Computer Assignment

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1 Market Setup

1.1 Interest Rate Setting

Consider an economy where the instantaneous OIS forward rate term structure is observed at time 0 to be flat at 100bps. That is,

$$f_{OIS}(0,T) = 1\%$$

for all $T > 0$. The OIS curve will be used for all discounting purposes. In addition, the time 0 spread between the Libor forward (f_L) curve and the OIS forward curve is flat at 50bps, such that

$$f_L(0,T) = 1.5\%$$

for all T. The Libor curve will be used as a reference for swap floating leg fixings.

Suppose that PCA analysis in our economy has demonstrated that a two-factor model is adequate to characterize OIS and Libor forward curve moves. Also, observations of the swaption skew reveal that it is not unreasonable to assume that forward rates are Gaussian. Consequently, the concrete two-factor Gaussian model in [1] is deemed an appropriate model for the OIS forward curve. The estimation of the model results in the following constant parameters for the OIS curve (see Section 2.3 in [2] for notation)

$$\sigma_r = 2\%, \quad c = 35\%, \quad \kappa_1 = 0, \quad \kappa_2 = 10\%, \quad \rho_\infty = 40\%.$$

Finally, the Libor and OIS curves are assumed to be related through the deterministic relationship

$$f_L(t,T) = f_{OIS}(t,T) + f_L(0,T) - f_{OIS}(0,T) = f_{OIS}(t,T) + 0.5\%.$$

1.2 Credit Curve Setting

We consider two counterparties engaging in trading with each other: firms B and C. Based on quoted CDS market spreads along with a recovery rate estimation of 40% for both firms, we determine (by bootstrapping) that the default intensity forward curves for the two firms are flat at

$$\lambda_f^B(0,T) = 1\%, \quad \lambda_f^C(0,T) = 3\%,$$

respectively. Here we have defined

$$X_B(t,T) = \exp\left(-\int_t^T \lambda_f^B(t,u)\,du\right)$$

where $X_B(t,T)$ is the time t survival probability for firm B to time T. Similarly for firm C.

Options on CDSs rarely trade, so there is limited observability into the volatilities of the default intensity curves. As such, we contend ourselves with simple one-factor Gaussian models for each default intensity, with the following parameters:

$$\sigma_B = 0.5\%, \quad \kappa_B = 10\%, \quad \sigma_C = 1\%, \quad \kappa_C = 10\%.$$

Here σ_B and σ_C are the volatilities of the short end of the intensity curves for B and C, respectively; and κ_B and κ_C are the corresponding mean reversion speeds.

1.3 Correlations

We assume the following correlation structure:

$$\begin{aligned}
\mathrm{corr}(d\lambda_B(t), df_{OIS}(t,\infty)) &= 10\%,\\
\mathrm{corr}(d\lambda_B(t), dr_{OIS}(t)) &= 25\%,\\
\mathrm{corr}(d\lambda_C(t), df_{OIS}(t,\infty)) &= 10\%,\\
\mathrm{corr}(d\lambda_C(t), dr_{OIS}(t)) &= 25\%,\\
\mathrm{corr}\,(d\lambda_B(t), d\lambda_C(t)) &= 75\%.
\end{aligned}$$

See Section 2.4 in [2] for more details on how to incorporate this into the modeling framework.

2 Portfolio & Credit Mitigation Setup

Assume that B and C have traded a \$150MM 10-year semi-annual swap where Libor is exchanged against a coupon of 1.75%; this trade is the only one that B and C have on the books with each other.

To protect against default, the parties have agreed to a bilateral downgrade protection clause where the deal will be unwound at the risk-free market value should either counterparty fall below an "A" rating; for our purposes, we assume that an "A" rating maps to a credit intensity of 3.75%. In addition, there is a bilateral collateral agreement with a threshold of \$5MM. On default, we assume that the recovery rate for both B and C is 40% of the exposure.

3 CVA/DVA Exercise

1. Assume first that neither the collateral nor the downgrade protection clauses in Section 2 are in effect. Using Monte Carlo simulation with 50,000 paths, compute and graph the present value as seen from B of the present value of expected exposure[1] $PVEE(T)$, for all T on a monthly schedule out to 10 years. Do this first assuming that B receives the fixed coupon (receiver swap), and then assuming that B pays the coupon (payer swap).

2. Use the results of Exercise 1 to compute the (unilateral) CVA from the perspective of B (for both payer and receiver swap).

3. Compute the unilateral DVA, for both payer and receiver swap. Also compute the net unilateral CVA.

4. For the receiver swap, graph the unilateral CVA, DVA, and net CVA against the interest rate model parameters σ_r and κ_2 (two separate graphs). Explain the results.

5. Some of the correlations in Section 1.3 control wrong- and right-way risk in the unilateral CVA computation. Construct a test to demonstrate (e.g., via a graph) the effects of these correlations (receiver swap only).

6. Now consider the credit mitigants in Section 2. Repeat Exercise 1 with a) the collateral agreement in place; b) the termination agreement in place; c) both agreements in place. (Three separate graphs). Compare the results to those in Exercise 1.

7. Turn off the credit mitigants again, and now compute the bilateral CVA, DVA, and net CVA for the naked receiver swap position. Compare against the results in Exercises 2 and 3. Explain.

4 IMM Exercise

Assume that historical averages of interest rate and default intensity volatilities are similar to the implied (risk-neutral) volatilities in Section 1. Also assume that IMM simulations, rather than using drifts consistent with a risk-neutral probability measure, aim to distribute intensities and interest rates around their time 0 forward values. For instance, for the two-factor interest rate model in [2], we set $y(t) = 0$ in equation (15) which ensures (see equation (17)) that $r(t)$ will be centered around $f(0,t)$.

1. Turn off all credit mitigants, and compute and graph the expected exposure profile (as in Section 4.2 of [1]) for both the payer and receiver swap. Contrast the results with those of Exercise 1 in Section 3 above.

[1] See formula (23) in [1]

2. Using the IMM formulas in [3], compute the EEPE, the EAD and the weighted maturity (M) the payer and receiver swaps (as seen from B's perspective).

3. Assuming that the 1-year (historical) default probability for firm C is $PD = 1\%$, use the results of Exercise 3 to compute firm B's regulatory credit capital for the receiver and payer swap positions, respectively.

References

[1] Andersen, L. (2014), "Background Material: Cox Default Processes, CVA, and Expected Exposure," Lecture Notes.

[2] Andersen, L. (2014), "Background Material: Gaussian and Quasi-Gaussian Models," Lecture Notes.

[3] Andersen, L. (2016), "Regulation, Capital, and Margining: Quant Angle ," Lecture Notes.