Supplementary Information

Cross-Platform Noise Thresholds in Quantum Teleportation:
Trapped-Ion and Neutral-Atom Architectures

v5.0

Celal Arda
Independent Researcher
celal.arda@outlook.de

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Contents

1 Data Availability and Code Repository

All simulation code, analysis scripts, raw data, and hardware circuit inputs/outputs associated with this paper are publicly available at:

GitHub: unearthlyimprint/teleportation-noise-thresholds

The repository contains the following components:

- **`code/`** — Core experiment scripts for the dephasing sweep and teleportation protocols.
- **`data/`** — All experimental data in CSV format, including dephasing sweeps, hardware results, and qubit scaling datasets.
- **`hardware_qpu_input/`** — Raw circuit submission (input JSON) and measurement results (output JSON) from the IonQ Forte-1 QPU.
- **`scripts/`** — Extended analysis, sweep, and plotting scripts, including Trotter-step scaling and control experiments.
- **`pasqal_native/`** — Complete Pasqal neutral-atom implementation: Pulser sequence builder, cloud submission scripts, emulator results, and generated figures.
- **`manuscript/`** — Full LaTeX source of the main manuscript and this Supplementary Information document.

Instructions for reproducing all results are provided in the repository's `README.md`.

2 Quantum Circuit Architecture

2.1 3-Qubit Teleportation Protocol

The core protocol employs 3 qubits: Alice (A), Bob (B), and one message qubit (M).

Stage 1: Entanglement. A Bell pair is created between Alice and Bob:

$$|A, B\rangle \xrightarrow{H\otimes I} \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{S1}$$

followed by $R_z(\pi)$ and $R_z(-\pi)$ phase kicks on A and B respectively.

Stage 2: Message Injection. A test message state $|+\rangle = H|0\rangle$ is prepared and swapped onto Alice's register via a SWAP gate (3 CNOT decomposition).

Stage 3: Parametric Dephasing. Controlled phase rotations of tunable strength γ are applied:

$$\mathcal{D}(\gamma) = R_z(\gamma\pi \cdot \xi_A) \otimes R_z(\gamma\pi \cdot \xi_B) \tag{S2}$$

where $\xi_A = 1.0$ and $\xi_B = -1.5$ distribute the dephasing asymmetrically. At $\gamma = 0$, no perturbation is applied. This is a *unitary* operation modeling coherent phase errors.

Stage 4: Bridge Evolution. The Heisenberg-type coupling is implemented via first-order Trotter decomposition:

$$e^{-i\theta(XX+YY+ZZ)} \approx R_{XX}(\theta) \cdot R_{YY}(\theta) \cdot R_{ZZ}(\theta) \tag{S3}$$

with $\theta = \pi/4$. Each R_{PP} decomposes into 2 CNOT gates, giving 6 CNOTs total.

Stage 5: Measurement. Bob's qubit is measured in the Hadamard basis. Fidelity: $F = 2P(|0\rangle) - 1$.

Total entangling gate count: 1 (Bell) + 3 (SWAP) + 6 (bridge) = 10 CNOTs.

2.2 Extended 9-Qubit Architecture

The original 9-qubit architecture used two 4-qubit boundary registers (Alice: A_0–A_3, Bob: B_0–B_3) plus one message qubit (M). As documented in Section 5, this architecture suffers from spectator qubits: only A_0, B_0, and M participate in information transfer.

3 Extended Experimental Data

3.1 IonQ Simulator — Full Dephasing Sweep

Table S1 presents the complete dephasing sweep results from the Azure Quantum IonQ simulator (100 shots per point).

Table S1: Full dephasing sweep on IonQ simulator.

γ	$P(\|0\rangle)$	F	σ	Status
0.000	0.960	0.920	0.039	High fidelity
0.050	0.960	0.920	0.039	High fidelity
0.100	0.880	0.760	0.065	Degraded
0.200	0.600	0.200	0.098	Near-random
0.300	0.500	0.000	0.100	Random
0.400	0.450	−0.100	0.099	Below random
0.535	0.460	−0.080	0.100	Collapsed
0.600	0.460	−0.080	0.100	Collapsed
0.800	0.490	−0.020	0.100	Collapsed
1.000	0.500	0.000	0.100	Random

3.2 IonQ Forte-1 Hardware — Shot-Level Statistics

The hardware experiments were performed with 1000 shots for the baseline and 200 shots for the control:

3.3 Pasqal Neutral-Atom — Full Sweep Data

Table S3 presents the fine-grained dephasing sweep on the Pasqal neutral-atom platform using the `EMU_FREE` emulator (42-atom layout, 8 γ points × 500 shots = 4 000 total shots). The transition region is clearly resolved between $\gamma = 0.15$ and $\gamma = 0.25$.

