Regulation, Capital, & Margin: Quant Angle - IV

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Part IV: Various Topics on IMM

Backtesting - (1)

- We discussed model choice and drift (= probability measure) in the previous lecture, how about variances (and co-variances)?
- In theory, these are measure invariant and can often be constructed without ambiguity from observable option prices.
- In practice, however, this is *not* what regulators want. Instead, they want the choice of variances and co-variances to be based on historical data.
- Mathematicians tend to be driven up a wall about this...
- Determining whether a model is suitable for IMM is done through *backtesting* procedures.
- In a nutshell, backtesting is the process of testing whether forecasts done by a stochastic model match realized values.

Backtesting - (2)

- For instance, for some risk factor X and some time grid $\{t_i\}$ we can consider the collection of novations $\epsilon_{ij} = X(t_i + \Delta_j) - X(t_i)$ for $i, j = 1, 2, \ldots$, where Δ_j are time horizons.
- The collection of $\epsilon_{\cdot j}$ for fixed j forms an empirical distribution that can be compared by the distribution generated by a parametric model.
- There is quite a bit of language around backtesting requirements in the Basel rules, as well as a dedicated BIS publication exclusively dealing with backtesting guidance ("Sound Practices for Backtesting Counterparty Credit Risk Models," *Basel Committee on Banking Supervision*, December 2010).
- Testing should be done at both the level of individual risk factors (e.g., an FX rate) and over relevant aggregations, most notably through "representative portfolios".

Backtesting - (3)

- Per regulatory guidance, backtesting must:
  - Be done regularly as part of ongoing model performance measurement, and as part of ongoing model validation
  - Be subject to governance, especially w.r.t. to remediation of exceptions.
  - Test not only the potential exposure percentile but the whole distribution (e.g., 1%, 5%, 25%, 50%, 75%, 95%, 99%)
  - Test a representative sample of time horizons (Δ_j), incl. 1 year or more
  - Test correlations as well as volatilities
  - Be representative of the bank's exposure
  - Monitor not only frequency of exceptions but also severity
  - Consider materiality of exceptions
  - Be designed with statistical significance in mind
  - Be based on historical calibrations using > 3 years of data (Basel 3).

Backtesting - (4)

- The complex (and thankless) task of calibrating and statistically backtesting models is normally handled by the counterparty credit risk team, rather than by quants. The models in question have many 100s, if not 1,000s of risk factors.
- Often the statistical framework relies on a "cascade" of tests, starting with BIS "traffic light" test.
- Details are very much bank specific, so let us just describe the traffic light test, the only concrete test that regulators have put forth.
- Assume that our model predicts that ϵ falling outside some range H takes place with probability q:

$$P(\epsilon \notin H) = q.$$

Backtesting - (5)

- In a historical data series with n realizations, we can count the random number of times N that ϵ is outside H ("exceptions")
- IF the model is *correct*, the distribution of N is (under suitable assumptions) binomial:

$$P(N = x) = \left(\begin{array}{c} x \\ n \end{array} \right) q^x (1-q)^{n-x} \triangleq B(q, x).$$

- Suppose we use a cut-off of x_c as a rejection of the model, in the sense that we will discard the model if $N \geq x_c$.
- The probability of rejecting a correct model (a type I error) is

$$p_I = \sum_{x \geq x_c} B(q, x).$$

Backtesting - (6)

- IF the model is wrong, in the sense that really $P(\epsilon \notin H) = w \neq q$, the probability of erroneously accepting a wrong model (a type II error) is

$$p_{II} = \sum_{x<x_c} B(w, x).$$

- Based on tables with various values of q and w, BIS has come up with zones of outcomes for N that it considers green (all OK), yellow (could be a problem), and red (no good).
- The BIS approach is quite simplistic, and really only done for low values of q (1 %).
- One would often supplement the traffic light tests with more sophisticated tests (Kupiec's POF test, Jarque-Bera, Pearson Q, etc).

