MANEJO DE LA INCERTIDUMBRE

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- INCERTIDUMBRE: Falta de información adecuada para tomar una decisión
- Generalmente se hace la ***acepción del mundo cerrado (close world assumption)***. Si no sabemos que una proposición es verdadera, la proposición se asume como falsa. Si no se hace dicha acepción aparece una tercera categoría considerada como desconocida ***(clear-cut world)***
- **Razonamiento monotónico.** La verdad se puede deducir con igual seguridad. Se mueve en una sola dirección. El número de hechos nunca decrece
- **Razonamiento no monotónico.** Las suposiciones que se hagan están sujetas al cambio de acuerdo a la información que se proporcione
- **Reglas**
  - *De bajo nivel*, conciernen a los sensores de datos y se eligen generalmente para examinarse
  - *De alto nivel*, reglas que conducen a una solución
  - *De transición* reglas intermedias

- Tipos de incertidumbre comunes en dominios de expertos:
  - *Conocimientos inciertos*. Con frecuencia el experto tendrá solamente conocimiento heurísitico con relación a algunos aspectos del dominio. P. ej. Si el experto podría saber que solamente cierto tipo de evidencia implicaría una conclusión, es decir, existe incertidumbre en la regla
  - *Datos inciertos*. Aún cuando tengamos la certidumbre en el conocimiento del dominio, puede haber incertidumbre en los datos que describen el ambiente externo. P. ej. Cuando intentamos deducir una causa específica a partir de un efecto observado, debido a que la evidencia puede provenir de una fuente que no es totalmente confiable, o la evidencia puede derivarse de una regla cuya conclusión fue probable en lugar de cierta y por lo mismo proporciona
  - *Información incompleta*. Toma de decisiones basados en información incompleta debido a múltiples sucesos:
    - Toma de decisiones en el curso de la información adquirida en forma incremental.

- La información disponible está incompleta en cualquier punto de decisión
- Las condiciones cambian en el tiempo
- Necesidad de lograr una “adivinación” eficiente, pero posiblemente incorrecta, cuando el razonamiento alcance un callejón sin salida

– *Uso del lenguaje vago (coloquial).* Nuestra forma de hablar presenta mucha ambigüedades

– *Azar.* El dominio tiene propiedades estocásticas. Hay situaciones cuya naturaleza es aleatoria y cuya ocurrencia, aunque incierta, puede ser anticipada por medios estadísticos

