Regulation, Capital, & Margin: Quant Angle - II

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Part II: IMM

IMM & EAD Basics

- Starting with EAD, the purpose of IMM is to find a way to take an arbitrary derivatives portfolio (including its collateral) and replace it with a “representative” loan notional.
- The loan notional need only be representative for a one-year period, since IRB works with this horizon.
- The cumulative loss on $[0, T]$ generated for a portfolio $V(t)$ with collateral $C(t)$ is

$$L(T) = l \cdot (V(\tau) - C(\tau))^{+} 1_{\tau \leq T} = l \cdot E(\tau) 1_{\tau \leq T}, \tag{1}$$

where τ is the counterparty default time and $E(t) = (V(t) - C(t))^{+}$ is the *exposure* at time t.

EAD by Expectations - (1)

- If we only had a loan with notional N (and maturity $> T$), we would have
$$L_N(T) = l \cdot N 1_{\tau \leq T}. \tag{2}$$
- How do we pick N such that (1) and (2) are "close"?
- We could try to align expectations (p is default probability, $T = 1$yr):
$$N \mathrm{E}\left(1_{\tau \leq T}\right) = \mathrm{E}\left(E(\tau) 1_{\tau \leq T}\right) \Rightarrow N = \frac{\mathrm{E}\left(E(\tau) 1_{\tau \leq T}\right)}{p},$$
where we have assumed that l is non-random.
- This can be rewritten as
$$N = \frac{\mathrm{E}\left(E(\tau) | \tau \leq T\right) \mathrm{E}\left(1_{\tau \leq T}\right)}{p} = \mathrm{E}\left(E(\tau) | \tau \leq T\right). \tag{3}$$

EAD by Expectations - (2)

- In the computation of the r.h.s. of (3), it is clear that *right- and wrong-way risk matters:* conditioned on an early default ($\tau \leq T$), is the exposure higher (wrong-way) or lower (right-way) than normal?
- Building models that cleanly allow for dependence between exposure and τ is non-trivial, so regulators want a way out of this.
- For this, suppose that exposure and τ are *independent*, in the sense that

$$\begin{aligned}\mathrm{E}\left(E(\tau)|\tau \leq T\right) &= \int_0^T \mathrm{E}\left(E(t)\right)\mathbb{P}\left(\tau \in dt|\tau \leq T\right)\\ &= \int_0^T EE(t)\mathbb{P}\left(\tau \in dt|\tau \leq T\right),\end{aligned}$$

where $EE(t)$, $t \leq T$, is the *expected exposure profile*.

EAD by Expectations - (3)

- What can we make of the term $\mathbb{P}(\tau \in dt | \tau \leq T)$? In a simple Poisson model where default arrives at an intensity of λ, we have, for $t \leq T$,

$$\mathbb{P}(\tau \in dt | \tau \leq T) = \frac{\mathbb{P}(\tau \in dt, \tau \leq T)}{\mathbb{P}(\tau \leq T)} = \frac{\mathbb{P}(\tau \in dt)}{\mathbb{P}(\tau \leq T)} = \frac{e^{-\lambda t}\lambda}{1 - e^{-\lambda T}}\, dt.$$

- If λ is smallish, we have

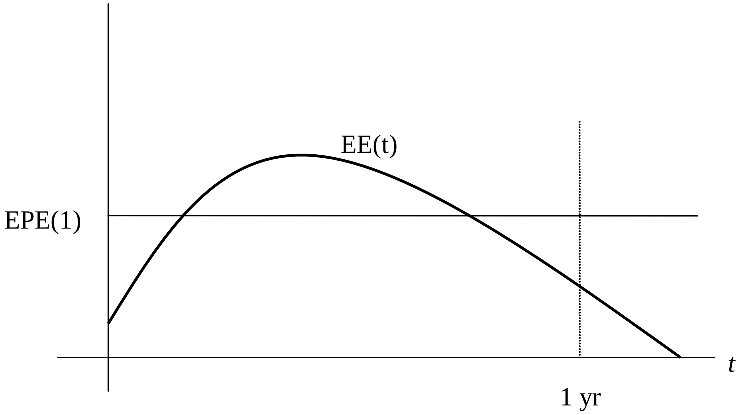
$$\mathbb{P}(\tau \in dt | \tau \leq T) \approx \frac{(1 - \lambda t)\lambda}{\lambda T}\, dt \approx \frac{1}{T}\, dt.$$

