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LÓGICA DIFUSA

Lógica difusa

- Bayesian updating and certainty theory are techniques for handling the uncertainty that arises, or is assumed to arise, from statistical variations or randomness. Possibility theory addresses a different source of uncertainty, namely vagueness in the use of language.
- Possibility theory, or fuzzy logic, was developed by Zadeh and builds upon his theory of fuzzy sets. Zadeh asserts that while probability theory may be appropriate for measuring the likelihood of a hypothesis, it says nothing about the *meaning of the* hypothesis.

Lógica difusa (cont.)

- Fuzzy logic is a **many-valued logic**, replacing the two classical truth values true (= 1) and false (= 0) by a continuum of truth values, usually represented by the unit interval [0,1]. Fuzzy sets, based on this many-valued logic, can be used to model linguistic vagueness which is intrinsically hidden in attributes like "large" and "small" and, in particular, the gradual transition between them. A main application of fuzzy logic is **human-like reasoning** in situations where vague, incomplete and/or (partially) contradictory knowledge is available, often in the form of rule-based systems as in fuzzy control

Fuzzy variables, fuzzy sets, operations in fuzzy sets

- The theory of fuzzy sets expresses imprecision quantitatively by introducing characteristic membership functions that can assume values between 0 and 1 corresponding to degrees of membership from "not a member" through to "a full member."
- If *F is a fuzzy set, then the membership function* $\mu_F(x)$ *measures the degree to* which an absolute value *x belongs to F*
- This degree of membership is sometimes called the *possibility that x is described by F.*

Fuzzy variables, fuzzy sets, operations in fuzzy sets

- A fuzzy variable is one that can take any value from a global set (e.g., the set of all temperatures), where each value can have a degree of membership of a fuzzy set (e.g., low temperature) associated with it.

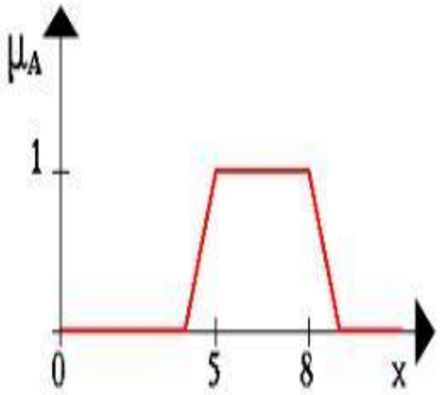
$$\mu_{X \text{ AND } Y}(x) = \min[\mu_X(x), \mu_Y(x)]$$

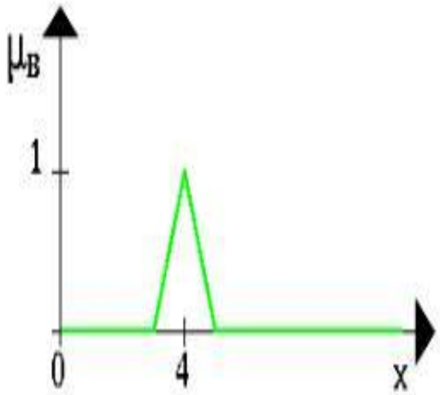
$$\mu_{X \text{ OR } Y}(x) = \max[\mu_X(x), \mu_Y(x)]$$

$$\mu_{\text{NOT } X}(x) = 1 - \mu_X(x)$$

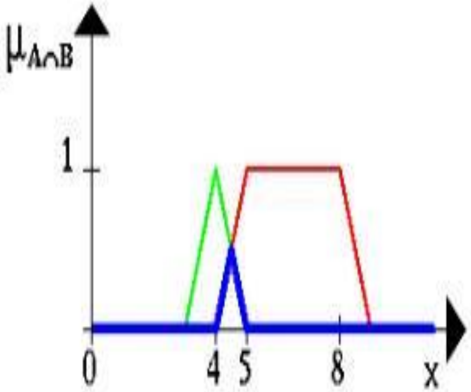
$$\mu_{LT \text{ DOR } MT}(x) = \min[1,\ \mu_{LT}(x) + \mu_{MT}(x)]$$

In order to clarify this, we give a few examples. Let *A* be a fuzzy interval *between* 5 *and* 8 and *B* be a fuzzy number *about* 4. The corresponding figures are shown below.