Teorema de Bayes

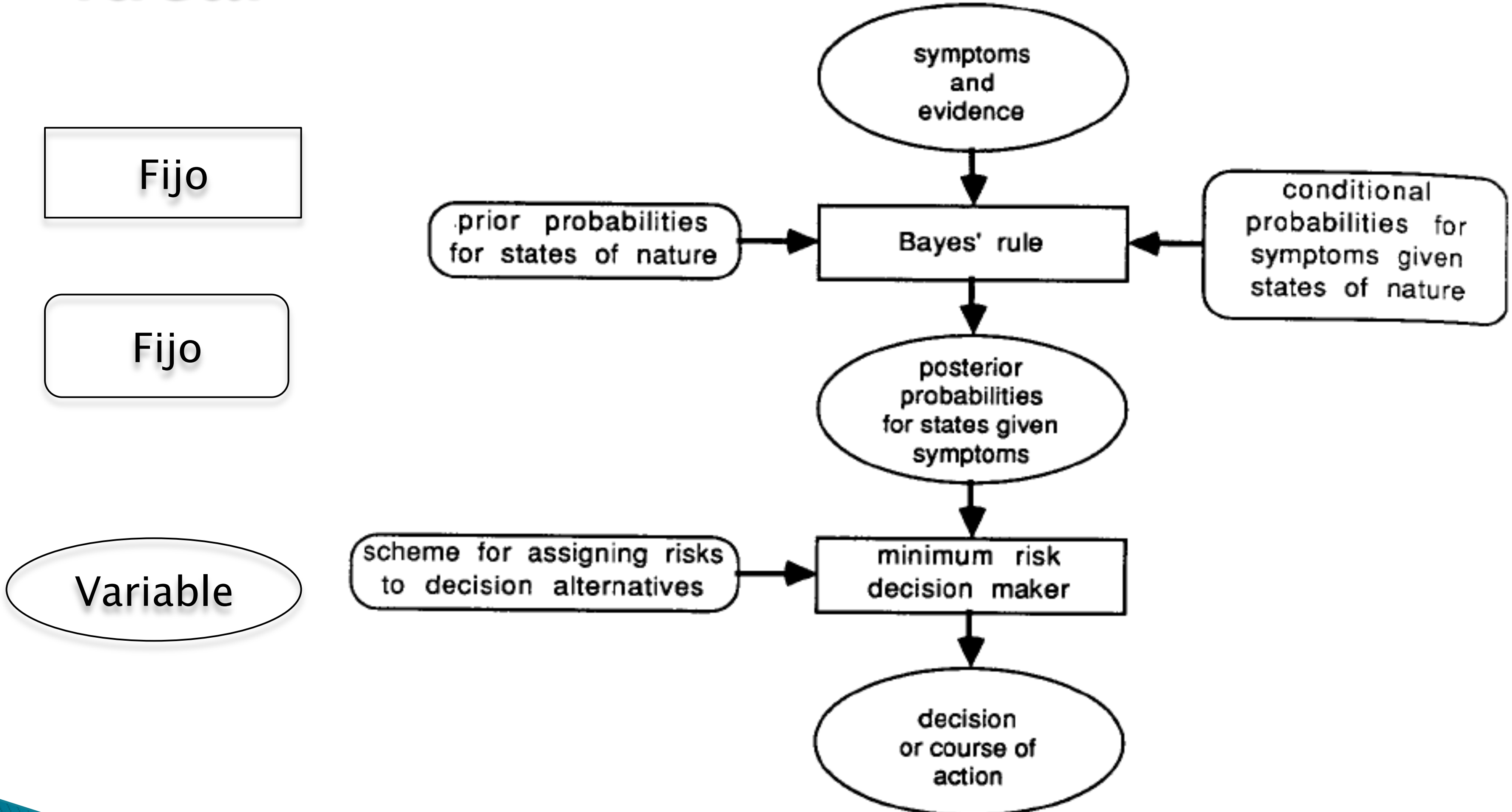
- Los sistemas de tipo probabilístico intentan medir la probabilidad con la que una evidencia sustenta una conclusión

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

$$P(E) = P(E|H)P(H) + P(E|\neg H)P(\neg H)$$

$$\mathrm{P(H|E)} = \frac{\mathrm{P(H) \times P(E|H)}}{\mathrm{P(H) \times P(E|H) + P(\sim H) \times P(E|\sim H)}}$$

Sistema de toma de decisiones ideal



- El sistema se puede adaptar a cualquier aplicación cambiando solamente dos cuadros:
  - las probabilidades tanto a priori como condicionales
  - y el esquema de asignación de riesgos

Red de inferencias

- La compleja relación entre la evidencia y las conclusiones finales puede ser expresada como una red de relaciones simples que involucren no solo la evidencia y las conclusiones finales sino las afirmaciones intermedias, tales como las conclusiones parciales y las

If we always had accurate general knowledge for such inference problems, we could make simple and clean machines to compute probabilities for various things considering all the evidence. Unfortunately, we usually do not have accurate knowledge of the conditional probabilities of sets of symptoms (or evidence) given the state of health (the hidden truth), so that the ideal, all-Bayesian system of Fig. 7.1 cannot be successfully built. However, heuristic modeling tools can be used to represent known relationships between evidence and conclusion. The complex relationship between evidence and final conclusions can be expressed as a network of simpler relationships involving not only the evidence and final conclusions, but also intermediate assertions: partial conclusions and close consequences of the evidence. Such networks are called "probabilistic inference

