# Studies of ERM Models with Correlated Disorder

**by Tom Folgmann**

Bachelor Thesis Presentation, 2024

# Upfront: Goals.

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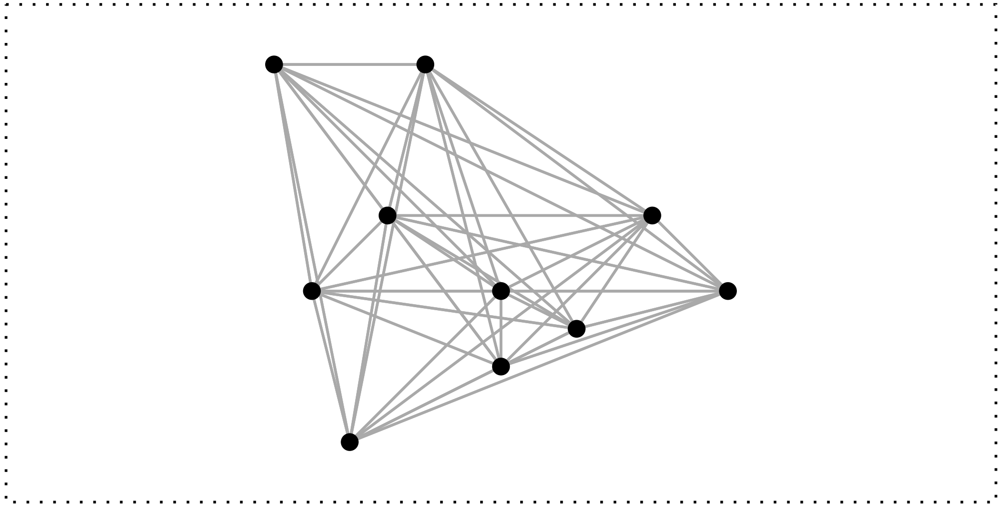

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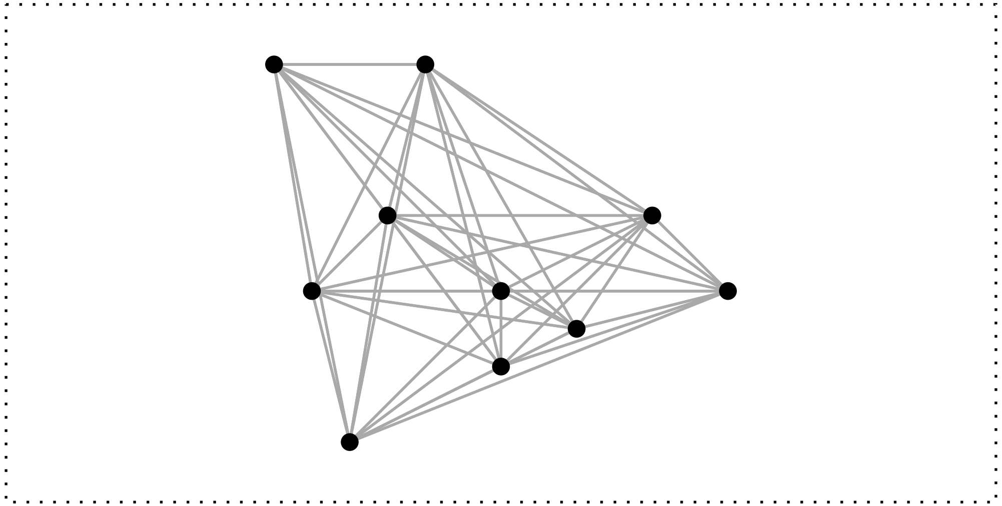

A system with $N \in \mathbb{N}$ (related) particles can be described by a mathematical *Graph*.

Fundamentally, the properties of such graphs can be rendered in a *Laplacian Matrix* $L$.

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.. special case is the *Adjacency Matrix* $A$, where $w_{i,j} \in \{0, -1, 1\}$.

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- *Interaction strength* given by $f_{ij} \overset{\mathrm{m}}{=} f\left(r_i - r_j\right)$
- *Self-interaction* given by $\Sigma(f,i) \overset{\mathrm{m}}{=} \sum_{j\in[N]\setminus\{i\}} f_{ij}$

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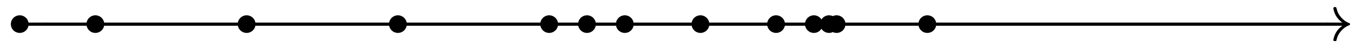

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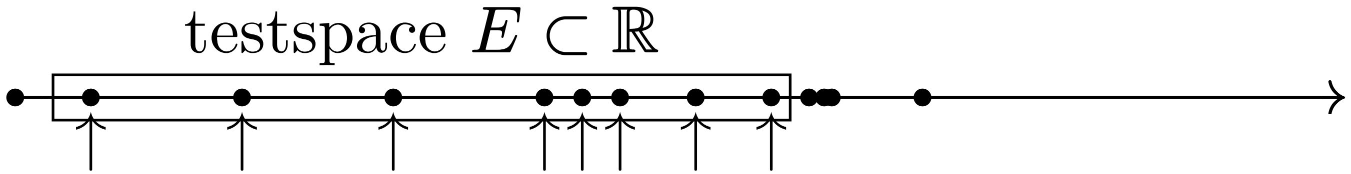