BofA Models and Simulation

- Like most banks, BofA's capital systems were built around counterparty credit risk systems.
- For IMM purposes, the models here are generally too simplistic, so a convergence towards the CVA system has taken place.
- Large parts of our CVA and IMM systems are merged, with a branch on the model configuration. CVA: market calibration (quants). IMM: statistical backtesting (quants, CCRA).
- The resulting engine is effectively a large "what-if" machinery, and in principle could handle VaR and IM (through additional configurations).
- Target state for most banks.

IMM Carveouts - (1)

- For a variety of reasons, it is likely that some trades in a netting set will not qualify for IMM treatment.
- This can happen if there are no (or inadequate, in the sense of too high pricing errors) pricing models implemented on the IMM platform.
- This, in turn, might be the case if an exotic security pricer is too slow to embed in the IMM simulation loop.
- Note: exotic securities can often be priced efficiently using regression-based methods, but these can take a while to develop, test, and validate – validation tends to be product specific.
- In this case, it becomes necessary to split the portfolio in two pieces: one piece (V_1) that is IMM compliant, and one piece (V_2) that is not.

IMM Carveouts - (2)

- V_1 will attract credit capital according to Basel 2, and V_2 will attract credit capital according to Basel I (CEM). Total credit capital is the sum of the two contributions.
- We notice that EE is always sub-additive when a portfolio and its collateral is split:

$$
\begin{aligned}
EE(t) = E\left((V(t) - C(t))^+\right) &= E\left((V_1(t) + V_2(t) - C_1(t) - C_2(t))^+\right) \\
&\leq E\left((V_1(t) - C_1(t))^+\right) + E\left((V_2(t) - C_2(t))^+\right),
\end{aligned}
$$

if $C(t) = C_1(t) + C_2(t)$ and $V(t) = V_1(t) + V_2(t)$.

- This is easily seen to carry through to EEPE and therefore to EAD. So *EAD is sub-additive*.

IMM Carveouts - (3)

- Note: when carving out trades, a concrete mechanism is required for splitting the collateral. Normally a heuristic is needed, as there is no unique way to do this.
- Is regulatory capital RC sub-additive? For this, we recall that

$$RC = EAD \cdot f(l,p) \cdot k(M,p) \cdot 1.06$$

  with $f(l,p)$ being a prescribed function of LGD (l) and PD (p), and $k(M,p)$ being a prescribed “transition risk” function of effective maturity M and PD.
- M depends on EE, so we must write, for the split portfolio,

$$RC_1 + RC_2 = f(l,p) \cdot \left(EAD_1 \cdot k(M_1,p) + EAD_2 \cdot k(M_2,p)\right),$$

  where we know that $EAD_1 + EAD_2 \geq EAD$.

IMM Carveouts - (4)

- Unfortunately, a careful analysis of the "black-box" function k reveals that it has a design-flaw: it can depend on exposure profiles in such a way that sometimes $RC_1 + RC_2 \leq RC$. Basically a consequence of the cap that sits in the definition of M.
- So regulatory capital is not always sub-additive: splitting the portfolio can result in less capital.
- While this might one doubt the coherence of the formulas, in practice strict subadditivity virtually never happens.
- Moreover, since the non-IMM portfolio gets subject to Basel 1 – which is typically much more "expensive" than Basel 2 – splitting a netting set involves a pretty severe capital cost.
- In Basel 3 this penalty gets extremely high, as we shall see later.

Margin Loans - (1)

- Securities that are subject to margin requirements are currently in a little bit of a vacuum when it comes to regulatory capital.
- One relevant business is margin lending, as executed in, say, the prime brokerage business. In the future most non-cleared securities will require margin posting. More about this later.
- In margin lending, a client holds a portfolio of cash and securities with value $\pi(t)$. Not all of this portfolio has been financed by the client; a certain amount of its value, $D(t)$, has been lent to the client by the bank.
- Writing $\pi(t) = D(t) + E(t)$, the quantity $E(t)$ is the client's *equity*.
- The lending bank can use the entire portfolio $\pi(t)$ as collateral for its loan $D(t)$ if the client defaults. As long as $E(t) > 0$ the position is *over-collateralized*.