- So, therefore, with independence,

$$\mathrm{E}(E(\tau) | \tau \leq T) \approx \frac{1}{T} \int_0^T EE(t)\, dt \triangleq EPE(T). \qquad (4)$$

- EPE ("expected positive exposure") is the *time-average* of EE.

EPE



- Note that EPE is an intuitive definition of loan-equivalent notional.

Reinvestment Risk - (1)

- Combining (3) and (4) would suggest that a reasonable (and intuitive) estimate for EAD could be 1-yr EPE.
- In reality, things are more complicated. First, we ignored wrong-way risk. Second, our principle of matching just the first moment would tend to ignore the volatility of the exposure, and could be less than conservative. Third, we have ignored *re-investment risk*.
- Implicit in the way we drew our exposure profile was an assumption that the portfolio is static: after time 0, we just leave it alone to age and, ultimately, die.
- For portfolios with maturities $\ll 1$ year (e.g. repos, or short-dated FX), this is not conservative, since in reality the portfolio will be "refreshed" with new trades as part of the on-going trading business.

Reinvestment Risk - (2)

- To capture this, Basel mandates that for RC computations one does not use the EE profile directly, but instead an alternative profile EE* ("effective EE") that is found as a running maximum of the EE profile:

$$EE^*(t_i) = \max\left(EE^*(t_{i-1}), EE(t_i)\right)$$

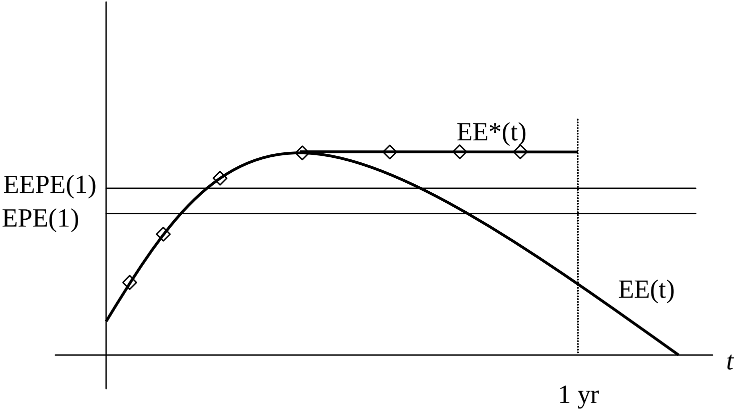
  where $\{t_i\}$ is some sampling grid.
- The time-average of this "modified" EE profile is called the *effective EPE*, or *EEPE*:

$$EEPE(T) = \frac{1}{T}\int_0^T EE^*(t)\,dt.$$

- If the longest deal maturity in the portfolio is less than T, then:

$$EEPE(T) = \frac{1}{T_{\max}}\int_0^{T_{\max}} EE^*(t)\,dt.$$

Reinvestment Risk - (3)



- Note that EEPE $>$ EPE. Not easy to justify EEPE formally, though.

Alpha Multiplier - (1)

- What about the independence assumption and zero variance?
- Originally, the Basel committee attempted to cover the missing variance by replacing expected exposure (EE) by *potential exposure* (PE), where PE is, say, the 95th percentile of the exposure $E(t)$.
- This, however, was met with protests from industry which demonstrated (using simulation studies) that using PE would lead to large overestimates for economic capital. This is not surprising.
- Instead, the Basel committee proposed to introduce a scale $\alpha > 1$:

$$EAD = \alpha \cdot EEPE(1) \tag{5}$$

- The "Alpha" multiplier is meant to adjust for a variety of effects: wrong-way risk, variance of the exposure, finite granularity of portfolios, noise in $E(t)$ simulation,...

