

Benchmark Analysis for Quantifying Urban Vulnerability to Terrorist Incidents

Walter W. Piegorsch,^{1*} Susan L. Cutter,² and Frank Hardisty³

We describe a quantitative methodology to characterize the vulnerability of U.S. urban centers to terrorist attack, using a place-based vulnerability index and a database of terrorist incidents and related human casualties. Via generalized linear statistical models, we study the relationships between vulnerability and terrorist events, and find that our place-based vulnerability metric significantly describes both terrorist incidence and occurrence of human casualties from terrorist events in these urban centers. We also introduce benchmark analytic technologies from applications in toxicological risk assessment to this social risk/vulnerability paradigm, and use these to distinguish levels of high and low urban vulnerability to terrorism. It is seen that the benchmark approach translates quite flexibly from its biological roots to this social scientific archetype.

KEY WORDS: Benchmark index; complementary log-log model; homeland security; urban vulnerability analysis

1. INTRODUCTION: RELATING URBAN VULNERABILITY TO TERRORIST ACTIVITY

Urban terrorism is by its very nature a multidimensional process, with many factors underlying the occurrence and incidence of each event (Bogen & Jones, 2006; Deisler, 2002; Reid *et al.*, 2004). While it is difficult to distill such a process down to a few, or even one, explanatory component(s), efforts to do so can provide guidance to numerous interested parties—urban officials, emergency managers, insurance underwriters, banking and property administrators, tax adjusters, among many others—when assessing their

locality's level of vulnerability to a terrorist event. Toward this end, we exploit a series of previously developed vulnerability indices (Borden *et al.*, 2007; Cutter *et al.*, 2003) whose quantitative characteristics allow us to benchmark an urban area's susceptibility to adverse events, including terrorist attacks. The measures are based on statistical reduction of the multidimensional factors (built environment and social characteristics, and the physical impact) that describe a locality's vulnerability to hazards. These can then be linked to data on terrorist incidents and related human casualties, in order to quantify the locality's risk of experiencing the terrorist event/outcome. Our goal is to study how place-based factors of urban vulnerability may relate to terrorist activity, and to use statistical features of the relationship to indicate and characterize the different levels of urban vulnerability to terrorist events. Within an "all hazards" framework, the vulnerability of an urban area's social systems and built environment exists independent of an adverse event. Thus, we can understand the underlying characteristics that contribute to such vulnerability and then relate these characteristics to likely impacts, be they from natural,

¹ Interdisciplinary Program in Statistics, BIO5 Institute, University of Arizona, Tucson, AZ 85721, USA.

² Hazards & Vulnerability Research Institute, Department of Geography, University of South Carolina, Columbia, SC 29208, USA.

³ GeoVista Center, Department of Geography, Pennsylvania State University, University Park, PA 16802, USA.

* Address correspondence to Walter W. Piegorsch, BIO5 Institute, University of Arizona, Tucson, AZ 85721, USA; tel: 520-621-2357; fax: 520-626-4824; piegorsch@math.arizona.edu.

technological, or human-induced threats such as terrorism.

To construct the terrorism database, we culled records from two different sources of data on terrorist incidents in the United States. The Terrorism Knowledge Base (<http://www.tkb.org>), produced by the Memorial Institute for the Prevention of Terrorism (MIPT), is an online database of domestic and international terrorist incidents dating from 1968. Domestic events (domestic groups attacking targets within their own country) are only available from 1998. To augment this coverage, we also collected terrorist incident data from the Global Terrorism Database (GTD), compiled by researchers at the U.S. National Center for the Study of Terrorism and Responses to Terrorism (START) (LaFree *et al.*, 2006). This database covers the time period 1970–1998 and includes both domestic and international incidents. For our study, we limited attention to only the domestic (U.S.) data. We used these data to manipulate the resulting incidence and casualty information into forms that allow for easy interpretation when linked with our vulnerability indices. Via statistical modeling and analysis, we studied the relationships between vulnerability and terrorist outcome, and borrowing quantitative benchmark methodologies from health risk assessment (Piegorsch & West, 2005), used these relationships to estimate *benchmark indices*—analogous to toxicological *benchmark doses* (Falk Filipsson *et al.*, 2003)—that specify an urban locality's vulnerability to the terrorist event. In Section 2 below, we detail our methods and in Section 3 we present the results. We conclude with a brief discussion in Section 4.

2. METHODOLOGY

2.1. Database

To build the foundation for our study of urban vulnerability to terrorist events, we focused attention on U.S. urban locations taken from an earlier study our research team conducted on natural hazards vulnerability (Borden *et al.*, 2007). Therein, we studied the 132 urban areas deemed at greatest risk to terrorist hazards by the U.S. Federal government (Table I). We defined the spatial footprint of these 132 urban areas using digital orthoquarter quadrangles (DOQQs), which resulted in cities with rigid, block-like shapes. Because the DOQQs are not standard enumeration units for demographic data, we found it difficult to calculate a metric of hazard vulnerability based on

this geography. Instead, we approximated the urban extents of the 132 cities using United States Census-defined counties, and took all those counties that intersected with the DOQQ-based urban boundaries. Although a perfect spatial match is lacking between these two urban definitions, our approach nonetheless does ensure complete coverage of the original urban boundaries while allowing for an enumeration unit for which Census data were available.

Our data are derived from two fundamental sources. As introduced in Section 1, the first is a database of terrorist incidents and casualties, while the second is a database of vulnerability metrics for urban centers across the United States.

The terrorist incidence database was compiled from the GTD and from MIPT's Terrorism Knowledge Base. U.S. incidents were selected from both sources (854 from GTD, 573 from MIPT) and then combined into one database for the United States for the 35-year period 1970–2004. Duplicates were removed based on repetitive date, target, and location. After removal of the duplicate records, 1,175 unique incidents remained. Of these, 1,098 were located in one of the urban areas in our study, while the remaining 75 incidents were located in rural areas. The rural events were not considered further. For each of the 1,098 urban incidents the following data were included: name of the group, ID number (provided in the original data source), date of the incident, city, deaths, injuries, damage, target, latitude and longitude (geocoded to the city level), county, state, and county and state FIPS code.

For each of the 132 urban centers listed in Table I, we used the combined terrorist database to identify the incidence of terrorist attacks. Of interest is the probability, π , that an urban center will have experienced a terrorist incident during the time span under study. Our goal is to relate π to the center's place-based vulnerability, using a vulnerability index we describe in Section 2.2. For simplicity, the incidence data were taken as binary indicators: $Y_{\text{incidence}} = 0$ if no terrorist incidents were recorded at that urban center and $Y_{\text{incidence}} = 1$ if any terrorist incident was recorded. In those cases where more than one incident was recorded, this represents a truncation of the response into a dichotomous outcome. When urban centers experienced more than one terrorist incident, the actual counts often skewed wildly; in some cases, the larger values could also fall victim to over/underreporting, making the final values unreliable. Both effects have the potential to increase variation in the data and hence decrease interpretability/

Table I. One Hundred Thirty-Two Urban Centers Studied for Vulnerability to Terrorist Events

