

Last time: abstractions of the physical layer

- Elasticity buffer: mediate between packets @ bitrate r_1 (r_{sender}) and packets @ bitrate r_2 (r_{receiver})
- Why would r_1 be different from r_2 ?
 - r_1 may be different from r_2 because clocks in the internet is different
 - Say the sender is sending at 10 Mbit/s (with a **clock** of 10 Mhz and 1 bit per cycle)
 - The receiver also has a **clock** of 10 Mhz and reads 1 bit per cycle, this is also 10Mbit/s
 - However, the two **clocks** have a different 10Mhz $\Rightarrow r_1$ is 10 Mbit/s from the sender's perspective and r_2 is Mbit/s from the receiver's perspective $\Rightarrow r_1$ is not equal to r_2 .
- Although r_1 may be different from r_2 , they can't be too different from each other because of they are both a clock of 10Mhz and there is a limited clock tolerance
 - $r_1 \in [10 \text{ Mbit/s} \pm 1000 \text{ ppm}]$ and $r_2 \in [10 \text{ Mbit/s} \pm 1000 \text{ ppm}]$
- Since r_1 is different from r_2 , some bad things may happen.
 - Case 1: $r_{\text{sender}} < r_{\text{receiver}} \Rightarrow$ buffer underflows (the receiver would try to drain things from the buffer when it's empty)
 - Case 2: $r_{\text{sender}} > r_{\text{receiver}} \Rightarrow$ buffer overflows
- Communications interface parameters
 - $r_{\text{sender}}, r_{\text{receiver}} \in [10 \text{ Mbit/s} \pm 1000 \text{ ppm}]$
 - MTU = 10 kbit
 - Inter-packet gap must be $>$ (intuitively the receiver should be able to drain enough during this gap)
 - Proposal one: $\frac{MTU}{r_{\text{max}} - r_{\text{min}}}$
 - Proposal two: $\frac{MTU}{r_{\text{min}}} \Rightarrow$ The receiver can always get back to zero state after the Inter-packet gap
 - The real number: look at the end of this notes
- Case 1: Underflow
 - r_{sender} is 9.99 Mhz and r_{receiver} is 10.01 Mhz
 - Every one second, the elasticity buffer is drained by $r_{\text{receiver}} - r_{\text{sender}} = 0.02 \text{ Mbits} = 20 \text{ Kbits}$.
 - If the receiver only tells the system to start draining when there is at least 1 Mbytes in the buffer, it takes 400s to drain the buffer.
 - If the sender has a packet that is 4000 exabytes, it takes way more than 400s to drain the buffer, and therefore we need something to limit that — **MTU**
- Case 1: Underflow with MTU and a large buffer
 - Sender has packet that is 10 kbits
 - Receiver is manufactured with buffer that is 10 kbits
 - Receiver strategy:
 - 1) receive entire packet

- 2) then tell its client to start reading
 - This works, but expensive
- Case 1: Underflow with MTU and a small buffer
 - Sender has packet that is 10 kbits
 - Receiver is manufactured with buffer that is 1 kbits
 - Receive strategy:
 - 1) receive first 1 kbit
 - 2) then tell its client to start reading
 - Starting at $t=0$, the buffer gets the first 1 kbit after (1 kbit / 9.99 Mbit/s is roughly 0.1 milliseconds)
 - Starting at $t = 0.1$ ms, it takes (1 kbit / 20 Kbits/s = 50 milliseconds) to drain
 - But, at $t = 1$ ms for the sender to send the whole packet
 - Thus, there is no underflow.
- Given this, what is the smallest elasticity buffer size?
 - Sender has packet that is 10 kbits
 - Receiver is manufactured with buffer that is 20 bits
 - Receive strategy:
 - 1) receive first 20 bits
 - 2) then tell its client to start reading
 - The buffer gets the first 20 bit after roughly 2 microsecond
 - The sender needs another (1 ms - 2 microsecond = 998 microsecond) to send the whole packet
 - So there is (20 bit - 998 microsecond * 20 Kbits /s ~ 0.02 bits)
 - It works!
- So the smallest elasticity buffer X is such that $\frac{X}{r_{receive} - r_{send}} = \frac{MTU}{r_{send}}$
- Case 2: Overflow
 - Sender has packet that is 10 kbits
 - Receiver is manufactured with buffer that is 10 kbits
 - Strategy:
 - 1) Receiver tells its client to start reading asap
 - 2) Sender needs to wait between packets - **Inter-packet gap**
- Case 2: Overflow with Inter-packet gap
 - Sender has packet that is 10 kbits
 - Receiver is manufactured with buffer that is 10 kbits
 - Strategy:
 - 1) Receiver tells its client to start reading asap
 - When the packet is done, there is (~ 1 millisecond * 20 Kbits/s = 20 bits) in the buffer size.
- So 20 bit buffer works for both cases with a different strategy. So this policy would work for both the cases:
 - Sender has packet that is 10 kbit
 - Receiver with buffer that is 40 bits
 - Receiver Strategy:

- If there are greater than or equal to 20 bits in the buffer
- Then tell client to start reading
- And the buffer size is (2 * the number of smallest buffer size we calculated at the end of Case 1) = $2 \times \frac{MTU}{r_{max}} \times (r_{max} - r_{min}) = 4 \times \frac{MTU}{r} \times \text{clock tolerance}$
- Last piece: let's go back to the minimum number of inter packet gap: the inter packet gap needs to be enough for the go back to a buffer size of $\frac{MTU}{r_{max}} \times (r_{max} - r_{min})$ from either buffer size 0 or buffer size $2 \times \frac{MTU}{r_{max}} \times (r_{max} - r_{min})$. Therefore, we need at least
$$\frac{MTU}{r_{max}} \times (r_{max} - r_{min}) / r_{min} = MTU \times \frac{r_{max} - r_{min}}{r_{max} \times r_{min}} \sim MTU \times \frac{2 \times \text{clock tolerance}}{r^2}$$
- **Summary:** given a clock rate r and a clock tolerance ϵ , a max transmission unit MTU , we have elasticity buffer size $4 \times MTU \times \frac{\epsilon}{r}$ and the proposed strategy works if the inter-packet gap is at least $2 \times MTU \times \frac{\epsilon}{r^2}$. (We could also motivate these results with a little intuition: the more accurate clocks are, the smaller $\frac{\epsilon}{r}$ will be, in other words, we require smaller inter-packet gap and elasticity buffer size for more accurate clocks. If the clock is ideal (no error at all), inter-packet gaps and elasticity buffers are not needed, and the senders and the receivers can just operate at their own clocks.)