Journal of Financial Economics 133 (2019) 42–63

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Liquidity windfalls: The consequences of repo rehypothecation☆

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ARTICLE INFO

Article history:
Received 22 December 2017
Revised 10 July 2018
Accepted 14 August 2018
Available online 6 February 2019

JEL classification:
G23
G24
G33

Keywords:
Rehypothecation
Repo
Dealer
Liquidity
Default
Collateral

ABSTRACT

This paper presents a model of repo intermediation in which dealers intermediate secured financing between lenders and borrowers using the same collateral. Lenders are insulated from dealers through their repo's collateral, but borrowers are exposed to dealers through the loss of their collateral. This makes lenders' repo terms insensitive to dealers' default, while borrowers' repo terms are not. The model shows that when repos serve to intermediate collateral, haircuts are negative. This paper explains the difference in haircuts between the bilateral and tri-party repo market and the different run dynamics observed across these markets during the financial crisis.

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1. Introduction

The 2007–2009 financial crisis highlighted the risks associated with intermediaries' reliance on wholesale short-term funding. This reliance is an important shift from more traditional forms of financial intermediation, such as deposits and loans, and has changed how intermediaries manage their liquidity and how it can come into question. One such shift has been the increase in intermediaries' use of repurchase agreements (repos)—secured loans in which the collateral consists of financial assets. In the context of repos, a borrower's liquidity can evaporate in different ways. For example, a lender's refusal to roll over existing repos can imply an abrupt withdrawal of funding, similar to a withdrawal of deposits. Alternatively, an increase in haircuts—the contracting term determining the amount of overcollateralization relative to the value of the collateral—forces borrowers to seek alternate sources of funding or to sell the underlying collateral.

However, despite the considerable literature that emerged following the financial crisis, it is unclear how repo markets contributed to intermediaries' shortage of liquidity. For example, while Krishnamurthy et al. (2014) and Copeland et al. (2014) show a mild variation

☆ I would like to thank an anonymous referee, Markus Brunnermeier, Mark Carlson, Adam Copeland, Matthew Eichner, Giovanni Favara, Jonathan Goldberg, Ron Kaniel, Antoine Martin, Stefan Nagel, Jason Wu, Egon Zakrajsek, and conference participants at the European Winter Meeting of the Econometric Society, the Workshop on the Risks of Wholesale Funding (FRB NY), the Workshop on Nonfinancial Firms and Financial Stability (FRB Atlanta), and the Santiago Finance Workshop for their helpful comments. Thanks to Amy Lorenc and Blake Phillips for excellent research assistance. This paper was previously distributed as "Money for nothing: the consequences of repo rehypothecation." The views of this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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https://doi.org/10.1016/j.jfineco.2019.02.004
0304-405X/Published by Elsevier B.V.

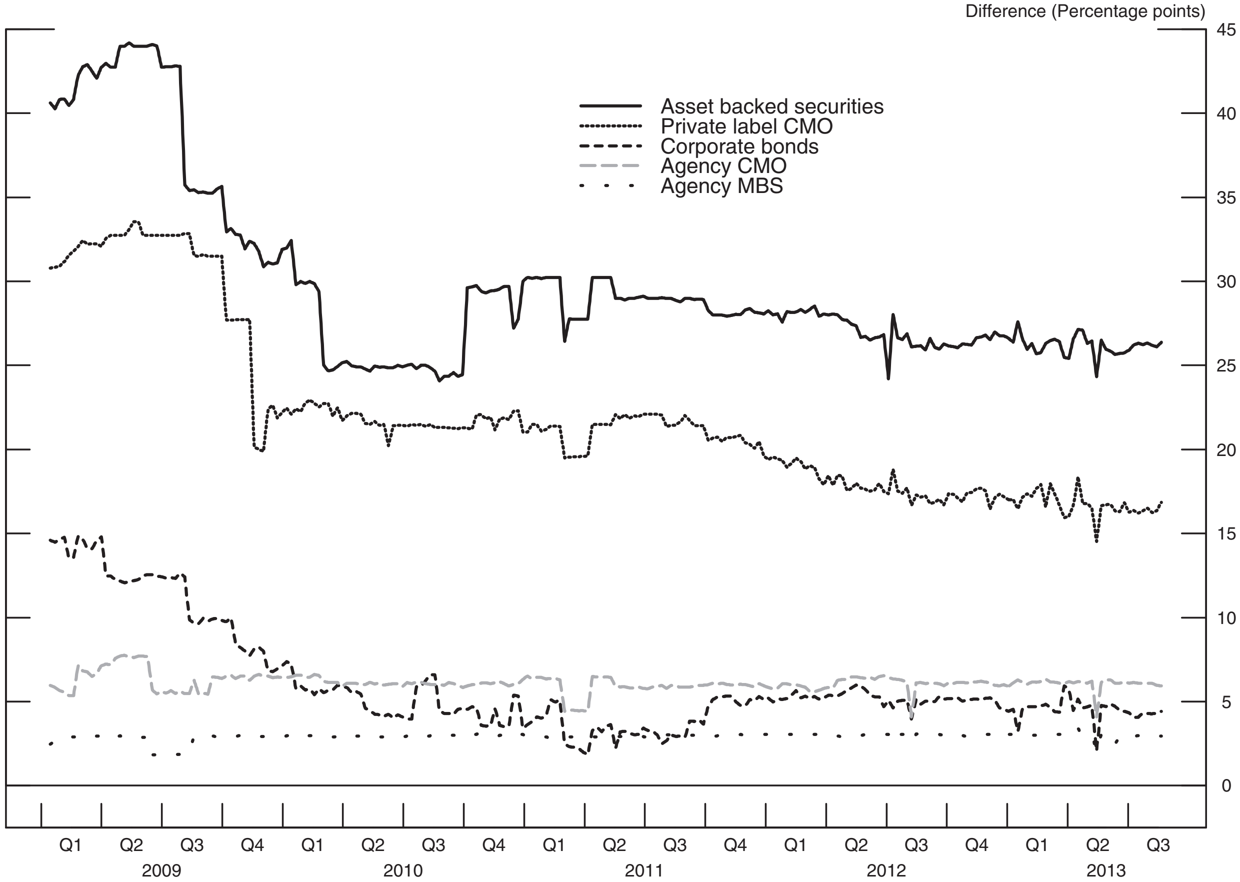

Fig. 1. Differences in median repo haircuts across the bilateral and tri-party repo market. The figure shows the time series of the difference in median haircuts across the bilateral repo market and the tri-party repo market for different collateral classes. Median haircuts are based on primary dealer surveys conduced the Federal Reserve Bank of New York. CMO is collateralized mortgage obligations and MBS is mortgage-backed securities. Data are from 1/14/2009 to 8/21/2013.

in haircuts and a sharp reduction in secured lending with riskier collateral classes (such as private-label asset-backed securities), Gorton and Metrick (2012) show a dramatic increase in haircuts, which forced cash borrowers to use more of their internal funds to finance their existing position. This paper aims to reconcile these different findings with a stylized model of repos from the perspective of a financial intermediary bringing borrowers and lenders together.

The model consists of a financial intermediary (i.e., a dealer) who borrows and lends funds through repos, using the same underlying collateral provided by the cash borrower for both contracts—a process known as *rehypothecation*.[1] Therefore, a difference in haircuts between repo counterparties implies a liquidity windfall for the dealer. The model characterizes how such a windfall arises in equilibrium and how it can change with different market conditions. Specifically, equilibrium repo contracting terms between dealers and cash lenders are stable, while repo contracting terms between dealers and cash borrowers are flexible and depend on the joint distribution of the dealer's default and the collateral's payoff. From this perspective, the observed differences between repo markets during the crisis can be understood by acknowledging that dealers stand between two segmented markets of borrowers and lenders. Fig. 1 supports this interpretation, showing the difference in repo haircuts for segments of the market where dealers are likely to be borrowers (tri-party repo market) and lenders (bilateral repo market).

The underlying frictions driving the results are the dealer's time preferences over payments and the dealer's resolution under bankruptcy. The first friction assumes that the dealer places a higher value on a cash inflow today than a cash inflow in the future. This assumption captures the idea that an initial influx of liquidity can allow the dealer to finance other activities or supplant any liquidity shortfalls. The second friction assumes that the dealer defaults exogenously, for reasons unrelated to repo rehypothecation, and that counterparties consider unsecured claims on the defaulted dealer to be worthless. This as-

[1] Previous literature has stressed US regulatory restrictions on rehypotyhecation. These restrictions only apply to customer assets, which are entrusted to a dealer—that is, assets that the dealer can use on the customer's behalf. The initial leg of a repo is an assets sale, where the buyer (i.e., cash lender) has full ownership of the asset. This gives buyers the right to use the underlying collateral for their own benefit, exempting repos from any limits on rehypothecation. The institutional details of rehypothecation in the United States are discussed further in Section 1.2.1.

sumption captures the idea that the resolution of a dealer's default is a complex and time-consuming process. These two frictions pin down each repo's haircut and final repurchase price—the contracting term determining the final debt repayment—balancing the dealer's preference for cash up front with counterparties unsecured exposures.

The resulting equilibrium predicts a higher haircut for cash borrowers than for cash lenders, implying a windfall for the dealer in the initial period. Given cash lenders' limited recourse to the dealer's assets upon default, it is optimal to set their repurchase price equal to the collateral's worst case outcome—the zero value-at-risk contract. This contract insulates lenders from the dealer's default, suggesting that haircuts between dealers and cash lenders are relatively stable and only depend on the underlying collateral.[2] In contrast, cash borrowers are directly exposed to the dealer's default because they risk losing their collateral, which has been rehypothecated, and can be more valuable than the initial cash loan. The dependence of cash borrowers' payoff on the dealer's solvency suggests flexible contracting terms between the two. If conditions deteriorate sufficiently, cash borrower's have incentives to withdraw their collateral, shutting down the repo rehypothecation process altogether, implying an abrupt withdrawal of funds to the dealer. From this perspective, the withdrawal of funds does not ensue because of lenders' unwillingness to provide cash but rather because of the borrowers' unwillingness to provide collateral.

The model is able to characterize the sensitivity of the cash borrower's margin—the contracting term determining the dollar amount of overcollateralization—to risks associated with the dealer's default and the collateral's outcome. Specifically, the model shows that as the correlation between the dealer's default and a low asset outcome increases, the cash borrower's margin increases. Intuitively, the main risk borrowers face is the loss of their collateral, and thus borrowers' exposure is proportional to their repo's margin. If they are more likely to lose an asset at a time when its valuation is low, their expected loss is reduced. Therefore, in equilibrium, borrowers would be more willing to increase their exposure to the dealer, that is, to increase the margin.

Interestingly, the model shows that whenever the probability of the dealer's default increases in isolation, the cash borrower's margin decreases. This result seems contradictory to the motivating facts of the paper, but the intuition is simple: because the borrower's exposure is proportional to their repo's margin, as the probability of a loss increases, it is optimal to reduce said exposure. In this intuition the collateral does not play a meaningful role and the sensitivity can be interpreted as being relevant for other types of borrowing contracts. For example, Ivashina and Scharfstein (2010) find evidence that at the height of the financial crisis, corporate borrowers drew down existing credit lines because of concerns over their banks' solvency. Also, Duffie (2013) discusses how prime brokerage clients drew down credit lines from their brokerage accounts to reduce exposures to an ailing dealer, implying a withdrawal of cash from the dealer. That is, when studying unsecured claims in isolation, the model produces sensitivities seen in other markets.

Having characterized the sensitivities of repo margins, this paper then translates margins into haircuts to compare with the existing empirical literature. The model predicts that in markets in which the dealer is likely to be a cash borrower (tri-party repo market), haircuts are collateral specific and equal to the lowest possible asset return. Thus, assuming stationary returns, haircuts in the tri-party market do not vary, a result that is consistent with Krishnamurthy et al. (2014) and Copeland et al. (2014). In contrast, in markets in which the dealer is likely to be a cash lender (bilateral repo market), haircuts depend on changes in margins and changes in collateral values. The increase in bilateral repo haircuts documented by Gorton and Metrick (2012) is more consistent with an increase in margins, which the model ascribes to an increase in the correlation between dealers' solvency risk and low collateral values. This paper cites evidence suggesting that during the financial crisis, the most troubled dealers had increased their exposure to the mortgage market, which, in the context of the model, implies an increase in repo margins for mortgage-related securities, consistent with the haircut increase of those collateral classes. However, the increase in haircuts does not preclude a mild decrease in margins, as lower collateral values can also drive haircuts higher. This insight highlights the importance of taking collateral values into account when examining haircut sensitivities.

The main focus of this paper is to study repos as a means to intermediate funds between cash lenders and cash borrowers; however, some repo markets serve to intermediate collateral. In these markets, repos are used to borrow specific assets, which is economically equivalent to a securities lending agreement. An extension of the model shows how contracting terms differ when clients source specific assets to take on short positions. The model extension gives conditions under which repo rates can be lower than prevailing risk-free rates, an empirical fact first documented by Duffie (1996) and known as *repo specialness*. The model also shows that haircuts in a securities lending context are negative, implying an overcollateralization of cash when borrowing a specific security. Overcollateralization with cash collateral is common practice in securities lending markets and has been confirmed by Baklanova et al. (2016) in parts of the bilateral repo market. In the context of securities lending, the haircut protects the collateral provider rather than the cash lender.

The paper is organized as follows. The remainder of the introduction discusses the related literature and relevant institutional details of repo markets in the United States. Section 2 presents the setup of the baseline model. Section 3 characterizes the model's unique equilibrium, its main comparative statics, and how the results relate to the financial crisis. Section 4 presents a model extension where repos are used to short securities. Section 5 discusses the model assumptions driving the results. Finally, Section 6 concludes.

[2] Risk-free contracting terms are a common equilibrium outcome in models of collateralized markets such as those described in Fostel and Geanakoplos (2015).

1.1. Literature review

The literature on repo has grown considerably since the 2007–2009 financial crisis. For example, Krishnamurthy et al. (2014) and Copeland et al. (2014) empirically study the US tri-party market and show that while funding for some firms dried up precipitously, there was no market-wide collapse of the market. In addition, the authors show that repo haircuts were fairly stable during the crisis. However, Gorton and Metrick (2012) show that haircuts in the bilateral market increased sharply during the crisis. The model in this paper reconciles these seemingly contradictory findings by highlighting that contracting terms depend on dealers' role in different segments of the repo market; in other words, as borrowers in the tri-party market and lenders in the bilateral market.

Despite the relative opaqueness of the bilateral market, there is empirical literature exploring the market's size and its contracting terms. For example, Duffie (1996) shows that repo rates backed by scarce collateral can be negative. More recently, Baklanova et al. (2016) confirm that dealers' main counterparties in these markets are other dealers, banks, and hedge funds, supporting the structure of the model in this paper. The authors find substantial haircut heterogeneity within certain collateral classes, suggesting that haircuts in the bilateral market are not solely collateral dependent. Baklanova et al. (2016) also show that haircuts in the bilateral market can be negative, which, in the model extension of this paper, arises because of the economic motive behind the trade: to borrow or lend securities. Auh and Landoni (2015) find that an important fraction of haircut variability in the bilateral market depends on lender characteristics, a key insight derived from the model in this paper.

This paper is also related to theoretical work studying optimal contracting terms for collateralized loans. Fostel and Geanakoplos (2015) show that in a binomial distribution setting, the optimal contract completely insulates the cash lender from the borrower's default—that is, the zero value-at-risk contract. In this paper, cash lenders' repos have the same feature, but cash borrowers' repos have higher margins that are borrower/lender specific. In Ewerhart and Tapking (2008), cash borrowers also risk taking a loss when a lender defaults, but that model does not capture the different contracting terms observed across the tri-party and bilateral repo markets. Martin et al. (2014) argue that these haircut differences stem from institutional arrangements; in other words, tri-party terms are "sticky" (fixed custodial agreements), whereas bilateral terms are negotiated trade by trade. The model in this paper offers an alternative explanation that stems from dealers' role as intermediaries between these two markets.