Albany, NY	Fresno, CA	Orlando, FL
Albuquerque, NM	Ft. Wayne, IN	Oxnard, CA
Allentown/Bethlehem, PA	Grand Rapids, MI	Palm Bay/Melbourne, FL
Amarillo, TX	Greensboro/Winston Salem, NC	Pensacola, FL
Anchorage, AK	Harrisburg, PA	Philadelphia, PA
Atlanta, GA	Hartford, CT	Phoenix/Glendale/Mesa, AZ
Augusta, GA	Helena, MT	Pierre, SD
Augusta, ME	Honolulu, HI	Pittsburgh, PA
Austin, TX	Houston, TX	Portland, OR
Bakersfield, CA	Huntsville, AL	Poughkeepsie/Newburgh, NY
Baltimore/Annapolis, MD	Indianapolis, IN	Providence, RI
Barre/Montpelier, VT	Jackson, MS	Raleigh/Durham, NC
Baton Rouge, LA	Jacksonville, FL	Reno, NV
Birmingham, AL	Jefferson City, MO	Richmond, VA
Bismarck, ND	Juneau, AK	Riverside/San Bernardino, CA
Boise, ID	Kansas City, MO	Rochester, NY
Boston, MA	Knoxville, TN	Sacramento, CA
Bridgeport/Stamford, CT	Lancaster, PA	Salem, OR
Buffalo, NY	Lansing, MI	Salt Lake City/Ogden, UT
Cape Coral, FL	Las Vegas, NV	San Antonio, TX
Carson City, NV	Lexington, KY	San Diego, CA
Charleston, SC	Lincoln, NE	San Francisco, CA [†]
Charleston, WV	Little Rock, AR	Santa Fe, NM
Charlotte, NC	Los Angeles, CA*	Sarasota/Bradenton, FL
Chattanooga, TN	Louisville, KY	Scranton, PA
Cheyenne, WY	Lubbock, TX	Seattle/Tacoma, WA
Chicago, IL	Madison, WI	Shreveport, LA
Cincinnati, OH	McAllen, TX	Spokane, WA
Cleveland/Akron, OH	Memphis, TN	Springfield, IL
Colorado Springs, CO	Miami/Ft. Lauderdale, FL	Springfield, MA
Columbia, SC	Milwaukee, WI	St. Louis, MO
Columbus, GA	Minneapolis/St. Paul, MN	Stockton, CA
Columbus, OH	Mission Viejo, CA	Syracuse, NY
Concord, NH	Mobile, AL	Tallahassee, FL
Corpus Christi, TX	Modesto, CA	Tampa/St. Petersburg, FL
Dallas/Ft. Worth, TX	Montgomery, AL	Toledo, OH
Dayton, OH	Nashville, TN	Topeka, KS
Denver, CO	New Haven, CT	Trenton, NJ
Des Moines, IA	New Orleans, LA	Tucson, AZ
Detroit/Warren, MI	New York/Newark, NY	Tulsa, OK
Dover, DE	Norfolk, VA [‡]	Washington, DC
El Paso, TX	Oklahoma City, OK	Wichita, KS
Flint, MI	Olympia, WA	Worcester, MA
Frankfort, KY	Omaha, NE	Youngstown, OH

*Los Angeles region also includes Glendale, Long Beach, and Huntington Beach, CA.

[‡]Norfolk region also includes Chesapeake, Newport News, and Virginia Beach, VA.

[†]San Francisco region also includes Oakland, San Jose, and Fremont, CA.

precision in the statistical analysis. By moving to binary indicators, we avoid some of this potential disturbance. Unfortunately, we can also suffer a loss of information, but to compensate for this we incorporated the truncation effect into our model for the response probability π , as described in the Appendix.

Of course, use of incidence data—truncated or not—fails to quantify the severity of the event(s). Incidence-level data assign equal importance both to

terrorist events involving only slight property damage and to those where a large number of individuals were injured or killed. Recognizing this, we also used the combined terrorist database to identify how often casualties were experienced when terrorist incidents occurred at our 132 urban centers. Specifically, we constructed another indicator variable, Y_{casualty} , and let $Y_{\text{casualty}} = 0$ if no terrorist-related casualties were experienced by that urban center, and $Y_{\text{casualty}} = 1$

if any terrorist-related casualty was experienced. We defined a casualty as occurrence of human deaths or injuries. Notice that Y_{casualty} cannot equal 1 if $Y_{\text{incidence}}$ is zero. In effect, the underlying casualty probability, π_{casualty} , is *conditional* on observing a terrorist event in that urban center: we view π_{casualty} as the probability of observing at least one casualty *given* that a terrorist incident has occurred. While it is important to keep the conditional nature of the casualty data in mind, this feature does not prevent us from developing statistical models for π_{casualty} , nor from constructing valid inferences from them.

Table II gives the resulting database of terrorism indicators for our 132 urban centers. Our goal is to employ these data in studying U.S. urban vulnerability to terrorist events/casualties via statistical analysis of their relationship with quantitative vulnerability measures. For expediency, Table II also gives data on our vulnerability metrics, which we describe in the next section.

2.2. Vulnerability Indices

Hazard/vulnerability indexing for populations, localities, ecosystems, and other entities at risk to adverse events is an emergent endeavor in 21st century risk assessment, which has only recently seen substantial development (Harner *et al.*, 2002; Peng *et al.*, 2005; Srebotnjak, 2008; Tsuzuki, 2006). Our research into vulnerability metrics traces its roots to Cutter *et al.* (2003), who introduced an approach for quantifying a geographical locality's vulnerability to hazards based on its underlying socioeconomic and demographic profile. Called SoVI—for *Social Vulnerability Index*—the method employed U.S. county-level data to construct an index of social vulnerability to environmental hazards. Using a factor analytic approach, 42 variables were reduced to 11 independent factors that accounted for about 76% of the explainable variance in these data. These factors were combined into an additive model to create the final univariate SoVI score, which was then used to quantify relative levels of social vulnerability and to further examine the distinctive spatial patterns of this vulnerability each locality exhibited. Note that SoVI is a unitless measure, and has importance for its *comparative* value across geographic locations: larger values indicate greater social vulnerability.

In Borden *et al.* (2007), we extended this approach by viewing vulnerability as a tripartite process, linking the social aspects of vulnerability with additional place-based components based on natural hazards

and built-environment characteristics. We mimicked the factor-analytic SoVI construction to also assemble a natural hazards vulnerability index, HazVI, and a built-environment vulnerability index, BEVI. Thus beyond those socioeconomic factors and processes that hinder, or enable, a location's response to and recovery from hazardous events, our HazVI and BEVI measures also capture the frequency, diversity, and impact of natural hazards and of built environment factors, respectively, which either amplify or attenuate the effects of such hazards on urban places. The HazVI measure is viewed as surrogate for community experience in responding to extreme events, which is an important factor in preparedness levels, while the BEVI measure takes into account vulnerable characteristics of the built environment such as water and transportation infrastructure, property value, and age of housing.

For purposes of summarizing the overall vulnerability burden, it seemed natural to combine our SoVI, HazVI, and BEVI measures into a single, place-based vulnerability index, PVI. A simple additive model could be used here again, but our experience suggests that these three different measures will exhibit differential patterns of variability. A common tactic for combining information in such a situation is to weight each component inversely to its observed variance (Piegorsch & Bailer, 2005, Section 7.3.1). That is, suppose that S_{sov} , S_{haz} , and S_{bev} are the standard deviations (the positive square roots of each variance) calculated for each index within a particular study or among a specified collection of localities. Then, inverse variance weighting leads to a combined vulnerability measure of the form

$$\text{PVI} = \frac{\text{SoVI}}{S_{\text{sov}}^2} + \frac{\text{HazVI}}{S_{\text{haz}}^2} + \frac{\text{BEVI}}{S_{\text{bev}}^2}.$$

We include in Table II the individual SoVI, HazVI, and BEVI measures for each urban center from Table I, along with their combined PVI measure. Values of the empirical standard deviations that led to final calculation on the weighted PVI in Table II were $S_{\text{sov}} = 2.844$, $S_{\text{haz}} = 1.732$, and $S_{\text{bev}} = 2.646$.