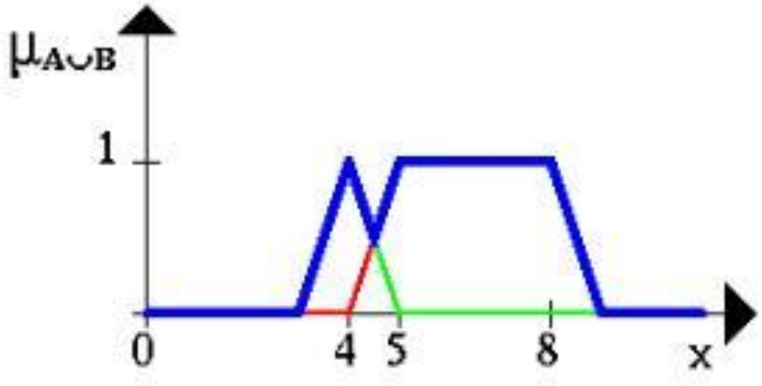




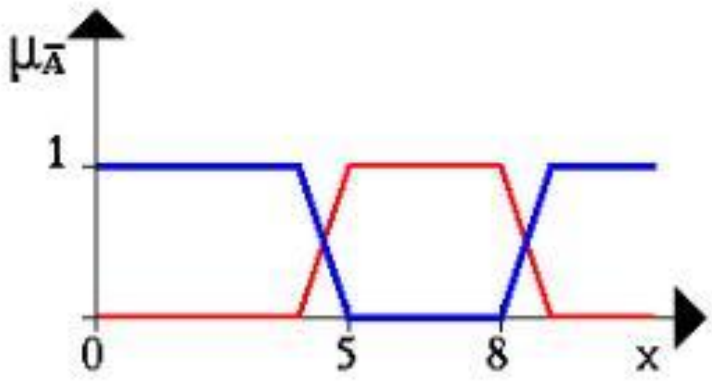
The following figure shows the fuzzy set *between* 5 *and* 8 AND *about* 4 (notice the blue line).



The Fuzzy set *between* 5 *and* 8 **OR** *about* 4 is shown in the next figure (again, it is the blue line).



This figure gives an example for a negation. The blue line is the **NEGATION** of the fuzzy set *A*.



Fuzzy expert systems

- A fuzzy expert system is an expert system that uses fuzzy logic instead of Boolean logic.
- A fuzzy expert system is a collection of membership functions and rules that are used to reason about data.
- Unlike conventional expert systems, which are mainly symbolic reasoning engines, fuzzy expert systems are oriented toward numerical processing.

Fuzzy rules

- The rules in a fuzzy expert system are usually of a form similar to the following:
  - if x is low and y is high then z = medium
- where x and y are input variables, z is an output variable, low is a membership function (fuzzy subset) defined on x, high is a membership function defined on y, and medium is a membership function defined on z.
- The antecedent describes to what degree the rule is applicable; the consequent assigns a membership function to each of one or more output variables.

Fuzzy rules (cont.)

- Most tools for working with fuzzy expert systems allow more than one conclusion per rule.
- A typical fuzzy expert system has more than one rule.
- Instead of assigning a single value to the output variable z, each rule assigns an entire fuzzy subset

Fuzzy rules (cont.)

- If a variable is set to a value by crisp rules, its value will change in steps as different rules fire. The only way to smooth those steps would be to have a large number of rules. However, only a small number of fuzzy rules is required to produce smooth changes in the outputs as the input values alter.
- The number of fuzzy rules required is dependent on the number of variables, the number of fuzzy sets, and the ways in which the variables are combined in the fuzzy rule conditions.
- The initial possibility values are assumed to be zero if these are the first rules to fire
- If several rules affect the same fuzzy set of the same variable, they are equivalent to a single rule whose conditions are joined by the disjunction OR

The Inference Process

- With the definition of the rules and membership functions in hand, we now need to know how to apply this knowledge to specific values of the input variables to compute the values of the output variables. This process is referred to as **Inference Process**
- In a fuzzy expert system, the inference process is a combination of four subprocesses: fuzzification, inference, composition, and defuzzification.
- The defuzzification subprocess is optional.