Monnet (2011) and Duffie (2013) are among the first papers to describe the risk that collateral providers face when posting securities that can be rehypothecated. Gottardi et al. (2017) show that collateral is a useful commitment device and that rehypothecation allows investors to maximizes debt capacity by generating a "collateral multiplier." Gottardi et al. (2017) also show that whenever cash borrowers are too risky, repo rehypothecation through a more creditworthy dealer is optimal. Maurin (2017) shows under what conditions rehypothecation is beneficial, balancing the trade-off between how rehypothecation relaxes borrowing constraints when collateral is scarce and how it makes the market more fragile because of lenders' option to keep the collateral. The model in this paper focuses on a specific market structure and characterizes the drivers behind contracting terms across different repo markets. In particular, the model provides clear predictions on the relative size, variability, and sign of different haircuts, all of which are qualitatively supported by the data. Andolfatto et al. (2017) shows how restrictions on rehypothecation can affect overall market liquidity and inflation, which differs from the objective of this paper to understand different repo contracting terms.

Finally, this paper is closely related to models in which dealers operate in different repo markets to bring together cash lenders with cash borrowers. Infante and Vardoulakis (2018) focus on the coordination problem faced by cash borrowers when posting collateral to a dealer and characterize panic-based runs on the dealer's assets. Eren (2015) argues that dealers rehypothecate collateral to supplant a fixed liquidity shortfall. Both of these papers assume that the contracting terms between dealers and cash lenders are the zero value-at-risk contract, an outcome that arises endogenously in this paper.

1.2. Institutional setting

The model in this paper captures key aspects of US repo markets, which can be separated into two distinct markets: the tri-party repo market and the bilateral repo market. These markets differ by the types of counterparties that participate in them, their economic motive to do so, and the relevant risks they face.

The tri-party market itself can be divided into two sub markets: the general tri-party market and the GCF Repo market. The general tri-party market is where large dealers raise funds from cash investors such as money market funds and corporate cash managers. Dealers use these funds to either finance their own securities' positions or to invest in riskier borrowers through bilateral repos, the latter of which reflects the type of intermediation chain modeled in this paper. The GCF Repo market is a smaller blind-brokered interdealer market that is primarily used to manage collateral or to finance smaller less creditworthy dealers.[3] However, the repos in this paper are conditional on knowing the identity of each counterparty, making the model inadequate to study margins in the GCF Repo market. Thus, the focus of the paper is on the general tri-party market.[4] Since 2010, the total size of the tri-party market has been on the order of $1.5 trillion.[5] The tri-party market

[3] See Fleming and Garbade (2003) for details on the GCF Repo market.

[4] For the rest of the paper, I shall refer to the general tri-party market simply as the tri-party market.

[5] An operational feature of the tri-party market is the presence of two clearing banks that provide clearing and settlement services. Prior to the crisis, clearing banks extended large amounts of intraday credit so that cash borrowers could access their securities throughout the day. The system's reliance on intraday credit was deemed a significant risk factor, and

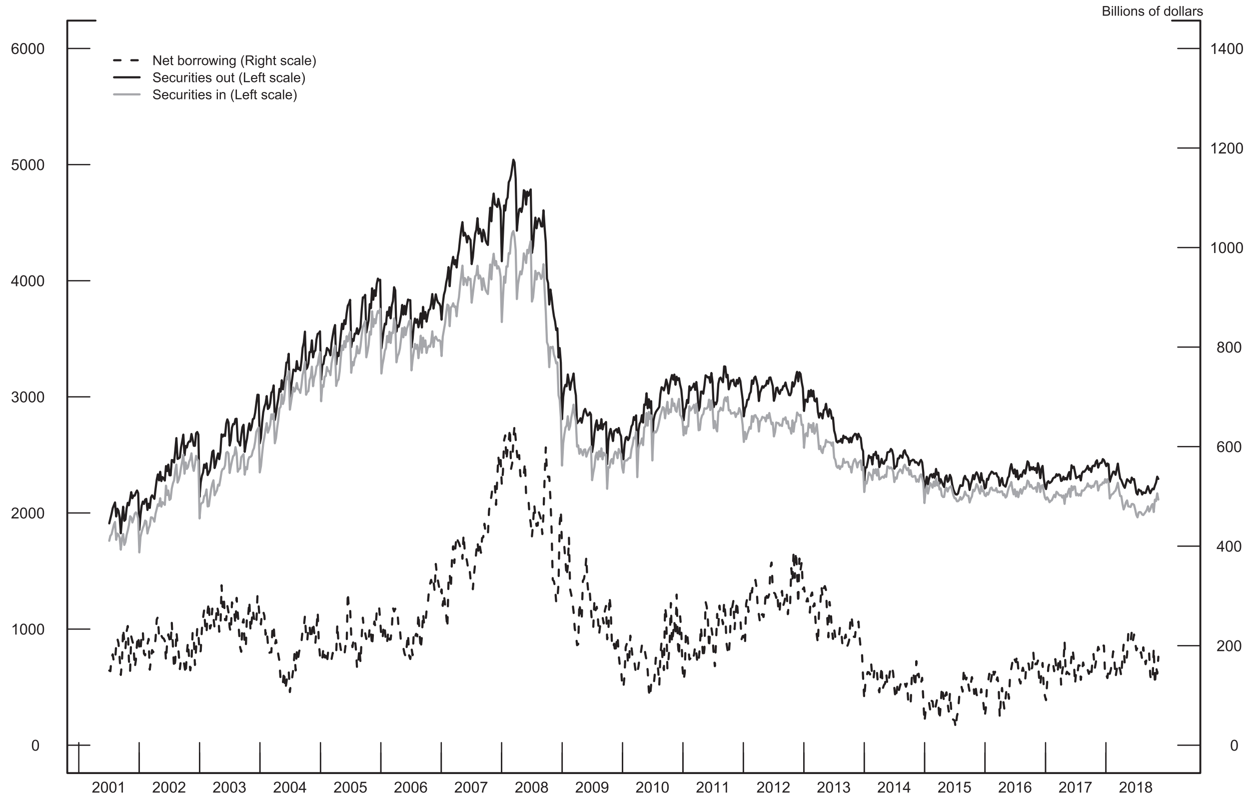

Fig. 2. Total primary dealer secured lending and secured borrowing borrowing. The figure shows the weekly time series of the total amount of dealer secured lending (Securities in), secured borrowing (Securities out), and net borrowoing (Net borrowing) for all securities. Securities in include all reverse repo transactions and securities borrowing transactions. Securities out includes all dealer repo transactions and securities lending transactions. Dealer net borrowing is calculated by securities out minus securities in. Data are from 4/03/2001 to 11/14/2018. Source: FR 2004.

is just one market among a broader class of money markets, making it a competitive market for cash lending.

The bilateral repo market is where more opaque and less creditworthy investors, such as hedge funds, finance their security positions. This market is also used to source and deliver specific securities to either intermediate collateral or facilitate short positions. As detailed in Adrian et al. (2013) and Baklanova et al. (2015), securities lending and borrowing transactions are economically equivalent to repos and reverse repos, respectively.[6] The limited amount of data on the bilateral market makes it difficult to know the market's precise size, but some estimates are in the order of $1.8 trillion.[7]

Raising funds from one repo market to invest in another is the type of rehypothecation chain modeled in this paper. Because of the opacity of their operations, and counterparty credit concerns, hedge funds cannot raise funding directly from the tri-party market. Thus, dealers play a critical role in providing secured financing to speculative investors through rehypothecation. Evidence on the importance of dealers' role as repo intermediaries can be gleaned through the Federal Reserve Bank of New York's weekly Survey of Primary Dealers (FR 2004). This survey captures dealers' securities positions and secured financing transactions. In particular, respondents report the total amount of cash received (securities out) and cash lent (securities in) through securities financing transactions, which include repos/securities lending and reverse repos/securities borrowing, respectively. The difference between securities out and securities in is a proxy for the total amount of cash the dealer receives through their secured financing activity. Fig. 2 shows time series of primary dealers' securities financing activity for all collateral classes reported in the FR 2004.

The first observation is that the majority of securities in transactions (grey line) seem to be financed with securities out transactions (black line), underscoring the importance of the rehypothecation chain modeled in this paper.[8] The second observation is that the amount of liquidity that

since then, tri-party repo reform has significantly reduced the amount of intraday credit. This operational feature is not present in the model, as it is assumed that dealers and money market funds are able to clear and settle trades between themselves seamlessly. However, the model's exogenous probability of a dealer default may be interpreted as a clearing bank's unwillingness to extend intraday credit, resulting in the dealer's bankruptcy.

[6] A reverse repo is a repo from the perspective of a cash lender.

[7] See Copeland, A., Davis, I., LeSueur, E., Martin, A., 2014a. Lifting the veil on the US bilateral repo market. FRB of New York Liberty Street Economics blog, July.

[8] Though a fraction of these repos are between dealers in the GFC Repo market, evidence reported by Baklanova et al. (2016) suggests that the bilateral market makes up a sizable fraction of dealers' repo activity.

dealers obtain through their repo activity (dashed line) is large. The sheer size of these markets, and dealers' role as intermediaries between them, imply that any difference in contracting terms can have important implications for dealers' access to liquidity. Using data similar to those used in Copeland et al. (2014), Fig. 1 shows that the difference in haircuts between the bilateral and tri-party market can be large for riskier collateral classes.

1.2.1. Repo rehypothecation in the United States

In the United States, dealers' use of customer assets is limited. Securities Exchange Act Rule 15c3-3 allows dealers to rehypothecate up to 140% of a customer's total loan balance and prohibits dealers from financing their own activity with client assets.[9] These types of restrictions were enacted in response to the 1968 crisis, known as the "Paperwork Crunch," when an increase in trading volume from brokerage firms' reuse of client assets overwhelmed the financial system. This increase in trading caused a breakdown in the processing of securities and resulted many brokerage firms to come under stress. This crisis prompted the creation of the Securities Investor Protection Act (SIPA) to shield customers from brokerage firm defaults and to restore confidence in financial markets.[10] An integral part of this legislation was the creation of the Securities Investor Protection Corporation (SIPC), designed to insure customers' securities and cash from brokerage firm failures.

Some economists have argued that these restrictions apply to repos. However, this interpretation is incorrect. The SIPA was designed to protect investors who were deemed to be customers of a broker-dealer. Customers are investors who entrust their securities to a dealer, and these securities can only be used on customers' behalf, granting them protection under SIPA. The courts have ruled that repo counterparties are not customers and therefore are not protected by SIPA. Even though the cash borrower of a repo is entitled to the collateral's interest payments, is subject to the collateral's risk, and has to account for the collateral on its balance sheet, the Master Repurchase Agreement (MRA), which governs standard repo terms, transfers full legal title to the cash lender (see paragraph 8 of the MRA). Because cash lenders own the collateral, they can use it for their own interests, effectively leaving them unrestricted to rehypothecate. This interpretation was recently upheld by the US Court of Appeals for the Second Circuit in In re: Lehman Brothers, Inc., 14-890 (2d Cir. 2015).[11] Therefore, the intermediation process described in this paper is permitted under US law and, given the total size of primary dealers' repo activities, appears to be common practice.[12]

[9] See Mitchell and Pulvino (2012) for a description of broker limits on client asset use.

[10] See Wells (2000) for more details on the Paperwork Crunch.

[11] In addition, the SIPC Modernization Task Force's 2012 report details SIPC's coverage of repos. The one exemption is a particular class of repos known as *hold-in-custody* repos, which currently represent a small fraction of the total US market (see footnote 24).

[12] I'd like to thank Matthew Eichner for detailing the role of SIPC and their treatment of repo.

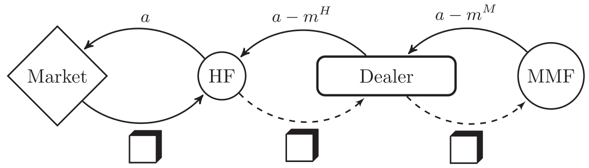

Fig. 3. Initial leg of rehypothecation with no default. Illustration of the money market fund's initial loan $a-m^M$ to the dealer, the dealer's initial loan to the hedge fund $a-m^H$, and the hedge fund's initial purchase of the underlying collateral. The collateral is purchased by the hedge fund from the market, passed on to the dealer as repo collateral (dashed line), and then posted it with the money market fund as repo collateral (dashed line).

2. Model setup

The model consists of two periods, $t \in \{0, 1\}$. In the first period, a cash borrower (hedge fund) enters into a repo with a dealer, who in turn enters into a repo with cash investors (money market funds). In the final period, the repos are repaid. I assume the cash borrower cannot bypass the dealer to receive funds directly from cash investors.[13] A detailed characterization of the model's setup is provided below.

2.1. Assets and contracts

There is one risky asset with a stochastic payoff $\tilde{a}$ in the second period. The asset has a binomial payoff distribution, taking a value of $\bar{a}$ with probability α and $\underline{a}$ with probability $1-\alpha$, where $\bar{a} > \underline{a}$. Under this probability measure, the asset's value in period 1 is $a = \mathbb{E}(\tilde{a})$. I assume the risky asset's trading price is equal to a because of an external (unmodeled) competitive investor sector pricing the risky asset.

The hedge fund enters into a repo with the dealer to purchase the risky asset $\tilde{a}$, which also serves as collateral on the repo. The amount lent on the initial leg of the repo is denoted $a-m^H$, and the final repurchase price is F^H, making the pair (m^H, F^H) the relevant contracting terms between the hedge fund and the dealer. Simultaneously, the dealer receives the asset as collateral and posts it with a money market fund though a second repo to finance the hedge fund's repo, a process known as rehypothecation. The amount lent on the initial leg of the second repo is denoted $a-m^M$, and the final repurchase price is F^M, making the pair (m^M, F^M) the relevant contracting terms between the dealer and the money market fund. Fig. 3 shows the initial leg of the rehypothecation channel, in which the dealer receives $a-m^M$ from the money market fund and distributes $a-m^H$ to the hedge fund, netting m^H-m^M. Fig. 4 shows the closing leg, in which the dealer receives F^H from the hedge fund and distributes F^M to the money market fund, netting F^H-F^M.

An important friction in the model is the assumption that if the dealer were to default, neither the hedge fund nor the money market fund have recourse to the dealer's

[13] This could be due to regulatory restrictions or a severe "lemons" problem that money market funds face when dealing with hedge funds directly.

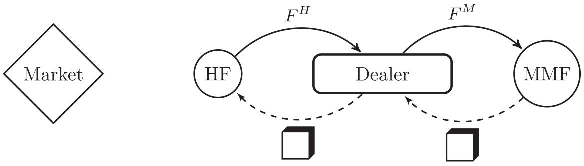

Fig. 4. Closing leg of rehypothecation with no default. Illustration of the hedge fund's loan repayment F^H to the dealer and the dealer's loan repayment F^M to the money market fund whenever the dealer does not default. The collateral is returned to the dealer (dashed line) and then returned to the hedge fund (dashed line).

