condition is weaker when the third party acts as the overall first mover.*

Proposition 3 shows that the timing of the third-party action matters. Recall that θ is the effectiveness of third party support and B_1 is how much benefits the third party receives from Faction 1's dominance. Given the moving order of the three players, the conditions in (8) and (9) are more likely to be satisfied when (i) the support is more effective and (ii) the third party's stake with Faction 1 is larger. When the third party acts first in aiding the faction it supports, deterrence could be avoided with a weaker condition, as given by Equation (8), compared to the case where the third party acts second, as given by Equation (9). That is, the presence of the third party strategically weakens the first-mover advantage by the unsupported faction. At the same time, the third party may establish their first-mover advantage by moving first. Whether it is the third party's optimal choice to move first remains to be analyzed which is the central question of the next section.

3.4. Timing of an Effective Third-party Intervention

In this section, we focus on the following two sequential games: 3–2–1 versus 2–3–1.[13] The question is: should the third party be the overall first mover (as in the 3–2–1 game) or wait until after observing the action of Faction 2 (as in the 2–3–1 game)? We show detailed derivations in Appendix A-4. Without additional conditions like other scenarios, we have the following proposition:

Proposition 4. *When the third party is the overall first mover as in the* 3–2–1 *game compared to the* 2–3–1 *game, the supported faction finds it beneficial to increase its optimal level of efforts. In equilibrium, the supported faction has relatively higher winning probability and expected payoff than the unsupported faction.*

As for the third party, we show in Appendix A-5 the following results:

$$I^{3\text{–}2\text{–}1} > I^{2\text{–}3\text{–}1}; \tag{10a}$$

$$Y_3^{3\text{–}2\text{–}1} > Y_3^{2\text{–}3\text{–}1}. \tag{10b}$$

The results in Equations (10) lead to

Proposition 5. *When the third party is the overall first mover, as in the* 3–2–1 *game, it provides relatively stronger support than the* 2–3–1 *game when it moves after observing the action of the unsupported faction. Consequently, the third party's expected payoff is relatively higher in the* 3–2–1 *game than in the* 2–3–1 *game.*

Propositions 4 and 5 show the importance of not only the presence of third party's support in conflict but also the timing of it. Our results show that, with effective support from the third party, the supported faction (Faction 1) allocates more effort against the unsupported faction when the third party acts as the overall first mover. From the perspective of the third party, there is a strategic advantage to act earlier (before Faction 2 acts) rather than later. Acting as the overall first mover, the third party is optimally offering a greater amount of financial support to Faction 1. Moreover, such a moving strategy brings in a higher expected payoff to the third party. This outcome emerges because (i) higher efforts by both the third party and Faction 1 significantly increase the winning probability of Faction 1, and (ii) the third party takes away Faction 2's first-mover advantage. Since the third party's intervention is biased, Party 3 enjoys a higher payoff when Faction 1 prevails.

It appears that the two Gulf Wars serve as interesting examples upon the timing of third-party intervention. The first Gulf War (1990–1991) can be considered as a 2–3–1 (Iraq-UN-Kuwait) game. Initially, Iraq cracked Kuwait. Kuwaiti land forces were demolished abruptly, but its royal government and air forces fled to Saudi Arabia; then, UN and especially the United States launched the first Gulf War against Iraq; latterly, Kuwaiti royal

[13]We omit the discussions over the 3–1–2 game because we wish to keep the same sequence for the two fighting factions.

government returned home, repelled Iraqi invaders, and rewarded the United States with financial and energy bestows.

The second Gulf War (2003–2011), on the other hand, can be considered as a 3–2–1 (US-Saddam-Iraqi dissidents) game. After losing the first Gulf War, Iraqi dictator Saddam Hussein still usurped the power, fending away the less organized dissidents. In 2003, US-led international troops[14] attacked Iraqi government army, which fought back but failed. The second Gulf War ended in the death of Saddam and the birth of a new Iraqi government led by Shi-ah Iraqis. This newborn pro-US government has been running Iraq since then.

The two Gulf Wars may help us think about issues concerning when and how a potential intervening party can move first versus second, despite the fact that the third party would choose to be the provoker if the decision is determined endogenously. In the first Gulf War, potential intervener(s) did not have a choice to 'provoke' because the action by Iraq to invade Kuwait was unanticipated. After 11 September 2001, it was thought that Saddam Hussein could become an imminent threat so the United States took action to intervene which ultimate led to the establishment of the new Iraqi government.