On the surface, it may seem difficult to imagine that urban vulnerability to physical, social, or built-environment hazards as summarized in PVI would relate naturally to the terrorist incidence data in Table II. Admittedly, the mechanisms underlying the components of PVI, especially the social vulnerability features incorporated in SoVI, may be governed differently from those underlying vulnerability to terrorist attack. Deeper in the mix, however, are

Table II. Terrorist Incidence and Casualty Indicators, and Vulnerability Measures for the 132 Urban Centers in Table I

Urban Center	Incidence, Y _{incidence}	Casualty, Y _{casualty}	SoVI	HazVI	BEVI	PVI
Albany, NY	1	1	-0.487	1.385	0.546	0.480
Albuquerque, NM	1	0	2.631	-2.062	-2.906	-0.777
Allentown/Bethlehem, PA	0	0	1.133	-0.161	0.891	0.214
Amarillo, TX	0	0	4.241	-1.454	-2.376	-0.300
Anchorage, AK	0	0	-1.605	-0.815	-3.440	-0.962
Atlanta, GA	1	1	3.713	3.810	-0.319	1.683
Augusta, GA	0	0	4.695	0.813	-0.841	0.731
Augusta, ME	0	0	-4.031	-0.953	0.040	-0.810
Austin, TX	0	0	0.421	0.056	-1.254	-0.109
Bakersfield, CA	1	0	-0.999	-0.864	-2.993	-0.839
Baltimore/Annapolis, MD	1	0	0.665	1.143	5.321	1.223
Barre/Montpelier, VT	0	0	-4.699	-1.357	-2.642	-1.411
Baton Rouge, LA	1	0	3.417	7.116	1.553	3.016
Birmingham, AL	1	1	1.600	0.128	-0.381	0.186
Bismarck, ND	0	0	0.850	-0.783	-3.237	-0.618
Boise, ID	1	0	0.116	6.019	-2.276	1.696
Boston, MA	1	1	-2.039	-0.055	6.120	0.604
Bridgeport/Stamford, CT	1	1	-3.811	-1.614	-0.066	-1.019
Buffalo, NY	1	1	2.112	-0.709	1.778	0.279
Cape Coral, FL	0	0	6.220	-0.876	0.462	0.543
Carson City, NV	1	0	-0.775	-1.675	-3.396	-1.139
Charleston, SC	0	0	3.592	4.770	3.561	2.543
Charleston, WV	0	0	2.320	-1.213	-0.314	-0.162
Charlotte, NC	0	0	1.911	1.709	1.679	1.046
Chattanooga, TN	0	0	0.169	1.644	-0.737	0.464
Cheyenne, WY	0	0	0.446	-1.980	-2.965	-1.028
Chicago, IL	1	1	0.077	1.589	6.051	1.404
Cincinnati, OH	1	0	1.189	-0.509	1.713	0.222
Cleveland/Akron, OH	1	1	-1.059	1.844	4.956	1.192
Colorado Springs, CO	0	0	-4.082	-0.755	-3.539	-1.262
Columbia, SC	0	0	3.497	1.335	1.816	1.137
Columbus, GA	1	1	1.901	-0.290	-1.705	-0.105
Columbus, OH	0	0	-1.646	0.439	0.151	-0.036
Concord, NH	0	0	-4.164	-2.145	-1.779	-1.484
Corpus Christi, TX	0	0	3.978	-0.296	-1.145	0.230
Dallas/Ft. Worth, TX	1	1	1.418	0.760	0.128	0.447
Dayton, OH	0	0	-0.081	-1.119	1.136	-0.221
Denver, CO	1	1	-0.935	1.036	-0.414	0.171
Des Moines, IA	0	0	-0.781	1.396	-1.479	0.157
Detroit/Warren, MI	1	1	-1.105	-0.284	2.931	0.188
Dover, DE	0	0	2.652	-0.536	-1.426	-0.055
El Paso, TX	0	0	7.588	-1.322	-2.158	0.189
Flint, MI	0	0	0.504	-0.950	-0.647	-0.347
Frankfort, KY	0	0	-1.253	-1.783	-1.617	-0.980
Fresno, CA	1	0	-0.310	0.274	-3.148	-0.397
Ft. Wayne, IN	0	0	-1.747	-1.914	-0.594	-0.939
Grand Rapids, MI	0	0	-1.632	-0.919	-0.899	-0.636
Greensboro/Winston Salem, NC	1	1	1.117	2.449	-0.983	0.814
Harrisburg, PA	0	0	-0.772	0.152	1.838	0.218
Hartford, CT	0	0	-3.091	-1.093	2.836	-0.341
Helena, MT	0	0	-4.260	-2.102	-3.511	-1.729
Honolulu, HI	0	0	-1.659	-0.671	1.340	-0.237
Houston, TX	1	0	2.049	4.299	1.102	1.844
Huntsville, AL	0	0	0.761	0.369	-0.030	0.213
Indianapolis, IN	0	0	-0.381	-1.339	0.799	-0.379
Jackson, MS	1	0	6.295	0.133	-1.843	0.559
Jacksonville, FL	1	0	1.636	-0.104	-0.538	0.091

