Final Project Report

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Link	Length (cm)
1	80
2	55
3	20

Included Scripts and Simuscape Models:

/

planararm_workspace.m : Using link lengths/transformation characteristics to show the workspace of the arm at different joint angles w/ FK

inCircle.m : Helper function to determine if a 2D cartesian point is inside, outside, or on a circle

gripperarm.slx : Simscape model of model to show that certain joint angles do place the gripper correctly

gripper_arm_traj_ctrl.slx : Simscape model to show that certain trajectories do translate the gripper to perform tasks

trajector_planning/

traj_design_posX.mlx : Design of the trajectory of the end effector position in the x-direction over T=3

traj_design_posY.mlx : Design of the trajectory of the end effector position in the y-direction over T=3

traj_design_speeds.mlx : Design of the trajectory of the end effector speed over T=3

traj_design_joint_angles.mlx : Script to design the trajector of any joint angle piece-by-piece, given the desired starting and ending angle of 1 joint from a desired starting and ending time

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Question 1

$$R_1^0 = R_z(q_1) = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad r_1^0 = \begin{bmatrix} 80c_1 \\ 80s_1 \\ 0 \end{bmatrix} \qquad A_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 80c_1 \\ s_1 & c_1 & 0 & 80s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^1 = R_z(q_2) = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad r_2^1 = \begin{bmatrix} 55c_2 \\ 55s_2 \\ 0 \end{bmatrix} \qquad A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & 55c_2 \\ s_2 & c_2 & 0 & 55s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3^2 = R_z(q_3) = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad r_3^2 = \begin{bmatrix} 20c_3 \\ 20s_3 \\ 0 \end{bmatrix} \qquad A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & 20c_3 \\ s_3 & c_3 & 0 & 20s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_E^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad r_E^3 = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \qquad A_E^3 = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_E^0 = A_1^0 A_2^1 A_3^2 A_E^3$$

$$A_E^0 = \begin{bmatrix} \sigma_1 & -c_3\sigma_3 - s_3\sigma_2 & 0 & 80c_1 + 55c_1c_2 - 55s_1 + 30c_3\sigma_2 - 30s_3\sigma_3 \\ c_3\sigma_3 + s_3\sigma_2 & \sigma_1 & 0 & 80s_1 + 55c_1s_2 + 55c_2s_1 + 30c_3\sigma_3 + 30s_3\sigma_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

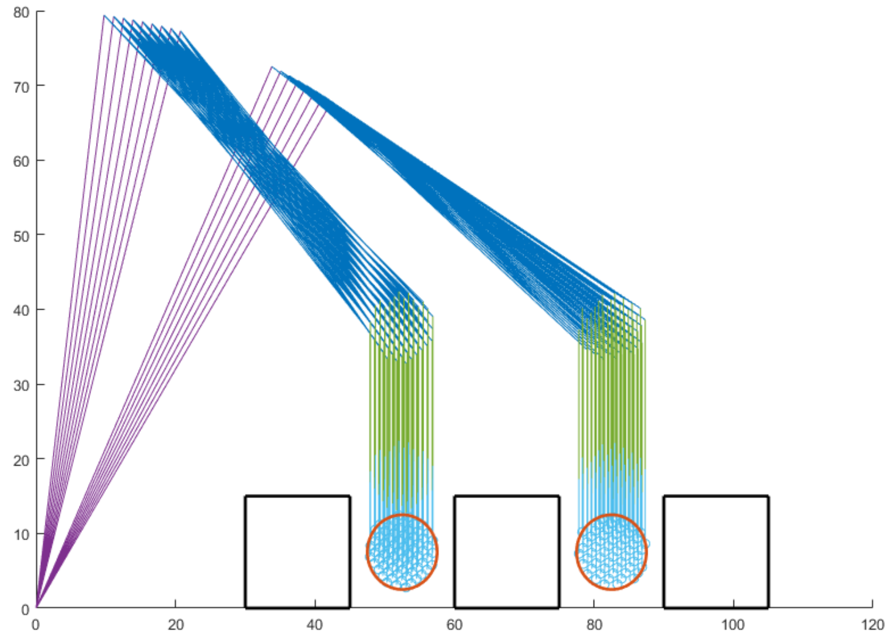
$$\sigma_1 = c_3\sigma_2 - s_3\sigma_3$$
$$\sigma_2 = c_1c_2 - s_1s_2$$
$$\sigma_3 = c_1s_2 + c_2s_1$$

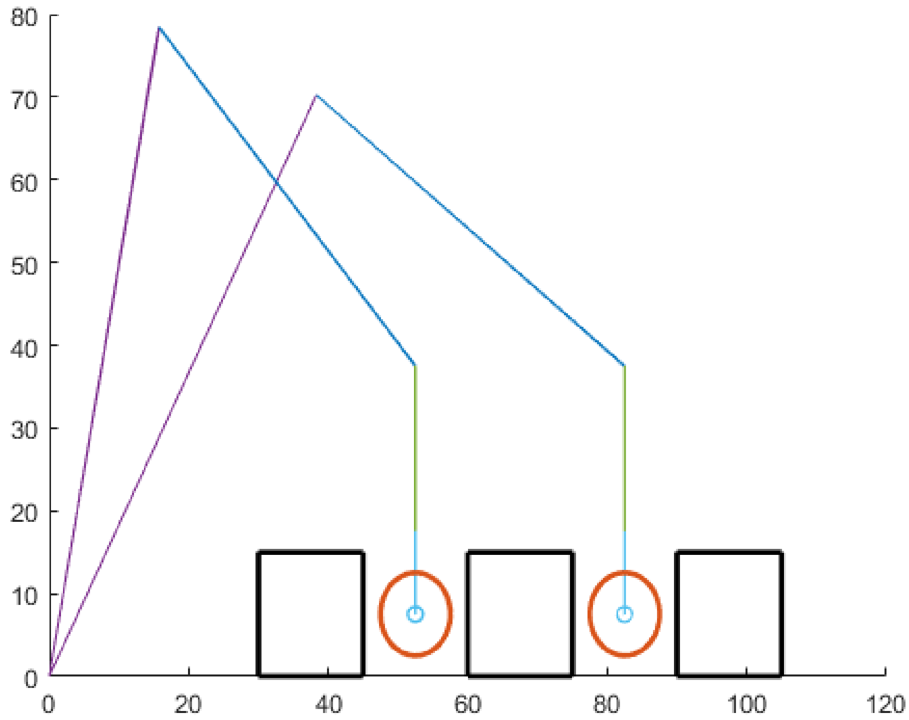
Question 2

See on Matlab Script

Question 2

See planarArm_workspace.m





Question 3 See planararm_workspace.m obj1 = [52.5 5 0]

① $80c_1 + 55c_1c_2 + 55s_1 + 30c_3\sigma_2 + 30s_3\sigma_3 = 52.5$ cm

② $80s_1 + 55c_1s_2 + 55c_2s_1 + 30c_3\sigma_2 + 30s_3\sigma_2 = 5$ cm

③ $q_1 + q_2 + q_3 = 270°$

let $c_1 = \cos(q_1)$ $s_1 = \sin(q_1)$

$c_2 = \cos(q_2)$ $s_2 = \sin(q_2)$

$c_3 = \cos(q_3)$ $s_3 = \sin(q_3)$

We have 3 independent equations and 3 variables (q_1, q_2, q_3). Therefore we can solve the equations, and get the following values:

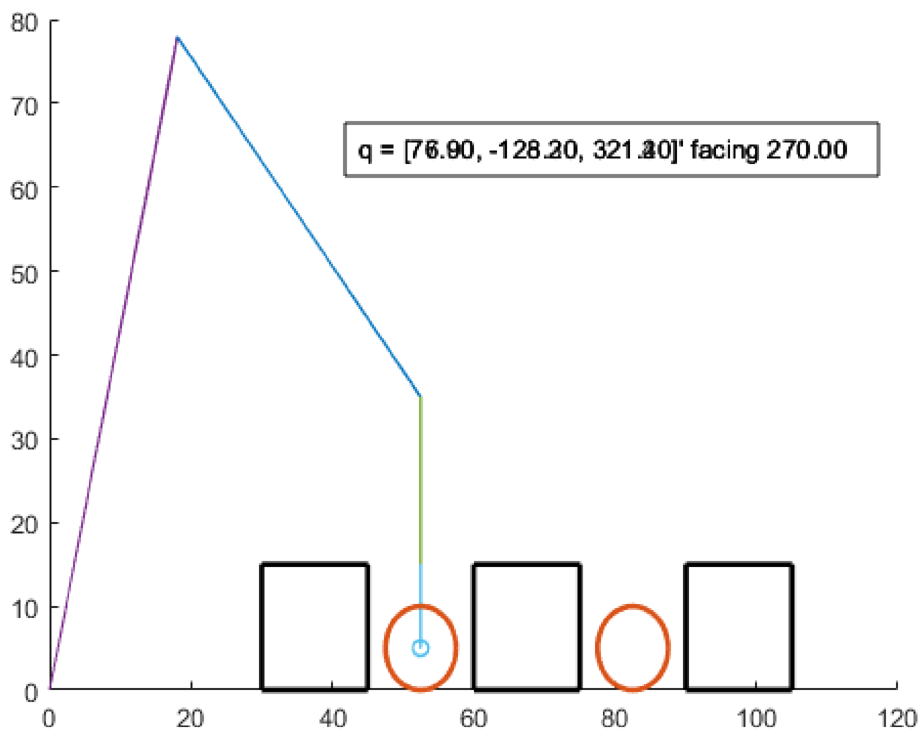
$\sigma_1 = c_3\sigma_2 - s_3\sigma_3$

$\sigma_2 = c_1c_2 - s_1s_2$

$\sigma_3 = c_1s_2 + c_2s_1$

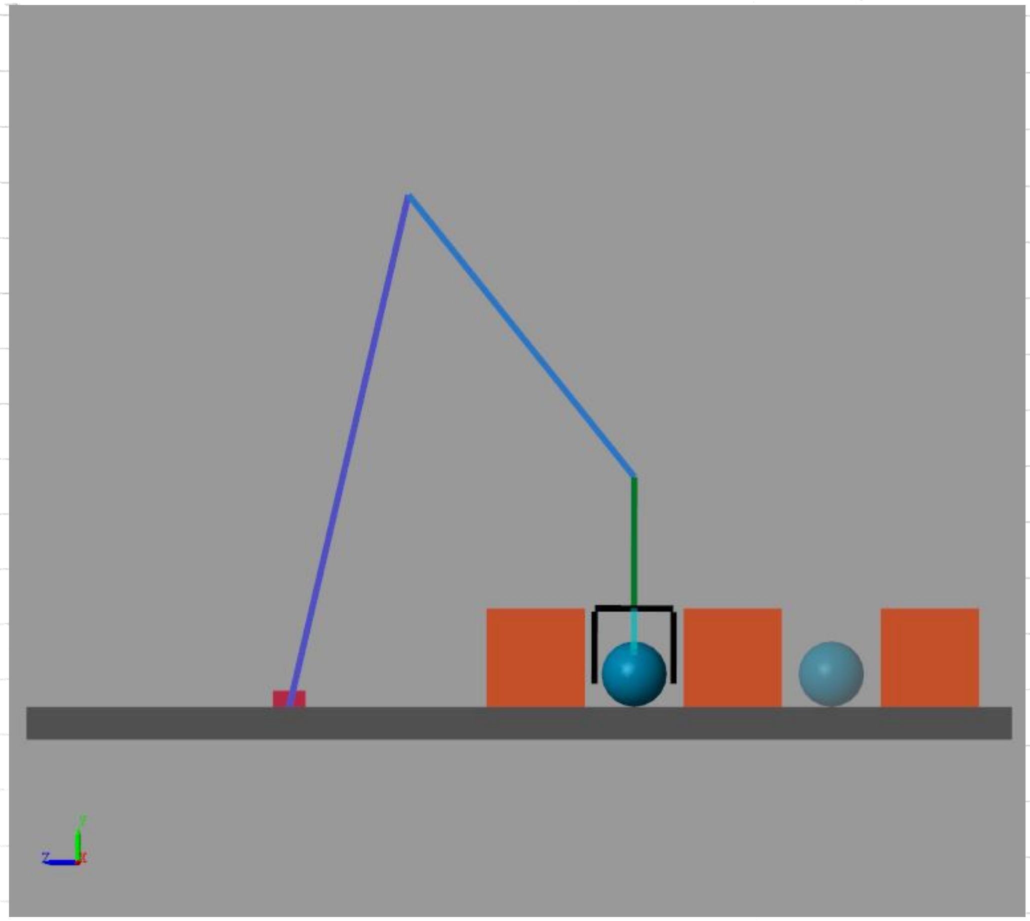
$\boxed{q_1 = 76.9° \quad q_2 = -128.2 \quad q_3 = 321.2°}$

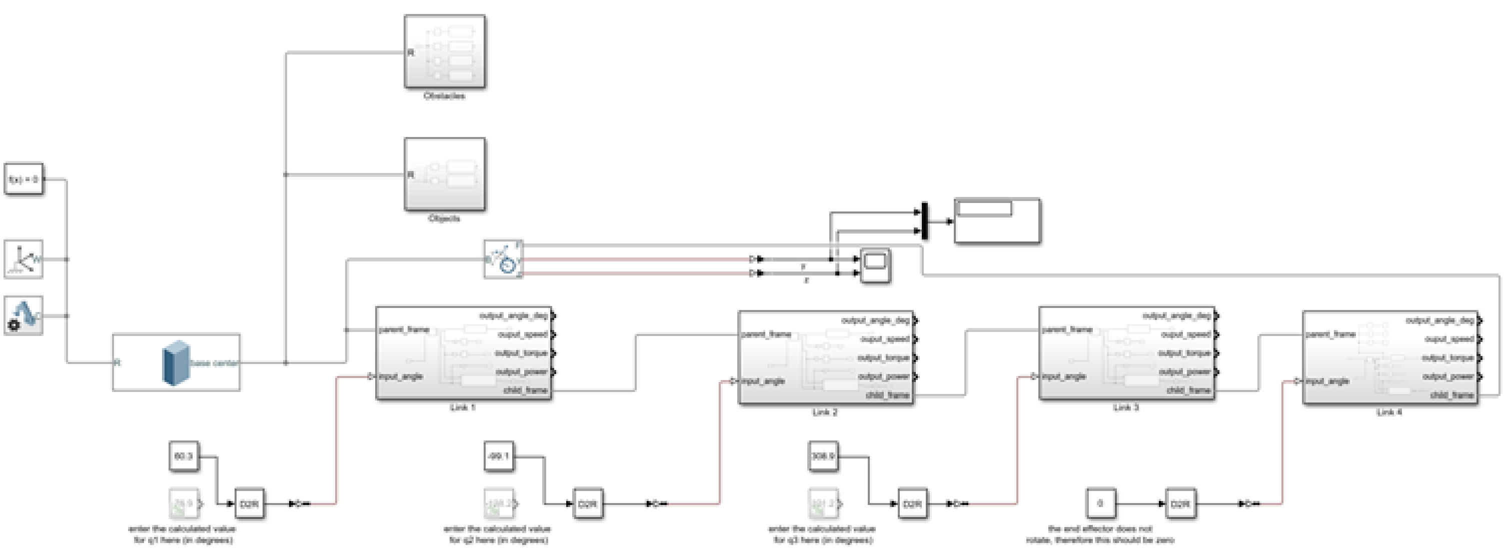
Below is a visual showing the work space that avoids the obstacles and reaches location 1