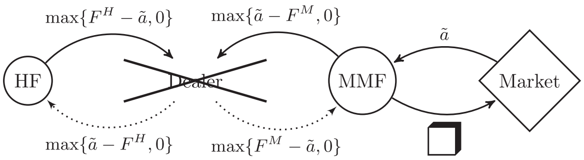

Fig. 5. Cash flows upon dealer default. Illustration of cash flows in case the dealer defaults and the asymmetric recourse assumption. The money market fund liquidates the asset to the market to recuperate its loan repayment, returning any excess to the defaulted dealer $\max\{\tilde{a} - F^M, 0\}$. The money market fund does not expect any unsecured claim on the dealer to be repaid $\max\{F^M - \tilde{a}, 0\}$ (dotted line). The hedge fund pays any unsecured claim it owes to the dealer $\max\{F^H - \tilde{a}, 0\}$. The hedge fund does not expect any unsecured claim on the dealer to be repaid $\max\{\tilde{a} - F^H, 0\}$ (dotted line).

remaining assets. Fig. 5 highlights the relevant cash flows after a dealer's default. The money market fund sells the asset to the market, and both the hedge fund and the money market fund pay the defaulted dealer any positive balance that they may owe (black lines). However, the nonrecourse assumption implies that any obligation owed by the dealer to its counterparties will not be fulfilled (dotted lines).

The money market fund sector is assumed to be competitive and prices the initial leg of the repo and final debt repayment to break even. The outcome between the hedge fund and the dealer is the result of a Nash bargaining game, in which the dealer and the hedge fund have market power θ and $1-\theta$, respectively.[14]

2.2. Agents' payoff

The three agents in the model are the original cash borrower (the hedge fund), the ultimate cash lender (the money market fund), and the intermediary between them (the dealer). For simplicity, I assume there is no time discounting.

2.2.1. Hedge fund's payoff

The hedge fund is risk neutral, has an initial endowment of W, and is optimistic about the asset's payoff. Specifically, the hedge fund believes the asset will have a high payoff with probability $\hat{\alpha} > \alpha$. This assumption generates gains from trade, creating incentives for the hedge fund to participate. The hedge fund is the residual claimant of the asset after repaying the repo, that is, the hedge fund has limited liability. The hedge fund will default whenever the value of its portfolio is lower than the repurchase price it owes to the dealer.[15] The hedge fund also internalizes the possibility that a dealer may default for reasons unrelated to its repo activity, which happens with probability $(1-p)$. In such a case, the hedge fund loses the asset, and because of the nonrecourse assumption, any unsecured claim with the dealer is worthless. This assumption captures a key incentive friction in the model: the hedge fund risks losing the asset precisely when it's most valuable, that is, when $\tilde{a} = \bar{a}$.

When $t=0$, the hedge fund starts with W, receives $(a - m^H)$ from the dealer, and pays a to purchase the asset, leaving it with $W - m^H$. When $t=1$, the asset's payoff is realized, and the hedge fund's payoff depends on the portfolio value relative to the repurchase price F^H and the dealer's solvency. Specifically, the hedge fund's payoff is given by

$$\hat{\mathbb{E}}(U^H(m^H, F^H)) = \begin{cases} p(W - m^H + \hat{a} - F^H) \\ \quad + (1-p)(W - m^H) & \text{if } F^H \le \underline{a} \\ p(W - m^H + \hat{a} - F^H) \\ \quad + (1-p)(W - m^H \\ \quad + (1-\hat{\alpha})(\underline{a} - F^H)) & \text{if } \underline{a} < F^H \le W - m^H + \underline{a} \\ p\hat{\alpha}(W - m^H + \bar{a} - F^H) \\ \quad + (1-p)\hat{\alpha}(W - m^H) & \text{if } W - m^H + \underline{a} < F^H, \end{cases} \tag{1}$$

where $\hat{\mathbb{E}}$ is the expectation under the hedge funds optimistic beliefs. To understand the hedge fund's payoff, it is useful to consider the outcome when the dealer is solvent (top lines) and the outcome when the dealer is in default (bottom lines) separately. When the dealer is solvent, the hedge fund is only subject to limited liability. That is, it receives the asset's payoff in both states if it can always repay; in other words, if $W - m^H + \underline{a} \ge F^H$. And it only receives the asset's payoff in the high state if it cannot repay in the low state; in other words, if $W - m^H + \underline{a} < F^H$. When the dealer is in default, the hedge fund is still subject to limited liability, but it loses the asset when its value is higher than the repurchase price F^H. That is, if a dealer defaults, the hedge fund can lose a valuable asset.

The hedge fund's outside option is simply to retain its initial endowment W. Also, it is implicitly assumed that the hedge fund's margin is lower than it's initial endowment, allowing the hedge fund to purchase the asset with the incoming funds on the initial leg of the repo.

[14] The relative competitiveness between markets highlights the institutional arrangement of the US market. The dealer-money market fund market (tri-party market) is deep and liquid, with dealers having access to multiple cash lenders. The dealer-hedge fund market is more opaque, with hedge funds concentrating their activities with a few dealers. More details on the institutional setting are in Section 1.2.

[15] The hedge fund defaults when its portfolio value is lower than its outstanding repurchase price. That is, the hedge fund defaults whenever it is insolvent. This type of event is markedly different than a repo fail, where a solvent counterparty fails to deliver cash or collateral, either strategically or because of an error in the clearing or settlement process. In this model, if an agent fail to repay or return collateral, they are in default.

2.2.2. Dealer's payoff

The dealer has quasi-linear utility in $t = 1$, no initial wealth, and a higher marginal value for funds in $t = 0$ denoted by $v(\cdot)$ with $v' > 1$ and $v'' < 0$. This assumption is the second main friction in the model and captures dealers' ability to use extra funds to finance other investments or supplant any liquidity shortfalls in other activities (unmodeled). That is, in the initial leg, the dealer nets $m^H - m^M$, getting a payoff of $v(m^H - m^M)$. This gives the dealer incentives to have a higher margin with the hedge fund than with the money market fund. Specifically, the dealer's payoff can be expressed as

$$\mathbb{E}(U^D(m^H, F^H, m^M, F^M)) = \begin{cases} v(m^H - m^M) \\ \quad + F^H - F^M & \text{if } F^H \leq W - m^H + \underline{a} \\ \\ v(m^H - m^M) \\ \quad + \alpha F^H + (1-\alpha) \\ \quad \times (W - m^H + \underline{a}) - F^M & \text{if } W - m^H + \underline{a} < F^H, \end{cases} \tag{2}$$

where in the down state the dealer gets paid in full if the hedge fund is solvent or gets whatever is left in the hedge fund's portfolio if the hedge fund defaults. For simplicity, the dependence of the dealer's payoff to the probability of default, in which case it is assumed the dealer gets nothing, is omitted. Recall that the dealer's default is independent of its rehypothecation activity. Thus, if p were to be included in the payoff $p\mathbb{E}\big(U^D(m^H, F^H, m^M, F^M)\big)$, the outside option (i.e., not intermediating the repo) would also be affected; in other words, the outside option would be $pv(0) := pv_0$. Therefore, p can be omitted from the dealer's payoff.

The dealer will face a budget constraint in $t = 0$, ensuring its ability to finance both repos in the initial leg; that is $m^H - m^M \geq 0$. The dealer will also face a global financing constraint in $t = 1$, ensuring its ability to pay back the money market fund whenever it doesn't default; that is $m^H - m^M + F^H - F^M \geq 0$. This last restriction implies the dealer cannot default because of the rehypothecation process. In the unique equilibrium of the model, both of these constraints are slack. This implies that even if the dealer could default in the rehypothecation process, in equilibrium, it does not.[16]

2.2.3. Money market fund's payoff

The money market fund is (possibly) risk averse, consuming its final wealth in $t = 1$ with a utility function $u^{M'} > 0$ and $u^{M''} \leq 0$. The money market fund gives the initial loan to the dealer, internalizing the possibility that it may default. If the dealer defaults, the money market fund has immediate access to the collateral, allowing it to liquidate the collateral to make its claim whole. In other words, repo is exempt from automatic stay.[17] All cash flows above and beyond the face value of the original repo must be returned to the borrower. Therefore, the money market fund's final payoff is given by

$$\mathbb{E}(U^M(m^M, F^M)) = \begin{cases} u^M(F^M - (a - m^M)) & \text{if } F^M \leq \underline{a} \\ \\ pu^M(F^M - (a - m^M)) \\ \quad + (1-p)[\alpha u^M(F^M - (a - m^M)) \\ \quad + (1-\alpha)u^M(\underline{a} - (a - m^M))] & \text{if } \underline{a} < F^M, \end{cases} \tag{3}$$

where the money market fund gets paid in full if the repurchase price is lower than $\tilde{a} = \underline{a}$, completely insulating it from the dealer through the collateral, but may take a loss otherwise. If the repurchase price is higher than $\tilde{a} = \underline{a}$, the money market fund will still get paid in full if the dealer is solvent, but it will be exposed to the asset's down side if the dealer defaults. The money market fund sector is assumed to be competitive; thus, in equilibrium, money market funds break even. Their outside option is the payoff from not investing: $u^M(0) := u_0^M$.[18]

3. The intermediary problem and equilibrium

This section formalizes the intermediation problem and characterizes the resulting equilibrium. The dealer and hedge fund enter a Nash bargaining problem, setting the terms of their repo to split the surplus between them, while the dealer also chooses the money market funds contracting terms so that it breaks even.

$$\max_{\{m^H, m^M, F^H, F^M\}} \Big(\mathbb{E}\big(U^D(m^H, F^H, m^M, F^M)\big) - v_0\Big)^{\theta} \times \Big(\hat{\mathbb{E}}\big(U^H(m^H, F^H)\big) - W\Big)^{1-\theta}$$

subject to

$m^H \geq m^M$	Initial financing constraint
$m^H + F^H \geq m^M + F^H$	Global financing constraint
$\mathbb{E}\big(U^M(m^M, F^M)\big) \geq u_0^M$	Money market fund break even
$m^H \leq W$	Hedge fund down payment constraint.

The first constraint ensures that the initial loan from the money market fund is enough to finance the hedge fund's repo. The second constraint is that the dealer does not default because of the rehypothecation process. The third constraint ensures the money market fund participates. The fourth constraint ensures that the hedge fund has a large enough endowment to purchase the asset with its initial repo. Note that the model does not put restrictions on the sign of the contracting terms.[19] The following theorem characterizes an equilibrium of this model.

Theorem 1 (Solution to the intermediary problem). *If the following parameter assumptions*

[16] Alternatively, it can be assumed that the dealer does not have limited liability in $t = 1$, implying that it can have a negative cash flow to repay the money market fund, giving the same result.

[17] Automatic stay is a US bankruptcy provision that prohibits creditors from collecting payments from a borrower who files for bankruptcy. Under current US law, repos are exempt from automatic stay, allowing lenders to immediately sell their collateral in case of default.

[18] I am implicitly assuming that the money market fund has enough wealth to issue the loan in the first place.

[19] This flexibility is crucial to study an extension of the model, analyzed in Section 4, in which a pessimistic hedge fund reverses the intermediation process. Specifically, the hedge fund borrows the asset to take a short position.

(i) $p\left(\frac{1-\alpha}{\alpha}\right) > \max\left\{\frac{\alpha}{1-\alpha}, \frac{1-\hat{\alpha}}{\hat{\alpha}}\right\}$
(ii) $p(\hat{a}-\underline{a}) \geq W$
(iii) m^* *that solves* $v'(m^* - (a-\underline{a})) = \frac{\alpha}{p} + (1-\alpha)$ *is less than* W

hold, then there exists a θ_1 *and* θ_2 *such that*

1. *For* $\theta \in [\theta_1, 1)$, *the intermediary problem has a unique equilibrium* $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ *given by* $m^{H*} = m^*, F^{H*} = \theta F_{MonoD} + (1-\theta)F_{MonoH}$, *where*

$$F_{MonoD} = \bar{a} - \frac{((1-\hat{\alpha})W + \hat{\alpha}m^*)}{\hat{\alpha}},$$

$$F_{MonoH} = \underline{a} - \frac{v(m^* - (a-\underline{a})) - v_0 + (1-\alpha)(W-m^*)}{\alpha},$$

and $m^{M*} = a - \underline{a}, F^{M*} = \underline{a}$.

2. *For* $\theta \in [\theta_2, \theta_1)$, *the intermediary problem has a unique equilibrium* $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ *given by*

$$m^{H*} + F^{H*} = W + \underline{a} \quad \text{and} \quad \frac{(1-\theta)\left(\mathbb{E}\left(U^D\right) - v_0\right)}{\theta\left(\hat{\mathbb{E}}\left(U^H\right) - W\right)} = \frac{v'(m^{H*} - m^{M*}) - 1}{\hat{\alpha}(1-p)},$$

and $m^{M*} = a - \underline{a}, F^{M*} = \underline{a}$.

Proof. See Appendix. □

The unique optimal contracting terms of the intermediation problem completely insulate the money market fund from the dealer's default and exposes the hedge fund to the potential loss of its collateral. The hedge fund's contracting terms are separated into two cases. For relatively high levels of dealer market power (i.e, $\theta \in [\theta_1, 1)$), the optimal repurchase price is high, making it more difficult for the hedge fund to repay its repo and thus forcing it to default in the event of a bad asset outcome. For relatively low levels of dealer market power (i.e, $\theta \in [\theta_2, \theta_1)$), the optimal repurchase price is low, allowing for the hedge fund to pay off it's outstanding repo irrespective of the asset outcome. The equilibrium is determined by the two main frictions of the model: the dealer's preference for cash in $t = 0$ and the nonrecourse assumption on the defaulted dealer. Intuitively, the dealer's preference for cash in $t = 0$ creates incentives to widen the difference in contract margins, increasing the initial cash windfall. In contrast, the nonrecourse assumption incentivizes counterparties to reduce their exposure to the dealer, decreasing the initial cash windfall. In effect, under the nonrecourse assumption, the hedge fund's exposure is proportional to the size of its margin, while the money market fund's exposure is inversely proportional to the size of its margin; thus, the friction creates incentives to reduce the difference in contract margins.

The precise way in which these frictions give rise to the theorem's unique equilibrium can be understood by inspecting each agent's marginal rate of substitution (MRS) between the relevant contracting terms.[20] To understand the forces that shape the equilibrium, it is instructive to study how each friction in isolation affects agents' incentives, and then see how both frictions together result in the final equilibrium outcome. The mechanisms at play can be understood by studying each contracting relationship separately.

Starting with the dealer-money market market fund contract, for $\theta \in [\theta_2, 1)$ the optimal contract imposes the lowest margin such that the asset fully insures the money market fund in case of a dealer default; in other words, the optimal contract is the zero value-at-risk contract. The top and middle panels of Fig. 6 show the dealer-money market fund indifference curves in (m^M, F^M) under each friction in isolation, and the bottom panel puts both frictions together. In all panels of Fig. 6, the money market fund is risk neutral. For these indifference curves, the dealer prefers smaller contracting terms, and the money market fund prefers larger ones. The top panel illustrates how the dealer's preference for liquidity generates curvature in the dealer's MRS while leaving the money market fund's MRS equal to one, that is, for the money market fund, contracting terms are perfect substitutes. This pushes the equilibrium to maximize the dealer's initial cash windfall, driving the money market fund margin to zero, $m^M = 0$. The middle panel illustrates how the nonrecourse assumption puts a kink in the money market fund's MRS while leaving the dealer's MRS equal to one. The need to compensate the money market fund in case of a dealer's default pushes the equilibrium to any contracting term where the money market fund is completely insulated from the dealer, that is, $F^M \leq \underline{a}$. The bottom panel of Fig. 6 puts both of these two frictions together, resulting in the following optimal contracting terms: $(m^{M*}, F^{M*}) = (a-\underline{a}, \underline{a})$. These contracting terms maximize the initial cash windfall while maintaining the zero value-at-risk contract.