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Urban Center	Incidence, Y _{incidence}	Casualty, Y _{casualty}	SoVI	HazVI	BEVI	PVI
Jefferson City, MO	0	0	-0.761	-1.159	-1.466	-0.690
Juneau, AK	0	0	-6.683	-3.032	-4.087	-2.421
Kansas City, MO	1	0	2.520	-0.726	1.329	0.259
Knoxville, TN	0	0	2.069	-0.974	0.281	-0.029
Lancaster, PA	0	0	-0.350	-0.736	1.215	-0.115
Lansing, MI	1	0	-0.785	-1.213	-0.920	-0.633
Las Vegas, NV	1	0	6.691	-2.120	-3.578	-0.391
Lexington, KY	0	0	1.199	-1.100	-1.124	-0.379
Lincoln, NE	1	0	1.535	-1.769	-1.013	-0.545
Little Rock, AR	1	0	1.889	-0.173	-1.358	-0.018
Los Angeles, CA	1	1	2.192	-0.650	2.566	0.421
Louisville, KY	0	0	0.955	0.119	-0.248	0.122
Lubbock, TX	0	0	0.746	-1.056	-2.442	-0.609
Madison, WI	1	1	-0.885	-0.550	-0.860	-0.416
McAllen, TX	0	0	11.593	-1.438	-1.940	0.677
Memphis, TN	0	0	8.024	-0.089	-0.344	0.913
Miami/Ft. Lauderdale, FL	1	1	5.395	-0.104	-0.262	0.595
Milwaukee, WI	1	0	-2.610	0.926	2.623	0.361
Minneapolis/St. Paul, MN	1	0	0.611	-0.149	2.202	0.340
Mission Viejo, CA	1	1	-1.764	-1.021	4.988	0.154
Mobile, AL	0	0	4.199	-0.082	-0.470	0.425
Modesto, CA	0	0	0.439	-1.825	-1.776	-0.808
Montgomery, AL	1	0	3.611	-0.088	-2.183	0.105
Nashville, TN	1	0	0.422	-1.002	-0.486	-0.351
New Haven, CT	1	1	-0.146	-0.944	3.039	0.102
New Orleans, LA	1	1	4.567	4.777	6.734	3.119
New York/Newark, NY	1	1	-0.363	2.919	8.580	2.154
Norfolk, VA	0	0	0.673	4.431	5.362	2.326
Oklahoma City, OK	1	1	-0.455	0.676	-0.843	0.049
Olympia, WA	1	0	-2.480	-1.688	-1.482	-1.081
Omaha, NE	1	1	1.248	0.498	0.072	0.331
Orlando, FL	0	0	4.038	1.037	-0.686	0.747
Oxnard, CA	1	0	-4.861	-0.953	-2.019	-1.207
Palm Bay/Melbourne, FL	0	0	3.144	-1.202	0.399	0.045
Pensacola, FL	1	1	2.498	1.389	-1.078	0.618
Philadelphia, PA	1	1	0.832	1.957	6.871	1.737
Phoenix/Glendale/Mesa, AZ	1	1	1.766	-2.154	-2.830	-0.904
Pierre, SD	0	0	-1.630	-2.359	-3.639	-1.508
Pittsburgh, PA	1	1	2.608	-0.053	2.537	0.667
Portland, OR	1	0	-0.747	0.482	-0.404	0.011
Poughkeepsie/Newburgh, NY	1	0	-1.086	1.323	-0.730	0.202
Providence, RI	0	0	-1.363	-0.905	1.330	-0.280
Raleigh/Durham, NC	0	0	1.904	2.919	-1.343	1.017
Reno, NV	1	0	-2.078	-0.627	-3.509	-0.967
Richmond, VA	1	0	1.772	2.942	5.150	1.935
Riverside/San Bernardino, CA	0	0	3.271	-0.399	-3.124	-0.175
Rochester, NY	1	0	-1.912	-0.186	0.249	-0.263
Sacramento, CA	1	1	-1.743	0.416	-1.244	-0.254
Salem, OR	1	0	-1.170	-1.569	-2.428	-1.015
Salt Lake City/Ogden, UT	1	1	1.714	0.378	-1.031	0.190
San Antonio, TX	0	0	1.994	2.160	-0.363	0.915
San Diego, CA	1	1	-0.708	-0.606	-1.605	-0.519
San Francisco, CA	1	1	-4.958	1.286	1.639	0.050
Santa Fe, NM	0	0	0.042	-2.226	-3.333	-1.213
Sarasota/Bradenton, FL	0	0	7.228	-0.282	-1.216	0.626
Scranton, PA	0	0	0.181	1.484	-0.550	0.438
Seattle/Tacoma, WA	1	0	-1.919	0.312	-1.272	-0.315

Table II. (Continued)

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Urban Center	Incidence, $Y_{\text{incidence}}$	Casualty, Y_{casualty}	SoVI	HazVI	BEVI	PVI
Shreveport, LA	0	0	4.907	-0.094	-0.315	0.530
Spokane, WA	1	0	-1.226	-0.768	-2.607	-0.780
Springfield, IL	0	0	-1.128	-1.207	-1.153	-0.707
Springfield, MA	0	0	0.633	-1.562	0.019	-0.440
St. Louis, MO	1	1	0.983	0.635	4.274	0.944
Stockton, CA	0	0	0.418	-1.370	-0.046	-0.412
Syracuse, NY	0	0	0.113	1.008	0.428	0.411
Tallahassee, FL	0	0	1.791	0.280	-2.180	0.003
Tampa/St. Petersburg, FL	1	1	5.563	0.333	2.460	1.150
Toledo, OH	0	0	-0.325	-0.191	5.743	0.717
Topeka, KS	1	1	-1.110	-0.601	-2.645	-0.716
Trenton, NJ	1	1	-0.454	-1.079	5.781	0.410
Tucson, AZ	0	0	3.728	-2.139	-3.609	-0.768
Tulsa, OK	1	0	-0.191	0.512	-1.198	-0.024
Washington DC	1	1	-0.505	3.375	6.410	1.978
Wichita, KS	1	1	-0.957	-0.873	-1.549	-0.631
Worcester, MA	0	0	-2.163	-1.609	-0.337	-0.852
Youngstown, OH	0	0	1.271	0.160	1.496	0.424

built-environment (BEVI) and physical (HazVI) factors that can prove more predictive of terrorist incidence; these include, for example, infrastructure vulnerability, particularly bridges, tunnels, and water/sewer systems; limited building/skyscraper robustness to sudden shock(s); aging roads and housing; topographical factors that affect evacuation efficiency; etc. By employing a broad-based summary measure such as our PVI, study of how these varied factors relate to terrorist activity in our 132 urban centers becomes possible, via the data in Table II. As we will see, the statistical (regression) methodology we utilize also provides an opportunity to characterize and “benchmark” a city’s terrorism vulnerability. The next section details our statistical approach.

2.3. Statistical Analysis

For data in the form of binary indicators—such as $Y_{\text{incidence}}$ or Y_{casualty} —statistical analysis typically proceeds by using external explanatory variables—such as our PVI—to model the corresponding, unknown response probability π . This is often referred to as a *quantal response model*. Here, there are two distinct probabilities of interest, $\pi_{\text{incidence}}$ corresponding to $Y_{\text{incidence}}$ and π_{casualty} corresponding to Y_{casualty} . In either case, the model relates π to PVI via a linear predictor, $\beta_0 + \beta_1 \text{PVI}$, where the β values are unknown parameters. The linear predictor is then linked to π via some nonlinear function whose mathematical and statistical properties describe a sensible model rela-

tionship between the two quantities. As we detail in the Appendix, a useful expression to exploit when binary indicators are based on truncated count data, as we have here, is known as the complementary log-log link function: $\log\{-\log(1 - \pi)\} = \beta_0 + \beta_1 \text{PVI}$, where \log indicates application of the natural logarithm. Notice that this can be inverted to describe π directly as a function of the PVI:

$$\pi(\text{PVI}) = 1 - \exp\{-\exp(\beta_0 + \beta_1 \text{PVI})\},$$

where \exp indicates exponentiation using the base, e , of the natural logarithm. This can be viewed as a member of a larger class of *generalized linear (regression) models* (Jørgensen, 2002), since it is a nonlinear generalization of the simple linear regression model for use with indicator variables such as $Y_{\text{incidence}}$ or Y_{casualty} .

Fitting the Y values to such a model is accomplished via the method of maximum likelihood (Piegorisch & Bailer, 2005, Section A.4.3). This results in a pair of point estimates, b_0 and b_1 , for the unknown β parameters, along with pertinent statistical quantities such as the standard errors of the b_j s, $se[b_0]$ and $se[b_1]$, or their covariance $\text{Cov}[b_0, b_1]$, that can be used for making inferences on the β_j s or on π . For instance, the estimated response probability is $\hat{\pi}(\text{PVI}) = 1 - \exp\{-\exp(b_0 + b_1 \text{PVI})\}$.

For testing the null hypothesis that there is no relationship between PVI and π , $H_0: \beta_1 = 0$, we employ a likelihood ratio (LR) test (Piegorisch & Bailer, 2005,

Section A.5.3). This produces a test statistic and corresponding p -value to assess H_0 against the alternative hypothesis, $H_A: \beta_1 \neq 0$, that a significant relationship exists between PVI and π . Unfortunately, closed-form expressions for these various statistical quantities are not available for our complementary log-log model, and so computer calculation is required. We will employ the SAS statistical package (SAS Institute Inc., 2000), via its GENMOD procedure.

