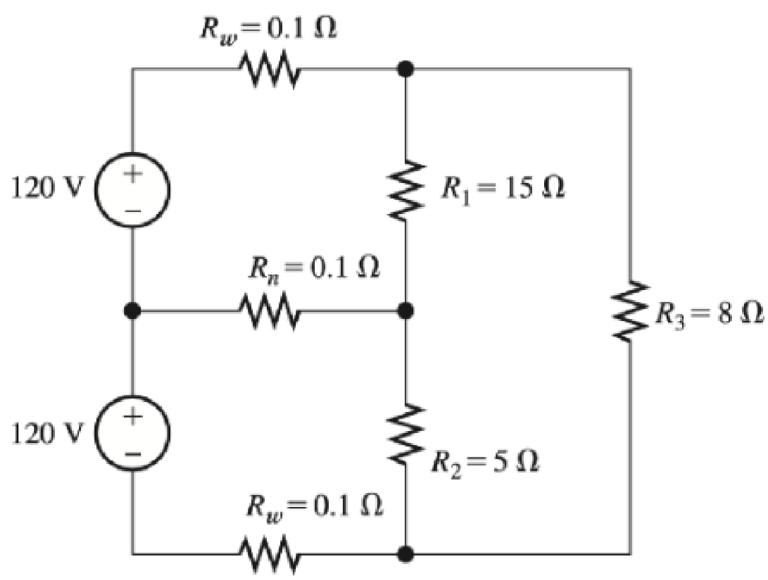
5. **(35 points)** The circuit shown in Figure 4.1 is the dc equivalent of a simple residential power distribution system. Each of the resistances labeled R_1 and R_2 represents various parallel-connected loads, such as lights or devices plugged into outlets that nominally operate at 120 V, while R_3 represents a load, such as the heating element in an oven that nominally operates at 240 V. The resistances labeled R_w represent the resistances of wires. R_n represents the "neutral" wire.

a) Use mesh-current analysis to determine the voltage magnitude for R_n **(10pts)**. Write the equations in matrix form **(5pts)**. Use Matlab (or any other software) to solve the equations **(5 pts).**

b) Now suppose that due to a fault in the wiring at the distribution panel, the neutral wire becomes an open circuit. Again compute the voltages across the loads **(10pts)** and comment on the probable outcome for a sensitive device such as a computer or plasma television that is part of the 15Ω load **(5pts)**.



Circuit labeling (so our equations match)

- Top "hot" node on the right: A
- Middle node on the right (where the neutral wire connects): B
- Bottom "hot" node on the right: C

Components:

- Wire resistances $R_w = 0.1\ \Omega$ on the top and bottom rails
- Neutral wire $R_n = 0.1\ \Omega$
- Loads: $R_1 = 15\ \Omega$ (between A and B), $R_2 = 5\ \Omega$ (between B and C), $R_3 = 8\ \Omega$ (between A and C
)
- Two 120-V sources in series on the left (split-phase DC model), so top-left node is at 240 V, middle-left at 120 V, bottom-left at 0 V.

(a) Mesh analysis to get the neutral voltage V_n

Choose clockwise mesh currents

- i_1: top-left rectangular loop (through top R_w, R_1, R_n, and the upper 120-V source)
- i_2: bottom-left rectangular loop (through bottom R_w, R_2, R_n, and the lower 120-V source)
- i_3: right rectangular loop (through R_1, R_2, R_3)

KVL for each mesh

(Shared elements appear with opposite signs; the source term is **+120** for both left meshes because, traversed clockwise, each source is crossed from $-$ to $+$.)

$$\begin{aligned}
&(\text{Top mesh}) && (R_w + R_1 + R_n)i_1 \; - \; R_n i_2 \; - \; R_1 i_3 = +120 \\
&(\text{Bottom mesh}) && -R_n i_1 \; + \; (R_w + R_2 + R_n)i_2 \; - \; R_2 i_3 = +120 \\
&(\text{Right mesh}) && -R_1 i_1 \; - \; R_2 i_2 \; + \; (R_1 + R_2 + R_3)i_3 = 0
\end{aligned}$$

Matrix form

$$\underbrace{\begin{bmatrix} R_w + R_1 + R_n & -R_n & -R_1 \\ -R_n & R_w + R_2 + R_n & -R_2 \\ -R_1 & -R_2 & R_1 + R_2 + R_3 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 120 \\ 120 \\ 0 \end{bmatrix}}_{\mathbf{b}}$$

Plugging $R_w = R_n = 0.1\,\Omega$, $R_1 = 15\,\Omega$, $R_2 = 5\,\Omega$, $R_3 = 8\,\Omega$ and solving $\mathbf{Ai} = \mathbf{b}$ gives

$$\boxed{i_1 = 36.75\ \text{A}}, \quad \boxed{i_2 = 51.56\ \text{A}}, \quad \boxed{i_3 = 28.89\ \text{A}}$$

These are the currents circulating in the three meshes; note that i_1 equals the current in the **top** R_w, i_2 equals the (left-to-right reference) current in the **bottom** R_w, and i_3 equals the current in R_3.

Neutral voltage.

The current through the neutral resistor is the difference of the two left meshes across the shared R_n: $i_n = i_2 - i_1$.

Therefore

$$\boxed{V_n = R_n(i_2 - i_1) = 0.1(51.56 - 36.75) = 1.48 \text{ V}}$$

(magnitude). This matches the key.

Sanity check via node voltages (optional but reassuring):

$$V_A = 240 - i_1 R_w = 236.33 \text{ V}, \quad V_B = 120 - V_n = 118.52 \text{ V}, \quad V_C = i_2 R_w = 5.157 \text{ V}.$$

(b) Open neutral (fault): compute voltages across the loads

With R_n **open**, node B floats; R_1 and R_2 become a series string between A and C. Solve by nodal KCL at A, B, C:

$$\frac{V_A - 240}{R_w} + \frac{V_A - V_B}{R_1} + \frac{V_A - V_C}{R_3} = 0$$
$$\frac{V_B - V_A}{R_1} + \frac{V_B - V_C}{R_2} = 0$$
$$\frac{V_C - 0}{R_w} + \frac{V_C - V_B}{R_2} + \frac{V_C - V_A}{R_3} = 0$$

Solving gives

$$V_A = 235.942 \text{ V}, \qquad V_B = 62.029 \text{ V}, \qquad V_C = 4.058 \text{ V}.$$

Hence the load voltages are

$$\boxed{V_{R_3} = V_A - V_C = 231.884 \text{ V}}, \qquad \boxed{V_{R_1} = V_A - V_B = 173.913 \text{ V}}, \qquad \boxed{V_{R_2} = V_B - V_C = 57.971 \text{ V}},$$

exactly as in the key.

What this means physically.

With the neutral open, the 120-V loads R_1 and R_2 no longer sit with respect to a stiff 120-V reference. They form a divider across the hot-to-hot potential, so the mid-node B "floats" to a value set by the ratio $R_1 : R_2 = 3 : 1$. That puts ~174 V across the 15-Ω branch and only ~58 V across the 5-Ω branch.

Power comparison (to see the danger):

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} \approx \frac{(173.9)^2}{15} \approx 2.02\ \mathrm{kW} \quad \text{vs.} \quad \frac{120^2}{15} = 0.96\ \mathrm{kW}\ (\text{normal}).$$

So the "120-V" devices on the $15\,\Omega$ branch are severely **over-volted** (~45% high) and can fail catastrophically. The $5\,\Omega$ branch is in brownout (~58 V), so devices there likely shut down or behave erratically. R_3 (the 240-V load) remains close to nominal voltage, aside from small drops in the hot conductors.