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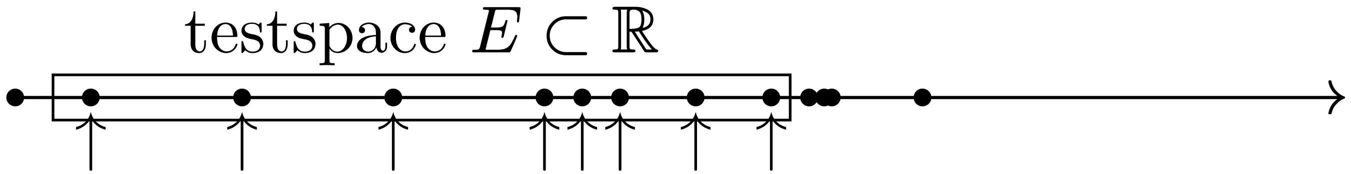


... results in an (unnormalized) density function

$$E \mapsto \sum_{i \in [k]} \delta_{\Lambda_i}(E) \qquad \in \{0, ..., k\}$$

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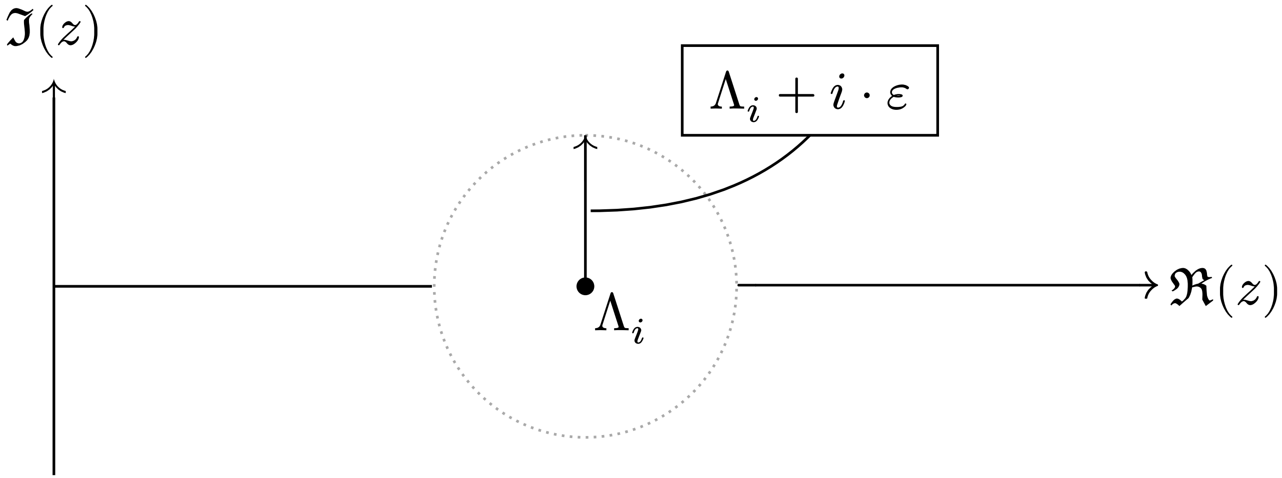

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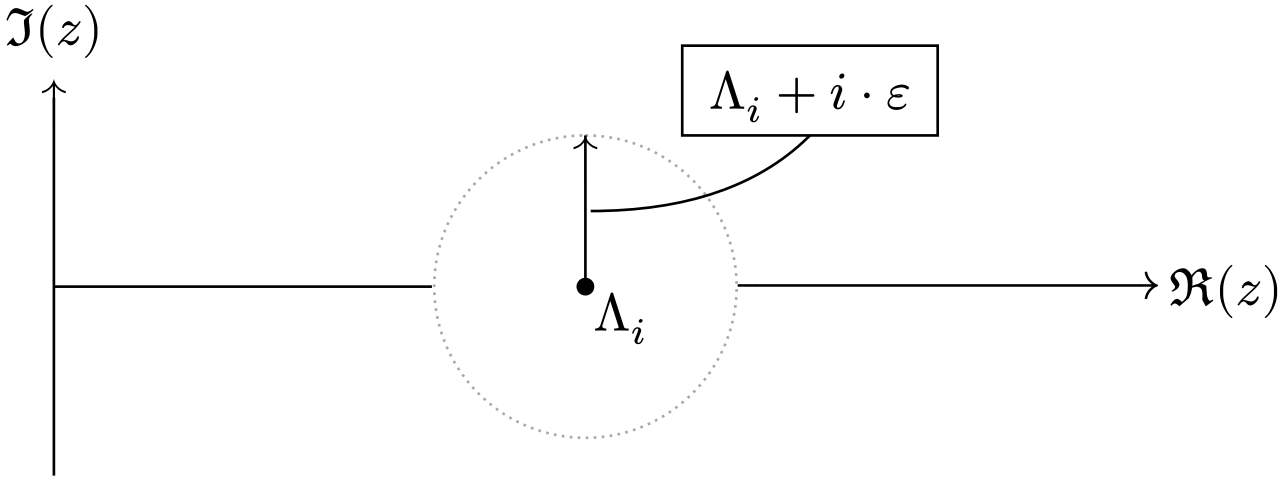


$\hookrightarrow$ Usecase is the resolvent with a singularity at $\Lambda_i$.

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This is just $f_{i,j}$!