Question 4

See gripper_arm.slx





Question 5 (Step 3 Repeated)

See gripper_arm.slx

$obj2 = [82.5 \quad 5 \quad 0]$

① $80c_1 + 55c_1c_2 + 55c_1c_2 + 30c_3\sigma_2 + 30s_3\sigma_3 = 82.5$

② $80s_1 + 55c_1s_2 + 55c_2s_1 + 30c_3\sigma_2 + 30s_3\sigma_3 = 5$ cm

③ $q_1 + q_2 + q_3 = 270°$

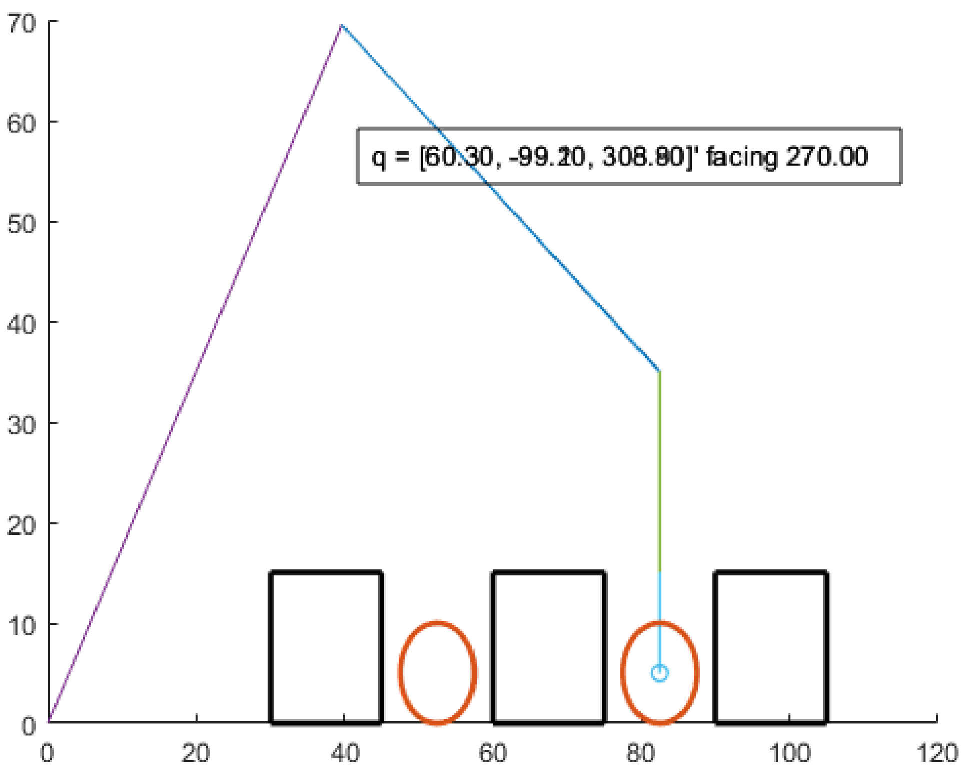
Using same variables as Q3

With 3 equations that are independent and 3 variables, we know that we can solve for q_1, q_2, and q_3.

Using Matlab, we know one solution is:

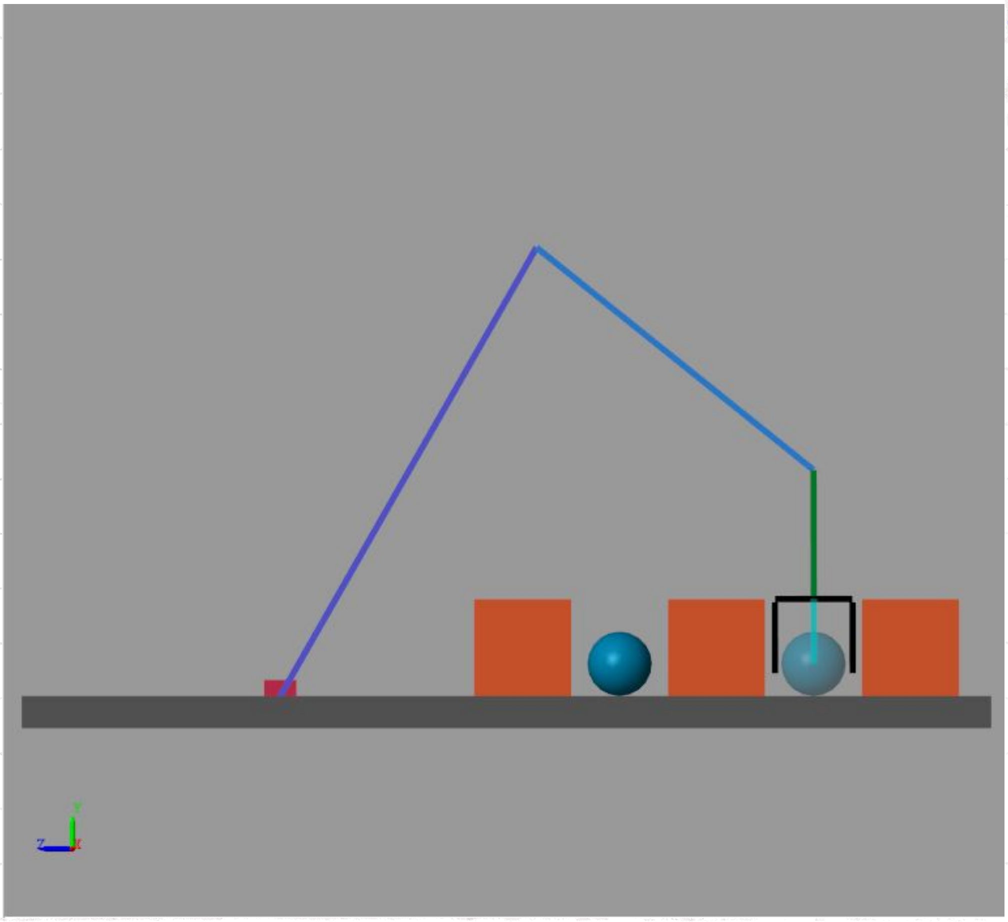
$$\boxed{q_1 = 60.3° \qquad q_2 = -99.1° \qquad q_3 = 308.9°}$$