Actualización Bayesiana

- Bayesian updating has a rigorous derivation based upon probability theory, but its underlying assumptions, e.g., the statistical independence of multiple pieces of evidence, may not be true in practical situations.
- Bayesian updating assumes that it is possible to ascribe a probability to every hypothesis or assertion, and that probabilities can be updated in the light of evidence for or against a hypothesis or assertion.
- This updating can either use Bayes' theorem directly, or it can be slightly simplified by the calculation of likelihood ratios.
- The Bayesian approach is to ascribe an *a priori probability (sometimes simply* called the *prior probability) to the hypothesis*

Actualización Bayesiana

- Bayesian updating is a technique for updating this probability in the light of evidence for or against the hypothesis.
- Habíamos establecido previamente que la evidencia conduce a la deducción con absoluta certeza, ahora solamente podemos decir que sólo se sustenta tal deducción
- Bayesian updating is cumulative, so that if the probability of a hypothesis has been updated in the light of one piece of evidence, the new probability can then be updated further by a second piece of evidence.

Actualización Bayesiana (Bayes' theorem directly)

- Suponiendo que se nos da una probabilidad a priori P(H) de la hipótesis. Las reglas se pueden reescribir como:
  - `Si E`
  - `Entonces actualiza P(H)`
- La observación de la evidencia E requiere que P(H) se actualice
- The technique of Bayesian updating provides a mechanism for updating the probability of a hypothesis P(H) in the presence of evidence E. Often the evidence is a symptom and the hypothesis is a diagnosis. The technique is based upon the application of Bayes' theorem (sometimes called Bayes' rule).
- Bayes' theorem provides an expression for the conditional probability P(H|E) of a hypothesis H given some evidence E, in terms of P(E|H), i.e., the conditional probability of E given H:

$$P(H|E) = \frac{P(H) \times P(E|H)}{P(H) \times P(E|H) + P(\sim H) \times P(E|\sim H)}$$

$$P(\sim H) = 1 - P(H)$$

$$O(H) = \frac{P(H)}{P(\sim H)} = \frac{P(H)}{1 - P(H)}$$

$$P(\sim H|E) = \frac{P(\sim H) \times P(E|\sim H)}{P(E)}$$

$$P(H) = \frac{O(H)}{O(H) + 1}$$

$$\frac{P(H|E)}{P(\sim H|E)} = \frac{P(H) \times P(E|H)}{P(\sim H) \times P(E|\sim H)}$$

$$O(H|E) = \frac{P(H|E)}{P(\sim H|E)} = A \times O(H)$$

$$A = \frac{P(E|H)}{P(E|\sim H)}$$

$$\mathrm{O(H|{\sim}E)} = D \times \mathrm{O(H)}$$

$$\mathrm{O(H|E_1\&E_2\&E_3 \ldots E_{\mathit{n}})} = A \times \mathrm{O(H)}$$

$$D = \frac{\mathrm{P({\sim}E|H)}}{\mathrm{P({\sim}E|{\sim}H)}} = \frac{1-\mathrm{P(E|H)}}{1-\mathrm{P(E|{\sim}H)}}$$

$$A = \frac{\mathrm{P(E_1\&E_2\&E_3 \ldots E_{\mathit{n}}|H)}}{\mathrm{P(E_1\&E_2\&E_3 \ldots E_{\mathit{n}}|{\sim}H)}}$$

$$\mathrm{P(H|B)} = \mathrm{P(H|E)} \times \mathrm{P(E|B)} + \mathrm{P(H|{\sim}E)} \times [1 - \mathrm{P(E|B)}]$$

$$A' = [2(A - 1) \times \mathrm{P(E)}] + 2 - A$$

$$D' = [2(1 - D) \times \mathrm{P(E)}] + D$$

$$P(H|E) = \frac{P(H) \times P(E|H)}{P(H) \times P(E|H) + P(\sim H) \times P(E|\sim H)}$$