.. for $i \neq j$.

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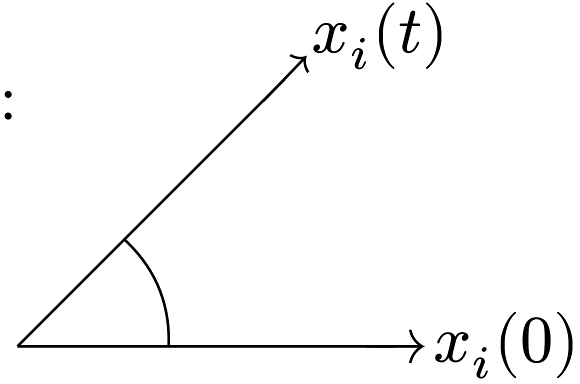

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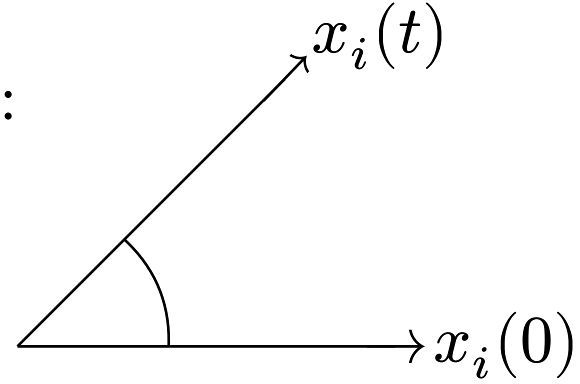


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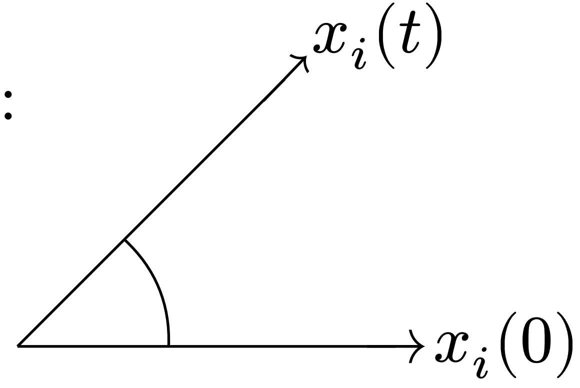


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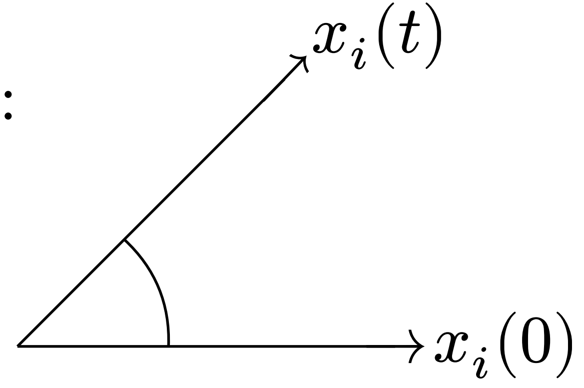


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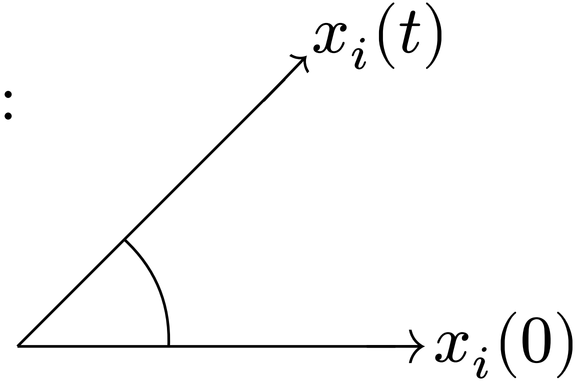


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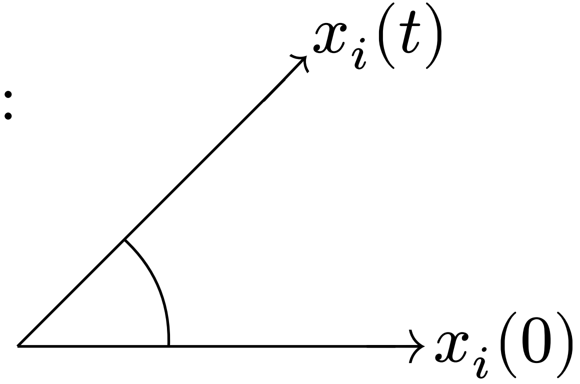


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$$\left(\mathcal{L}F_{j,i}\right)(s) = \pm\frac{1}{\tilde{U}\left(f, x^*(t)\right)_{i,j} - \delta_{ij}\cdot\lambda_i^2}.$$

[6]With $x^*(t) = \left(i \mapsto x_{i(t)}\right)$ and $F_{j,i}(t) := \langle x_j(t), x_i(0)\rangle$.

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Q: What are we integrating over?

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| **Ev. step** | **Meaning** |
|---|---|
| $R$ | Random variable, abstract |
| $R(\omega)$ | Vector of time dep. pos. |
| $R(\omega)_i$ | $i$-th particle position, time dep. path |
| $R(\omega)_i(t)$ | Position of $i$-th particle at time $t$ (fixed for us.) |

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Value in $\mathbb{R}$ ..
(simplification!)