In theory, the curvature of the dealer's MRS could be so large that it would be optimal to expose the money market fund to the dealer's default. That is, the desire to increase the initial cash windfall could be large enough to reduce the money market fund's margin further, exposing it to a possible loss. But in the unique equilibrium characterized in Theorem 1, the dealer's MRS is strictly lower than the money market fund's MRS when $F^M \geq \underline{a}$, even when the money market fund is risk neutral. That is,

$$v'(m^{H*} - m^{M*}) < \frac{1}{p + \alpha(1-p)} \leq \frac{(p+\alpha(1-p))u^{M'}(F^M - (a-m^M)) + (1-\alpha)(1-p)u^{M'}(\underline{a} - (a-m^M))}{(p+\alpha(1-p))u^{M'}(F^M - (a-m^M))}. \tag{4}$$

The first inequality in Eq. (4) stems from the optimality condition for $\theta \in [\theta_1, 1)$, as well as the parameter assumption in i), and is sustained for $\theta \in [\theta_2, \theta_1)$.[21]

[20] The MRS between contracting terms can be simply calculated by inspecting agents' payoff functions. Specifically, MRS for agent j is $\frac{\partial F^i}{\partial m^i} = -\frac{\partial U^j}{\partial m^i} \Big/ \frac{\partial U^j}{\partial F^i}$.

[21] In effect, θ_2 is pinned down by $v'(m^* - (a-\underline{a})) = \min\left\{\frac{1}{p+\alpha(1-p)}, \frac{1}{p+(1-\hat{\alpha})(1-p)}\right\}$. Details in are in the proof of Theorem 1.

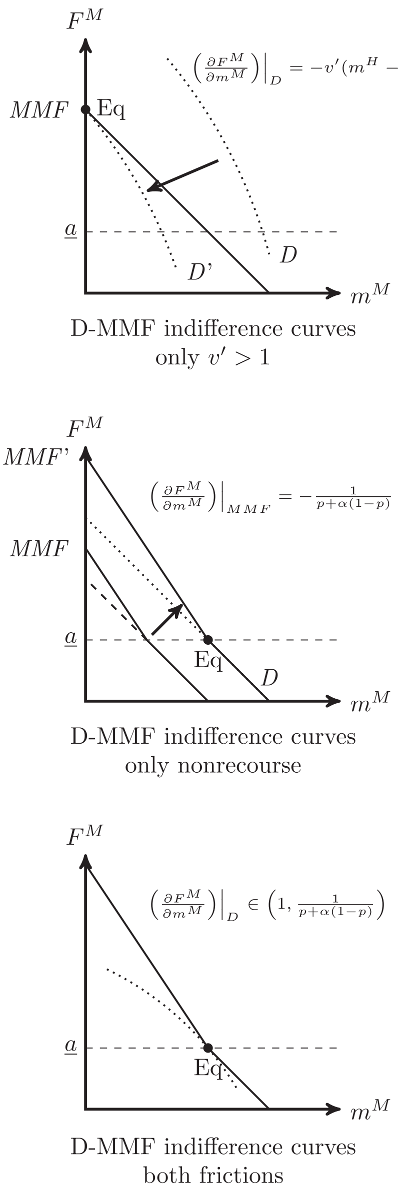

Fig. 6. Indifference curves with dealer-money fund contracting terms. The figure shows the D-MMF indifference curves for a risk neutral MMF where the MMF prefers larger contracting terms and the D prefers smaller ones. The top panel illustrates how D's preference for liquidity in isolation affects the optimal contracting terms: MMF is indifferent between the initial margin and final repurchase price, while D's preference for liquidity induces it to decrease the margin to maximize the initial cash windfall—the *D* indifference curve shifts down to *D*', resulting in $m^M = 0$. The middle panel illustrates how the nonrecourse assumption in isolation affects the optimal contracting terms: D is indifferent between the initial margin and final repurchase price, while MMF can increase it's payoff by accepting any risk-free contract ($F^M \leq \underline{a}$)—the *MMF* indifference curve shifts up to *MMF*', resulting in $m^M \geq a - \underline{a}$. The bottom panel shows the equilibrium outcome between D-MMF with the two frictions put together. The optimal contract is the lowest possible margin that makes the MMF repo risk free, that is, $m^M = a - \underline{a}$.

Parameter assumption i) is strong enough so that it is optimal to insulate the money market fund from the dealer's default, even with a risk neutral money market fund. Specifically, the portion of condition i) that ensures a risk-free dealer-money market fund contract is $p(1-\alpha)/\alpha > \alpha/(1-\alpha)$, which can be interpreted as assuming that the probability of a bad asset outcome relative to a good one must be sufficiently higher than the probability of a good asset outcome relative to a bad one.[22] This condition suggests that when facing a risk neutral money market fund, the result is more likely to hold for assets that, for a given *p*, the probability of a bad outcome is large relative to a good one.

If the money market fund were risk averse, it would be easier to sustain the zero value-at-risk contract. In effect, in the case of a risk-averse money market fund, $u^{M''} < 0$, the second inequality in Eq. (4) stems from the concavity of its payoff function. That is, if $F^M > \underline{a}$ the MRS of a risk-averse money market fund would be even larger than that of the dealer, making it suboptimal to expose the money market fund to the dealer's default. This second inequality shows that the conditions on the asset's distribution can be relaxed with higher money market fund risk aversion. For example, with an infinitely risk averse money market fund, any distribution of $\tilde{a}$ with a finite support would give the same result. In this sense, the money market fund's risk aversion makes these contracting terms more likely.[23,24]

Turning to the dealer-hedge fund contract, for $\theta \in [\theta_1, 1)$, the optimal contract equalizes the dealer and hedge fund's MRS, with the hedge fund risking default in the event of a bad asset outcome. The top and middle panels in Fig. 7 shows the dealer-hedge fund indifference curves in (m^H, F^H) under each friction in isolation, and the bottom panel puts both frictions together. For these indifference curves, the dealer prefers larger contracting terms, and the hedge fund prefers smaller ones. The top panel illustrates how the dealer's preference for liquidity generates curvature in the dealer's MRS while leaving the hedge fund's MRS is equal to one. The dealer's preference for liquidity pushes the equilibrium to maximize the initial cash windfall, that is, it places the initial margin as high as possible. The middle panel illustrates how the nonrecourse assumption puts two kinks in the hedge fund's MRS, while the dealer's MRS is equal to one. The two kinks in the hedge fund's MRS occur when the contracting terms force the hedge fund to default in case of a bad asset outcome ($F^H = \underline{a} + W - m^H$) and when the repurchase

[22] Note that a necessary condition to have zero value-at-risk contracts is that the asset payoff distribution has finite support.

[23] If there is positive mass on the worst case outcome, a sufficiently risk averse money market fund will lead to the zero value-at-risk contract. Note that if there is no mass on the worst case outcome, the dealer-money market fund contract may not be absolutely risk free, but the risk can be arbitrarily low for a high enough risk aversion.

[24] Although not explored in Theorem 1, there could be equilibrium outcomes that result in a risky dealer-money market fund contract. These contracting terms would involve equalizing the dealer and money market fund's MRS as well as the dealer and hedge fund's MRS. As will be discussed in Section 3.3, the zero value-at-risk contract between the dealer and money market fund is more empirically relevant.

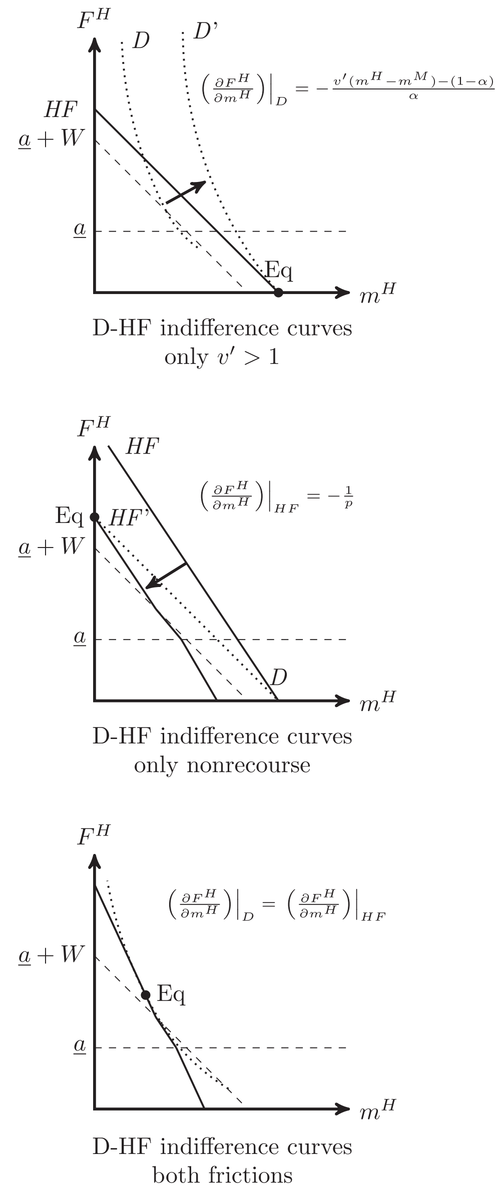

Fig. 7. Indifference curves for dealer-hedge fund contracting terms. The figure shows the D-HF indifference curves where the D prefers larger contracting terms and the HF prefers smaller ones. The top panel illustrates how D's preference for liquidity in isolation affects the optimal contracting terms: HF is indifferent between the initial margin and final repurchase price, while D's preference for liquidity induces the it to increase the margin to maximize the initial cash windfall—the *D* indifference curve shifts up to *D*', resulting in the highest possible m^H. The middle panel illustrates for the nonrecourse assumption in isolation affects the optimal contracting terms: D is indifferent between the initial margin and final repurchase price, while HF can increase it's payoff reducing the initial margin to minimize the loss of a good asset—the *HF* indifference curve shifts down to *HF*', resulting in $m^H = 0$. The bottom panel shows the equilibrium outcome between D-HF with the two frictions put together. The optimal contract involves equalizing D and HF's marginal rate of substitution in the range where the HF defaults in case of a bad outcome ($m^H + F^H > \underline{a} + W$), that is, $\frac{v'(m^H - m^M) - (1-\alpha)}{\alpha} = \frac{1}{p}$.

price is equal to the worst outcome ($F^H = \underline{a}$). Because the dealer is indifferent between the two contracting terms, the optimal contract reduces the burden of a dealer's default on the hedge fund, eliminating the hedge fund's exposure to the dealer, that is, $m^H = 0$.

The bottom panel of Fig. 7 puts these two frictions together for large levels of dealer market power ($\theta \in [\theta_1, 1)$), where the optimal dealer-hedge fund margin equalizes the dealer and hedge fund's MRS. Specifically, for $F^H > \underline{a} + W - m^H$, the hedge fund defaults after a bad asset outcome and its MRS is $1/p$, while the dealer's MRS is equal to $(v' - (1-\alpha))/\alpha$. In this case, the dealer internalizes that a higher hedge fund margin reduces part of the recovery value in case the dealer defaults (with probability $1-\alpha$), but a higher repurchase price only pays off after a good asset outcome (with probability α). Thus, for $F^H > \underline{a} + W - m^H$, the dealer is exposed to the asset's risk, creating a difference between the margin and repurchase price, and leveraging the dealer's preference for liquidity today. That is, this leverage effect increases the relative benefit of the initial cash windfall. The optimal margin is pinned down by equalizing the MRS of the dealer and hedge fund.[25] Having determined the optimal margin, the optimal repurchase price is pinned down by splitting the surplus between the dealer and hedge fund, depending on each agents relative market power. A higher level of dealer market power (i.e., higher θ) corresponds to a higher dealer-hedge fund repurchase price F^H.

For smaller levels of dealer market power ($\theta \in [\theta_2, \theta_1)$), F^H reaches $\underline{a} + W - m^H$ at which point it is optimal to decrease the dealer-hedge fund margin to increase the marginal benefit of a liquidity windfall in $t = 0$. The equilibrium is sustained until the v' is so large that it becomes optimal to expose a risk neutral money market fund to a possible loss (i.e., condition (4) does not hold), which is one of the conditions that determine θ_2.

To understand how the optimal dealer-money market fund and dealer-hedge fund contracting terms interact, it is useful to compare the hedge fund's and money market fund's MRS directly. When $F^H > \underline{a} + W - m^H$, the hedge fund's MRS is $1/p$, and a risk neutral money market fund's MRS is $1/(p + \alpha(1-p))$. Comparing the MRS of the two may lead one to think that it is optimal to decrease the money market fund's margin rather than increase the hedge fund's because the money market fund is protected by the underlying collateral.[26] However, although this observation is true, it does not take into account how the dealer values altering both contracts. In effect, when interacting with the money market fund, the dealer's MRS is simply v'. But when interacting with the hedge fund, the dealer accounts for the hedge fund's default after a

[25] The equality would be optimal for any distribution with finite support. Because $v' > 1$, by increasing the repurchase price for the initial margin, the leverage of the dealer's exposure increases, amplifying the effect of the initial cash windfall, making it arbitrarily large. The hedge fund's trade-off between contracting terms is driven by the possibility that it loses its margin when the dealer defaults; i.e., $\approx 1/p$. This ensures that the MRS of both can be equalized for arbitrary asset distributions.

[26] Formally, because $1/p > 1/(p + \alpha(1-p))$, $m^H - m^M$ could be larger by equalizing $v' = 1/(1 - (1-\alpha)(1-p))$.

bad asset outcome, altering the dealer's trade-off between contracting terms and increasing the relative benefit for a higher m^H. This can be interpreted as a leverage effect. Thus, from a dealer's perspective, it is preferable to increase the hedge fund's margin rather than decrease the money market fund's margin, resulting in the unique equilibrium characterized by Theorem 1.

3.1. Equilibrium comparative statics

Having characterized the repo intermediation equilibrium, it is important to understand how the feasibility of such an equilibrium can change with p. The following proposition highlights how the interval of acceptable equilibrium changes with a dealer's solvency probability.

Proposition 1 (Feasibility as dealer risk changes). *Given the equilibrium characterized in Theorem 1 with p sufficiently close to 1, the lower bound threshold θ_2 is decreasing in p.*

Proof. See Appendix. □

Proposition 1 shows that the interval of acceptable equilibrium in Theorem 1 shrinks as the dealer's probability of default increases (i.e., p decreases). Because the unique optimal contracts exposes the hedge fund to a dealer's default through the loss of a highly valued asset while leaving the money market fund insulated through the collateral, a higher probability of default reduces the hedge fund's incentives to participate. In other words, a higher dealer default probability increases the hedge fund's incentives to withdraw its collateral. This effectively decreases the feasible set where the dealer engages in rehypothecation.

It is also important to understand how the contracting terms change for different levels of the dealer's counterparty risk.

Proposition 2 (Margins as dealer risk changes). *Given the equilibrium characterized in Theorem 1 for $\theta \in [\theta_1, 1)$*

$$\frac{\partial m^{H*}}{\partial p} = -\frac{\alpha}{p^2 v''(m^{H*} - m^{M*})} > 0,$$

and for $\theta \in [\theta_2, \theta_1)$,

$$\frac{\partial m^{H*}}{\partial p} = \frac{(1-\theta)\left(\mathbb{E}\left(U^D\right) - v_0\right) + \theta\left(v'(m^{H*} - m^{M*}) - 1\right)\hat{\alpha}(\overline{a} - F^{H*})}{\hat{\alpha}(1-p)(v'(m^{H*} - m^{M*}) - 1) - \theta\left(\hat{\mathbb{E}}\left(U^H\right) - W\right)v''(m^{H*} - m^{M*})} > 0.$$

Proof. See Appendix. □

Proposition 2 indicates that as the dealer's probability of remaining solvent increases, so does the dealer-hedge fund margin. Intuitively, as the dealer's probability of default decreases (i.e., p increases), the hedge funds unsecured claims are less important, which increases the surplus in the relationship. That is, the trade-off between the hedge fund's nonrecourse to the dealer and the dealer's preference for liquidity is altered because the expected loss is smaller. Therefore, a larger margin is optimal.