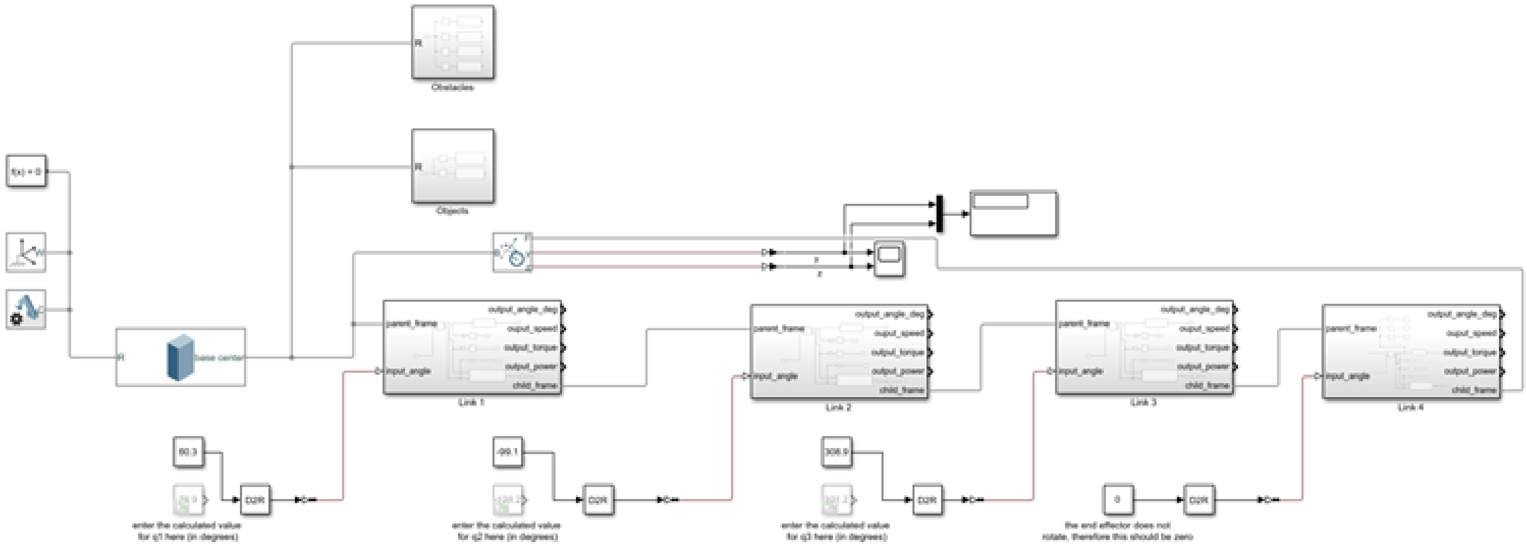
Below is the visual again that shows the obstacles being avoided.



Question 5 (step 4 repeated)

See gripper-arm.slx





See the following scripts:

trajectory planning / traj-design_posX.mlx
traj-design_posY.mlx
traj-design_speeds.mlx
traj-design_joint_angles.mlx

Question 6.

End Effector Waypoints	Joint Waypoints
$P(0) = [52.5 \quad 5 \quad 0]$	$q = [76.4,\ -128.2,\ 321.2]'$
$P(1) = [52.5 \quad 30 \quad 0]$ @270° (vert. down)	$q = [89.1,\ -110.5,\ 291.40]'$
$P(2) = [82.5 \quad 25 \quad 0]$ @275°	$q = [69.1,\ -90.3,\ 296.1]'$
T=3 $P(3) = [82.5 \quad 5 \quad 0]$	$q = [60.3°,\ -99.1°,\ 308.9°]'$