This is already a good starting point to understand our *Correlated Disorder* modification!

# .. missing key elements:

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- The *moment generating function* $Z_{z,R_\omega}[J]$. It requires the *force field* $J$.

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.. coming from ..

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This needs explanation.

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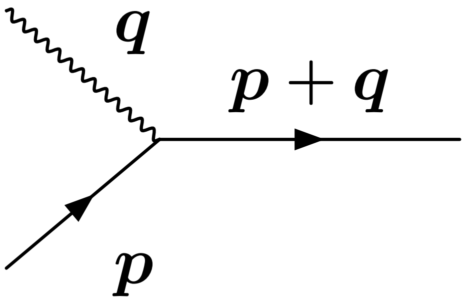
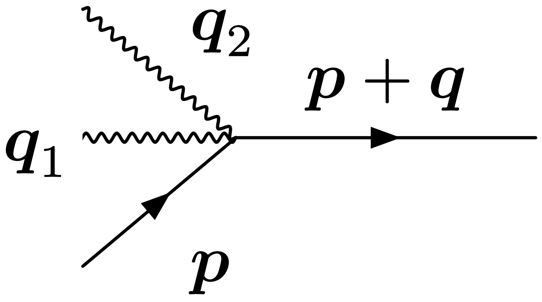



$$:= \mu_z(\boldsymbol{p}, \boldsymbol{q}) \quad \Rightarrow \quad \text{Three-point Vertex}$$

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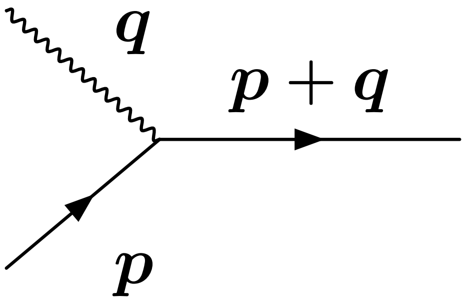


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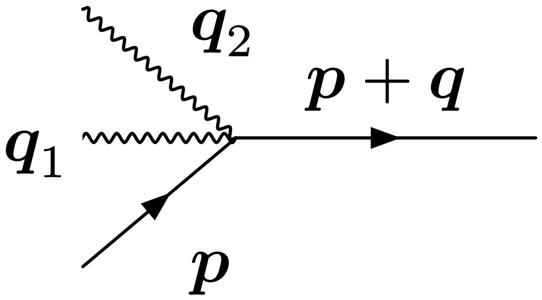


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.. which completes the set of Feynman rules.

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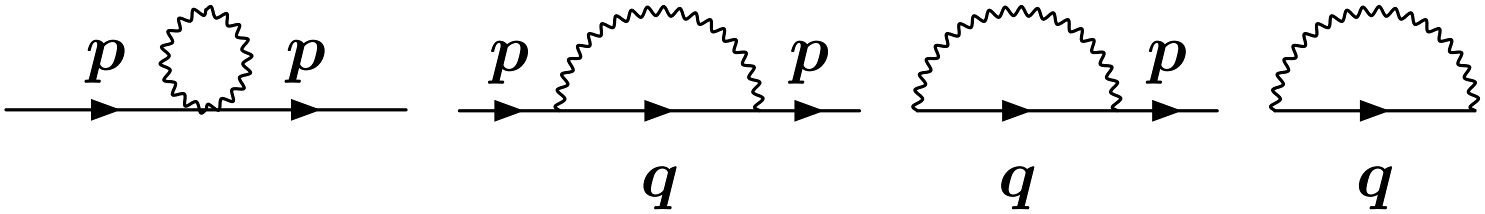

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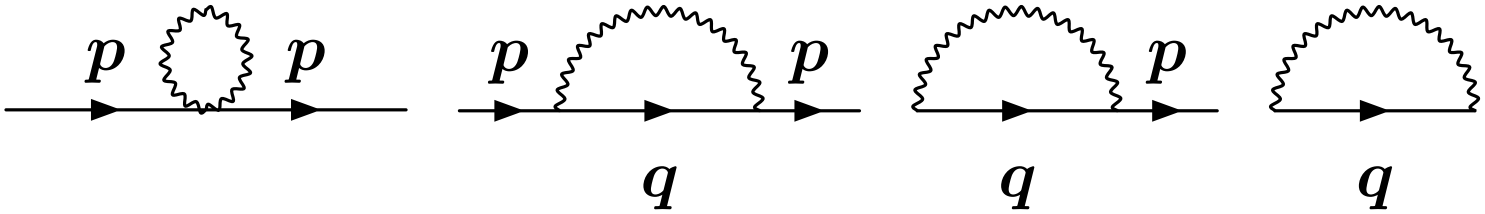


.. represented diagrams are *irreducible*: $Z_{z,R_\omega}[J] \propto \exp\Big(\sum_{C \in \mathcal{C}} C\Big)$.

# Integral representations[16]

[16]Attention! The terms have been simplified. For more details see Thesis sec. 2.4.2.