4. CONCLUDING REMARKS

Applying the standard rent-seeking game with asymmetric valuations, we have investigated how the equilibrium outcomes of social conflicts between two factions are strategically altered by an intervening third party. The sequential game framework permits us to determine appropriate strategies for a third party in terms of the optimal intensity and timing of intervention.

We show that, irrespective of the sequence of moves in a three-stage game, an intervening third party is able to increase the winning probability for its supported faction in contest. As a result, the expected payoff for the supported faction is higher with third-party intervention than without it. We also show possible situations that third-party intervention may be ineffective in improving an ally's position, depending on the intrinsic values of political dominance to the contending factions and the stakes the third party holds with each of the two factions. Fighting against the odds with a relatively higher value attached to political dominance than its opponent, the unsupported faction may have a higher winning probability when it moves first as a provoker. This result holds when the stakes the third party holds with each of the two contending factions do not differ significantly.

When a third party is the overall first mover in a sequential game, relative to the situation when it moves after observing the action of the unsupported faction, its supported faction has an incentive to raise its effort or combative input in contest. In this case, intervention subsidies and fighting effort may constitute 'strategic complements' for the supported faction. In equilibrium, the supported faction has a higher winning probability and a higher expected payoff. At the same time, the third party also has a higher expected payoff when it acts as the overall first mover.

Some caveats about the present paper, and hence possible extensions of the simple model, should be mentioned. First, as we assume that combative inputs of the two conflating parties are equally effective in their contest success functions, our analysis does not allow for asymmetry in conflict technology. Second, we do not take into account

[14] These international troops included Iraqi Kurd repellants. However, after the second Gulf War, it was Shi-Ah Iraqis rather than Kurds who dominated the new Iraqi government. Hence, we still suggest a 3–2–1 game.

imperfectness in terms of the information structure. Third, like many theoretical papers in the literature on armed conflicts, our analysis abstracts from the possibilities of destruction or damage caused by fighting.[15] Incentives of a third party and its decision to intervene may be affected by such factors as asymmetric technology of conflict, the asymmetry and imperfectness of information, as well as the destructiveness of armaments used in conflicts. These are potentially interesting topics for future research.

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[15]Vahabi (2006, 2009) systematically examines the role of destructive power in affecting conflict over the 'rules of the game' or 'institutions.' Specifically, the author shows that capacities to destroy and produce constitute a key element in generating and maintaining institutions. Attempting to incorporate the endogeneity of destruction into the conflict literature, Chang and Luo (2013) discusses implications of endogenous destruction for contending parties in making their optimal choices between negotiating a settlement and fighting a war.

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APPENDIX

A-1 PROOF OF PROPOSITION 1

We first compare (i) the 1–2 sequential game of two-faction conflict without intervention to (ii) the 3–1–2 sequential game with intervention when Party 3 is the overall first mover. Solving for the equilibrium allocations of combative inputs by the two contenders, G_i^r ($i = 1, 2$; $r = 1\text{–}2, 3\text{–}1\text{–}2$) and the third party's optimal intervention intensity, I^{3-1-2}, we have:

$$G_1^{1-2} = \frac{V_1^2}{4V_2}; \tag{a1}$$

$$G_2^{1-2} = \frac{V_1}{2} - \frac{V_1^2}{4V_2}; \tag{a2}$$

$$G_1^{3-1-2} = \frac{V_1^2}{4V_2}\left[\frac{\theta B_1}{2}\frac{V_1}{V_2}\right]^{\frac{2\theta}{1-\theta}}; \tag{a3}$$

$$G_2^{3-1-2} = \frac{V_1}{2}\left[\frac{\theta B_1}{2}\frac{V_1}{V_2}\right]^{\frac{\theta}{1-\theta}} - \frac{V_1^2}{4V_2}\left[\frac{\theta B_1}{2}\frac{V_1}{V_2}\right]^{\frac{2\theta}{1-\theta}}; \tag{a4}$$

$$I^{3-1-2} = \left[\frac{\theta B_1}{2}\frac{V_1}{V_2}\right]^{\frac{1}{1-\theta}} - 1. \tag{a5}$$

It follows from (a5) that for the existence of intervention, $I^{3-1-2} > 0$, we must have

$$\frac{\theta B_1}{2}\frac{V_1}{V_2} > 1. \tag{a6}$$

We assume that this condition holds throughout the paper.