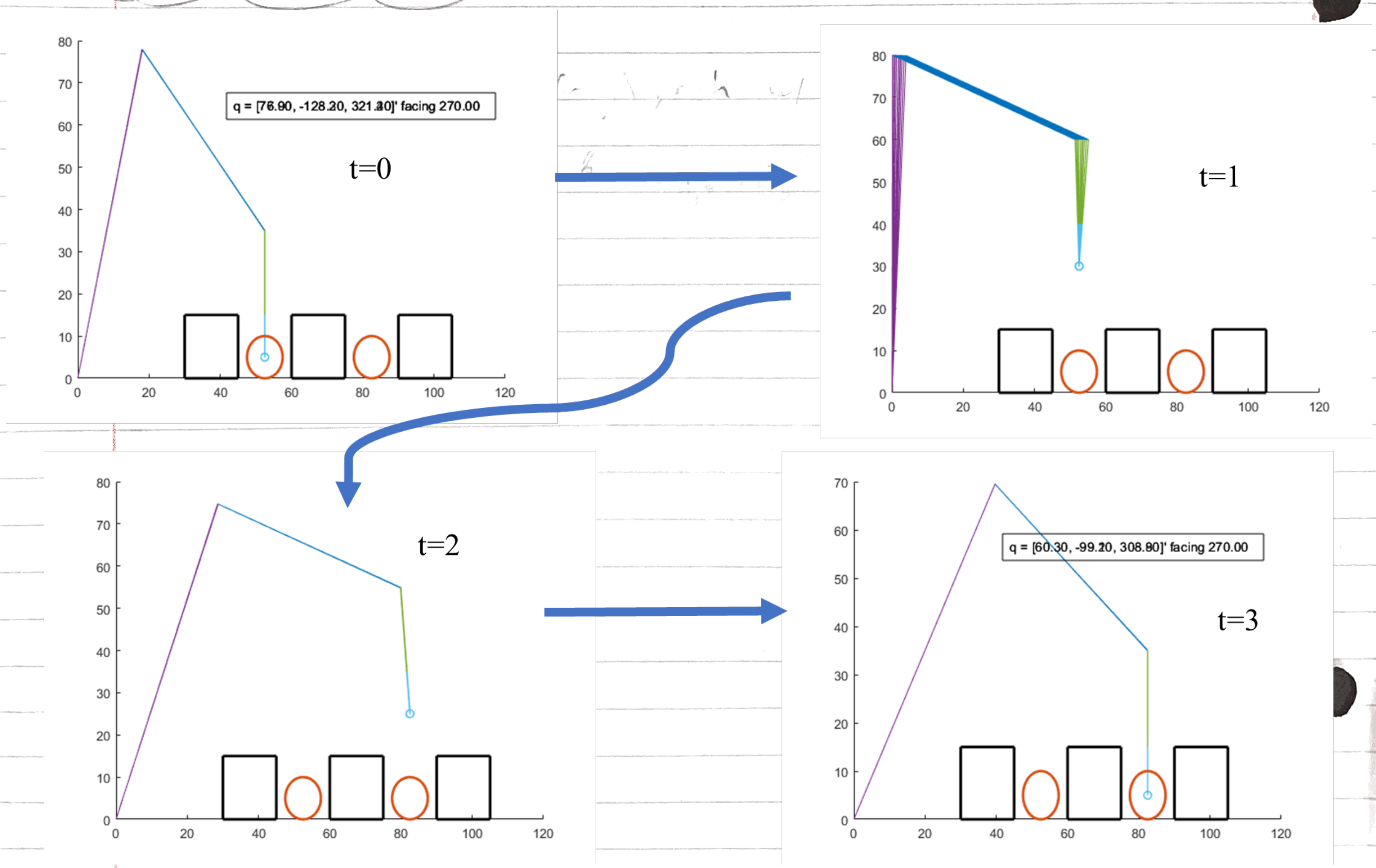
$\dot{P}(0) = 0 \quad \ddot{P}(0) = 0$

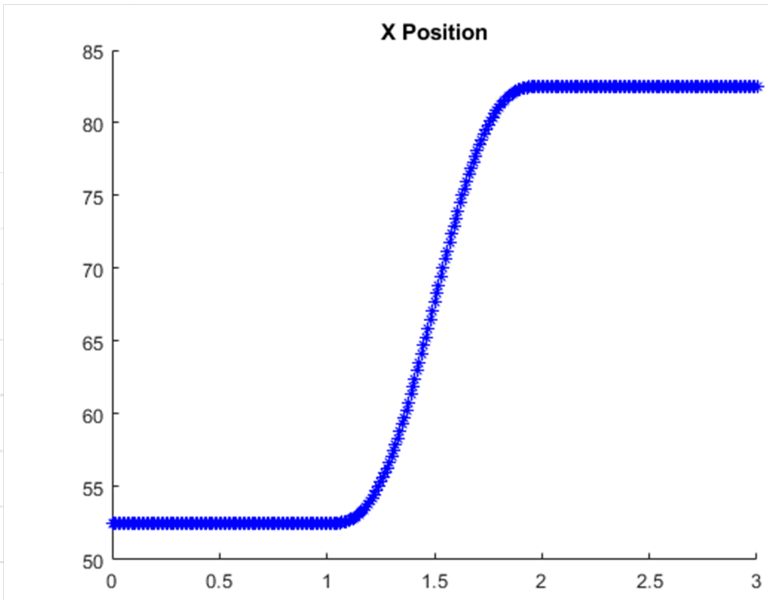
$\dot{P}(T_3) = 0 \quad \ddot{P}(T_3) = 0$

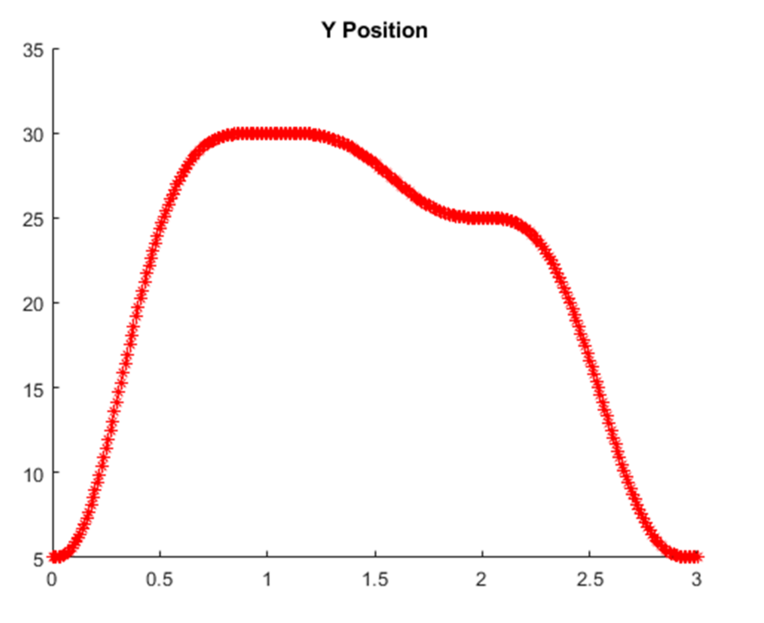
$\dot{P}(T) = 0 \quad \ddot{P}(T) = 0$

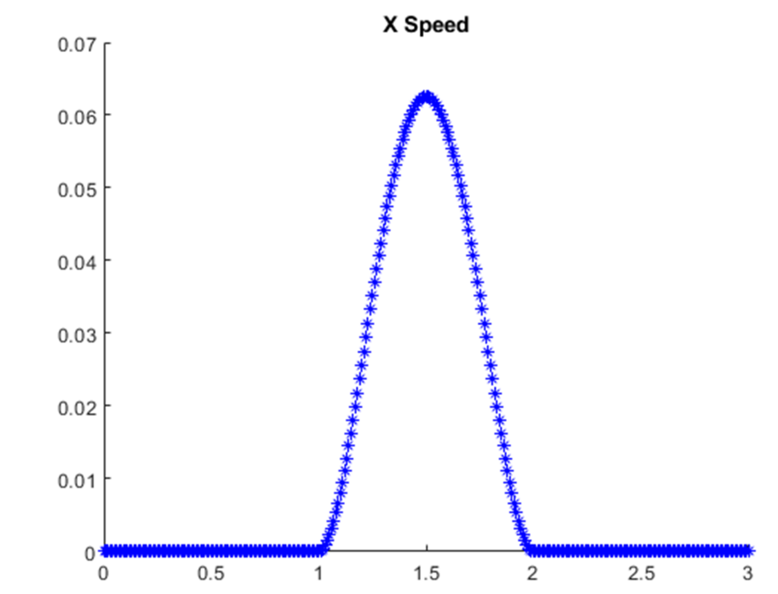
$P_x(0) = 52.5 \quad P_x(1) = 52.5 \quad P_x(2) = 82.5 \quad P_x(3) = 82.5$

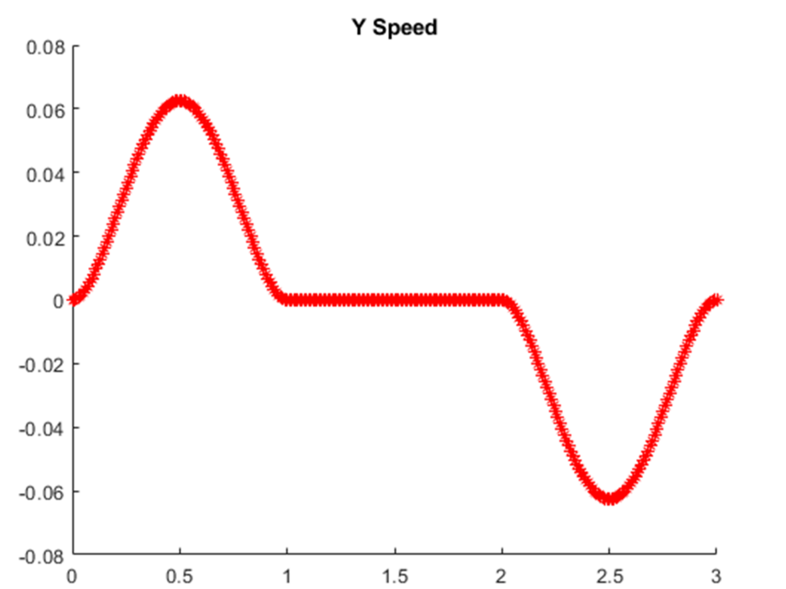
$P_y(0) = 5 \quad P_y(1) = 30 \quad P_y(2) = 25 \quad P_y(3) = 5$