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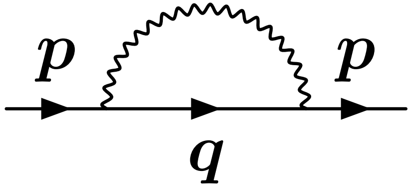


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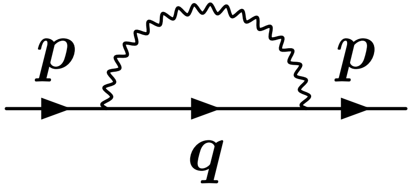
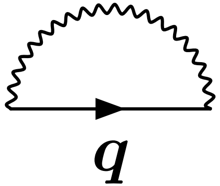



$$= -\frac{2 \cdot G_0(\boldsymbol{p}, z)}{\rho_*} \cdot \int_{\mathbb{R}^d} G_0(\boldsymbol{p} - \boldsymbol{q}, z) \cdot \mu_{z(\boldsymbol{p}, -\boldsymbol{q})} \, d\boldsymbol{q},$$




$$= \frac{1}{\rho_*} \cdot \int_{\mathbb{R}^d} G_0(\boldsymbol{p} - \boldsymbol{q}, z) \, d\boldsymbol{q}.$$

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- Utilization of vertex' and propagator symmetries.

# What is Correlated Disorder?

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**How can we include *structure* in our probability density?**

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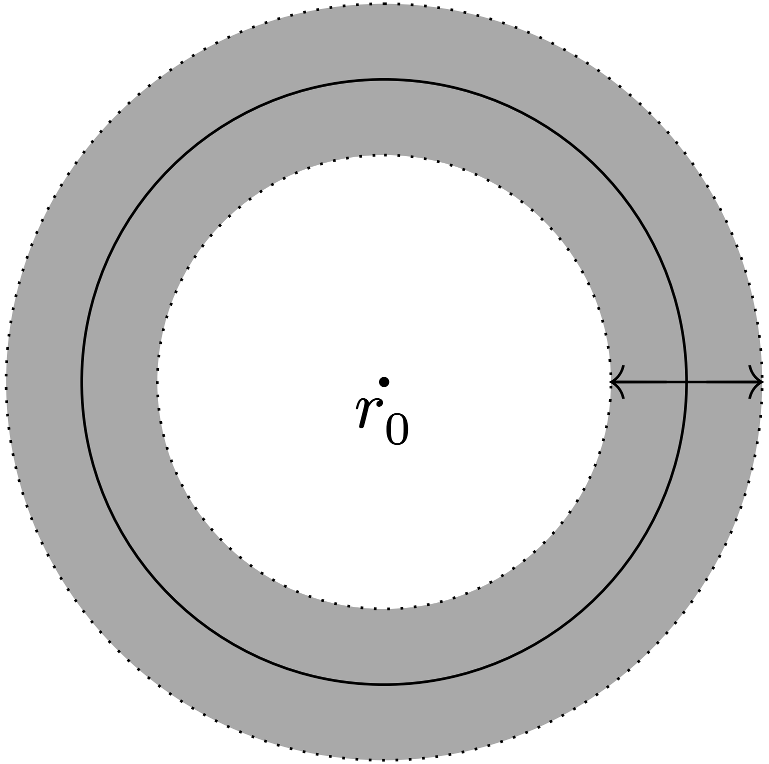

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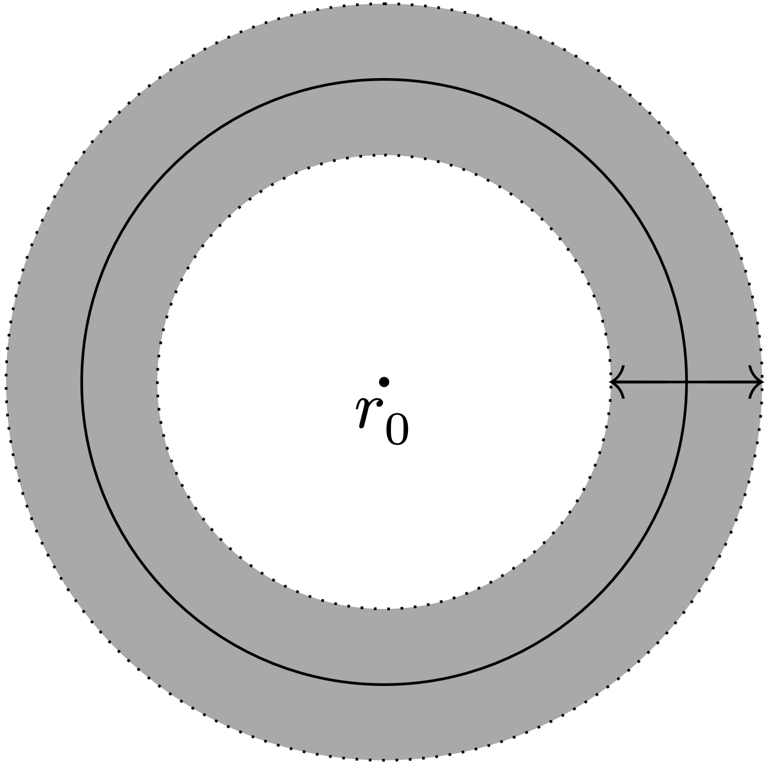
while $\rho_N^{(2)}$ reflects integration of $\exp(-\beta \cdot H(r, \cdot))$ for remaining particles.