Note that nothing in our statistical model precludes use of different measures for the vulnerability index. Thus we could apply SoVI, HazVI, and/or BEVI in place of the PVI measure in any of the above equations. Doing so would simply refocus attention on that specific component of the vulnerability metric.

2.4. Benchmark Analysis

The use of a quantal response model in the statistical analysis (above) allows us to characterize the urban vulnerability to terrorist events via quantitative risk-analytic methodologies. Increasingly popular in toxicological and health risk assessment is an approach known as *benchmark analysis* (Crump, 2002; Piegorsch & West, 2005). This employs results from the quantal response analysis to estimate benchmark levels of a predictive measure, such as a toxic benchmark dose, past which the risk of an adverse event becomes unacceptable (Crump, 1984; Starr *et al.*, 2005). Risk-benchmark prediction is not commonly practiced with nonbiological endpoints, but in fact the quantitative technology can be transferred to our setting in a straightforward manner. The goal is to identify values of PVI past which an urban locality's vulnerability to, here, a terrorist event exceeds reasonable benchmark levels.

To assemble a benchmark vulnerability analysis for our terrorist incidence data, we begin by defining the core benchmarking component: the vulnerability function, $V(\cdot)$. This represents a locality's vulnerability to the adverse event as a function of some input variable. Here, the input variable is the PVI, while the adverse outcome can be either occurrence of terrorist incidents ($Y_{\text{incidence}} = 1$) or of casualties after a terrorist event ($Y_{\text{casualty}} = 1$). By analogy with toxicological benchmark analysis—where risk functions are often defined in terms of some probability of an adverse event—it is natural to use the (estimated) response probability for $V(\cdot)$; that is, simply set $V(\text{PVI})$ equal to $\hat{\pi}_{\text{incidence}}(\text{PVI})$ if terrorist incidence is the outcome of interest or to $\hat{\pi}_{\text{casualty}}(\text{PVI})$ if terrorist-related casualties are under consideration.

At its fundamental level, the benchmark approach uses information in $V(\text{PVI})$ to estimate the value of PVI past which a locality's vulnerability exceeds a benchmark level. This latter quantity is the *benchmark vulnerability*, BMV. Since $V(\text{PVI})$ represents a probability in our setting, we have $0 < \text{BMV} < 1$. Given a value of BMV, we then set $\text{BMV} = V(\text{PVI})$ and solve for PVI. This is a simple inversion of the nonlinear regression relationship, sometimes called *inverse dose estimation* or *effective dose estimation* in the biological sciences, or *statistical calibration* in chemometric analysis. The solution is the *benchmark index*, BMI, at that level of BMV; we denote this as $\text{BMI}_{100\text{BMV}}$.

Specification of BMV must be made in advance by the risk assessor/risk manager, and will depend upon the application intended for the BMI. For example, $\text{BMV} = 1/2$ produces a median effect level, analogous to the median lethal dose, LD_{50} , common in toxicity testing (Piegorsch & Bailer, 2005, Section 4.1.1): the BMI_{50} serves as an index point that separates the data into two equally-probable strata of vulnerability. In effect, we are asking what value of our PVI measure corresponds to a vulnerability benchmark of no more than 50% that a terrorist incident, using $V(\text{PVI}) = \hat{\pi}_{\text{incidence}}(\text{PVI})$, or casualty from a terrorist attack, using $V(\text{PVI}) = \hat{\pi}_{\text{casualty}}(\text{PVI})$, can occur.

Since they represent a sort of “central” level of the response curve, median effect levels can also be employed for comparative purposes; for example, comparing vulnerability profiles between two geographically distinct regions. A lower BMI_{50} would suggest greater vulnerability to the hazardous outcome under study, since median effect levels are inverse measures of hazardous potency/potential.

For applications with severely adverse outcomes, a lower, more conservative value of BMV is often employed, for example, $\text{BMV} = 0.10$ or $\text{BMV} = 0.01$. Multiple BMVs can be assigned if proper statistical adjustment is made for the multiplicity in the consequent inferences (see below). Indeed, when the adverse outcome is of special interest, such as a highly lethal form of cancer, BMVs as low as 10^{-5} might be considered. Unless the data provide exceptionally precise information at these low levels of response, however, use of very small BMVs will often represent true extrapolations, and the corresponding statistical estimates may be unstable. Users must proceed with due caution in such instances.

Given a value of BMV, to find the corresponding $\text{BMI}_{100\text{BMV}}$ under our complementary log-log model

we solve $BMV = 1 - \exp\{-\exp(b_0 + b_1 PVI)\}$, leading to

$$BMI_{100BMV} = \frac{C_{BMV} - b_0}{b_1}, \quad (1)$$

where for notational simplicity we let $C_{BMV} = \log\{-\log(1 - BMV)\}$; for instance, at $BMV = 1/2$, $C_{0.5} = -0.3665$. Of course, Equation (1) represents only an estimate of the actual benchmark vulnerability index. To account for statistical uncertainty in the point estimate, we also report $100(1 - \alpha)\%$ confidence limits for the BMI. (Any small value for α is reasonable; we operate below at $\alpha = 0.05$.) When interest concerns an adverse outcome, as it does here, it is common in quantitative risk assessment to ask only for lower confidence limits on benchmark values (U.S. EPA, 2000, Section 2); following standard terminology for such, we say the *benchmark index lower limit* at benchmark level BMV is $BMIL_{100BMV}$ (Crump, 1995). To construct this, we employ a popular confidence construction known as a lower $100(1 - \alpha)\%$ “Wald” limit

$$BMIL_{100BMV} = BMI_{100BMV} - Z_\alpha se[BMI_{100BMV}], \quad (2)$$

where Z_α is the upper- α critical point from a standard normal (‘Z’) distribution, and $se[BMI_{100BMV}]$ is the standard error of BMI_{100BMV} . The standard error is found by viewing $BMIL_{100BMV}$ as a function of the point estimates b_0 and b_1 , then manipulating a low-order Taylor series expansion of this function to approximate $se[BMI_{100BMV}]$, an application of the statistical *delta method* (Piegorisch & Bailer, 2005, Section A.6). Here, we have

$$se[BMI_{100BMV}] = \frac{\sqrt{se^2[b_0] + 2BMI_{100BMV}Cov[b_0, b_1] + BMI_{100BMV}^2 se^2[b_1]}}{|b_1|} \quad (3)$$

for use in the Wald limit in Equation (2).

As we mention above, more than one level of benchmark vulnerability may be considered when estimating the BMI. For example, a multi-BMV suite of values seen increasingly in toxicological risk assessment is $BMV = 0.10, 0.05, 0.01$ (Gaylor & Aylward, 2004; Sand *et al.*, 2003; Schlosser *et al.*, 2003), or, if interest lies in characterizing the breath of possible BMIs across symmetric cut-points, one might consider the four quartiles separated by $BMV = 1/4, 1/2, 3/4$. Point estimation is unaffected by use of multiple BMVs, but for inferences associated with the BMILs some correction must be applied. Construc-

tion of multiple BMILs from the same set of data constitutes a simultaneous inference and without correction for the multiplicity, the overall coverage level of the resulting inferences drops below $1 - \alpha$, sometimes well below it. For such a correction one can apply the well-known Bonferroni Inequality, which modifies the Z-critical point in (2) to $Z_{\alpha/M}$ when M is the number of multiple BMVs employed in the calculations (M must be a positive integer and must be specified in advance). More complex adjustments are also possible when correcting for multiplicity in BMIL constructions, but the Bonferroni approach has been seen to provide useful, relatively simple, simultaneous confidence limits in benchmark analysis (Nitcheva *et al.*, 2005).