Comparing G_1^{1-2} in (a1) to G_1^{3-1-2} in (a3) for the supported faction, taking into account the intervention condition in (a6), we have

$$G_1^{3-1-2} > G_1^{1-2}. \tag{a7}$$

Next, a comparison between G_2^{1-2} in (a2) and G_2^{3-1-2} in (a4) for the unsupported faction reveals that

$$G_2^{3-1-2} < G_2^{1-2}. \tag{a8}$$

From the findings in (a7) and (a8), we have the equilibrium winning probabilities of Faction 1 and Faction 2 for the two alternative cases:

$$p_1^{3-1-2} > p_1^{1-2}; \tag{a9}$$

$$p_2^{3-1-2} < p_2^{1-2}. \tag{a10}$$

The next step is to calculate Faction 1's expected payoff, Y_1^r. Substituting the optimal combative input allocations of Faction 1 into its expected payoff functions for the games yields

$$Y_1^{1-2} = \frac{V_1^2}{4V_2}. \tag{a11}$$

$$Y_1^{3-1-2} = \frac{V_1}{2}\left[\frac{\theta^\theta B_1^\theta}{2}\frac{V_1}{V_2}\right]^{\frac{1}{1-\theta}}. \tag{a12}$$

Under the intervention condition in (a6), we infer that $Y_1^{3-1-2} > Y_1^{1-2}$ for the sequential games 1–2 and 3–1–2.

We now analyze and compare the sequential games of 2–1 and 3–2–1. Solving for the equilibrium combative input allocations and the optimal intervention intensity, we have:

$$G_1^{2-1} = \frac{V_2}{2}\left(1 - \frac{V_2}{2V_1}\right); \tag{a13}$$

$$G_2^{2-1} = \frac{V_2^2}{4V_1}; \tag{a14}$$

$$G_1^{3-2-1} = \frac{V_2}{2}\left\{1 - \left[\frac{1}{2\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}\right\}; \tag{a15}$$

$$G_2^{3-2-1} = \frac{V_2}{2}\left[\frac{1}{2\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}; \tag{a16}$$

$$I^{3\text{–}2\text{–}1} = \left[\frac{\theta B_1}{2}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}} - 1. \tag{a17}$$

To analyze the role of third-party intervention, we assume an interior solution that $I^{3\text{–}2\text{–}1} > 0$. This implies that the following inequality relationship is satisfied:

$$\frac{\theta B_1}{2}\frac{V_2}{V_1} > 1. \tag{a18}$$

Under the inequality condition in (a18), it can be verified that

$$\left[\frac{1}{2\theta^{\theta}B_1^{\theta}}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}} < \frac{1}{2}\frac{V_2}{V_1},$$

which implies the following two sets of results for the optimal combative input allocations and the equilibrium winning probabilities:

$$G_1^{3\text{–}2\text{–}1} > G_1^{2\text{–}1} \text{ and } G_2^{3\text{–}2\text{–}1} < G_2^{2\text{–}1}; \tag{a19}$$

$$p_1^{3\text{–}2\text{–}1} > p_1^{2\text{–}1} \text{ and } p_2^{3\text{–}2\text{–}1} < p_2^{2\text{–}1}. \tag{a20}$$

We further calculate the corresponding expected payoffs as follows:

$$Y_1^{2\text{–}1} = \left(1 - \frac{1}{2}\frac{V_2}{V_1}\right)^2 V_1; \tag{a21}$$

$$Y_1^{3\text{–}2\text{–}1} = \left\{1 - \left[\frac{1}{2\theta^{\theta}B_1^{\theta}}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}\right\}^2 V_1 = (p_1^{3\text{–}2\text{–}1})^2 V_1. \tag{a22}$$

It follows from (a21) and (a22) that

$$Y_1^{3\text{–}2\text{–}1} > Y_1^{2\text{–}1}.$$

Before proceeding to the third and last cases, from the above results, we have the following:

LEMMA 1. *When the intervening third party is the overall first mover in a sequential game, the supported (unsupported) faction allocates more (less) resources to the contest as compared to the scenario without intervention. Consequently, the supported (unsupported) faction has higher (lower) winning probability.*

In addition, an examination of Equations (a1), (a2), (a13), and (a14) permits us to construct the following

LEMMA 2. (Nti 1999) *In a two-faction conflict without third-party intervention, the faction whose value of political dominance is relatively higher allocates more resources to the contest, with the results that its winning probability is higher.*

We now evaluate and compare the sequential games of 2–1 and 2–3–1. Solving for the optimal combative input allocations and the optimal intervention intensity for the 2–3–1 game, we have

$$G_1^{2\text{–}3\text{–}1} = \frac{V_2}{2+\theta}\left\{1 - \left[\frac{2^\theta}{(2+\theta)\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}\right\}; \tag{a23}$$

$$G_2^{2\text{–}3\text{–}1} = \frac{V_2}{2+\theta}\left[\frac{2^\theta}{(2+\theta)\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}; \tag{a24}$$

$$I^{2\text{–}3\text{–}1} = \left[\frac{\theta B_1}{2(2+\theta)}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}} - 1. \tag{a25}$$

We further calculate the equilibrium winning probabilities:

$$p_1^{2\text{–}3\text{–}1} = 1 - \left[\frac{2^\theta}{(2+\theta)\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}; \tag{a26}$$

$$p_1^{2\text{–}1} = 1 - \frac{1}{2}\left(\frac{V_2}{V_1}\right). \tag{a27}$$

It can be verified that the sufficient condition for $p_1^{2\text{–}3\text{–}1}$ in Equation (a26) to be greater than $p_1^{2\text{–}1}$ in Equation (a27) is

$$B_1 > 2^{\frac{1+2\theta}{\theta}}(2+\theta)^{-\frac{1}{\theta}}\left(\frac{V_1}{\theta V_2}\right). \tag{a28}$$

For the existence of intervention in the 2–3–1 sequential game, $I^{2\text{–}3\text{–}1} > 0$, we must have

$$B_1 > 2(2+\theta)\frac{V_1}{\theta V_2}. \tag{a29}$$

Comparing the inequalities in Equations (a28) and (a29), noting the fact that

$$2(2+\theta) > 2^{\frac{1+2\theta}{\theta}}(2+\theta)^{-\frac{1}{\theta}},$$

we infer that

$$p_1^{2\text{–}3\text{–}1} > p_1^{2\text{–}1}.$$

Substituting the optimal combative input allocations into the expected payoffs yields, noting $p_1^{2\text{–}3\text{–}1}$ in Equation (a26) and $p_1^{2\text{–}1}$ in Equation (a27), we have

$$Y_1^{2\text{–}3\text{–}1} = \left\{1 - \left[\frac{2^\theta}{(2+\theta)\theta^\theta B_1^\theta}\frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}\right\}^2 V_1 = (p_1^{2\text{–}3\text{–}1})^2 V_1; \tag{a30}$$

$$Y_1^{2-1} = \left(1 - \frac{1}{2}\frac{V_2}{V_1}\right)^2 V_1 = (p_1^{2-1})^2 V_1. \tag{a31}$$

Since $p_1^{2-3-1} > p_1^{2-1}$, we have from Equations (a30) and (a31) that $Y_1^{2-3-1} > Y_1^{2-1}$. This completes the entire proof of Proposition 1.

A-2 PROOF OF PROPOSITION 2

For the 3–1–2 sequential game, we have from Equation (a6) the intervention condition

$$B_1 > \frac{2}{\theta}\left(\frac{V_2}{V_1}\right). \tag{a32}$$

A comparison between Equations (a3) and (a4) indicates that

$$B_1 < \frac{2}{\theta}\left(\frac{V_2}{V_1}\right)^{\frac{1}{\theta}} \tag{a33}$$

is the condition for $G_2^{3-1-2} > G_1^{3-1-2}$. Note that for Equations (a32) and (a33) to hold simultaneously, we need $V_2 > V_1$.