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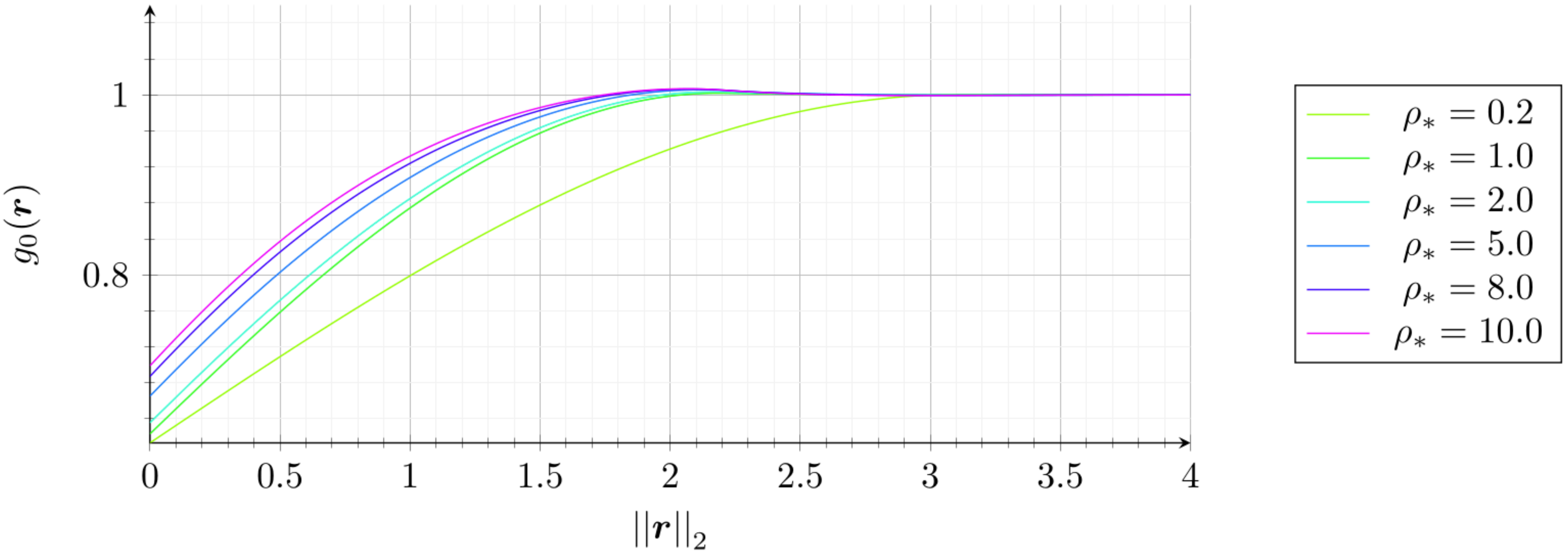

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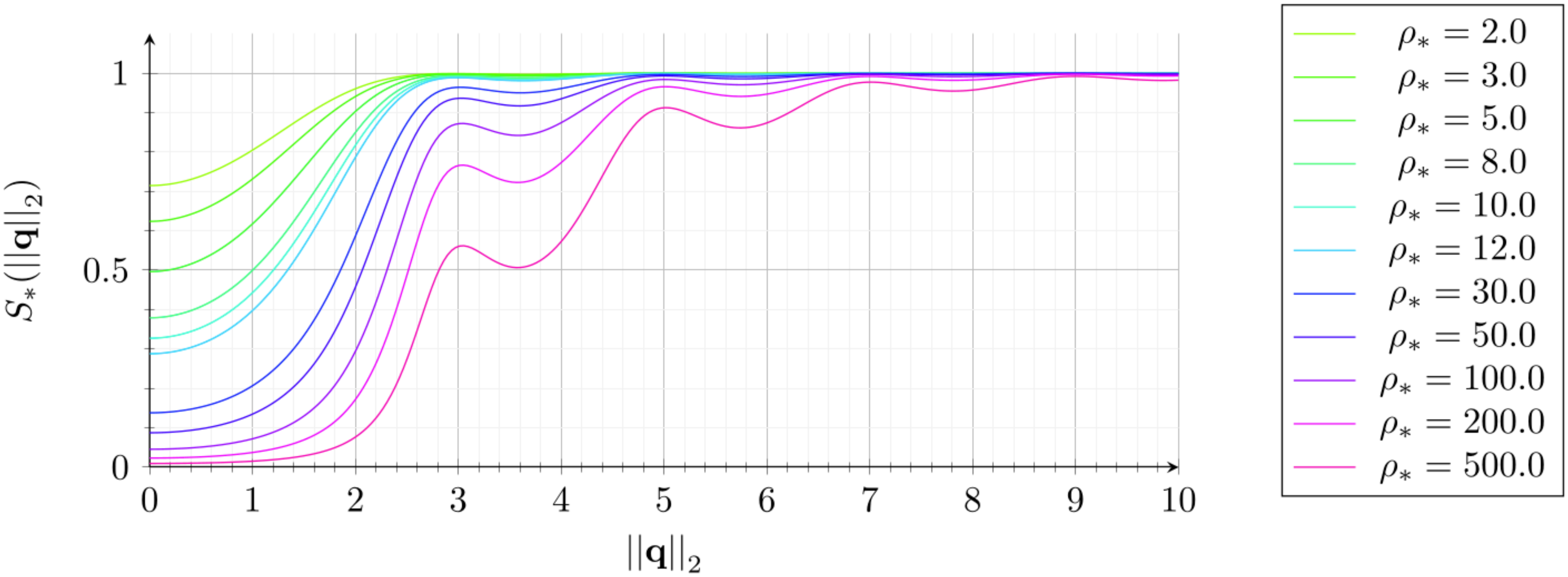