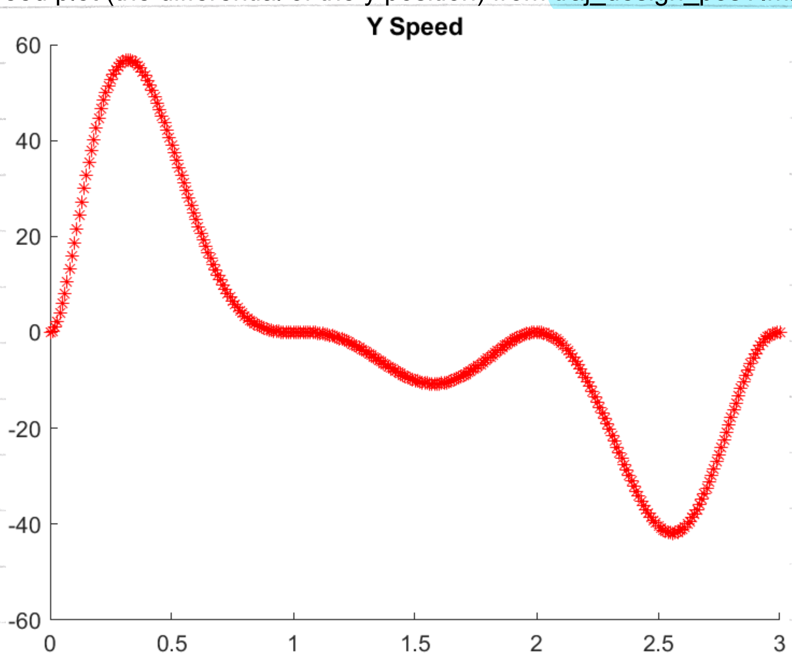






Note that these speed plots show a general plan that was laid out earlier in the project with traj_design_speeds.mlx

Below is the true speed plot (the differential of the y position) from traj_design_posY.mlx



Question 7

See the traj-design_joint_angles.mlx

Below are the fully calculated joint angle trajectories

$$P_{q_1}(t) = \begin{cases} u^6 + \frac{351}{5}u^5 - 180u^4 + 121u^3 + \frac{769}{10} & 0 \le t < 1 \\ u^6 - 129u^5 + 933u^4 - 2663u^3 + 3666u^2 - 2436u + 7171/10 & 1 \le t \le 2 \\ u^6 - \frac{339}{5}u^5 + 753u^4 - 3561u^3 + 8478u^2 - 10044u + 47907 & 2 < t < 3 \end{cases}$$

$$P_{q_2}(t) = \begin{cases} u^6 + \frac{516}{5}u^5 - \frac{525}{2}u^4 + 176u^3 - \frac{641}{5} & 0 \le t < 1 \\ u^6 + \frac{561}{5}u^5 - 876u^4 + 2563u^3 - 3570u^2 \\ \quad 2386u - \frac{7287}{10} & 1 \le t \le 2 \\ u^6 - \frac{339}{5}u^5 + 783u^4 - 3561u^3 + 8478u^2 - 10044u + 46313/10 & 2 < t < 3 \end{cases}$$

$$P_{q_3}(t) = \begin{cases} u^6 - \frac{909}{5}u^5 + 450u^4 - 299u^3 + \frac{1606}{5} & 0 \le t < 1 \\ u^6 + \frac{96}{5}u^5 - 357u^4 + 548u^3 - 780u^2 \\ \quad 528u + \frac{1537}{10} & 1 \le t \le 2 \\ u^6 + \frac{304}{5} - 867u^4 + 4431u^3 - 10962u^2 \\ \quad 13284u - 12083/2 & 2 < t < 3 \end{cases}$$

These solve for the following waypoints/constraints (shown in Q6) (position waypoints obtained w/ planararm_workspace.m)

$P_q(0) = [76.9^\circ \ \ -128.2^\circ \ \ 321.2^\circ]'$ $\quad \dot{P}_q(0) = 0$ $\quad \ddot{P}_q(0) = 0$

$P_q(1) = [89.1^\circ \ \ -110.5^\circ \ \ 291.4^\circ]'$ $\quad \dot{P}_q(1) = 0$ $\quad \ddot{P}_q(1) = 0$

$P_q(2) = [69.1^\circ \ \ -90.3^\circ \ \ 296.1^\circ]'$ $\quad \dot{P}_q(2) = 0$ $\quad \ddot{P}_q(2) = 0$

$P_q(3) = [60.3^\circ \ \ -99.1^\circ \ \ 308.9^\circ]'$ $\quad \dot{P}_q(3) = 0$ $\quad \ddot{P}_q(3) = 0$