Fuzzification

- In the fuzzification subprocess, the membership functions defined on the input variables are applied to their actual values, to determine the degree of truth for each rule premise.
- The degree of truth for a rule's premise is sometimes referred to as its **alpha**.
- If a rule's premise has a nonzero degree of truth (if the rule applies at all...) then the rule is said to **fire**.

Inference

- In the inference subprocess, the truth value for the premise of each rule is computed, and applied to the conclusion part of each rule.
- This results in one fuzzy subset to be assigned to each output variable for each rule.
- Exists two inference methods: **MIN** and **PRODUCT**
- In **MIN** inferencing, the output membership function is clipped off at a height corresponding to the rule premise's computed degree of truth. This corresponds to the traditional interpretation of the fuzzy logic AND operation.
- In **PRODUCT** inferencing, the output membership function is scaled by the rule premise's computed degree of truth.

Composition

- In the composition subprocess, all of the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable.
- Exists two composition methods: **MAX composition** and **SUM composition.**
- In **MAX composition**, the combined output fuzzy subset is constructed by taking the pointwise maximum over all of the fuzzy subsets assigned to the output variable by the inference rule.
- In **SUM composition** the combined output fuzzy subset is constructed by taking the pointwise sum over all of the fuzzy subsets assigned to the output variable by the inference rule.

Composition (cont.)

- Note that this can result in truth values greater than one! For this reason, **SUM composition** is only used when it will be followed by a defuzzification method, such as the **CENTROID method**, that doesn't have a problem with this odd case.

Defuzzification

- Sometimes it is useful to just examine the fuzzy subsets that are the result of the composition process, but more often, this **fuzzy value** needs to be converted to a single number (a **crisp value)**. This is what the **defuzzification** subprocess does.
- Defuzzification takes place in two stages:
  - *SCALING THE MEMBERSHIP FUNCTIONS:* adjust the fuzzy sets in accordance with the calculated possibilities:
  - Larsen's product operation rule: the membership functions are multiplied by their respective possibility values. The effect is to compress the fuzzy sets so that the peaks equal the calculated possibility values. An alternative approach in which the fuzzy sets are truncated

Defuzzification (cont.)

- *FINDING THE CENTROID. The most commonly used method of defuzzification is the centroid method, sometimes called the center of gravity, center of mass, or center of area method.*
- If there are N membership functions with centroids ci and areas ai then the combined centroid C, i.e., the defuzzified value, is:

$$C = \frac{\sum_{i=1}^{N} a_i c_i}{\sum_{i=1}^{N} a_i}$$

*Esta fórmula se emplea en el método de truncamiento

Defuzzification

- When the fuzzy sets are compressed using Larsen's product operation rule, the values of *ci are unchanged from the centroids of the uncompressed shapes, Ci,* and *ai is simply μi Ai where Ai is the area of the membership function prior to* compression.

$$C = \frac{\sum_{i=1}^{N} a_i c_i}{\sum_{i=1}^{N} a_i}$$

- donde ai = μi * Ai

Deffuzzificación en los extremos

- En el caso de los extremos, se puede optar por 2 alternativas: Considerando el centroide del conjunto difuso involucrado o por la regla del espejo.
- Por la regla del espejo y el producto de Larsen, la obtención del centroide se simplifica a
-

$$C = \frac{\sum_{i=1}^{N} \mu_i C_i}{\sum_{i=1}^{N} \mu_i}$$

Glossary

- **Def 2.1 Intersection of Sets** We call a new set generated from two given sets *A* and *B intersection of A and B*, if the new set contains exactly those elements that are contained in *A* and in *B.*
- **Def 2.2 Unification of Sets** We call a new set generated from two given sets *A* and *B unification of A and B*, if the new set contains all elements that are contained in *A* or in *B* or in both.
- **Def 2.3 Negation of Sets** We call a new set containing all elements which are in the universe of discourse but not in the set *A* the *negation of A*.
- **Def 3.1 Linguistic Variable** A *linguistic variable* is a quintuple (*X*,*T*(*X*),*U*,*G*,*M*,), where *X* is the name of the variable, *T*(*X*) is the term set, i.e. the set of names of linguistic values of *X*, *U* is the universe of discourse, *G* is the grammer to generate the names and *M* is a set of semantic rules for associating each *X* with its meaning.

References

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