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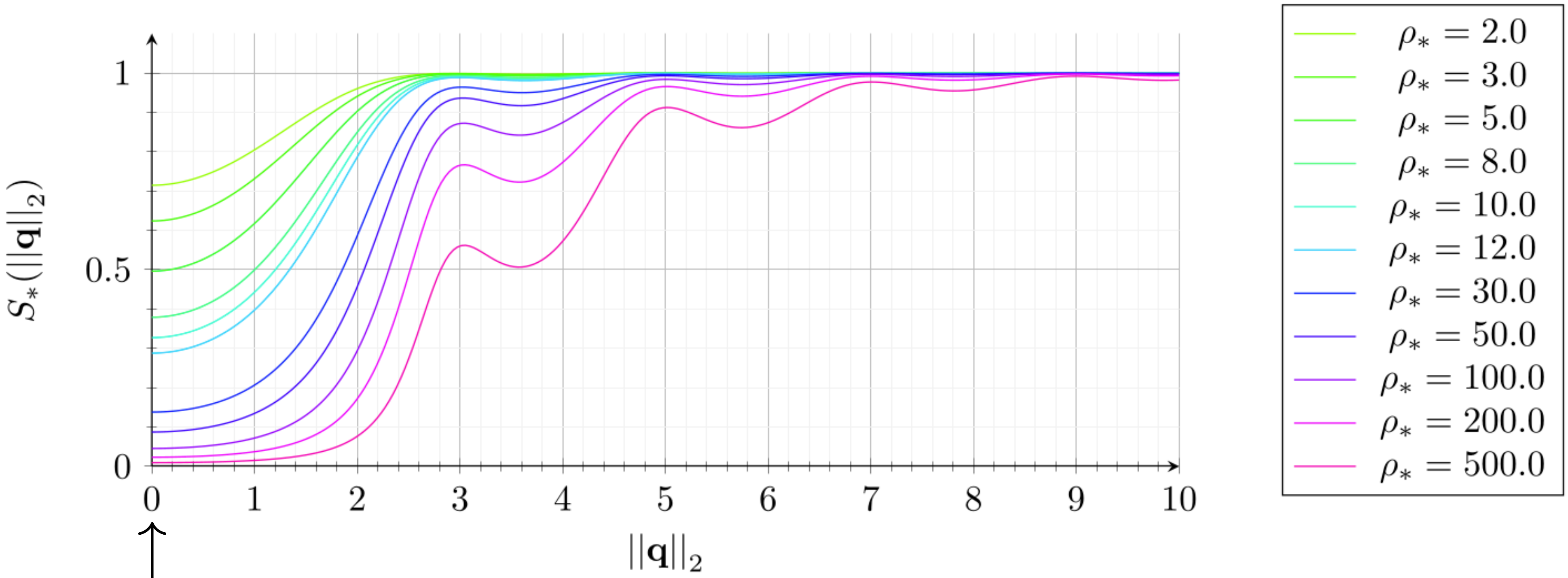


$$S_*(\mathbf{0}) = \rho_* k_B T \cdot \kappa$$

**Where did we implement this, what did it change?**

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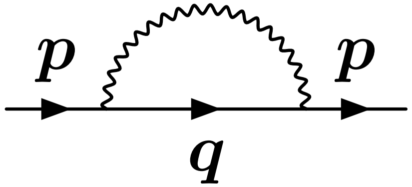


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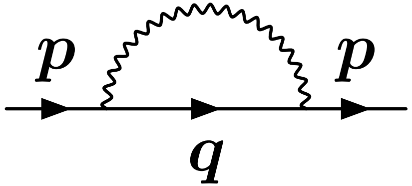


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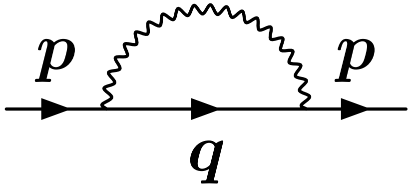


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This also affects the Self-Energy:

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# Can we in any way compare our results?

# The Approach of Martin-Mayor

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Here, a *superposition approximation* was used:

$$\frac{1}{|V_{d,N}|} \cdot \exp(-\beta \cdot U(r)) \approx \frac{1}{|V_{d,N}|} \cdot \exp\left(-\beta \cdot \sum_{i \in [N-1]} u(r_i - r_{i+1})\right)$$

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This has an explicit approximation built into the spring function!

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- We did not change the zeroth order term in the propagator.

