1 Evaluation of polynomial

Let $A(x) = a_0 + a_1x^1 + a_2x^2 + \cdots + a_{n-1}x^{n-1}$ be a polynomial of degree $(n-1)$, and let $a = (a_0, a_1, \ldots, a_{n-1})$ be the coefficient array of A.

We assume that, without loss of generality, n is an even number. (If not, let $n' = n + 1$ and consider A' such that $a' = (a_0, a_1, \ldots, a_{n-1}, 0)$.) Then we have

$$\begin{aligned} A(x) &= a_0 + a_2x^2 + \cdots + a_{n-2}x^{n-2} + a_1x^1 + a_3x^3 + \cdots + a_{n-1}x^{x-1} \\ &= \sum_{i=0}^{n/2-1} a_{2i}x^{2i} + x \cdot \sum_{i=0}^{n/2-1} a_{2i+1}x^{2i} \,. \end{aligned}$$

By introducing two polynomial coefficient arrays of A_0 with $(a_0, a_2, \ldots, a_{n-2})$ and A_1 with $(a_1, a_3, \ldots, a_{n-1})$, the above can be rewritten to

$$A(x) = A_0(x^2) + xA_1(x^2) \,.$$

Therefore, if we calcurate $\log n$ terms $x^2, x^4, \ldots, x^{n/2}$, time complexity $T(n)$ to obtain the value $A(x)$ of polynomial degree $n-1$ is

$$T(n) = \begin{cases} 2 & n \leq 1 \\ 2T(n/2) + 2 & \text{otherwise.} \end{cases}$$

2 String matching

Let Σ be a finite alphabet, t a text $t = t_0 \cdot t_1 \cdot \cdots \cdot t_{n-1}$ over Σ^*, and p a pattern string $p \in \Sigma^*$.

Let ω^i be a coefficient in $\mathbb{C}$. We define a text and a pattern in complex vector

$$T(i) = t_i \cdot \omega^{n-1-i},$$
$$P(i) = p_i \cdot w_p^i$$

where

$$w_p^i = \begin{cases} \omega^i & 0 \leq i < |p| \\ 0 & i \geq |p| \end{cases} \,.$$

$$M(i) = \sum_{k=0}^{n-1} T(i+k) \cdot P(k) = \sum_{k=0}^{k<|p|} t_{i+k} \cdot p_i \cdot \omega^{n-1-(i+k)+k} = \sum_{k=0}^{k<|p|} t_{i+k} \cdot p_i \cdot \omega^{n-1-i}$$

For $t = t_0 \cdot t_1 \cdot t_2 \cdot t_3 \cdot t_4 \cdots t_{n-1}$ and $p = p_0 \cdot p_1 \cdot p_2$, we examine whether p occurs at position 2 by $M(2) = t_2p_0\omega^{n-1-2+0} + t_3p_1\omega^{n-1-3+1} + t_4p_2\omega^{n-1-4+2}$, thus $M(2) = \omega^{n-1-3}(t_2p_0 + t_3p_1 + t_4p_2)$.