$$P(\sim H) = 1 - P(H)$$

$$P(\sim H|E) = \frac{P(\sim H) \times P(E|\sim H)}{P(E)}$$

$$P(H|B) = P(H|E) \times P(E|B) + P(H|\sim E) \times [1 - P(E|B)]$$

$$P(Y \mid X) = \frac{P(X \mid Y) P(Y)}{P(X)}$$

$$P(Y \mid X \wedge E) = \frac{P(X \mid Y \wedge E) P(Y \mid E)}{P(X \mid E)}$$

Condicionalización

$$P(A \wedge B \mid E) = P(A \mid B \wedge E) P(B \mid E)$$

$$P(A \wedge B \mid E) = P(A \mid E) P(B \mid E)$$

$$P(A \mid B \wedge E) = P(A \mid E)$$

$$P(B \mid A \wedge E) = P(B \mid E)$$

Independencia condicional

$$P(A \mid B \wedge C) = \frac{P(B \mid A \wedge C) P(A \mid C)}{P(B \mid C)}$$

$$P\left(X \mid Y \wedge Z\right) = P\left(Y \mid Z\right)$$

$$P\left(Z \mid X \wedge Y\right) = \frac{P\left(Z\right)P\left(X \mid Z\right)P\left(Y \mid Z\right)}{P\left(X \wedge Y\right)}$$

- Bayesian updating is also critically dependent on the values of the prior probabilities. Obtaining accurate estimates for these is also problematic.
- Even if we assume that all of the data supplied in the above worked example are accurate, the validity of the final conclusion relies upon the statistical independence from each other of the supporting pieces of evidence.

- The principal *advantages of Bayesian updating are:*
  - (i) The technique is based upon a proven statistical theorem.
  - (ii) Likelihood is expressed as a probability (or odds), which has a clearly defined and familiar meaning.
  - (iii) The technique requires deductive probabilities, which are generally easier to estimate than abductive ones. The user supplies values for the probability of evidence (the symptoms) given a hypothesis (the cause) rather than the reverse.
  - (iv) Likelihood ratios and prior probabilities can be replaced by sensible guesses. This is at the expense of advantage (i), as the probabilities subsequently calculated cannot be interpreted literally, but rather as an imprecise measure of likelihood.
  - (v) Evidence for and against a hypothesis (or the presence and absence of evidence) can be combined in a single rule by using *affirms and denies* weights.
  - (vi) Linear interpolation of the likelihood ratios can be used to take account of any uncertainty in the evidence (i.e., uncertainty about whether the condition part of the rule is satisfied), though this is an *ad hoc solution.*
  - (vii) The probability of a hypothesis can be updated in response to more than one piece of evidence.

- The principal *disadvantages of Bayesian updating are:*
  - (i) The prior probability of an assertion must be known or guessed at.
  - (ii) Conditional probabilities must be measured or estimated or, failing those, guesses must be taken at suitable likelihood ratios. Although the conditional probabilities are often easier to judge than the prior probability, they are nevertheless a considerable source of errors. Estimates of likelihood are often clouded by a subjective view of the importance or utility of a piece of information [4].
  - (iii) The single probability value for the truth of an assertion tells us nothing about its precision.
  - (iv) Because evidence for and against an assertion are lumped together, no record is kept of how much there is of each.
  - (v) The addition of a new rule that asserts a new hypothesis often requires alterations to the prior probabilities and weightings of several other rules. This contravenes one of the main advantages of knowledge-based systems.
  - (vi) The assumption that pieces of evidence are independent is often unfounded. The only alternatives are to calculate *affirms and denies* weights for all possible combinations of dependent evidence, or to restructure the rule base so as to minimize these interactions.
  - (vii) The linear interpolation technique for dealing with uncertain evidence is not mathematically justified.
  - (viii) Representations based on odds, as required to make use of likelihood ratios, cannot handle absolute truth, i.e., odds = infinito.