# What did a numerical model show?

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$$V_{d,N} \ni R \mapsto U_a^{(num)}(R) = \sum_{(i,j)\in[N]^2} \begin{cases} \frac{1}{2} \cdot \left(\|R_i - R_j\| - a\right)^2 \text{ if } \|R_i - R_j\| < a, \\ 0 \text{ else.} \end{cases}$$

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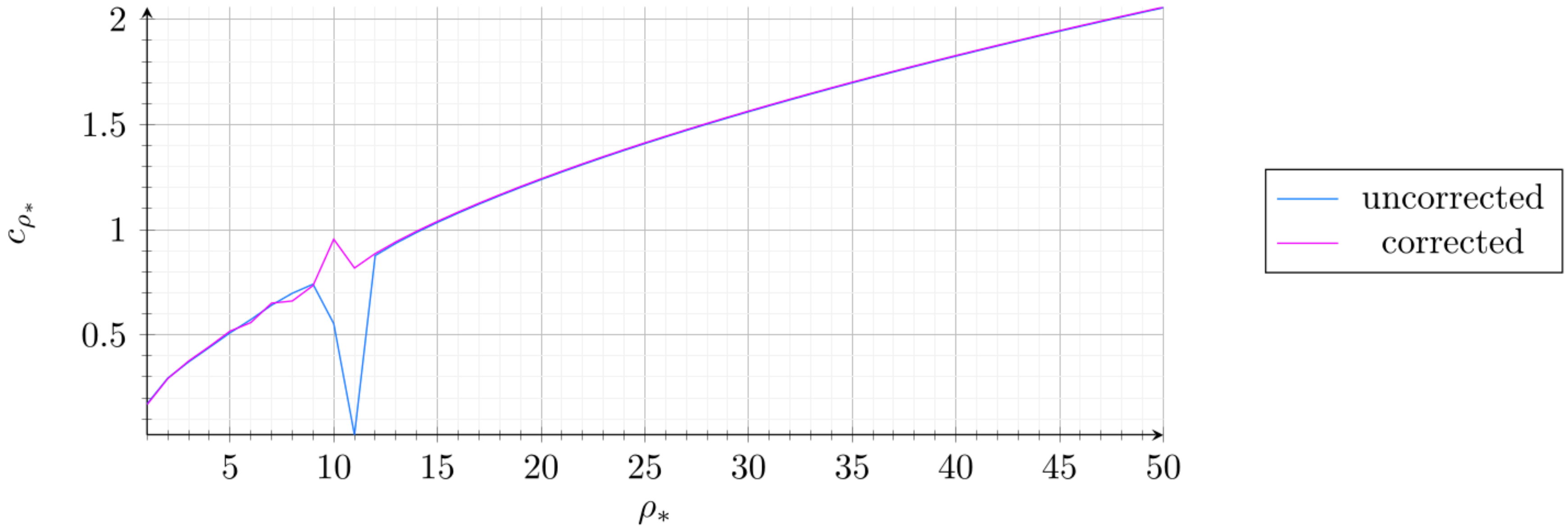
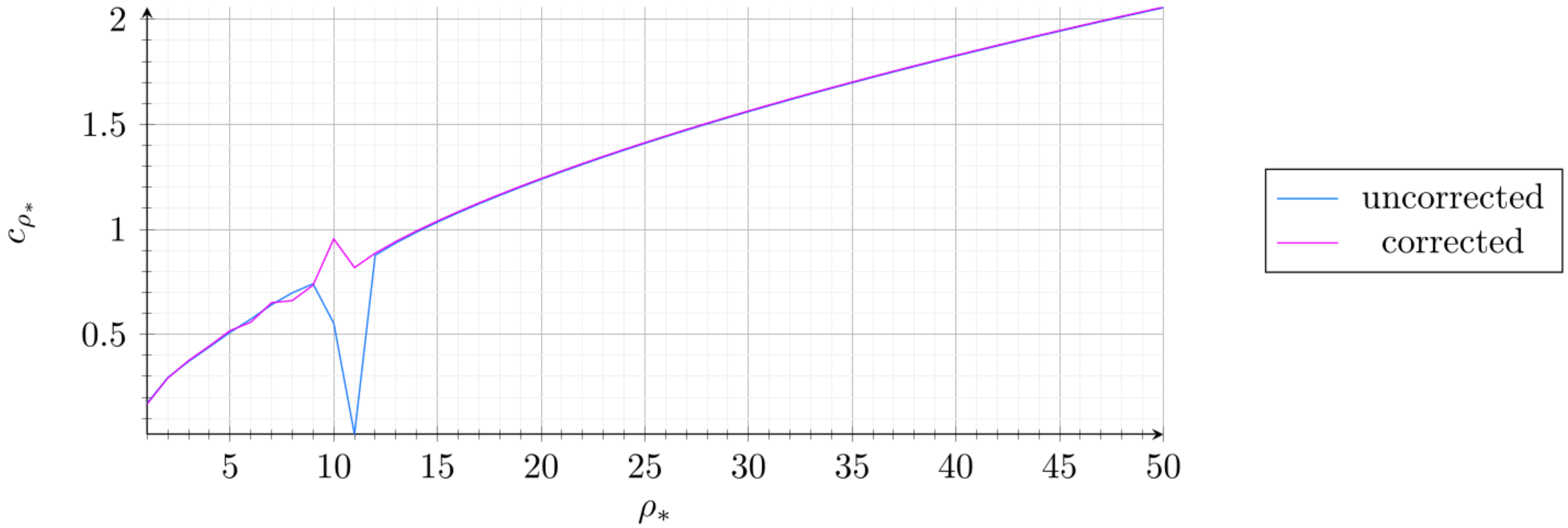


# Results using the Hypernetted Chain




→ No major differences in the velocity of sound noticeable.

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$$\exp(-\beta \cdot U(r)) \approx \exp\Big(-\beta \cdot (r - \nu)^{\perp} \cdot A \cdot (r - \nu)\Big), (\rho \text{ mediocre})$$

# Thank you for your attention!

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`unb3rechenbar/BA24-CorDis.git`

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