The collateral plays no meaningful role in this result. Proposition 2 can be interpreted as borrowers adjusting their unsecured claims to changes in a dealer's default. In other words, as a dealer's probability of default increases, a borrower would want to withdraw more cash from the dealer to reduce its overall exposure, which in this context translates into a lower m^H. This result is in line with empirical evidence documenting that corporate cash borrowers and prime brokerage clients draw down credit lines from dealers that are perceived to have increased credit risk (see Ivashina and Scharfstein, 2010; Duffie, 2013).

3.2. Equilibrium with correlated dealer default and asset outcome

The model can be enriched by considering the possibility of a correlation between the dealer's solvency and the asset's outcome. The idea is to capture a situation in which the dealer is heavily invested in the underlying collateral that it's rehypothecating. Specifically, I assume that

$$\mathbb{P}(\text{dealer solvent}|\tilde{a} = \underline{a}) = \rho, \tag{5}$$

with $\rho \in [0, p]$. If $\rho = p$, then the asset outcome is independent of the dealer's default. As ρ decreases, the dealer's solvency depends more on the asset's payoff. This can be taken to the extreme, where the dealer defaults whenever $\tilde{a} = \underline{a}$, that is, $\rho = 0$.

In this section, I analyze how the contracting terms change whenever the dealer's default is correlated with the repo's collateral, specifically how the dealer-hedge fund margin changes as ρ decreases.

Proposition 3 (Margins as correlation changes). *There exists a unique equilibrium to the intermediary problem when $\rho \in [0, p]$ that exposes the hedge fund to, and insulates the money market fund from, the dealer's default. In this equilibrium, if p is sufficiently close to 1, then the optimal dealer-hedge fund margin is decreasing in ρ.*

Proof. See Appendix. □

Proposition 3 shows that an increase in correlation between the dealer's solvency and the asset's outcome (i.e., ρ decreases) implies a higher dealer-hedge fund margin. The intuition behind the result stems from the observation that when a dealer defaults, the hedge fund's loss only materializes after a good asset outcome. Therefore, if states of the world when the dealer defaults coincide with a bad asset outcome, then the ex-ante expected hedge fund loss is smaller.

This result suggests that when the dealer is heavily exposed to the underlying collateral, the hedge fund is willing to increase its exposure to the dealer, which, in this context, translates into a higher m^H. In other words, if the hedge fund is more likely to lose an asset when it's value is low, then it's more willing to increase its exposure.

3.3. Model predictions and their relation to the financial crisis

The equilibrium in Theorem 1 and the subsequent comparatives statics characterized in Propositions 1–3 provide a number of important insights that are relevant for repo rehypothecation. To better interpret the model outcome, it is useful to translate the model's contracting terms into the ones observed empirically. The haircut is defined as one minus the ratio of cash lent over the collateral's value. Using the optimal contracting terms from Theorem 1, and denoting ϕ_t^{*M} as the equilibrium dealer-money market fund haircut at time t, changes in haircuts translate to

$$\phi_{t+1}^{*M} - \phi_t^{*M} = \frac{a_t - m_t^{*M}}{a_t} - \frac{a_{t+1} - m_{t+1}^{*M}}{a_{t+1}} = \frac{\underline{a}_t}{a_t} - \frac{\underline{a}_{t+1}}{a_{t+1}}. \quad (6)$$

In other words, changes in model implied dealer-money market fund haircuts are equal to changes in the worst possible asset return, which is the zero value-at-risk contract. Under the assumption of stationary returns, this implies that $\phi_{t+1}^{*M} - \phi_t^{*M} = 0$. This is consistent with the empirical evidence from the US tri-party market, which brings together dealers and cash lenders. Krishnamurthy et al. (2014) and Copeland et al. (2014) show that, conditional on the underlying collateral's asset class, during the financial crisis, haircuts in the tri-party repo market are relatively homogeneous, suggesting that margins in this market are largely asset specific. Copeland et al. (2014) also argue that there was no system-wide run on repo, suggesting that on aggregate, the stable haircuts in the tri-party were large enough to protect cash lenders, consistent with the zero value-at-risk contract.

Turning to the dealer-hedge fund contract, denoting ϕ_t^{*H} as the equilibrium dealer-hedge fund haircut at time t, changes in haircuts translate to

$$\phi_{t+1}^{*H} - \phi_t^{*H} = \frac{a_t - m_t^{*H}}{a_t} - \frac{a_{t+1} - m_{t+1}^{*H}}{a_{t+1}} = \frac{a_t m_{t+1}^{*H} - a_{t+1} m_t^{*H}}{a_t a_{t+1}}. \quad (7)$$

In other words, dealer-hedge fund haircuts increase if and only if $a_t m_{t+1}^{*H} > a_{t+1} m_t^{*H}$. Recall that Propositions 2 and 3 have opposing predictions on how dealer-hedge fund margins should respond to changes in a dealer's risk profile. On the one hand, a decrease in a dealer's probability to remain solvent between t and $t+1$ (i.e., a lower p) implies $m_{t+1}^{*H} < m_t^{*H}$. On the other hand, a decrease in the dealer's probably to remain solvent conditional on a bad asset outcome between t and $t+1$ (i.e., a lower ρ) implies $m_{t+1}^{*H} > m_t^{*H}$. Intuitively, these two events can be characterized as an increase in dealer stress, yet the model predicts two opposing margin sensitivities. The increase in bilateral haircuts documented by Gorton and Metrick (2012), and shown in Fig. 1, are more consistent with a decrease in the dealer's probably to remain solvent conditional on a bad asset outcome, that is, the result in Proposition 3.

Therefore, one interpretation of the increase in haircuts during the 2007–2009 financial crisis is that dealers extending repo financing to hedge funds specializing in mortgage-backed securities (MBS) were themselves heavily invested in the asset class. As concerns over the MBS market grew, so did concerns over the solvency of dealers active in that market. These concerns, coupled with a decrease in valuations, implies that $a_t m_{t+1}^{*H} > a_{t+1} m_t^{*H}$, accounting for the increase in haircuts documented in Gorton and Metrick (2012).

The narrative of dealers heavily exposed to mortgage related products is consistent with the experience of the most emblematic dealers that came under stress during the financial crisis: Bear Stearns and Lehman Brothers. In the case of Bear Stearns, in addition to its reportedly large exposure to the mortgage market, prior to the firm's take over, it pledged \$3.2 billion dollars to support one of its troubled hedge funds invested in credit debt obligations linked to the subprime mortgage market.[27] Bear Stearns' own position, and its support for its internal hedge fund, implied a large exposure to the troubled mortgage market.[28] In the case of Lehman Brothers, the Valukas Report found that at the end of 2016, after the start of the subprime mortgage crisis, the firm's management strategically increased their exposure to the mortgage market. The report states that "...Lehman's management believe that while other financial institutions were retrenching and reducing their risk profile, Lehman had the opportunity to pick up ground and improve its competitive positions. Lehman had benefited from a similar 'countercyclical growth strategy' during prior market dislocations, and its management believed it could similarly benefit from the subprime lending crisis. Lehman miscalculated."[29] Both of these examples allude to a correlation between dealers' solvency and the performance of an underlying collateral class that saw large increases in haircuts, suggestive of the mechanism in the model.

However, it is important to note that the empirical results do not rule out an increase in the dealer's default probability, that is, the result in Proposition 2. In effect, a large drop in collateral values could still imply that $a_t m_{t+1}^{*H} > a_{t+1} m_t^{*H}$, accounting for the increase in bilateral haircuts. This insight cautions that empirical studies on repo haircut sensitivities should condition on changes in the value of the underlying collateral.

In addition to changes in dealers' risk profile, there may have been other forces that contributed to the increase in bilateral repo haircuts. For example, the solvency of dealers' hedge fund counterparties may have deteriorated for reasons unrelated to their repos. From a dealer's perspective, an increase in counterparty risk would incentivize an increase in the amount of overcollateralization on their secured lending. Although this mechanism could have contributed to the increase in bilateral repo haircuts, it does not directly speak of the stability of haircuts in the tri-party market. The model in the paper underscores the sizable role of dealers as repo intermediaries (see Fig. 2) and shows how frictions at the dealer level can help understand the differences in repo contracting terms across markets.

[27] See Kelly (2009) for a discussion on the firm's exposure to the mortgage market and detailed account of the firm's final days.

[28] While the relationship between a dealer and its own money management unit is interesting, this analysis is outside the scope of this paper. The example merely serves to highlight the exposure of a troubled dealer to an asset class that had a large increase in repo haircuts.

[29] Valukas, A.R., 2010. Lehman Brothers Holdings Inc. Chapter 11 proceedings examiner's report. US Bankruptcy Court Southern District of New York, Jenner and Block LLP.

Another interesting feature of the model comes from Proposition 1 that shows that an increase in the probability of a dealer default makes it harder to sustain a rehypothecation equilibrium. Empirically, a withdrawal of a hedge fund would imply a precipitous drop in dealer-money market fund repo volumes. This seemed to be the case for troubled dealers during the financial crisis. In effect, Copeland et al. (2014) show that Lehman's repo book shrunk significantly the week before its bankruptcy, consistent with an abrupt withdrawal of collateral. Infante and Vardoulakis (2018) also provide evidence that during Bear Stearns' final year, an important fraction of its repo funding stemmed from differences in repo margins. This suggests that Bear Stearns' secured borrowers had a relatively large exposure to the firm, giving them incentives to withdraw their collateral. In addition, Copeland et al. (2014) and Krishnamurthy et al. (2014) show a significant decline in repo backed by riskier collateral classes such as nonagency Asset Backed Securities (ABS) and corporate bonds. These authors interpret these declines as a reduction in money market funds willingness to accept repo backed by riskier collateral, but the results are also consistent with decline in repo rehypothecation driven by cash borrowers' unwillingness to post collateral.

Therefore, the insights that stem from the model are consistent with the empirical facts documented in US repo markets during the crisis: haircuts in the tri-party market were homogeneous and relatively stable, haircuts in the bilateral market were larger and more volatile, and disruptions in the tri-party market only materialized through sharp reductions in volumes.

4. Model extension of pessimistic hedge funds taking short positions

The model described thus far considers an optimistic hedge fund taking a levered long position in the risky asset. But there also may be a pessimistic hedge fund wanting to take a short position. To do so, the investor would have to borrow the asset and sell it to the market. This effectively reverses the dealer-hedge fund interaction. This extension considers a set up where the dealer needs to source the asset to deliver it to the pessimistic investor. The dealer will source the asset from a new agent called a securities lender. This new agent plays a role similar to the money market fund in the original model, but rather than lending cash, it lends a risky security.[30]

To alleviate notation, the dealer-hedge fund contracting terms and payoffs are expressed as in the original model. It should be understood that in this section, the variables represent contracting terms in a securities lending context. The dealer-securities lender contracting terms are denoted by (m^S, F^S). In this setup, the hedge fund's payoff is similar to the baseline model, but for a short asset position,

$$\check{\mathbb{E}}\big(U^H(m^H,F^H)\big)=\begin{cases} p(1-\check{\alpha})(W+m^H+F^H-\underline{a}) \\ +(1-p)(1-\check{\alpha})(W+m^H) & \text{if } F^H\le\bar{a}-(W+m^H) \\[1ex] p(W+m^H+F^H-\check{a}) \\ +(1-p)(W+m^H+\check{\alpha}(F^H-\bar{a})) & \text{if } \bar{a}-(W+m^H)<F^H\le\bar{a} \\[1ex] p(W+m^H+F^H-\check{a}) \\ +(1-p)(W+m^H) & \text{if } \bar{a}<F^H, \end{cases} \tag{8}$$

where $\check{\mathbb{E}}$ is the expectation under the hedge fund's pessimistic beliefs of the asset, with $\check{\alpha}<\alpha$ and $\check{\mathbb{E}}(\tilde{a})=\check{a}<a$. As before, to understand the hedge fund's payoff, it is useful to consider separately the outcome if the dealer is solvent (top lines) or if the dealer defaults (bottom lines). When the dealer is solvent, the hedge fund must buy the asset in period $t=1$ with its remaining wealth and the repurchase price it receives from the dealer. Again, the hedge fund is subject to limited liability; thus, when the asset value is too high to buy it back (i.e., if $F^H+(W+m^H)\le\bar{a}$) the hedge fund defaults and receives nothing. The nonrecourse assumption implies that if the dealer defaults, the hedge fund must return the difference between a high-valued asset and the repurchase price but does not get the repurchase price in the event of a low-valued asset. In other words, if the dealer defaults, the hedge fund must pay any funds it owes but does not expect to get paid on a profitable short position.

In this case, the dealer's payoff is given by

$$\mathbb{E}\big(U^D(m^H,F^H,m^S,F^S)\big)=\begin{cases} v(m^S-m^H)+ \\ F^S+\alpha(W+m^H-\bar{a})-(1-\alpha)F^H & \text{if } F^H\le\bar{a}-(W+m^H) \\[1ex] v(m^S-m^H)+F^S-F^H & \text{if } \bar{a}-(W+m^H)<F^H. \end{cases} \tag{9}$$

In the initial leg, the dealer receives $a-m^H$ from the hedge fund, distributes $a-m^S$ to the securities lender, and intermediates the underlying asset. In the final leg, when the dealer is solvent, it receives F^S and pays the hedge fund F^H. If the hedge fund were to default, the dealer would receive the hedge fund's remaining unencumbered assets, $W+m^H$, but would have to buy the asset to return it to the securities lender. In this way, the dealer is exposed to the asset's upside when the hedge fund defaults. As before, the dependence of the dealer's payoff to its probability of default is omitted.

[30] Note that securities lending contracts are economically equivalent to a reverse repos.

Similar to the money market fund in the original model, securities lender's payoff is given by

$$\mathbb{E}\big(U^S(m^S,F^S)\big) = \begin{cases} p\Big[\alpha u^S(\overline{a}-F^S+a-m^S)+(1-\alpha)u^S(\underline{a}-F^S+a-m^S)\Big]+ \\ (1-p)\Big[\alpha u^S(a-m^S)+(1-\alpha)u^S(\underline{a}-F^S+a-m^S)\Big] & \text{if } F^S \le \overline{a} \\ \alpha u^S(\overline{a}-F^S+a-m^S)+(1-\alpha)u^S(\underline{a}-F^S+a-m^S) & \text{if } \overline{a} < F^S, \end{cases} \tag{10}$$

where u^S is the securities lender's per period payoff with $u^{S'}>0, u^{S''}\le 0$. The securities lender receives the cash in the first leg of the loan and later receives the asset's payoff net of the dealer's repayment. As with the hedge fund, when the dealer defaults, there is an asymmetry in the contract's resolution: the securities lender owes the dealer in the event of a bad asset outcome, $\underline{a}-F^S$, but does not expect to get its asset back the event of a good asset outcome. In other words, the securities lender risks losing the asset when it's valuable, forfeiting $\overline{a}-F^S$.