Several early expert system projects (besides PROSPECTOR) attempted to adapt Bayesian techniques to their problem-solving needs. The independence assumptions, continuous updates of statistical data, and the calculations required to support statistical inference gradually stimulated the search for other measures of "confidence." The most important alternative approach was used at Stanford in developing the MYCIN program (Buchanan and Shortliffe 1984). Unlike Bayesian approaches, which attempt to measure the probability with which evidence supports a conclusion, certainty theory attempts to measure the confidence merited by a given heuristic. When reasoning with heuristic knowledge, human experts are able to give adequate, useful estimates of the confidence we are justified in having in their conclusions. They weight them with terms like "highly probable," "unlikely," "almost certainly," or "possible." These weights are clearly not based in careful analysis of probabilities. Instead, they are themselves heuristics derived from experience in reasoning about the problem domain. Certainty theory is an effort to formalize this heuristic approach to reasoning with uncertainty.

Stanford certainty theory is based on a number of observations. The first is that in traditional probability theory, the sum of confidence for a relationship and confidence against the same relationship must add to one. However, it is often the case that an expert might

confidence 0.7 (say) that some relationship is true and have no feeling of it being
We saw in Section 7.1.3 how Dempster–Shafer handled with this one-sum constra
Another assumption that underpins certainty theory is that the knowledge conten
ules is much more important than the algebra of confidences that holds the sys
her. Confidence measures correspond to the informal evaluations that human exp
h to their conclusions, such as "it is probably true," "it is almost certainly true" o
ghly unlikely."

The Stanford certainty theory makes some simple assumptions for crea
idence measures and has some equally simple rules for combining these confider
e program moves toward its conclusion. The first assumption is to split "confide
from "confidence against" a relationship:

Call MB(H | E) the measure of belief of a hypothesis H given evidence E.

Call MD(H | E) the measure of disbelief of a hypothesis H given evidence E.

Factores de certidumbre

- Certainty theory is an adaptation of Bayesian updating that is incorporated into the EMYCIN expert system shell. EMYCIN is based on MYCIN, an expert system that assists in the diagnosis of infectious diseases.
- The name EMYCIN is derived from “essential MYCIN,” reflecting the fact that it is not specific to medical diagnosis and that its handling of uncertainty is simplified.
- Certainty theory represents an attempt to overcome some of the shortcomings of Bayesian updating, although the mathematical rigor of Bayesian updating is lost.
- As this rigor is rarely justified by the quality of the data, this is not really a problem.

- Instead of using probabilities, each assertion in EMYCIN has a certainty value associated with it. Certainty values can range between 1 and -1.
- For a given hypothesis H, its certainty value C(H) is given by:

$C(H) = 1.0$ if H is known to be true;

$C(H) = 0.0$ if H is unknown;

$C(H) = -1.0$ if H is known to be false.

- There is a similarity between certainty values and probabilities, such that:

$$C(H) = 1.0 \quad \text{corresponds to } P(H)=1.0;$$
$$C(H) = 0.0 \quad \text{corresponds to } P(H) \text{ being at its } \textit{a priori} \text{ value};$$
$$C(H) = -1.0 \quad \text{corresponds to } P(H)=0.0.$$

- Each rule also has a certainty associated with it, known as its certainty factorCF. Certainty factors serve a similar role to the *affirms and denies weightings* in Bayesian systems:
  - IF <evidence> THEN <hypothesis> WITH certainty factor CF
- Part of the simplicity of certainty theory stems from the fact that identical measures of certainty are attached to rules and hypotheses.
- The certainty factor of a rule is modified to reflect the level of certainty of the evidence, such that the modified certainty factor CF' is given by:

$$CF' = CF \times C(E)$$

if $C(H) \geq 0$ and $CF' \geq 0$:

$$C(H|E) = C(H) + [CF' \times (1 - C(H))]$$

if $C(H) \leq 0$ and $CF' \leq 0$:

$$C(H|E) = C(H) + [CF' \times (1 + C(H))]$$

if $C(H)$ and CF' have opposite signs:

$$C(H|E) = \frac{C(H) + CF'}{1 - \min(|C(H)|, |CF'|)}$$

$$C(E_1 \text{ AND } E_2) = \min[C(E_1), C(E_2)]$$

$$C(E_1 \text{ OR } E_2) = \max[C(E_1), C(E_2)]$$

$$C(\sim E) = -C(E)$$

An interpretation of the certainty factors

Definitely not	−1.0
Almost certainly not	−0.8
Probably not	−0.6
Maybe not	−0.4
Unknown (neutral)	−0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

