Regulation, Capital, and Margin: Hints on Assignment

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Hints on Assignment

Swap Pricing - (1)

- Given a plain vanilla swap that pays on a schedule $\{T_i\}_{i=1}^N$ (say, quarterly or semi-annually). Payer swap: fixed coupon paid, floating coupon received. Receiver swap: vice-versa.
- At time T_i, the fixed payment is $c(T_i - T_{i-1})$, where c is the coupon rate.
- At time T_i, the floating payment is $L(T_{i-1})(T_i - T_{i-1})$ where L is the floating rate (set in arrears)
- Let $P_{OIS}(t, T)$ be the OIS zero-coupon bond price observed at time t, for \$1 delivery at time T.
- Assume that L sets off a discount curve $P_L(t, T)$.
- Then $L(T_{i-1}) = \left(P_L(T_{i-1}, T_i)^{-1} - 1\right) / (T_i - T_{i-1})$

Swap Pricing - (2)

- For a payer swap, the value at time t is simply

$$V(t) = \sum_{T_i > t} P_{OIS}(t, T_i) \cdot (T_i - T_{i-1}) \left(\mathbb{E}_t^{T_i}(L(T_{i-1}) - c) \right) \quad (1)$$

- Here $\mathbb{E}_t^{T_i}(L(T_{i-1}) = L(T_{i-1})$ for the first (past set) stub where $T_{i-1} < t$, and

$$\mathbb{E}_t^{T_i}(L(T_{i-1}) = \left(\frac{P_L(t, T_{i-1})}{P_L(t, T_i)} - 1 \right) (T_i - T_{i-1})^{-1}, \quad t \leq T_{i-1}. \quad (2)$$

- In the assignment, we have a simple relationship:

$$P_L(t, T) = P_{OIS}(t, T) e^{-0.5\%(T-t)}.$$

Swap Simulation - (1)

- Note that (1) and (2) then expresses the swap value only as a function of OIS discount factors $P_{OIS}(\cdot,\cdot)$. Let us simplify and write $P_{OIS}(t,T) = P(t,T)$.
- If $P(t,T) = P(t, x(t), T)$ where $x(t)$ is a vector stochastic drivers, then clearly

$$V(t) = V(t, x(t)),$$

  and simulation of $V(t)$ is easy.
- The specific form for $P(t, x(t), T)$ we want to use is derived in your background reading material. It is written as

$$P(t, x(t), T) = e^{-\int_t^T f(t, x(t), u)\, du}$$

  where

$$f(t, x(t), u) = f(0, u) + M(t, u)^\top \left(x(t) + y(t) G(t, u)\right).$$

Swap Simulation - (2)

- Here $f(0,u) = -d \ln P(0,u)/du$ (in your assignment, $f(0,u)$ is actually constant), and $M(t,u)$, $G(t,u)$, $y(t)$ are matrix- or vector-valued deterministic functions that are given in your reading material.
- You can easily find $P(t,T)$ as a function of $x(t)$ by analytical integration – do this. Hint: expression should be something like

$$P(t, x(t), T) = \frac{P(0,T)}{P(0,t)} \exp\left(a(t,T) - b(t,T)^\top x(t) \right). \quad (3)$$

- Note: for the time-homogenous case in the assignment, M, G, y can easily be calculated in closed form by evaluating some easy integrals. Same for a and b in (3). Do this, and show your work.

Swap Simulation - (3)

- Also, the process for $x(t) = (x_1(t), x_2(t))^\top$ is Gaussian:

$$dx(t) = (y(t)\mathbf{1} - \kappa x(t))\ dt + \sigma_x^{*\top}\ dW^*(t) \tag{4}$$

where

$$\kappa = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \sigma_x^* = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}.$$

and $W^*(t)$ is a 2-D BM with correlation ρ_x.

- The background paper ("Practical Considerations") show how to relate $\sigma_1, \sigma_2, \rho_x, \kappa_1, \kappa_2$ to observable quantities (such as the volatility of the short rate r and the correlation between short and long rates). These observable quantities are the ones given in the assignment.

Swap Simulation - (4)

- Since the SDE in (4) is linear, $x(t+\delta)|x(t)$ is Gaussian with easily computable moments; you should compute these moments and simulate the SDE **exactly** – do **not** discretize it naively (other than for tests and comparisons). Everybody should know how to solve a Gaussian SDE.
- Equation (4) can be combined with (3) and (1)-(2) to devise a scheme for simulation swap values $V(t_i)$ on a monthly grid for 10 years (as requested in assignment).
- You need 50,000 random paths, each containing 120 swap values (one per month).

Swap Simulation - (5)

- In the simulation, the bank exposure $E_B(t_i)$ is computed as $E_B(t_i) = (V(t_i) - C(t_i))^+$, where $C(t_i)$ is a threshold-type collateral agreement (which is turned on or off). See the assignment for details.
- The counterparty exposure E_C is computed the same way.
- Also, there is a ratings-driven termination, which kills the exposure the first time either of the default intensities of the bank and its counterparty exceed a given threshold.

CVA + default intensity simulation - (1)

- The bank's and the counterparty's intensities are denoted λ_B and λ_C, respectively.
- They are simulated as Gaussian one-factor models, with given vols (σ_B, σ_C) and mean reversions (κ_B, κ_C).
- The correlation of these intensities to each other and to the interest rate factors are given and must be honored.
- As before, you can simulate λ_B and λ_C on a monthly grid.
- The paths of intensities are used for two things: First, in combination with the short rate $r_{OIS}(t) = f_{OIS}(t, t)$, the intensities define stochastic discounting terms needed in the various definitions of CVA/DVA.

CVA + default intensity simulation - (2)

- These definitions (bilateral, unilateral,..) can be found in your background reading.
- Second, they are used to check the termination agreement in the exposure.
- You are asked to demonstrate how correlations can introduce *wrong-way effects*.

IMM Simulation

- For IMM work, the $x(t)$ process is simplified a bit (to move it into a historical measure), and exposure calculations are no longer discounted.
- This allows you to compute the EE, EPE, EEPE profiles/numbers, and in turn the EAD + M.
- And from this, you can compute regulatory credit capital for the swap position.
- PD and LGD are given in the assignment.