As in the original model, I assume the dealer-hedge fund problem is a Nash bargaining game with bargaining power θ and $1-\theta$, respectively. Similar to the model with the money market fund, I assume the securities lending market is competitive and their outside option is merely holding the asset: $\mathbb{E}\big(u^S(\tilde{a})\big) := u_0^S$.[31] In addition, I assume securities lenders' have deep pockets and do not default.[32] Therefore, the securities lending problem can be summarized as

$$\max_{\{m^H,m^S,F^H,F^S\}} \Big(\mathbb{E}\big(U^D(m^H,F^H,m^S,F^S)\big)-v_0\Big)^{\theta}$$
$$\times \Big(\check{\mathbb{E}}\big(U^H(m^H,F^H)\big)-W\Big)^{1-\theta}$$

subject to

$m^S \ge m^H$	Initial financing constraint
$m^S+F^S \ge m^H+F^H$	Global financing constraint
$\mathbb{E}\big(U^S(m^S,F^S)\big) \ge u_0^S$	Securities lender break even
$-m^H \le W$	Hedge fund down payment constraint.

To simplify the securities lender's outside option, I assume that it is risk neutral.[33] Note that the model allows for negative margins, that is, the hedge fund may have to post more cash collateral than the value of the asset to borrow it $-m^H \le W$. The problem gives rise to the following equilibrium:

Theorem 2 (Solution to the securities lending problem). *If the following parameter assumptions*

(i) $p\big(\frac{\alpha}{1-\alpha}\big) > \max\big\{\frac{1-\alpha}{\alpha}, \frac{\breve{\alpha}}{1-\breve{\alpha}}\big\}$
(ii) $W \le p(\overline{a}-\breve{a})$
(iii) *$-m^*$ that solves $v'((a-\overline{a})-m^*) = \frac{1-\alpha}{p}+\alpha$ is less than W*

hold, then there exits a θ_1^S and θ_2^S such that

(i) *For $\theta \in [\theta_1^S, 1)$, the securities lending problem has a unique equilibrium $(m^{H*}, F^{H*}, m^{S*}, F^{S*})$ given by $m^{H*}=m^*, F^{H*}=\theta F_{MonoD}+(1-\theta)F_{MonoH}$, where*

$$F_{MonoD} = \underline{a} + \frac{\breve{\alpha}W-(1-\breve{\alpha})m^*}{p(1-\breve{\alpha})},$$

$$F_{MonoH} = \overline{a} + \frac{v((\overline{a}-a)-m^*)-v_0+\alpha(W+m^*)}{(1-\alpha)},$$

and $m^{H*}=m^*, m^{S*}=a-\overline{a}$, and $F^{S*}=\overline{a}$.

(ii) *For $\theta \in (\theta_2^S, \theta_1^S]$, the securities lending problem has a unique equilibrium $(m^{H*}, F^{H*}, m^{S*}, F^{S*})$ given by*

$$m^{H*}+F^{H*} = \overline{a}-W \quad \text{and} \quad \frac{(1-\theta)\big(\mathbb{E}\big(U^D\big)-v_0\big)}{\theta\big(\hat{\mathbb{E}}\big(U^H\big)-W\big)} = \frac{v'(m^{H*}-m^{S*})-1}{(1-\breve{\alpha})(1-p)},$$

and $m^{H*}=m^*, m^{S*}=a-\overline{a}$, and $F^{S*}=\overline{a}$.

Proof. See Appendix. □

The unique optimal contracting terms of the securities lending problem can be considered as the mirror image of the optimal contracting terms in the intermediary problem. The equilibrium involves a risk-free contract with the securities lender, while the hedge fund is exposed to the dealer's default. In other words, the dealer posts sufficient cash collateral for the securities lender to be insulated from the risk of losing a high-valued asset. The hedge fund overcollateralizes the asset loan even further, placing more cash collateral than is distributed to the securities lender, effectively allowing the dealer to capture a liquidity windfall on the initial leg of the loan.

This result suggests that margins are negative whenever the motive to trade is to source a specific asset. This outcome can be interpreted as the cash margin that dealers' counterparties need to post to short an asset. The result is also consistent with the securities lending market convention that dealers overcollateralize their securities when borrowing from securities lenders.

This theorem also suggests that bilateral repo rates between dealers and their clients can be negative, an empirical fact first documented by Duffie (1996) and commonly referred to as repo specialness. In effect, if we assume the equilibrium repurchase price is F_{MonoD} then we have the following repo rate inequality,

$$\frac{F_{MonoD}}{a-m^{H*}} < \frac{F_{MonoD}}{a-m^{S*}} = \frac{\underline{a}}{\overline{a}} + \frac{\breve{\alpha}W-(1-\breve{\alpha})m^{H*}}{\overline{a}p(1-\breve{\alpha})} \le 1, \tag{11}$$

where the final inequality holds because $-m^{H*} \le W \le p(\overline{a}-\breve{a})$. Although the equilibrium repurchase price will be higher than F_{MonoD}, it can be arbitrarily close to the monopolist solution for a high enough level of dealer market

[31] Implicitly, I am assuming that the securities lender is endowed with the asset initially. Alternatively, I could assume that the securities lender purchases the asset with the funds received from lending out the security, which is equivalent.

[32] The equilibrium contracting terms will imply that the initial leg will cover any potential payments.

[33] Without this assumption, the securities lender break even condition would entail a risk premium. This would complicate the final contract without qualitatively contributing to the result.

power. Therefore, the frictions embedded in the rehypothecation process in a securities lending context also gives rise to repo specialness.

5. Discussion of model assumptions

This section discusses the main modeling assumptions driving the results. The two main frictions are the dealer's preference for liquidity in $t = 0$ and counterparties' nonrecourse to a defaulted dealer's remaining assets.

5.1. Dealer's preference for liquidity

In the model, dealers have a preference for holding liquidity in $t = 0$, captured by v. Their preference for cash upfront gives them incentives to have a higher spread between margins whenever they rehypothecate collateral. This preference can be motivated in several ways. For example, dealers may use these funds to finance other areas of their business, effectively using the cash windfall for more profitable activities.[34] It may also reflect dealers' need for liquidity in times of market stress. In effect, the cash windfall stemming from overcollateralization may be an important source of liquidity in a crisis scenario.

As discussed in Section 1.2.1, in the United States, there are no restrictions on dealers' use of counterparties' pledged assets. This can also be seen in Fig. 2, which shows that dealers' repo activity to raise funds is the same order of magnitude as their repo activity to distribute funds. This suggests that differences in repo contracting terms, as shown in Fig. 1, are an important source of liquidity for dealers.

5.2. Nonrecourse to a defaulted dealer

In the model, neither the hedge fund nor the money market fund have access to the dealer's balance sheet in case of its default. Because repos are recourse, in theory, if counterparties suffer a loss, they have an unsecured claim on the bankrupt dealer. Although counterparties are entitled to these payments, in reality, the costly and tedious task of resolving a large broker-dealer may significantly hinder the recovery process. For example, given the contracting nature between a money market fund and its own clients, awaiting the resolution of a bankruptcy process may trigger a run on the money market fund itself.[35] Because repos are exempt from automatic stay—that is, the money market fund can liquidate the collateral immediately after a default event—the money market fund would have a strong incentive to completely insulate itself from the dealer. Thus, even if the dealer had some remaining assets in bankruptcy, the money market fund would likely place a small value on any unsecured claims on the dealer.

Regarding hedge funds, if the dealer defaults in the rehypothecation process, the hedge fund loses the upside on its collateral relative to the repurchase price. The hedge fund's loss can be motivated by the same logic as the money market fund's: it may be costly for a hedge fund to wait for the resolution of a lengthy bankruptcy process to receive payments on unsecured claims. Note that the model's nonrecourse assumption is asymmetric. In other words, the hedge fund still has an obligation to pay any shortfall in case of its own default. This asymmetry can be interpreted as the dealer's ability to seize their counterparties' portfolio in case of any outstanding balance, irrespective of the dealer's bankruptcy status. Therefore, it is reasonable to assume that a hedge fund would not expect any unsecured claims on a defaulted dealer to be fulfilled (at least not immediately) but would expect to pay any outstanding balances if it were to default itself.

6. Conclusion

This paper studies how dealers intermediate cash and collateral across different repo markets. In the baseline model, dealers receive funds from money market funds through the tri-party market and distribute them to hedge funds through the bilateral market, while the collateral moves in the opposite direction. This paper argues that different contracting terms between these markets are an important source of liquidity for dealers and underscores the risk dealers face from a withdrawal of hedge fund counterparties.

The model predicts that conditional on the repo's collateral class, haircuts in the tri-party market are homogeneous and relatively stable, whereas haircuts in the bilateral market can depend on the dealer's risk profile. Specifically, if a hedge fund is more likely to lose a low-valued asset upon a dealer's default, it would be willing to increase its initial exposure. In contrast, if a hedge fund is more likely to lose an asset unconditionally upon a dealers default, it would want to reduce its initial exposure. This result suggests a pecking order exists among hedge funds to alter their repo terms depending on the underlying collateral: hedge funds may ask for more favorable contracting terms when borrowing against high quality collateral (i.e., a decrease in margins) that does not affect the dealer's default directly and riskier contracting terms when borrowing against collateral that is related to the dealer's default (i.e., an increase in margins). Hedge funds' incentives to withdraw different types of collateral is an area of future research.

The paper links the model results to the existing literature on repos and corroborates some of the stylized facts of these markets during the recent financial crisis. Specifically, there are stable contracting terms in the tri-party market with sharp volume reductions in times of market stress, while there are time varying contracting terms in the bilateral market.

The importance of dealers as repo intermediaries leaves an open question: why does the intermediation chain exist in the first place? Specifically, why do initial cash lenders not directly fund ultimate cash borrowers? Although beyond the scope of this paper, one reason may be the relatively opaque nature of the ultimate cash borrower. An important feature of the hedge fund industry is that its

[34] Infante and Vardoulakis (2018) explicitly model the incentive to use the liquidity windfall to invest in risky securities.

[35] In many cases, money market funds offer their clients a fixed net asset value, which creates an important source of run risk for money market funds.

activity is largely unknown to outsiders. Arguably, dealers do not suffer such a severe informational problem when dealing with a hedge fund because they have better information on the hedge fund's position. This informational friction would have similar implications as the difference in creditworthiness proposed by Gottardi et al. (2017) to show the efficiency of the repo rehypothecation chain.

Appendix

Proof of Theorem 1. The intermediation problem's Lagrangian takes the following form:

$$\begin{aligned}\mathcal{L} = {} & \theta \log\left(\mathbb{E}\left(U^D\right) - v_0\right) + (1-\theta)\log\left(\hat{\mathbb{E}}\left(U^H\right) - W\right) \\ & +\mu\left(\mathbb{E}\left(U^M\right) - u_0^M\right) + \xi_1(m^H - m^M) \\ & +\xi_2(m^H + F^H - m^M - F^M)) + \lambda(W - m^H). \end{aligned} \tag{12}$$

Taking the FOC gives

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial m^H} = {} & \left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial m^H} \\ & + \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)\frac{\partial\hat{\mathbb{E}}\left(U^H\right)}{\partial m^H} + \xi_1 + \xi_2 - \lambda = 0 \end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial F^H} = {} & \left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial F^H} \\ & + \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)\frac{\partial\hat{\mathbb{E}}\left(U^H\right)}{\partial F^H} + \xi_2 = 0 \end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial m^M} = {} & \left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial m^M} \\ & + \mu\frac{\partial\mathbb{E}\left(U^M\right)}{\partial m^M} - \xi_1 - \xi_2 = 0 \end{aligned} \tag{15}$$

$$\frac{\partial\mathcal{L}}{\partial F^M} = \left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial F^M} + \mu\frac{\partial\mathbb{E}\left(U^M\right)}{\partial F^M} - \xi_2 = 0, \tag{16}$$

where the partial derivatives of agents' contracting terms can be deduced by inspecting their payoff functions.

Using the dealer's marginal payoff, and the slope of it's iso-utility curves, it can be verified that it's payoff is is strictly quasi-concave. In effect, its upper contour sets are strictly convex because of the strict concavity of the payoff in $t = 0$ (i.e., v) and the quasi-linearity of the payoff in $t = 1$. Note that this holds even when crossing the hedge fund's default threshold. By inspecting the hedge fund's marginal payoff, and the slope of it's iso-utility curves, it can be verified that it's payoff is piecewise quasi-concave. In effect, for contracting terms (m^H, F^H) in which the hedge fund defaults (i.e., $W - m^H + \underline{a} \leq F^H$), the slope of its iso-utility curve is equal to $-1/p$, implying quasi-concavity. For contracting terms (m^H, F^H) in which the hedge fund avoids defaults (i.e., $W - m^H + \underline{a} \geq F^H$), the slope of its iso-utility curve takes on two values: $-1/(p + (1-p)(1-\hat{\alpha}))$ and $-1/p$. Because $p + (1-p)(1-\hat{\alpha}) > p$, the upper contour set is convex, implying quasi-concavity.

Therefore, when $W - m^H + \underline{a} \leq F^H$ or when $W - m^H + \underline{a} \geq F^H$, the dealer and hedge fund payoffs are strictly quasi-concave and quasi-concave, respectively. Thus, in each region, the solution to the Nash bargaining game is unique. The proof of the theorem consists in solving for the optimal solution when the dealer defaults (i.e., $W - m^H + \underline{a} \leq F^H$) and when the dealer is solvent (i.e., $W - m^H + \underline{a} \geq F^H$) separately and then picking the maximum solution between the two, resulting in a globally unique equilibrium. In each case, a Lagrange multiplier for the default condition is introduced, denoted by ξ_R. Given that the theorem proposes a globally unique equilibrium, the proof will concentrate on cases where the financing conditions are slack, that is, $\xi_1, \xi_2 = 0$.

To analyze the equilibria in different hedge fund default regions, it is useful to characterize general conditions under which $F^M = \underline{a}$ and $m^M = a - \underline{a}$ are optimal. Lemma 1 gives such conditions. Because $\mathbb{E}\left(U^D\right) - v_0$ will always be positive in equilibrium, if contracting terms are such that $v'(m^H - (a - \underline{a})(p + \alpha(1-p)) < 1$, then $F^M = \underline{a}$ and $m^M = a - \underline{a}$ solve Eqs. (16) and (15), making the proposed money market fund contracting terms optimal. Lemma 1 (below) allows the proof to focus on characterizing the optimal contracting terms between the hedge fund and dealer.

Case I: $W - m^H + \underline{a} \leq F^H$:

Consider the proposed equilibrium for $\theta \geq \theta_1$, Eqs. (13) and (14) can be rewritten as

$$\begin{aligned} & \left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\left(v'(m^* - (a - \underline{a})) - (1-\alpha)\right) \\ & -\left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)\hat{\alpha} = 0 \end{aligned} \tag{17}$$

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\alpha - \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)p\hat{\alpha} = 0. \tag{18}$$

Substituting the second equation in the first, we have the m^* solving $v'(m^* - (a - \underline{a})) = \frac{\alpha}{p} + (1-\alpha)$ is a solution to the first equation. The second equation holds if

$$\alpha\theta\left(\hat{\mathbb{E}}\left(U^H\right) - W\right) = p\hat{\alpha}(1-\theta)\left(\mathbb{E}\left(U^D\right) - v_0\right), \tag{19}$$

which is true for the proposed repurchase price $F^{H*} = \theta F_{MonoD} + (1-\theta)F_{MonoH}$. In effect, given the linearity of the dealer and hedge fund's payoff in F^H, the market power weighted convex combination of their monopolist solutions solves the above equation. Therefore, (m^{H*}, F^{H*}) satisfy the first two optimality conditions.

In addition, $v'(m^* - (a - \underline{a})) = \frac{\alpha}{p} + (1-\alpha) < \frac{1}{p + \alpha(1-p)}$ because $p\left(\frac{1-\alpha}{\alpha}\right) > \frac{\alpha}{1-\alpha}$ (i.e., assumption i). Thus, from Lemma 1, $(m^{M*}, F^{M*}) = (a - \underline{a}, \underline{a})$ satisfy the last two optimality conditions.