We can apply these various statistical methods for estimating BMIs and BMILs to the vulnerability index employed in our study. These values provide quantitative guidance for identifying which levels of the index the likelihood of terrorist events—incidents or casualties—resides in or moves into an unacceptable range. By doing so, we gain insights into the relative level(s) of vulnerability a select urban center or group of centers exhibits in terms of possible terrorist threats.

3. RESULTS

Primary to our study of terrorism and place-based vulnerability is the relationship between the PVI measure and the terrorist incidence/casualty data in Table II. Regressing $Y_{incidence}$ on PVI via our complementary log-log model produces the point estimates $b_0 = -0.3829$ and $b_1 = 0.3236$. These correspond to the estimated vulnerability function $V_{incidence}(PVI) = 1 - \exp\{-\exp(-0.3829 + 0.3236PVI)\}$. The LR p -value for testing $H_0: \beta_1 = 0$ here is $p = 0.0147$, suggesting that our weighted PVI measure can significantly explain the (increasing) relationship between PVI and terrorist incidence. We also calculated the LR test for a possible quadratic effect in the predictor; that is, testing $H_0: \beta_2 = 0$ in $\pi_{incidence}(PVI) = 1 - \exp\{-\exp(\beta_0 + \beta_1 PVI + \beta_2 PVI^2)\}$. For these data, the p -value was $p = 0.7469$, suggesting that addition of a quadratic term does not significantly improve the predictive capability of our complementary log-log model with these data. Similarly, regressing $Y_{casualty}$ on PVI leads to the estimated vulnerability function $V_{casualty}(PVI) = 1 - \exp\{-\exp(-1.2545 + 0.4742PVI)\}$; the LR p -value for β_1 is again significant, ($p = 0.0037$), while the additional quadratic term remains insignificant ($p = 0.0693$).

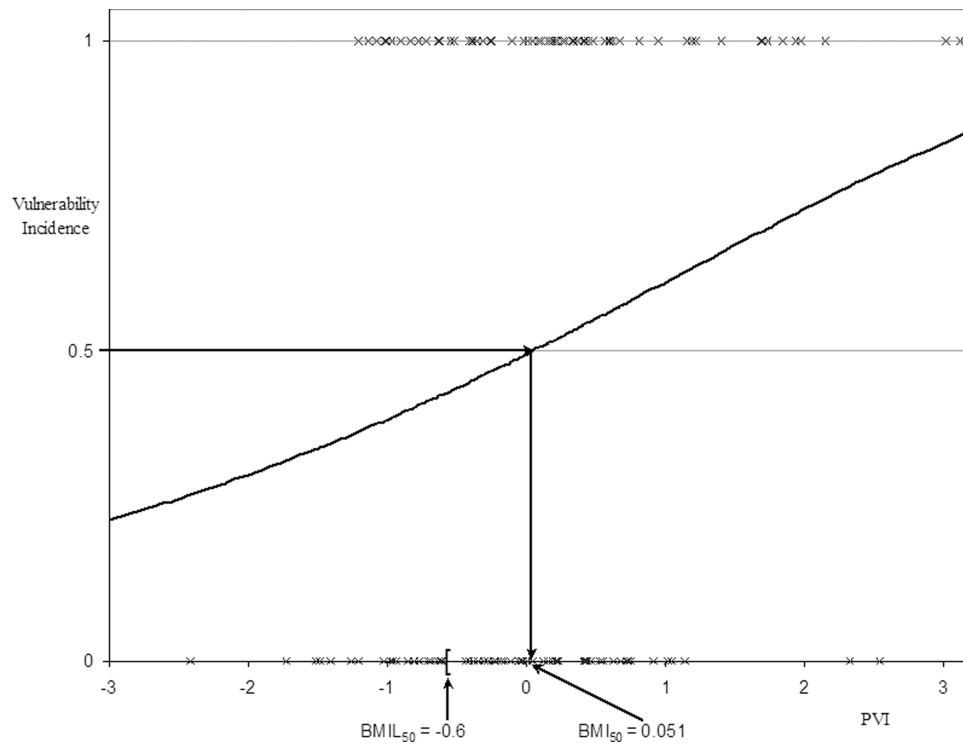


Fig. 1. Incidence data (x) and fitted complementary log-log $V_{\text{incidence}}(\text{PVI})$ function (solid curve) for 132 urban areas from Table II. Calculation of $\text{BMI}_{50} = 0.051$ is illustrated with reverse prediction arrows at $\text{BMV} = 0.50$. $\text{BMIL}_{50} = -0.60$ is marked by left bracket, [.

For the incidence metric, at $\text{BMV} = 1/2$ the corresponding BMI_{50} calculates from Equation (1) as $\text{BMI}_{50} = 0.051$. For purposes of statistical inference, we can continue with our BMI calculation and build a lower 95% confidence limit on BMI. We start with the standard error from Equation (3), for which we require the variance components $\text{se}^2[b_0] = 1.676 \times 10^{-2}$, $\text{se}^2[b_1] = 1.717 \times 10^{-2}$, and $\text{Cov}[b_0, b_1] = -4.080 \times 10^{-3}$. From (3), these yield $\text{se}[\text{BMI}_{50}] = 0.396$, and with this, Equation (2) gives the 95% lower bound as $\text{BMIL}_{50} = 0.051 - (1.645)(0.396) = -0.60$. Thus we are 95% confident that PVIs above at least -0.6 identify urban centers that reside in the higher tier of the vulnerability-to-terrorism metric.

For the casualty metric, repeating these calculations at $\text{BMV} = 1/2$ produces $\text{BMI}_{50} = 1.873$. For the lower 95% confidence bound we find $\text{se}^2[b_0] = 3.364 \times 10^{-2}$, $\text{se}^2[b_1] = 2.431 \times 10^{-2}$, $\text{Cov}[b_0, b_1] = -1.147 \times 10^{-2}$, and hence $\text{se}[\text{BMI}_{50}] = 0.581$. With this, Equation (2) gives $\text{BMIL}_{50} = 1.873 - (1.645)(0.581) = 0.917$. For those urban localities who consider casualties from terrorist attack to be of greater concern than simple incidence, the pertinent 50% benchmark separation point occurs at PVIs above at least 0.917. Localities uncomfortable with their PVI status above

either the incidence-based or casualty-based benchmark levels may wish to review the nature of their vulnerability and consider ways to decrease it.

Notice that the benchmark measures from the casualty analysis are larger—and by interpretation, less severe—than their corresponding values from the incidence analysis. This is expected, since as noted in Section 2 the casualty data are conditional on occurrence of a terrorist incident. Technically, $\pi_{\text{incidence}} \geq \pi_{\text{casualty}}$ at every PVI. The result is to, in effect, shift the casualty vulnerability curve to the right, increasing any benchmark index taken from it. Fig. 1 plots the binary incidence data against PVI, and overlays both the predicted $V_{\text{incidence}}(\text{PVI})$ function and the calculated BMI_{50} and BMIL_{50} ; Fig. 2 does the same for the binary casualty data. Visual comparison of the Figures corroborates the ‘right-shift’ phenomenon with our urban database.