Alpha Multiplier - (2)

- Note: the original draft of Basel 2 (from 2001) had an explicit granularity adjustment, which was subsequently dropped.
- Industry studies on real portfolios have shown that $\alpha \approx 1.1 - 1.2$.
- Basel 2 uses $\alpha = 1.4$, but does allow banks to argue for a lower α:

  > *"Banks may seek approval from their supervisors to compute internal estimates of alpha subject to a floor of 1.2, where alpha equals the ratio of economic capital from a full simulation of counterparty exposure across counterparties (numerator) and economic capital based on EPE (denominator), assuming they meet certain operating requirements."*

- All things considered, $\alpha = 1.4$ is probably reasonable.

But wait, there is more..

- On top of the 1.4 multiplier, the BCBS has issued language to add *another* fudge factor:

  > *"The Committee believes it is important to reiterate its objectives regarding the overall level of minimum capital requirements. These are to broadly maintain the aggregate level of such requirements, while also providing incentives to adopt the more advanced risk-sensitive approaches of the revised Framework. To attain the objective, the Committee applies a scaling factor to the risk-weighted asset amounts for credit risk under the IRB approach. The current best estimate of the scaling factor using quantitative impact study data is 1.06."*

- ???

CVA Adjustment

- The portfolio value V used in computing expected exposures are default-free versions of the real transactions
- But in reality the counterparty can, of course, default, so V is not the value that a bank actually recognizes – it adjusts V down by CVA.
- The exposure is therefore exaggerated if CVA is not accounted for.
- So, in Basel 3, the resulting formula is (with $T = 1$ year)

$$EAD = 1.06 \times \max(\alpha \cdot EEPE(T) - CVA, 0) \tag{6}$$

- Formula is not entirely obvious, and CVA allowance potentially overlaps with the EL allowance

Effective Maturity - (1)

- IMM also covers the computation of effective maturity M.
- M is meant to represent credit spread duration, so it makes sense to consider a contract that pays out an amount $EE(\tau)$ at time τ, on some interval $[0, T_{\max}]$.
- If the default intensity is λ, we have (ignoring discounting)

$$PV = \int_0^{T_{\max}} EE(t)\lambda e^{-\lambda t} dt$$

such that, for small λ,

$$PV01 = \frac{\partial PV}{\partial \lambda} = \int_0^{T_{\max}} EE(t)e^{-\lambda t} dt - \lambda \int_0^{T_{\max}} EE(t)te^{-\lambda t} dt$$
$$\approx \int_0^{T_{\max}} EE(t)\, dt.$$

Effective Maturity - (2)

- For a (loan) contract with a flat notional of EAD and maturity M, we would have

$$PV01 \approx EAD \int_0^M dt = EAD \cdot M.$$

- This suggests using (ignoring α)

$$M = \frac{\int_0^{T_{\max}} EE(t)\, dt}{EAD} = \frac{\int_0^{T_{\max}} EE(t)\, dt}{\int_0^1 EE^*(t)\, dt},$$

where T_{max} is the longest maturity in the book.

Effective Maturity - (3)

- The actual formula is similar to this, but a) adds discounting; b) splits the numerator into two pieces; and c) imposes a 5-year cap and a 1-year floor:

$$M = \max\left(1, \min\left(5, T\frac{\int_0^1 EE^*(t)\,P(t)dt + \int_1^{T_{\max}} EE(t)\,P(t)dt}{\int_0^1 EE^*(t)\,P(t)dt}\right)\right) \tag{7}$$

  where $P(t)$ is the discount factor to time t.

- The formula for M is quite a **nuisance**, as it requires one to establish EE-profiles all the way out to $T_{\max}$ (which could be 50 years for a swap portfolio). In contrast, computation of EAD only requires establishing out to $T = 1$ year.
- With formulas (7) and (5)-(6), the technical part of the IMM framework is **complete** .