Margin Loans - (2)

- To protect itself against the client not repaying the debt amount $D(t)$, the lending bank has a policy where it will issue a margin call if the riskiness of the position is too high.
- The call will require the client to top up the equity position with cash or eligible securities, to some level $E_{\min}(t)$.
- Often, $E_{\min}(t)$ is set by VaR methods. Specifically, if we assume that it will take a period of Δ to liquidate the portfolio after a client default, we might want, for some small number q,

$$P\left(\pi(t+\Delta) < D(t)\right) < q,$$

or

$$P\left(X(t) > 0\right) < q, \quad X(t) = \pi(t) - \pi(t+\Delta) - E_{\min}(t)$$

Margin Loans - (3)

- That is, the probability of the portfolio deterioration exceeding the equity over the liquidation horizon is very small.
- Often q is minuscule – much less than 1%. Besides VaR protection, many margin policies add protection (through add-ons to VaR) against downgrades, concentration risk, liquidity issues, etc.
- How do we define exposure for a margined portfolio?
- If default takes place at time τ, first assume (worst-case) that the client has no excess equity beyond the margin level $E_{\min}$.
- The loss to the lending bank associated with liquidation is

$$L(\tau) = X(\tau)^+.$$

- So, the expected exposure is $EE(t) = \mathrm{E}\left(X(t)^+\right)$.

Margin Loans - (4)

- Since margined portfolios tend to be quite dynamic with active trading and frequent margin calls, it is a challenge to predict what $\pi(t)$ and $E_{\text{min}}(t)$ will look like at time t. Some simplification is needed.
- One (overly?) sophisticated method would involve using kernel regression to estimate conditional moments of the portfolio, and then to approximate the computation of VaR and, subsequently, $E_{min}(t)$.
- This calculation would assume that the current portfolio is "static' 'and no new trades would ever be added. Often this is unreasonable – even though it is similar to regular IMM assumptions.
- A better assumption for, say, prime brokerage is that the margin policy aims to keep the tail distribution of the loss-variable X close to constant over time, in the sense that the VaR stays relatively fixed.

Margin Loans - (5)

- Equivalently, assume that the current ($t = 0$) portfolio composition and margin is a "representative" position.
- With this assumption, we write for all t,

$$EE(t) = \mathrm{E}\left(X(0)^{+}\right) = \mathrm{E}\left(\left(K - \pi(\Delta)\right)^{+}\right), \quad K = \pi(0) - E_{\mathrm{min}}(0). \tag{1}$$

- So, to compute the entire EE profile (and thereby EEPE and EAD), we "just" need to price a put on $\pi(\Delta)$.
- One complication here is that K is very small due to the overcollateralization feature, so Monte Carlo simulation is difficult to make operational without lots of tricks (importance sampling).
- For speed and clarity, one can use delta-gamma approximations, coupled with a Gaussian distribution assumption for the risk factors behind π. Justified here due to the short time-horizon Δ.

Margin Loans - (6)

- Specifically, we write

$$\pi(\Delta) = \pi(0) + D^\top \epsilon + \frac{1}{2}\epsilon^\top \Gamma \epsilon$$

  where $\epsilon \sim N(0, C)$ for some covariance matrix C.
- After a few rotations, it is possible to write the characteristic function for $\pi(\Delta)$ in closed form.
- The expectation in (1) can then be written as a Fourier integral.
- Due to the extreme OTM behavior of the integral, in practice we rely on saddlepoint techniques.
- The saddlepoint techniques can be extended to provide efficient hedge and attribution analysis, which is very convenient in practice. (Joint work with J. Kim, 2012).

Margin Loans - (6)

- It should be clear that capital requirements for margin business can be very low. This might be controversial, and some elements of the margin portfolios may, for this reason, stay with Basel I through carve-outs.
- On the other hand, regulators are generally skeptical about cherry-picking of accords, so we shall see.