4. DISCUSSION

We have described a statistical methodology that can characterize the vulnerability of U.S. urban centers to terrorist attack, using a place-based vulnerability index (PVI) and a database of terrorist

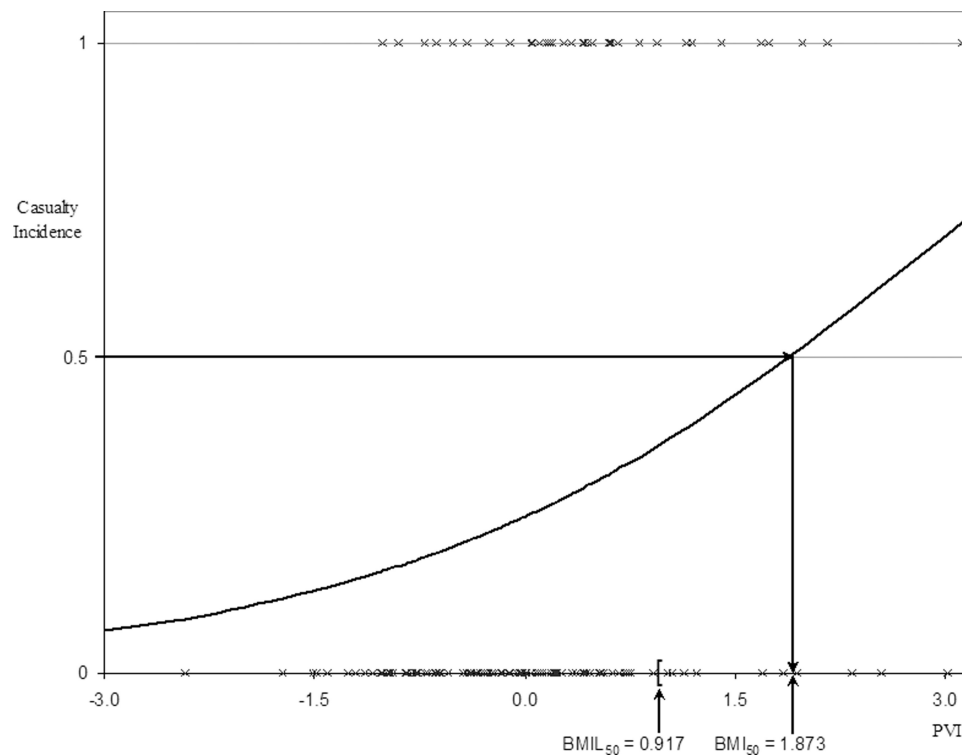


Fig. 2. Casualty data (×) and fitted complementary log-log $V_{\text{casualty}}(\text{PVI})$ function for 132 urban areas from Table II. Calculation of $\text{BMI}_{50} = 1.873$ is illustrated with reverse prediction arrows at $\text{BMV} = 0.50$. $\text{BMIL}_{50} = 0.917$ is marked by left bracket, [.

incidents and related human casualties. Via statistical modeling and analysis, we studied the relationships between vulnerability and terrorist outcome, and found that our vulnerability metric significantly described both terrorist incidence and occurrence of human casualties from terrorist attacks. The quantitative features of the PVI measure allowed us also to transfer benchmark analytic technologies from applications in toxicological and health risk assessment to this social risk/vulnerability paradigm. We found that the resulting benchmark indices could effectively delineate those U.S. urban centers whose vulnerability to terrorists attack exceeded a central 50% benchmark. To put this into a practical perspective, suppose city officials in, for example, Charleston, SC or Norfolk, VA were considering new or updated forms of coastal antiterrorist protection. According to our analysis, their relatively high PVI (cf. Table II) indicates a greater vulnerability to terrorist events at the community level. This could motivate increased funding allocation(s) or other heightened efforts to connect urban risk management programs with terrorism vulnerability assessments (Kunreuther, 2002b). Similarly, insurance adjusters considering underwriting ef-

forts to protect against future terrorist-related losses may find necessary cost/premium increases to be substantial, and some form of *ex ante* public-private partnership may prove more cost-effective than large *ex post* public relief expenditures (Kunreuther, 2002a; Kunreuther & Michel-Kerjan, 2004). By focusing attention on localities whose PVIs exceed defined benchmarks such as the BMIL_{50} , limited public and private resources can be targeted to areas of greatest need.

To illustrate the benchmark delineations geospatially, Fig. 3 maps the urban centers whose PVIs exceed our various 50% benchmarks. Centers whose PVIs exceed both the incidence-based benchmark ($\text{BMIL}_{50} = -0.60$) and the less-conservative casualty-based benchmark ($\text{BMIL}_{50} = 0.917$) are marked in red. Locations whose PVIs exceed only the incidence-based benchmark ($\text{BMIL}_{50} = -0.60$) are marked in yellow, while locations whose PVIs are below both benchmarks are marked in green. The figure shows that urban locations exhibiting high relative benchmark vulnerability—PVIs exceeding both BMIL_{50} s—are located primarily in the eastern half of the United States. There, the high-vulnerability locations

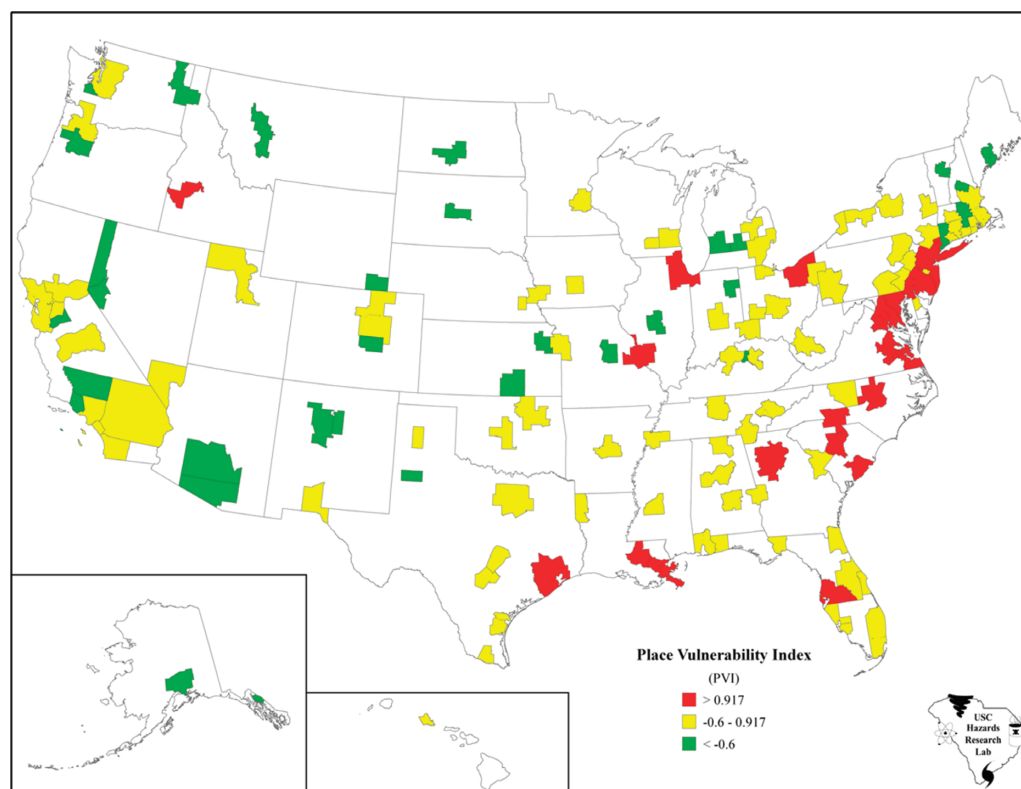


Fig. 3. Comparing urban centers from Table I using 50% benchmark vulnerability markers: Red locations exceed casualty-based benchmark ($PVI > 0.917$) and incidence-based benchmark ($PVI > -0.60$); yellow locations exceed incidence-based benchmark ($PVI > -0.60$) but not casualty-based benchmark ($PVI \not> 0.917$); and green locations do not exceed either benchmark.

are generally concentrated along shorelines (eastern seaboard, Gulf Coast, Great Lakes), and in the interior South. Interestingly, all of the top five cities most vulnerable to terrorist attack based on our analysis—New Orleans; Baton Rouge, LA; Charleston, SC; New York-Newark; and Norfolk, VA—are port cities. Few western locations exceed both $BMIL_{50}$ s, the only exception being Boise, ID. Indeed, lower (relative) vulnerability urban centers that lie *below* both $BMIL_{50}$ s appear almost exclusively in the West and along the northern tier of states; no such lower-vulnerability cities appear in the deep South.