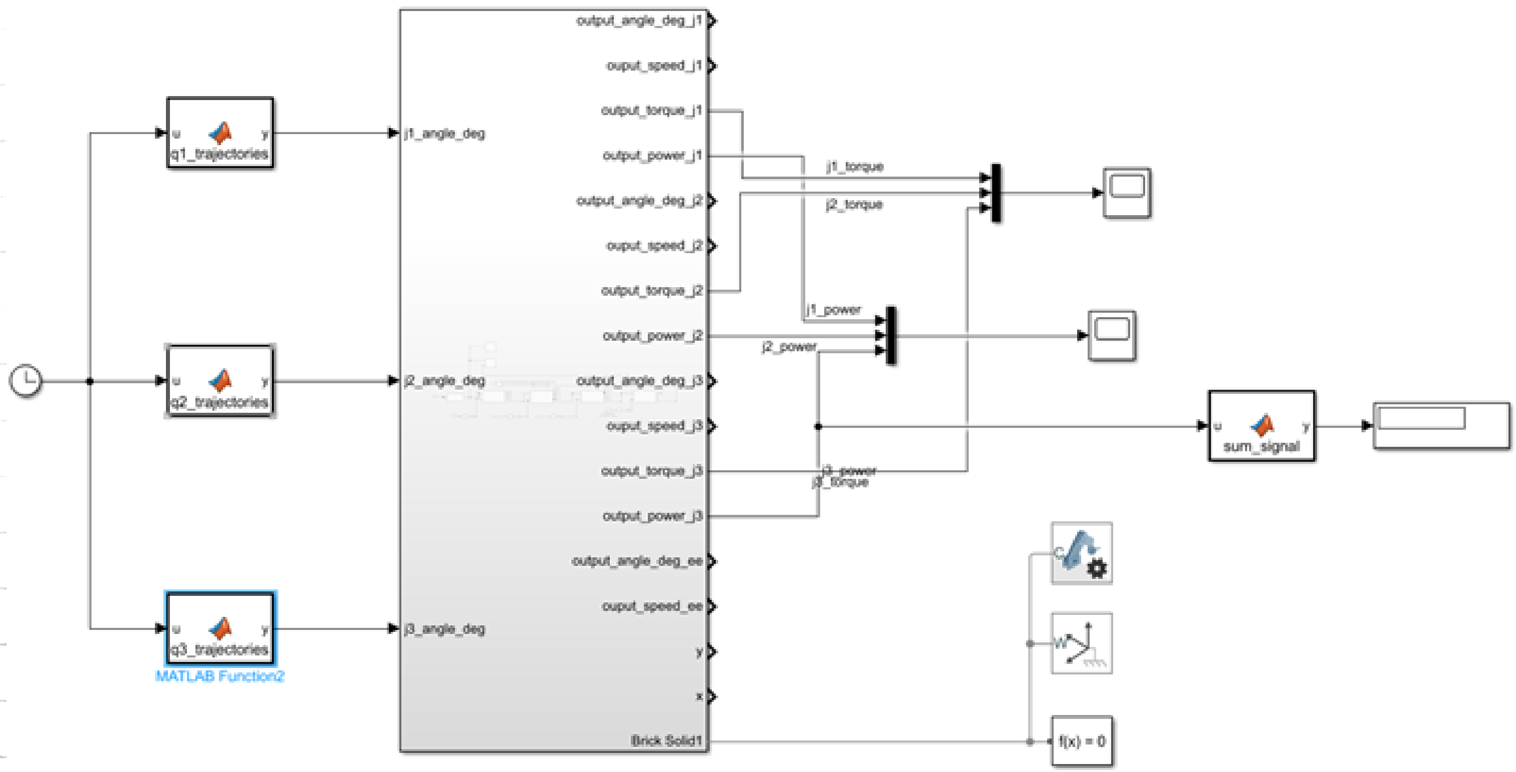
```matlab
function y = q1_trajectories(u)
% Question 7-8: below are the calcualted trajectories for q1. Based on the following
% waypoints for the angular position of q1:
% P1(t=0) = 76.9
% P1(t=1) = 89.1
% P1(t=2) = 69.1
% P1(t=3) = 60.3
% at t's 0,1,2, and 3, the joint should at be at rest and not accelerating
% input u is time
p1 = u^6 + (351/5)*u^5 - (180)*u^4 + (121)*u^3 + (769/10); % u=[0,1)
p2 = u^6 - (129)*u^5 + (933)*u^4 - (2663)*u^3 + (3666)*u^2 - (2436)*u + (7171/10); % u=[1,2)
p3 = u^6 - (339/5)*u^5 + (753)*u^4 - (3561)*u^3 + (8478)*u^2 - (10044)*u + (47907/10); % u=[2,3)

if (0 <= u) && (u < 1)
    y = p1;
elseif (1 <= u) && (u <= 2)
    y = p2;
elseif (2 < u) && (u < 3)
    y = p3;
else
    y = 0;
end
```

```matlab
function y = q2_trajectories(u)
% Question 7-8: below are the calcualted trajectories for q1. Based on the following
% waypoints for the angular position of q2:
% P1(t=0) = -128.2
% P1(t=1) = -110.5
% P1(t=2) = -90.3
% P1(t=3) = -99.
% at t's 0,1,2, and 3, the joint should at be at rest and not accelerating
p1 = u^6 + (516/5)*u^5 - (525/2)*u^4 + (176)*u^3 - (641/5); % u=[0,1)
p2 = u^6 + (561/5)*u^5 - (876)*u^4 + (2563)*u^3 - (3570)*u^2 + (2388)*u - (7287/10); % u=[1,2)
p3 = u^6 - (339/5)*u^5 + (753)*u^4 - (3561)*u^3 + (8478)*u^2 - (10044)*u + (46313/10); % u=[2,3)

if (0 <= u) && (u < 1)
    y = p1;
elseif (1 <= u) && (u <= 2)
    y = p2;
elseif (2 < u) && (u < 3)
    y = p3;
else
    y = 0;
end
```

```matlab
function y = q3_trajectories(u)
% Question 7-8: below are the calcualted trajectories for q1. Based on the following
% waypoints for the angular position of q2:
% P1(t=0) = 321.2
% P1(t=1) = 291.4
% P1(t=2) = 296.1
% P1(t=3) = 308.9
% at t's 0,1,2, and 3, the joint should at be at rest and not accelerating
p1 = u^6 - (909/5)*u^5 + (450)*u^4 - (299)*u^3 + (1606/5); % u=[0,1)
p2 = u^6 + (96/5)*u^5 - (357/2)*u^4 + (548)*u^3 - (780)*u^2 + (528)*u + (1537/10); % u=[1,2)
p3 = u^6 + (309/5)*u^5 - (867)*u^4 + (4431)*u^3 - (10962)*u^2 + (13284)*u - (12083/2); % u=[2,3)

if (0 <= u) && (u < 1)
    y = p1;
elseif (1 <= u) && (u <= 2)
    y = p2;
elseif (2 < u) && (u < 3)
    y = p3;
else
    y = 0;
end
```

Question 8

See gripper_arm_traj_ctrl.slx



Question 9

See gripper-arm-traj-ctrl.slx

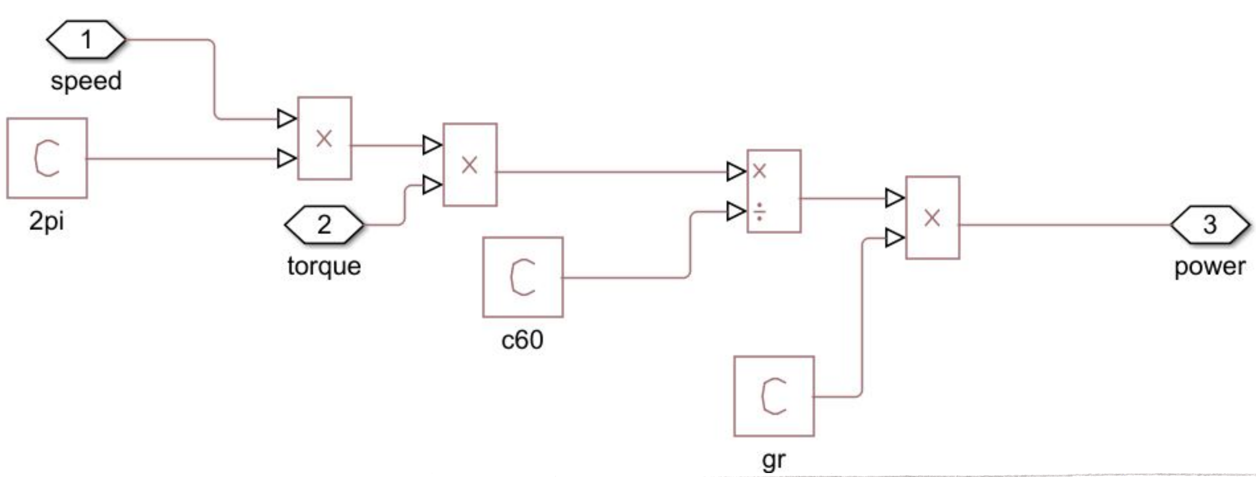
Signal Statistics   Density = 2700 kg/m^3 for all links