This equilibrium holds if in fact $W - m^{H*} + \underline{a} < F^{H*}$ for a high enough θ. Because $p(\hat{a} - \underline{a}) \geq W$, it is direct to show that $W - m^{H*} + \underline{a} < F_{MonoD}$, making the inequality hold for θ close to 1. This proves the proposed equilibrium is optimal when $W - m^{H*} + \underline{a} < F^{H*}$, until $\theta = \theta_1$.

Consider the proposed equilibrium for $\theta \le \theta_1$, Eqs. (13) and (14) can be rewritten as

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\left(v'(m^* - (a - \underline{a})) - (1-\alpha)\right)$$
$$-\left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)\hat{\alpha} + \xi_R = 0 \tag{20}$$

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\alpha - \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)p\hat{\alpha} + \xi_R = 0. \tag{21}$$

Replacing the value of ξ_R of the first equation in the second gives

$$\frac{(1-\theta)\left(\mathbb{E}\left(U^D\right) - v_0\right)}{\theta(\hat{\mathbb{E}}\left(U^H\right) - W)} = \frac{v'(m^{H*} - m^{M*}) - 1}{\hat{\alpha}(1-p)}, \tag{22}$$

which, along with $m^{H*} + F^{H*} = W + \underline{a}$, characterizes the optimal hedge fund contracting terms of the theorem.

To ensure that the proposed contracting terms are the equilibrium, we need to verify that conditions in Lemma 1 holds and that $\xi_R > 0$. The second condition can be reduced to

$$v'(m^H - m^M) - \frac{\alpha}{p} - (1-\alpha) > 0. \tag{23}$$

At $\theta = \theta_1$, the condition in Lemma 1 holds and the right-hand side of condition (23) is equal to zero. As the dealer's market power decreases, the hedge fund margin and repurchase price decreases and increases, respectively (maintaining $m^{H*} + F^{H*} = W + \underline{a}$). Because of v's concavity, a decrease in m^H implies that condition (23) holds. This equilibrium is sustained until the condition on v' in Lemma 1 becomes an equality, which is one of the conditions that pin down θ_2.

Case II: $W - m^H + \underline{a} \ge F^H$:

In this case, for $\theta \ge \theta_1$, I conjecture the following equilibrium: $W \ge m^{H*} > m^*$, $F^{H*} = \underline{a} + W - m^{H*}$, $m^{M*} = \underline{a} - a$, $F^{M*} = \underline{a}$, where both financing conditions are slack.[36] Eqs. (13) and (14) can be rewritten as

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)v'(m^H - m^M) - \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right) - \xi_R = \lambda \tag{24}$$

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right) - \left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)$$
$$\times (p + (1-p)(1-\hat{\alpha})) - \xi_R = 0, \tag{25}$$

replacing the value for ξ_R of the second equation in the first gives

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right) - v_0}\right)\left(v'(m^H - m^M) - 1\right)$$
$$-\left(\frac{1-\theta}{\hat{\mathbb{E}}\left(U^H\right) - W}\right)(1-p)\hat{\alpha} = \lambda. \tag{26}$$

If λ is equal to zero, $\xi_R > 0$ is reduced to

$$v'(m^H - m^M) < \frac{1}{p + (1-\hat{\alpha})(1-p)}. \tag{27}$$

Recall that $v'(m^* - (a - \underline{a})) = \frac{\alpha}{p} + (1-\alpha)$. Because $p\left(\frac{1-\alpha}{\alpha}\right) > \frac{1-\hat{\alpha}}{\hat{\alpha}}$ (i.e., assumption i), it is easy to show that $v'(m^* - (a - \underline{a})) < \frac{1}{p+(1-p)(1-\hat{\alpha})}$. Therefore, we have that $\xi_R > 0$ holds for $m^{H*} > m^*$ and $m^{M*} = (a - \underline{a})$ because of v's concavity, where m^{H*} either solves Eq. (26) with $\lambda = 0$ or is equal to W, and $F^{H*} = \underline{a} + W - m^{H*}$. This also implies that the condition in Lemma 1 also holds, making $F^{M*} = \underline{a}$ optimal.

Note that this equilibrium is feasible when the dealer defaults and $\theta \ge \theta_1$, i.e, case I. Because the analysis of case I gives a different optimal solution, it dominates the one in case II; implying that the equilibrium in case I is the unique equilibrium for $\theta \ge \theta_1$.

Considering the proposed equilibrium for $\theta \le \theta_1$, Eqs. (13) and (14) take the same expression as Eqs. (24) and (25) with $\lambda = 0$; however, in this case, $m^{H*} < m^*$. As before, $\xi_R > 0$ is reduced to the same condition (27). This equilibrium is sustained until condition (27) or the condition on v' in Lemma 1 become an equality. That is, θ_2 is pinned down when the conjectured equilibrium for $\theta < \theta_1$ satisfies

$$v'(m^* - (a - \underline{a}))$$
$$= \min\left\{\frac{1}{p + \alpha(1-p)}, \frac{1}{p + (1-\hat{\alpha})(1-p)}\right\}. \tag{28}$$

Finally, note that the equilibrium characterized in case II for $\theta < \theta_1$ is the same equilibrium characterized in case I, ensuring that in fact the unique equilibrium for $\theta_2 \le \theta \le \theta_1$, completing the proof. □

Lemma 1. If there exists an (m^H, F^H) *in the intermediary problem such that* $\mathbb{E}\left(U^D\right) - v_0 > 0$ *and*

$$v'(m^H - (a - \underline{a}))(p + \alpha(1-p)) < 1,$$

then $m^M = a - \underline{a}$ *and* $F^M = \underline{a}$ *solve Eqs. (15) and (16) when* $\xi_1 = \xi_2 = 0$.

Proof of Lemma 1. Note that $\mathbb{E}\left(U^M\right)$ takes different values depending on whether F^M is above or below $\underline{a}$. Separating these cases introduces an additional multiplier ξ_M associated with the restriction $F^M \ge \underline{a}$ or $F^M \le \underline{a}$, which can be analyzed in turn as follows:

When $F^M \le \underline{a}$, where Eq. (16) has $-\xi_M$, using the expressions for the partial derivatives of $\mathbb{E}\left(U^M\right)$ (which are identical in this case) and replacing the third equation in the fourth gives

[36] Note that this is not the equilibrium characterized in the theorem, but it will be dominated by the equilibrium that is.

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\left(v'(m^H-m^M)-1\right)=\xi_M, \tag{29}$$

which implies $\xi_M>0$ because v' is greater than 1 by assumption, making $F^M=\underline{a}$ optimal from below.

When $F^M\geq\underline{a}$, where Eq. (16) has $+\xi_M$, using the expressions for the partial derivatives of $\mathbb{E}\left(U^M\right)$, replacing the third equation in the fourth, and evaluating in $F^M=\underline{a}$ gives

$$\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\left(v'(m^H-m^M)(p+\alpha(1-p))-1\right)=-\xi_M. \tag{30}$$

Therefore, if $v'(m^H-m^M)(p+\alpha(1-p))<1$, then, necessarily, $F^M=\underline{a}$ is optimal from above. This implies that $F^{M*}=\underline{a}$ satisfies optimality conditions (15) and (16). From Eq. (15), it is clear that $\mu>0$; therefore, a binding participation constraint implies the optimal $m^{M*}=a-\underline{a}$. □

Proof of Proposition 1. The result comes from taking the implicit derivative of the equilibrium conditions when $\theta=\theta_2$. Recall that θ_2 can be defined by one of two conditions: either the money market fund contract ceases to be risk free or the hedge fund contract becomes risk free with slack; see Eq. (28). Therefore, the system of equations that determines the equilibrium (m^{H*}, F^{H*}) and θ_2 is given by

$$\begin{aligned}T_1 &= \hat{\alpha}(1-p)(1-\theta_2)\left(\mathbb{E}\left(U^D\right)-v_0\right)\\ &\quad -(v'-1)\theta_2\left(\hat{\mathbb{E}}\left(U^H\right)-W\right)=0\end{aligned} \tag{31}$$

$$T_2 = m^{H*}+F^{H*}-(W+\underline{a})=0 \tag{32}$$

$$T_3 = v'-\min\left\{\frac{1}{p+\alpha(1-p)},\frac{1}{p+(1-\hat{\alpha})(1-p)}\right\}=0. \tag{33}$$

The proof consists in considering both cases for T_3 separately. From the implicit function theorem we have

$$\begin{pmatrix}\frac{\partial m^{H*}}{\partial p}\\ \frac{\partial F^{H*}}{\partial p}\\ \frac{\partial \theta_2}{\partial p}\end{pmatrix}=-\underbrace{\begin{bmatrix}\frac{\partial T_1}{\partial m^{H*}} & \frac{\partial T_1}{\partial F^{H*}} & \frac{\partial T_1}{\partial \theta_2}\\ \frac{\partial T_2}{\partial m^{H*}} & \frac{\partial T_2}{\partial F^{H*}} & \frac{\partial T_2}{\partial \theta_2}\\ \frac{\partial T_3}{\partial m^{H*}} & \frac{\partial T_3}{\partial F^{H*}} & \frac{\partial T_3}{\partial \theta_2}\end{bmatrix}^{-1}}_{:=D^{-1}}\begin{pmatrix}\frac{\partial T_1}{\partial p}\\ \frac{\partial T_2}{\partial p}\\ \frac{\partial T_3}{\partial p}\end{pmatrix}, \tag{34}$$

where irrespective of the possible expressions of T_3, D^{-1} can be expressed as a function of the partial derivatives of T_1,

$$D^{-1}=\frac{1}{\left(-v''\frac{\partial T_1}{\partial \theta_2}\right)}\begin{bmatrix}0 & 0 & -\frac{\partial T_1}{\partial \theta_2}\\ 0 & -\frac{\partial T_1}{\partial \theta_2} & \frac{\partial T_1}{\partial \theta_2}\\ -v'' & v''\frac{\partial T_1}{\partial F^{H*}} & \frac{\partial T_1}{\partial m^{H*}}-\frac{\partial T_1}{\partial F^{H*}}\end{bmatrix}. \tag{35}$$

In addition, given that T_2 does not depend on p, the partial derivative of θ_2 takes the following form:

$$\frac{\partial \theta_2}{\partial p}=\frac{1}{\left(v''\frac{\partial T_1}{\partial \theta_2}\right)}\left[-v''\frac{\partial T_1}{\partial p}+\left(\frac{\partial T_1}{\partial m^{H*}}-\frac{\partial T_1}{\partial F^{H*}}\right)\frac{\partial T_3}{\partial p}\right], \tag{36}$$

where $\frac{\partial T_1}{\partial \theta_2}<0$ because both excess payoffs are positive. Using the expressions for the partial derivatives of $\mathbb{E}\left(U^D\right)$ and $\hat{\mathbb{E}}\left(U^H\right)$ whenever $F^{H*}=\underline{a}+W-m^{H*}$ and equation T_1 to replace the expression of the dealer's payoff results in

$$\frac{\partial T_1}{\partial p}=-\frac{\theta_2(v'-1)}{(1-p)}\left[(\hat{a}-\underline{a})-W\right]<0 \tag{37}$$

$$\begin{aligned}\frac{\partial T_1}{\partial m^{H*}}-\frac{\partial T_1}{\partial F^{H*}} &= \hat{\alpha}(1-p)(v'-1)\\ &\quad \underbrace{-\theta_2\left(p(\hat{a}-\underline{a})-W+(1-p)\hat{\alpha}(W-m^{H*})\right)}_{>0},\end{aligned} \tag{38}$$

where the above inequalities stem from condition $p(\hat{a}-\underline{a})>W$ and $m^{H*}<W$.

First, consider the case in which $\frac{1}{p+\alpha(1-p)}\leq\frac{1}{p+(1-\hat{\alpha})(1-p)}$. From T_3 this implies that

$$v'-1=\frac{(1-\alpha)(1-p)}{p+\alpha(1-p)} \quad\text{and}\quad \frac{\partial T_3}{\partial p}=\frac{(1-\alpha)}{(p+\alpha(1-p))^2}, \tag{39}$$

which from Eq. (36) gives

$$\begin{aligned}\frac{\partial \theta_2}{\partial p} &= \frac{1}{\left(v''\frac{\partial T_1}{\partial \theta_2}\right)}\left[\frac{v''\theta_2(1-\alpha)}{p+\alpha(1-p)}\left[(\hat{a}-\underline{a})-W\right]\right.\\ &\quad +\left(\frac{\hat{\alpha}(1-p)^2(1-\alpha)}{p+\alpha(1-p)}-\theta_2(p(\hat{a}-\underline{a})-W\right.\\ &\quad \left.\left.+(1-p)\hat{\alpha}(W-m^{H*}))\right)\frac{(1-\alpha)}{(p+\alpha(1-p))^2}\right],\end{aligned} \tag{40}$$

which is negative for p sufficiently close to 1.

Next, consider the case in which $\frac{1}{p+\alpha(1-p)}\geq\frac{1}{p+(1-\hat{\alpha})(1-p)}$. From T_3, this implies that

$$\begin{aligned}v'-1 &= \frac{\hat{\alpha}(1-p)}{p+(1-\hat{\alpha})(1-p)} \quad\text{and}\\ \frac{\partial T_3}{\partial p} &= \frac{\hat{\alpha}}{(p+(1-\hat{\alpha})(1-p))^2},\end{aligned} \tag{41}$$

which from Eq. (36) gives

$$\begin{aligned}\frac{\partial \theta_2}{\partial p} &= \frac{1}{\left(v''\frac{\partial T_1}{\partial \theta_2}\right)}\left[\frac{v''\theta_2\hat{\alpha}}{p+(1-\hat{\alpha})(1-p)}\left[(\hat{a}-\underline{a})-W\right]\right.\\ &\quad +\left(\frac{\hat{\alpha}^2(1-p)^2}{p+(1-\hat{\alpha})(1-p)}-\theta_2(p(\hat{a}-\underline{a})-W\right.\\ &\quad \left.\left.+(1-p)\hat{\alpha}(W-m^{H*}))\right)\frac{\hat{\alpha}}{(p+(1-\hat{\alpha})(1-p))^2}\right],\end{aligned} \tag{42}$$

again, which is negative for p sufficiently close to 1, completing the proof. □

Proof of Proposition 2. For $\theta\in[\theta_1, 1)$, the result is derived directly by taking the implicit derivative of the first order condition that pins down m^*. For $\theta\in[\theta_2, \theta_1)$, the result is derived by invoking the implicit function theorem on the two equations that pin down (m^{H*}, F^{H*}) as follows:

$$T_1 = (1-\theta)\big(\mathbb{E}\big(U^D\big) - v_0\big)\hat{\alpha}(1-p) \\ -\theta(\hat{\mathbb{E}}(U^H) - W)(v'(m^{H*} - m^{M*}) - 1) = 0 \quad (43)$$

$$T_2 = m^{H*} + F^{H*} - (W + \underline{a}) = 0. \quad (44)$$

From these equations, it is easy to show the proposition's result. □

Proof of Proposition 3. To the proof of the proposition consists of first characterizing the equilibrium when $\rho \in [0, p]$ and then studying its comparative statics to ρ. In this extension, the hedge fund, dealer, and money market fund's payoff take the following form[37]

$$\hat{\mathbb{E}}\big(U^H(m^H, F^H)\big)\begin{cases} p\Big[\big(1 - \frac{\rho(1-\hat{\alpha})}{p}\big)(W - m^H + \overline{a} - F^H) \\ + \frac{\rho(1-\hat{\alpha})}{p}(W - m^H + \underline{a} - F^H)\Big] \\ +(1-p)[W - m^H] & \text{if } F^H \le \underline{a} \\ \\ p\Big[\big(1 - \frac{\rho(1-\hat{\alpha})}{p}\big)(W - m^H + \overline{a} - F^H) \\ + \frac{\rho(1-\hat{\alpha})}{p}(W - m^H + \underline{a} - F^H)\Big] \\ +(1-p)\Big[W - m^H + \frac{(1-\rho)(1-\hat{\alpha})}{(1-p)}(\underline{a} - F^H)\Big] & \text{if } \underline{a} < F^H \le W - m^H + \underline{a} \\ \\ p\Big[\big(1 - \frac{\rho(1-\hat{\alpha})}{p}\big)(W - m^H + \overline{a} - F^H)\Big] \\ +(1-p)\big(1 - \frac{(1-\rho)(1-\hat{\alpha})}{(1-p)}\big)\big[W - m^H\big] & \text{if } W - m^H + \underline{a} < F^H. \end{cases} \quad (45)$$

$$\mathbb{E}\big(U^D(m^H, F^H, m^M, F^M)\big) \\ = \begin{cases} v(m^H - m^M) + F^H - F^M & \text{if } F^H \le W - m^H + \underline{a} \\ \\ v(m^H - m^M) \\ \quad +\big(1 - \frac{\rho(1-\alpha)}{p}\big)F^H \\ \quad + \frac{\rho(1-\alpha)}{p}(W - m^H + \underline{a}) - F^M & \text{if } W - m^H + \underline{a} < F^H. \end{cases} \quad (46)$$

$$\mathbb{E}\big(U^M(m^M, F^M)\big) \\ = \begin{cases} u^M(F^M - (a - m^M)) & \text{if } F^M \le \underline{a} \\ \\ pu^M(F^M - (a - m^M)) \\ \quad +(1-p)\Big[\big(1 - \frac{(1-\rho)(1-\hat{\alpha})}{(1-p)}\big) \\ \quad \times u^M(F^M - (a - m^M)) \\ \quad + \frac{(1-\rho)(1-\hat{\alpha})}{(1-p)} u^M(\underline{a} - (a - m^M))\Big] & \text{if } \underline{a} < F^M. \end{cases} \quad (47)$$