It is instructive to note here that use of a much smaller value of BMV in our analysis would depict a much more conservative vulnerability landscape. Suppose we set $BMV = 0.10$, a common level for use with severe-event risk analyses: then for the incidence data in Table II, Equation (1) produces $BMI_{10} = -5.771$. The corresponding standard error from Equation (3) is $se[BMI_{10}] = 2.463$, leading to a 95% lower confidence limit of $BMIL_{10} = -5.771 - (1.645)(2.463) = -9.823$. Thus, under a 10% BMV

criterion, the benchmark separation point between low- and high-vulnerability tiers occurs at PVIs above at least -9.823 . A quick examination of Table II indicates that this places *all* of the urban centers in our study into the high-vulnerability tier. Moving to the casualty-based metric does not affect this conclusion: the 95% lower confidence limit changes to $BMIL_{10} = -2.100 - (1.645)(0.917) = -3.608$, which remains below any of the observed PVIs in Table II. From this perspective, one is led to infer that essentially any large- and moderate-sized urban center in the 50 United States will exceed this (admittedly conservative) benchmark in terms of its place-based vulnerability to terrorist events! How this finding affects future strategies for vulnerability screening (Apostolakis & Lemon, 2005), urban disaster management, and allocation of homeland security resources is a question of potential public concern (Sarewitz *et al.*, 2003).

This $BMIL_{10}$ calculation also illustrates an important interpretation that both BMI and $BMIL$ possess: these quantities are based upon the fitted model and

the risk/vulnerability operations that are built into their construction. Thus, separation of the data at a certain BMV does not necessarily correspond to separation at a similar data quantile: well more than 50% of the actual data points can lie below BMI_{50} , well less than 10% of the data can lie below BMI_{10} , etc. Delineations based on BMI/BMIL calculations use the fitted model to incorporate the observed terrorist incidence or casualty data, and in this sense are more instructive than simple orderings of the original PVI's.

Of course, some caveats and qualifications are in order. As implied above, the entire notion of benchmark analysis and BMI estimation is model dependent, and other statistical models for $V(PVI)$ may produce different inferences on the BMI. This is of particular concern when employing very small values of BMV (Barnes *et al.*, 1995; Gephart *et al.*, 2001; Guess *et al.*, 1977). Also, our presentation has considered only the case of binary, dichotomous outcomes, truncated from more complex count data. A natural extension of the dichotomous case is that for proportion data, where benchmark analysis is fairly well developed (Nitcheva *et al.*, 2005; Parham & Portier, 2005). However, approaches for risk benchmarking with other forms of discrete observations are still evolving and in some cases difficult to implement; greater research is needed for developing ways to apply the benchmark approach across different models (Bailer *et al.*, 2005; Sand *et al.*, 2002) and with, e.g., nondichotomous count data (Morales & Ryan, 2005; See & Bailer, 1998). Nonetheless, studies have shown that the benchmark method can confer several significant advantages for risk/safety assessment (Faustman & Bartell, 1997) and, given the flexibility it exhibits here, may prove useful in a number of other risk/vulnerability settings.

Finally, our results illustrate a robust methodology for the potential allocation of national and regional funding to support homeland security preparedness and response in U.S. cities. Not all urban areas are equally at risk, nor do they have the same underlying vulnerabilities. Our capacity to adequately prepare for and respond to these vulnerabilities varies widely across the country, especially in urban areas. We see that "place matters," and so any one-size-fits-all strategy of resource allocation and training will ignore the reality of geographic differences in the social, built, and natural environments, and will correspondingly limit urban areas' abilities to prepare for and respond to terrorist events.

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APPENDIX: COMPLEMENTARY LOG-LOG REGRESSION MODEL FOR TRUNCATED COUNT DATA

Construction of the complementary log-log model employed above can be motivated from the truncation operation used to create the binary indicators in Table II. Using formal statistical notation, let the binary indicator at the i th urban center be Y_i , $i = 1, \dots, 132$. This indicator is computed from a possibly larger number representing the recorded count of events (incidents or casualties) at that urban center. Denote this larger count by U_i . A common statistical model for unbounded counts is the Poisson distribution, where the mean number of events per location is an unknown, positive rate parameter, λ_i . By truncating U_i into Y_i we recenter attention from modeling λ_i into modeling the occurrence probability $\pi_i = \Pr[Y_i = 1]$. But, under a Poisson model for U_i , π_i can be derived mathematically as

$$\begin{aligned}\pi_i &= P[Y_i = 1] = P[U_i \geq 1] = 1 - P[U_i = 0] \\ &= 1 - \exp\{-\lambda_i\},\end{aligned}$$

the latter equality following from the basic form of the Poisson probability mass function (Piegorsch & Bailer, 2005, Section A.2.4).

Now, suppose that the Poisson mean parameter varies as a function of our vulnerability index, PVI. A natural way to link PVI to λ is via a simple linear equation: $\lambda_i = \beta_0 + \beta_1 PVI_i$, where PVI_i is the PVI measure taken at the i th urban center. Given the observed pattern of response among the 132 recorded PVI's, a typical linear regression analysis would set the linear predictor $\beta_0 + \beta_1 PVI_i$ equal to the mean

response and analyze the relationship between them accordingly. Here, however, equating $\beta_0 + \beta_1 \text{PVI}_i$ to the mean event rate λ_i fails to account for the constraint that λ_i must be positive under a Poisson model. To overcome this, we can model λ_i via an exponential form:

$$\lambda_i = \exp\{\beta_0 + \beta_1 \text{PVI}_i\}.$$

Since the exponential function is always positive, λ_i is now guaranteed to be positive for any realization of the linear predictor's parameters, β_0 and β_1 .

Pulling all the pieces of this model together, we arrive at a direct expression that links π to the PVI: $\pi_i = 1 - \exp\{-\exp(\beta_0 + \beta_1 \text{PVI}_i)\}$. Inverting gives

$$\log\{-\log(1 - \pi_i)\} = \beta_0 + \beta_1 \text{PVI}_i,$$

which recovers the link function seen in the text. Since this relates the linear predictor to the log of the log of the complementary probability $1 - \pi_i$, this is often referred to as a complementary log-log link function.

It is possible to expand on this construction and consider probability models other than the Poisson for the distribution of U_i . When doing so, the key difference would occur in the expression for $P[U_i = 0]$, and this invariably introduces a variety of extra parameters that would themselves require some assumption on whether they depend upon PVI_i . The extra complications this induces in the overall model extend beyond the scope of our presentation here, although we do acknowledge that advanced modeling such as this could become an area of fruitful research in quantitative vulnerability analysis.

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