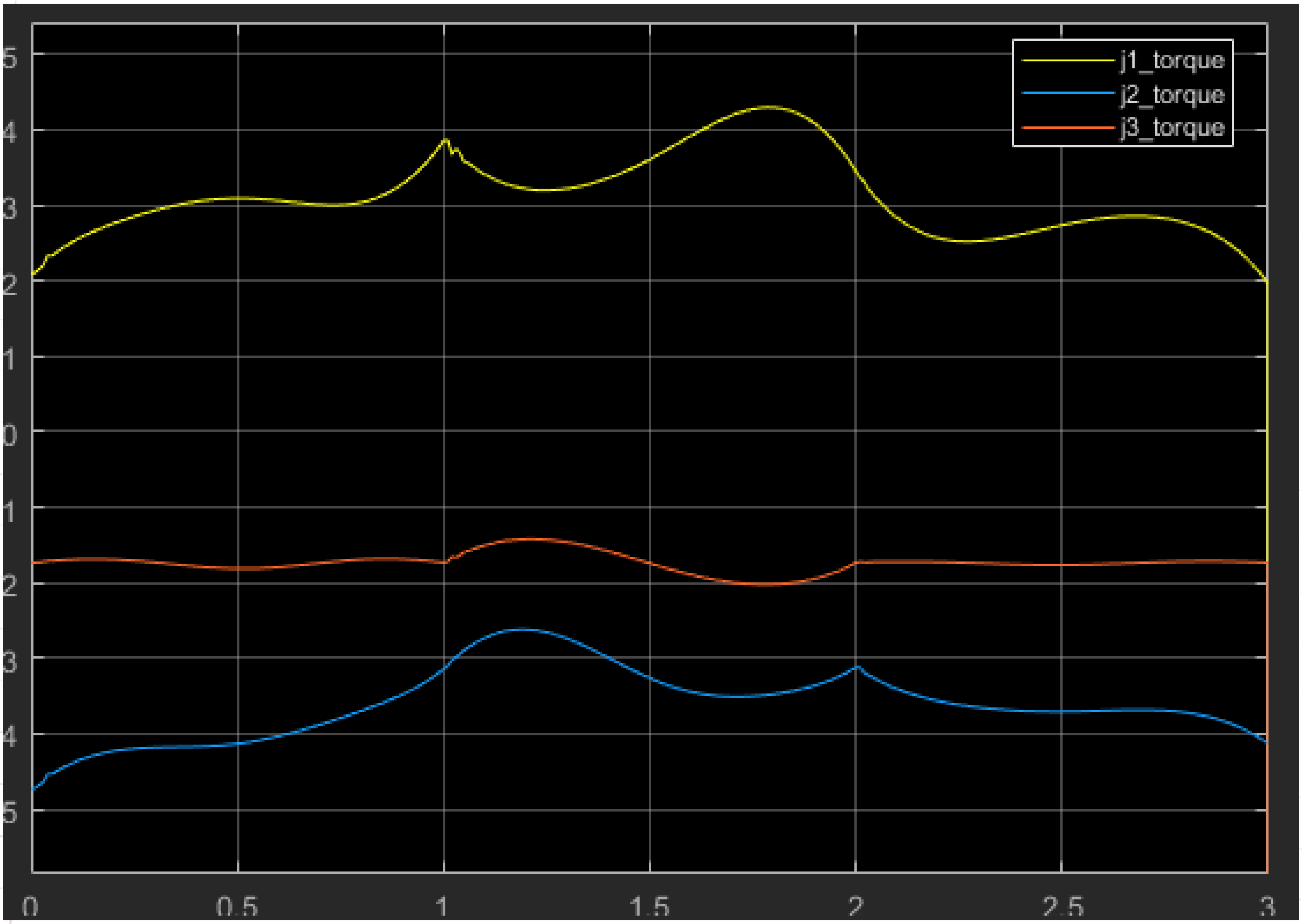
Joint 1: Min = -5.061 W   Total Power: 201.5 W over 3 secs
Max = 2.583 W

Joint 2: Min = -5.008   Total Power = -436.5 W over 3 sec
Max = 2.232

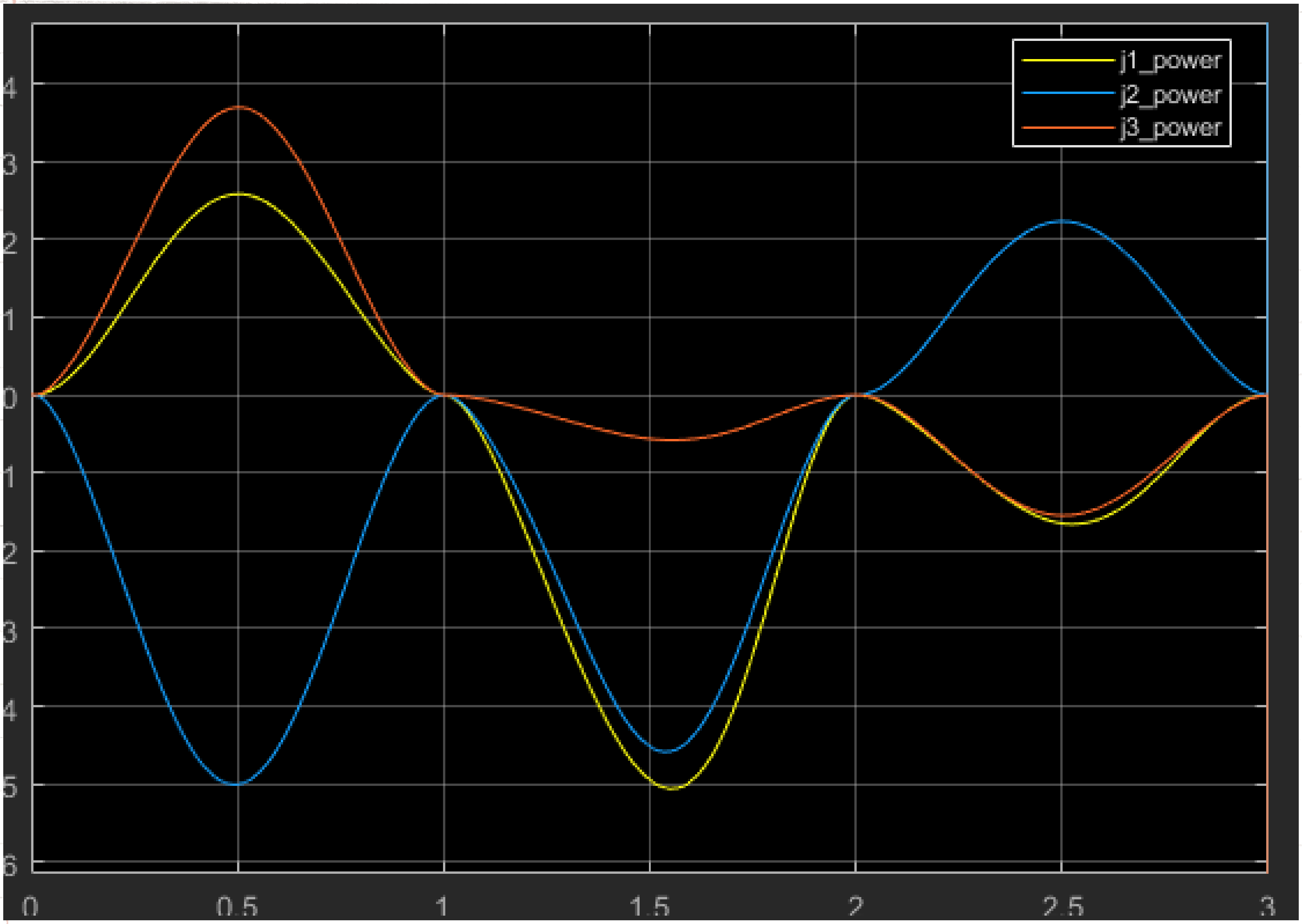
Joint 3: Min = -1.547   Total Power: 851.7 W over 3 sec
Max = 3.698

The power statistics show interesting features of the trajectory. Firstly, Joint 2 had the least amount of power needing to be exerted, because most of the time it was being held downwards so gravity was aiding it's acceleration/forces. Joint 3 had the shortest moment arm, which is likely why it exerted the most power, and why it had the largest peak.





j1_torque
j2_torque
j3_torque
5
4
3
2
1
0
1
2
3
4
5
0
0.5
1
1.5
2
2.5
3



j1_power
j2_power
j3_power
4
3
2
1
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3

Question 12

See gripper_arm_traj_ctrl.slx and Figures/gripper_arm_traj_ctrl.mp4 for a video of the Simscape model with the obstacles, fixed table, and objects for improved visualization