EE Computations - (1)

- At the heart of both (7) and (5)-(6) lies the expected exposure profile EE – once this has been established, computation of EE* and, finally, EAD and M is straightforward.
- Computation of EE is the *biggest challenge* of IMM.
- Recall that

$$EE(t) = \mathrm{E}\left((V(t) - C(t))^+\right) = \mathrm{E}\left(E(t)\right),$$

where V is the (netted) portfolio value and $C(t)$ the (dynamic) collateral.

EE Computations - (2)

- A possible Monte Carlo algorithm looks like this:
  1. Simulate in $\mathbb{P}$ a path of correlated market data (rates, equities, commodities, spreads, FX,...) out to time $T_{\max}$. Let $\omega(t)$ be the market data state at time t and prior.
  2. At some time-grid $\{t_i\}$, use $\omega(t_i)$ in pricers to establish value of all (netable) trades in the counterparty portfolio.
  3. Aggregate trade values to compute portfolio value $V(t_i)$.
  4. Use CSA and collateral data to establish $C(t_i)$. Compute $E(t_i) = (V(t_i) - C(t_i))^+$.
  5. Repeat for many paths, to establish $EE(t_i)$, $i = 1, \ldots$.

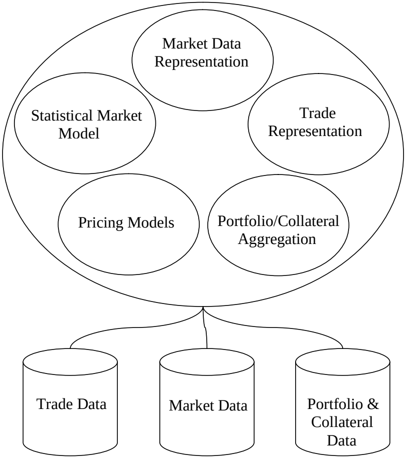
EE Computations - (3)

- When simulating collateral, we should aim to incorporate "margin delay" when financially important.
- Margin delays can mean two things: a) the frequency at which margin/collateral agreements get enforced; b) the delay associated with collateral disputes.
- In the exposure simulation, one can explicitly account for a margin period η. Adding also a δ closeout period, exposure can be written as something like $\max\left(V(t) - V(t-\delta-\eta), 0\right)$, for full collateralization with no thresholds.
- Requires adding extra points to the simulation (to make sure that for each t_i we also have $t_i - \delta - \eta$). There are Brownian Bridge methods available that can sometimes allow one to avoid doubling the grid.

Computer Systems for EE - (1)

- Functionally, a system to compute EE profiles must have:
  1. A representation of market data (rates, vols, spreads, equity prices, FX rates, etc.). Market data values at time 0 is the initial condition for the market data simulation.
  2. A simulation model for the joint evolution of the market data in the statistical measure. Must be supported by historical back-testing.
  3. A representation of trades. This representation must (unlike VaR) be able to **correctly age** trades through time.
  4. Pricing models, taking market and trade data as inputs.
  5. A concept of portfolios, netting sets, and collateral. Combined with the trade data and pricing models, this allows for the computation of exposures on the simulation path generated by the statistical model.

Computer Systems for EE - (2)



Computer Systems for EE - (3)

- In principle, it seems logical to use as many front office (FO) components for EE simulations as possible: FO market data, FO trade data, FO pricing models, and so forth.
- In practice, this may not be feasible due to enormous number of repricings that a capital system needs to perform.
- For instance, if we simulate 1,000,000 trades (BAC has much more than this) for 5,000 paths on a bi-monthly grid for 30 years, we need of the order of 10^{12} trade valuations (!).
- Many security valuations are complex, so typically FO systems can only handle in the order of $10^6 - 10^8$ trades per day.
- So strong simplifications are typically required on both trade representation, market data representation, and pricing models.