For the 3–2–1 sequential game, we have from Equation (a18) that

$$B_1 > \frac{2}{\theta}\left(\frac{V_1}{V_2}\right). \tag{a34}$$

A comparison between Equations (a15) and (a16) reveals that

$$B_1 < \frac{2}{\theta}\left(\frac{V_2}{V_1}\right)^{\frac{1}{\theta}} \tag{a35}$$

constitutes the condition for $G_2^{3-2-1} > G_1^{3-2-1}$. For the inequalities in (a34) and (a35) to hold simultaneously, we need $V_2 > V_1$. Note that the conditions of V_i for both the 3–1–2 and 3–2–1 games are the same.

For the 2–3–1 sequential game, a comparison between Equations (a23) and (a24) reveals that

$$B_1 < \frac{1}{\theta}\left[\frac{2^{1+2\theta}}{(2+\theta)}\frac{V_2}{V_1}\right]^{\frac{1}{\theta}} \tag{a36}$$

constitutes the condition for $G_2^{2-3-1} > G_1^{2-3-1}$. For the inequalities in (a29) and (a36) to hold simultaneously, we need $V_2 > (2+\theta)V_1/2$.

It is easy to verify that when the conditions in (a33) and (a35) hold, the condition in (a36) holds automatically. Also, it is straightforward to see that $(2+\theta)V_1/2 > V_1$. These together prove Proposition 2.

A-3 PROOF OF PROPOSITION 3

To find the non-deterrence condition under which $G_1^{3-2-1} > 0$, i.e. Faction 1 will not be deterred, we make use of Equations (a15) to get

$$\frac{V_1}{V_2} > \frac{1}{2\theta^\theta B_1^\theta}; \tag{a37}$$

The intervention condition from Equation (a18) can be rewritten as

$$\frac{V_1}{V_2} < \frac{\theta B_1}{2}. \tag{a38}$$

Equations (a37) and (a38) hold simultaneous when

$$\frac{1}{2\theta^\theta B_1^\theta} < \frac{V_1}{V_2} < \frac{\theta B_1}{2} \tag{a39}$$

which is the condition under which Faction 1, the ‘follower,’ will not be deterred with subsidy from the third party. For Equation (a39) to hold, it requires that

$$\theta B_1 > 1. \tag{a40}$$

Similarly, we obtain the ‘non-deterred’ condition from Equations (a21) and (a23) as

$$\frac{2\theta}{(2+\theta)\theta^\theta B_1^\theta} < \frac{V_1}{V_2} < \frac{\theta B_1}{2(2+\theta)}$$

which requires

$$\theta B_1 > 2. \tag{a41}$$

Equations (a40) and (a41) together complete the proof of Proposition 3.

A-4 PROOF OF PROPOSITION 4

Solving the deterrence condition for $G_2^{3-1-2} = 0$ from Equation (a4), we have

$$B_1 \geq \frac{1}{\theta}\left(\frac{2V_1}{V_2}\right)^{\frac{1}{\theta}},$$

which proves Proposition 4.

A-5 PROOF OF PROPOSITION 5

From Equations (a15) and (a23), for obtaining the result that $G_1^{3-2-1} > G_1^{2-3-1}$, it suffices to show that

$$\frac{1}{2} < \frac{2^\theta}{2+\theta}. \tag{a42}$$

One can verify that the RHS of Equation (a42) is monotonically increasing in θ and is equal to $1/2$ when $\theta = 0$. Since θ is assumed to be positive, we immediately have $G_1^{3\text{–}2\text{–}1} > G_1^{2\text{–}3\text{–}1}$.

With the condition in (a42), we derive $p_1^{3\text{–}2\text{–}1}$ from Equations (a15) and (a16), and obtain

$$p_1^{3\text{–}2\text{–}1} = 1 - \left[\frac{1}{2\theta^{\theta} B_1^{\theta}} \frac{V_2}{V_1}\right]^{\frac{1}{1+\theta}}.$$

Note that the equilibrium winning probability $p_1^{2\text{–}3\text{–}1}$ is given in Equation (a26). Given the relationship between G_1^r and p_1^r, we find that since Equation (a42) holds unconditionally, the result that $p_1^{3\text{–}2\text{–}1} > p_1^{2\text{–}3\text{–}1}$ must also hold unconditionally. Finally, given that $Y_1^r = (p_1^r)^2 V_1$ from Equations (a22) and (a30), we have $Y_1^{3\text{–}2\text{–}1} > Y_1^{2\text{–}3\text{–}1}$. This proves Proposition 5.