Propiedades de los valores de certidumbre

(i) the function is continuous and has no singularities or steps;

(ii) the updated certainty C(H|E) always lies within the bounds –1 and +1;

(iii) if either C(H) or CF' is +1 (i.e., definitely true) then C(H|E) is also +1;

(iv) if either C(H) or CF' is –1 (i.e., definitely false) then C(H|E) is also –1;

(v) when contradictory conclusions are combined, they tend to cancel each other out, i.e., if $C(H) = -CF'$ then $C(H|E) = 0$;

(vi) several pieces of independent evidence can be combined by repeated application of the function, and the outcome is independent of the order in which the pieces of evidence are applied;

(vii) if $C(H) = 0$, i.e., the certainty of H is at its *a priori* value, then $C(H|E) = CF'$;

(viii) if the evidence is certain (i.e., $C(E) = 1$) then $CF' = CF$.

(ix) although not part of the standard implementation, the absence of evidence can be taken into account by allowing rules to fire when $C(E) < 0$.

If evidence E supports hypothesis H, i.e., P(H|E) is greater than P(H), then:

$$\left.\begin{array}{ll} CF = \dfrac{P(H|E) - P(H)}{1 - P(H)} & \text{if } P(H) \neq 1 \\ CF = 1 & \text{if } P(H) = 1 \end{array}\right\} \tag{3.34}$$

If evidence E opposes hypothesis H, i.e., P(H|E) is less than P(H), then:

$$\left.\begin{array}{ll} CF = \dfrac{P(H|E) - P(H)}{P(H)} & \text{if } P(H) \neq 0 \\ CF = -1 & \text{if } P(H) = 0 \end{array}\right\} \tag{3.35}$$

$$MB(H,E) = \begin{cases} 1 & \text{if } P(H) = 1 \\ \dfrac{\max[P(H \mid E),\ P(H)] - P(H)}{\max[1,0] - P(H)} & \text{otherwise} \end{cases}$$

$$CF = \frac{MB - MD}{1 - \min(MB, MD)}$$

$$MD(H,E) = \begin{cases} 1 & \text{if } P(H) = 0 \\ \dfrac{\min[P(H \mid E),\ P(H)] - P(H)}{\min[1,0] - P(H)} & \text{otherwise} \end{cases}$$

Characteristics	Values
Ranges	$0 \leq MB \leq 1$ $0 \leq MD \leq 1$ $-1 \leq CF \leq 1$
Certain True Hypothesis $P(H \mid E) = 1$	$MB = 1$ $MD = 0$ $CF = 1$
Certain False Hypothesis $P(H' \mid E) = 1$	$MB = 0$ $MD = 1$ $CF = -1$
Lack of Evidence $P(H \mid E) = P(H)$	$MB = 0$ $MD = 0$ $CF = 0$

Table 5.1 Some Characteristics of MB, MD, and CF

Diferencia en el manejo de las evidencias en BU y CF

- En la actualización Bayesiana si existen dos reglas que conduzcan a una misma conclusión es posible manejarlas como una sola regla debido a que las evidencias se consideran independientes entre sí y que cada una de ellas posee sus propios pesos de afirmación y de negación.
- En los factores de certidumbre si existen dos reglas que conduzcan a una misma conclusión y sus evidencias son independientes entre sí, se debe manejar en forma separada debido a que el factor de certidumbre asociado con una regla se maneja como un todo. Cuando una nueva evidencia se añade es necesario volver a determinar el CF

Referencia

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- Nakatsu, Robbie T.