Pricing Errors - (1)

- Simplifications can cause problems, if done too coarsely. This can happen if price and/or data models are too simplistic, e.g., if the Capital systems get out of synch with FO developments.
- A key symptom of problems are discrepancies in the current market value (CMV) produced by FO and Capital systems.
- To be more precise, let the counterparty portfolio in question consist of R trades with values $v_1(t), v_2(t), \ldots v_R(t)$. That is,

$$V(t) = \sum_{j=1}^{R} v_j(t)$$

- Pricing errors cause (FO: front office; C: capital system)

$$V^{FO}(0) \neq V^{C}(0)$$

Pricing Errors - (2)

- This causes initial exposures to differ, $E^{FO}(0) \neq E^{C}(0)$. So, the EE profile computed by the capital system has the wrong starting point.
- Regulators like to measure pricing errors at the trade level, by adding absolute trade pricing errors:

$$e = \sum_{j=1}^{R} |v_j^{FO}(0) - v_j^{C}(0)|$$

- Regulators want a low value of e, which involves improving trade and market data fidelity, as well as improving pricing model precision. Very complex accuracy-efficiency tradeoffs are involved here
- Also, some part of e is unavoidable, due to timing effects.

Pricing Errors - (3)

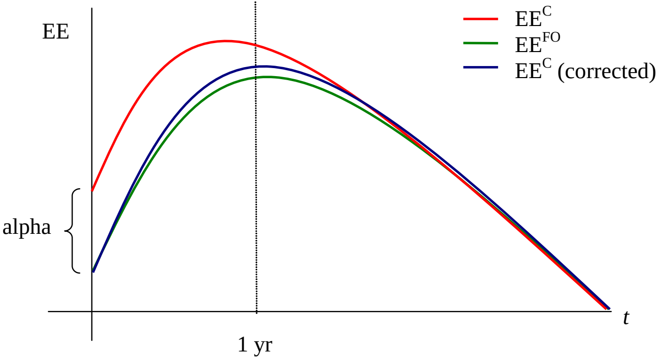
- For systems that generate pricing errors, it is possible to ad-hoc adjust for these errors. Not a substitute for fixing deeper problems, but can work remarkably well if errors are not too big.
- For instance, we can define an improved pricer with guaranteed zero error:

$$v_j^{C*}(t) = v_j^C(t) + \left(v_j^{FO}(0) - v_j^C(0)\right).$$

- This additive adjustment works well for European options, say, where EE profiles are pretty flat.
- For instruments (such as swaps) where errors naturally decline over time, one can amortize away the initial error. For instance, for an instrument with maturity T_j:

$$v_j^{C*}(t) = v_j^C(t) + \frac{T_j - t}{T_j}\left(v_j^{FO}(0) - v_j^C(0)\right)$$

Pricing Errors - (4)



Error correction procedure for a stylized swap.

But wait: the Collins floor

- As part of the Dodd-Frank Act, US senator Susan Collins instituted a floor on capital that, in effect, makes sure that no future regulation will result in less capital than what would have been required at the time of passing the DF Act (July 2010).
- Effectively, this floors all capital computations at Basel 1 capital levels.
- So, after spending $100MM's on the AIRB in Basel 2, the result could be "thrown away" and replaced by a number that everybody, including BCBS, agreed is less than meaningful.
- According to the American Banker's Association:

> *"Imposing a floor that is tied to Basel I rules raises the question of why any bank would want to undertake the expense and effort to convert to the advanced approaches rules if it has the option not to do so. Such rules become, in essence, very expensive risk management exercises."*

IMM Appendix:

Acronyms and Abbreviations

Appendix - (1)

- α: Basel 2 multiplier on EEPE, in computation of EAD
- CCR: counterparty credit risk
- CMV: current market value
- EAD: exposure-at-default (a.k.a. equivalent loan notional, ELN)
- E: random exposure profile
- EC: economic capital
- EE: expected exposure profile
- EE*: "effective" exposure profile (accounting for roll-over)
- EEPE: 1-year time average of EE*
- EL: expected loss (a.k.a. credit reserve, CR)

Appendix - (2)

- EPE: time average of EE
- IRB: Internal ratings-based approach (Basel 2)
- LGD: loss given default (percentage)
- L: credit loss
- M: effective maturity
- PD: 1-year default probability
- RC: regulatory capital
- RW: risk weight on equivalent loan notional
- RWA: risk weighted assets, RWA = RC * 12.5 (so RC = RWA * 8%)
- UL: unexpected loss