And the following theorem characterizes the equilibrium in the model extension with $\rho \in (0, p)$: □

Theorem 3 (Solution to the intermediary problem with. $\rho \in (0, p)$*) If the following parameter assumptions*

(i) $\frac{\hat{\alpha}}{p}\left\{\frac{p-\rho(1-\alpha)}{p-\rho(1-\hat{\alpha})}\right\} + \frac{\rho}{p}(1-\alpha) < \frac{1}{1-(1-\alpha)(1-\rho)} < \frac{1}{p+(1-\rho)(1-\hat{\alpha})}$

(ii) $p(\hat{a} - \underline{a}) \ge W$

(iii) m^* *that solves* $v'(m^* - (a - \underline{a})) = \frac{\hat{\alpha}}{p}\left(\frac{p-\rho(1-\alpha)}{p-\rho(1-\hat{\alpha})}\right) + \frac{\rho}{p}(1-\alpha)$ *is less than* W

hold with $\rho \in [0, p]$*, then there exits a* θ_1^ρ *and* θ_2^ρ *such that*

1. *For* $\theta \in [\theta_1^\rho, 1)$*, the intermediary with dealer solvency and asset correlation problem has a unique equilibrium* $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ *given by* $m^{H*} = m^*, F^{H*} = \theta F_{MonoD} + (1-\theta)F_{MonoH}$*, where*

$$F_{MonoD} = \overline{a} - \frac{((1-\hat{\alpha})W + \hat{\alpha}m^*)}{p - \rho(1-\hat{\alpha})},$$

$$F_{MonoH} = \underline{a} - \frac{v(m^* - (a - \underline{a})) - v_0 + \frac{\rho(1-\alpha)}{p}(W - m^*)}{\big(1 - \frac{\rho(1-\alpha)}{p}\big)},$$

and $m^{M*} = a - \underline{a}, F^{M*} = \underline{a}$.

2. *For* $\theta \in [\theta_2^\rho, \theta_1^\rho)$*, the intermediary with dealer solvency and asset correlation problem has a unique equilibrium* $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ *given by*

$$m^{H*} + F^{H*} = W + \underline{a} \quad \text{and} \quad \frac{(1-\theta)\big(\mathbb{E}\big(U^D\big) - v_0\big)}{\theta(\hat{\mathbb{E}}\big(U^H\big) - W)} \\ = \frac{v'(m^{H*} - m^{M*}) - 1}{(1-p) - (1-\hat{\alpha})(1-\rho)},$$

and $m^{M*} = a - \underline{a}, F^{M*} = \underline{a}$.

Proof of Theorem 3. The proof consists in following the same steps as in Theorem 1. See Online Appendix for details. □

Note that the equilibrium coincides with the equilibrium characterized in Theorem 1 when $\rho = p$. Having the optimal equilibrium for $\rho \in [0, p]$ allows to characterize its comparative statics with respect to ρ. For $\theta \in [\theta_1^\rho, 1)$, the comparative statics are given by the implicit derivative of the equilibrium equation $v'(m^* - (a - \underline{a})) =$

[37] To reduce the notational burden, I maintain the same notation for each agent's payoff function. It should be understood that throughout this proof, the expressions are relevant to the case in which $\rho \in [0, p]$.

$\frac{\hat{\alpha}}{p}\left(\frac{p-\rho(1-\alpha)}{p-\rho(1-\hat{\alpha})}\right)+\frac{\rho}{p}(1-\alpha)$, giving

$$\frac{\partial m^{H*}}{\partial \rho}=\frac{1}{v''(m^{H*}-m^{M*})}\left\{\frac{1-\alpha}{p}-\frac{\hat{\alpha}(\hat{\alpha}-\alpha)}{(p-\rho(1-\hat{\alpha}))^2}\right\}, \tag{48}$$

which is negative for p sufficiently close to 1.

For $\theta\in[\theta_2^\rho,\theta_1^\rho)$, the comparative statics are given by invoking the implicit function theorem on the equilibrium system of equations,

$$T_1=(1-\theta)\left(\mathbb{E}\left(U^D\right)-v_0\right)\left((1-p)-(1-\hat{\alpha})(1-\rho)\right)$$
$$-\theta\left(\hat{\mathbb{E}}\left(U^H\right)-W\right)(v'(m^{H*}-m^{M*})-1)=0 \tag{49}$$

$$T_2=m^{H*}+F^{H*}-(W+\underline{a})=0. \tag{50}$$

From these equations, it is direct to show that $\frac{\partial m^{H*}}{\partial \rho}<0$.

Proof of Theorem 2. The proof to the securities lending problem is very similar to the intermediary problem and so is the solution for the optimal contracting terms. Therefore, the proof shall just outline the solution strategy and refer to the proof of Theorem 1 for details. The securities lending Lagrangian takes the following form:

$$\mathcal{L}=\theta\log\left(\mathbb{E}\left(U^D\right)-v_0\right)+(1-\theta)\log\left(\check{\mathbb{E}}\left(U^H\right)-W\right)$$
$$+\mu\left(\mathbb{E}\left(U^S\right)-u_0^S\right)+\xi_1(m^S-m^H)$$
$$+\xi_2(m^S+F^S-m^H-F^H))+\lambda(W+m^H). \tag{51}$$

Taking FOC gives

$$\frac{\partial\mathcal{L}}{\partial m^H}=\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial m^H}$$
$$+\left(\frac{1-\theta}{\check{\mathbb{E}}\left(U^H\right)-W}\right)\frac{\partial\check{\mathbb{E}}\left(U^H\right)}{\partial m^H}+\xi_1+\xi_2+\lambda=0 \tag{52}$$

$$\frac{\partial\mathcal{L}}{\partial F^H}=\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial F^H}$$
$$+\left(\frac{1-\theta}{\check{\mathbb{E}}\left(U^H\right)-W}\right)\frac{\partial\check{\mathbb{E}}\left(U^H\right)}{\partial F^H}+\xi_2=0 \tag{53}$$

$$\frac{\partial\mathcal{L}}{\partial m^S}=\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial m^S}+\mu\frac{\partial\mathbb{E}\left(U^S\right)}{\partial m^S}-\xi_1-\xi_2=0 \tag{54}$$

$$\frac{\partial\mathcal{L}}{\partial F^S}=\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\frac{\partial\mathbb{E}\left(U^D\right)}{\partial F^S}+\mu\frac{\partial\mathbb{E}\left(U^S\right)}{\partial F^S}-\xi_2=0, \tag{55}$$

where the partial derivatives of agents' contracting terms can be deduced by inspecting their payoff functions.

As with the intermediary problem, by inspecting the dealer's marginal payoffs, it can be verified that its payoff is strictly quasi-concave because of the concavity of v and the quasi-linearity of the payoff in $t=1$. Note that the condition even holds when crossing the hedge fund's default threshold $\bar{a}-W-m^H=F^H$. Also, by inspecting the hedge fund's marginal payoffs, and slope of it's iso-utility curves, it can be verified that it's payoff is piecewise quasi-concave. Specifically, it's utility function is quasi-concave both for region $\bar{a}-W-m^H\geq F^H$ and $\bar{a}-W-m^H\leq F^H$. Therefore, if the solution to the Nash bargaining game is unique in each region, the global optimal of the theorem results from picking the largest between the two. To study each region separately, a Lagrange multiplier for the default condition is introduced, denoted by ξ_R.

To analyze the equilibria in different hedge fund default regions, it is useful to characterize general conditions under which $F^S=\bar{a}$ and $m^S=a-\bar{a}$ is optimal. The strategy consists in following the same steps as in Lemma 1. Specifically, observing Eqs. (54) and (55) when $\xi_1=\xi_2=0$, with a multiplier associated with F^S begin above or below $\bar{a}$, it is easy to show that if

$$v'((a-\bar{a})-m^H)(p+(1-p)(1-\alpha))<1, \tag{56}$$

then $F^S=\bar{a}$ and, from the participation constraint, $m^S=a-\bar{a}$. Condition (56) allows the proof to focus on characterizing the optimal contracting terms between the hedge fund and dealer.

Case I: $\bar{a}-W-m^H\geq F^H$:

Consider the proposed equilibrium for $\theta\geq\theta_1^S$, with $F^H<\bar{a}$, Eqs. (52) and (53) can be rewritten as,

$$-\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)\left(v'((a-\bar{a})-m^*)-\alpha\right)$$
$$+\left(\frac{1-\theta}{\check{\mathbb{E}}\left(U^H\right)-W}\right)(1-\check{\alpha})=0 \tag{57}$$

$$-\left(\frac{\theta}{\mathbb{E}\left(U^D\right)-v_0}\right)(1-\alpha)+\left(\frac{1-\theta}{\check{\mathbb{E}}\left(U^H\right)-W}\right)p(1-\check{\alpha})=0. \tag{58}$$

Putting the second equation in the first, we have the m^* solving $v'((a-\bar{a})-m^*)=\frac{1-\alpha}{p}+\alpha$ is a solution to the first equation. The second equation holds if

$$(1-\alpha)\theta\left(\check{\mathbb{E}}\left(U^H\right)-W\right)=p(1-\check{\alpha})(1-\theta)\left(\mathbb{E}\left(U^D\right)-v_0\right), \tag{59}$$

which is true for the proposed repurchase price $F^{H*}=\theta F_{MonoD}+(1-\theta)F_{MonoH}$. Therefore, (m^{H*},F^{H*}) satisfy the first two optimality conditions. Because of the parameter assumption i) of the theorem $v'((a-\bar{a})-m^*)=\frac{1-\alpha}{p}+\alpha<1/(p+(1-p)(1-\alpha))$, satisfying condition (56) and implying that $F^{S*}=\bar{a}$ and $m^{S*}=a-\bar{a}$ is optimal.

This equilibrium holds if $\bar{a}-W-m^{H*}>F^{H*}$, which is true for a high enough θ because from condition $W\leq p(\bar{a}-\check{a})$, $\bar{a}-W-m^{H*}>F_{MonoD}$, making the inequality hold for θ close to 1. This proves that the proposed equilibrium is optimal in the case when $W-m^{H*}+\underline{a}<F^{H*}$, until $\theta=\theta_1^S$.

Consider the proposed equilibrium for $\theta\leq\theta_1^S$; Eqs. (52) and (53) introduce a multiplier $-\xi_R$ for the restriction $\bar{a} - W - m^H \geq F^H$. It is direct to show that the proposed solution involves a lower F^{*H} and higher m^{*H} maintaining $\bar{a} - W = F^{H*} + m^{H*}$ until condition (56) no longer holds, which is one of the conditions that pin down θ_2^S.

Case II: $\bar{a} - W - m^H \leq F^H$:

In this case, for $\theta \geq \theta_1^S$, I conjecture the following equilibrium: $W \geq -m^{H*} > -m^*, F^{H*} = \bar{a} - W - m^{H*}, m^{S*} = \bar{a} - a$, and $F^{S*} = \bar{a}$, where both financing conditions are slack.[38] Eqs. (52) and (53) introduce a multiplier ξ_R, with both financing conditions slack (i.e., $\xi_1 = \xi_2 = 0$) and a potentially binding down payment constraint $-m^{H*} \leq W$ (i.e., $\lambda \geq 0$). For $\xi_R > 0$, we need the following condition

$$v'(m^S - m^H) < \frac{1}{p + \breve{\alpha}(1-p)}. \tag{60}$$

Recall that $v'((a - \bar{a}) - m^*) = \frac{1-\alpha}{p} + \alpha$. Because $\frac{1-\alpha}{p} + \alpha < \frac{1}{p+\breve{\alpha}(1-p)}$ (i.e., assumption i), it is easy to show that $v'((a - \bar{a}) - m^{H*}) < \frac{1}{p+(1-p)(1-\breve{\alpha})}$. Therefore, we have that $\xi_R > 0$ holds for $-m^{H*} > -m^*$ and $m^{S*} = \bar{a} - a$ because of v's concavity, where m^{H*} either solves a combination of Eqs. (52) and (53) that solves for ξ_R with $\lambda = 0$, or is equal to W, and $F^{H*} = \bar{a} - W - m^H$. This also implies that the condition in condition (56) also holds, making $F^{S*} = \bar{a}$ optimal. Because this equilibrium is also feasible for case I, which gives a different solution, the equilibrium proposed in the theorem dominates, making it the unique equilibrium for $\theta \geq \theta_1^S$.

Considering the proposed equilibrium for $\theta \leq \theta_1^S$, Eqs. (52) and (53) take the same expression as before—introducing a multiplier ξ_R in Eqs. (52) and (53)—but in this case, $-m^{H*} < -m^*$. As before, $\xi_R > 0$ is reduced to the same condition (60). This equilibrium is sustained until condition (60) or the condition (56) become an equality. That is, θ_2^S is pinned down when the conjectured equilibrium for $\theta < \theta_1^S$ satisfies

$$v'((a - \bar{a}) - m^*)) = \min\left\{\frac{1}{p + (1-p)(1-\alpha)}, \frac{1}{p + \breve{\alpha}(1-p)}\right\}. \tag{61}$$

Finally, note that the equilibrium characterized in case II for $\theta < \theta_1^S$ is the same equilibrium characterized in case I, ensuring that, in fact, the unique equilibrium for $\theta_2^S \leq \theta \leq \theta_1^S$, completing the proof. □

[38] Note that this is not the equilibrium characterized in the theorem, but will be dominated by the equilibrium that is.

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