Mills

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Chapter 1

Mills' constant

Definition 1.1. *A positive real number x is Mills if $1 < x$ and for all positive integers n, the number $\lfloor x^{3^n} \rfloor$ is prime.*

Theorem 1.2. *BHP*

Proposition 1.3. *There exists a Mills number.*

Proof. proof □

Proposition 1.4. *W has minimum.*

Proposition 1.5. *$\theta = 21/40$. There exists a natural number k_0 such that for all $k \geq k_0$, we have*

$$p_k^3 \leq p_{k+1} \leq p_k^3 + p_k^{3\theta}.$$

Lemma 1.6. *For all natural numbers k, we have*

$$p_k \leq p_{k+1} < (p_k^3 + 1)^3 - 1.$$

Lemma 1.7. *There exists $\gamma > 0$ and $k_1 > 1$ such that for all $k \geq k_1$, we have*

$$|A^{3k} - p_k| \leq e^{-3^k \gamma}.$$

Theorem 1.8. *Mahler*

Theorem 1.9. *The Mills' constant is irrational.*

Proof. proof □