Assignment4

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♠Q22 (1) When the input state to the quantum circuit represented by (110) is $|11\rangle$, the results for a step-by-step calculation are as follows.

$$|\Psi_{\mathrm{in}}\rangle = |11\rangle \tag{1}$$

$$|\Psi_{\mathrm{mid}}\rangle = \left(\hat{H}|1\rangle\right) \otimes |1\rangle \tag{2}$$

$$|\Psi_{\mathrm{out}}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \tag{3}$$

(2) Applying the CNOT gate to a single state $|S\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ changes it to

$$\frac{|01\rangle - |11\rangle}{\sqrt{2}}. \tag{4}$$

This can be rewritten as a separatable state

$$\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes |1\rangle \tag{5}$$

and proved to be not entangled.

(3) For example, consider the superposition of $|\Phi_+\rangle$ and $|\Psi_+\rangle$. If c_0 and c_1 are complex, the superposition of $|\Phi_+\rangle$ and $|\Psi_+\rangle$ can be written as

$$c_0|\Phi_+\rangle + c_1|\Psi_+\rangle. \tag{6}$$

If $c_0 = c_1$, then the above equation becomes

$$\begin{aligned} c_0(|\Phi_+\rangle + |\Psi_+\rangle) &= c_0\left(\frac{|00\rangle + |11\rangle + |01\rangle + |10\rangle}{\sqrt{2}}\right) \\ &= c_0\left(\frac{|00\rangle + |10\rangle + |01\rangle + |11\rangle}{\sqrt{2}}\right) \\ &= c_0\left(\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle + \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |1\rangle\right) \\ &= c_0\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes (|0\rangle + |1\rangle), \end{aligned} \tag{7}$$

which is the not entangled state. Thus, it is shown that even superposition of entangled states may not result in an entangled state.

♠Q23 (1) When the initial state of qubit A is $|\psi\rangle + c_0|0\rangle + c_1|1\rangle$, the three-qubit state before the process denoted by E is

$$c_0|001\rangle + c_1|110\rangle. \tag{8}$$

(2) When the three-qubit state is preserved by the proccess E, the final state of qubit A is $c_0|0\rangle + c_1|1\rangle$.

(3) the final three-qubit state : $c_0|110\rangle + c_1|010\rangle$
the final qubit A state : $c_0|1\rangle + c_1|0\rangle$

(4) the final three-qubit state : $c_0|011\rangle + c_1|011\rangle$
the final qubit A state : $c_0|0\rangle + c_1|0\rangle$

(5) the final three-qubit state : $c_0|000\rangle + c_1|100\rangle$
the final qubit A state : $c_0|0\rangle + c_1|1\rangle$

(6) the final three-qubit state : $c_0|010\rangle + c_1|110\rangle$
the final qubit A state : $c_0|0\rangle + c_1|1\rangle$

♠Q24