Experiment	Shots	Raw $P(\|0\rangle)$ at Bob	Corrected F
Teleport $\|0\rangle$	1000	0.509 (pre-correction)	0.987 ± 0.004
Teleport $\|1\rangle$	1000	0.494 (pre-correction)	0.988 ± 0.003
Control (no ent.) $\|0\rangle$	200	0.995 (Bob stays $\|0\rangle$)	n/a
Control (no ent.) $\|1\rangle$	200	0.995 (Bob stays $\|0\rangle$)	n/a

Table S2: Shot-level summary of all IonQ Forte-1 hardware experiments.

γ	Mean $\langle n \rangle$	Ground state P_0	Shannon entropy S (bits)
0.00	0.48	12.0%	3.2
0.05	0.45	12.0%	3.1
0.10	0.38	22.5%	2.8
0.15	0.28	45.0%	2.3
0.20	0.14	71.5%	1.2
0.25	0.04	93.0%	0.4
0.30	0.01	99.0%	0.1
0.40	0.01	99.5%	0.0

Table S3: Pasqal neutral-atom EMU_FREE sweep: fine-grained γ resolution.

3.4 Emulator Cross-Validation

To verify simulation accuracy, we cross-validated the `EMU_FREE` results against exact state-vector simulation (`EMU_SV`) at three representative checkpoints. Table S4 confirms agreement within 1.5% at all checkpoints, establishing that the computationally efficient `EMU_FREE` backend faithfully reproduces the quantum dynamics.

γ	`EMU_FREE` P_0	`EMU_SV` P_0	Δ
0.05	12.0%	11.0%	+1.0%
0.20	71.5%	72.5%	−1.0%
0.40	93.0%	94.0%	−1.0%

Table S4: Cross-validation of `EMU_FREE` against exact state-vector simulation (`EMU_SV`). Agreement is within 1.5% at all checkpoints.

3.5 FRESNEL_CAN1 QPU Hardware Results

The four-tier validation chain was completed with physical execution on Pasqal's FRESNEL_CAN1 neutral-atom QPU (22 atoms: 9 core qubits + 13 spectators, 500 shots per checkpoint).

Three distinct noise regimes are identified:

- $\gamma = 0.05$ **(noise suppression):** Hardware decoherence damps coherent Rydberg excitations, yielding *higher* ground-state probability than ideal (ratio < 1).
- $\gamma = 0.20$ **(close agreement):** QPU matches ideal simulation within 1.4%, indicating the dephasing-induced transition is robust against hardware noise.

γ	QPU P_0	EMU_FREE P_0	Noise Ratio	Unique States
0.05	19.6%	8.0%	0.67×	68
0.20	70.6%	72.0%	1.13×	28
0.40	79.0%	93.0%	2.84×	19

Table S5: FRESNEL_CAN1 QPU hardware results (core 9-qubit extraction). The noise ratio quantifies $(1 - P_0^{\mathrm{QPU}})/(1 - P_0^{\mathrm{ideal}})$. Note: Ideal EMU_FREE baseline values differ from Table S3 due to the reduced spectator atom count in the 22-atom QPU layout (vs. 42 atoms in the emulator configuration).

- $\gamma = 0.40$ **(noise floor):** Residual hardware noise creates a $\sim$2.6% excitation floor, preventing the QPU from reaching the ideal ground state.

4 Hardware Platform Specifications

Table S6 summarizes the specifications of both quantum platforms used in this study.

Parameter	**Value**
IonQ Forte-1 Trapped-Ion QPU	
Ion species	$^{171}\mathrm{Yb}^{+}$
Connectivity	All-to-all
Single-qubit gate fidelity	$> 99.7\%$
Two-qubit gate fidelity (MS gate)	$\sim 99.5\%$–99.7%
Two-qubit gate error	$\sim 0.3\%$–0.5% per gate
Readout fidelity	$> 99.5\%$
Coherence time (T_2)	> 1 s
Access Details	
Cloud provider	Microsoft Azure Quantum
Backend (simulation)	`ionq.simulator`
Backend (hardware)	`ionq.qpu.forte-1`
Pasqal Neutral-Atom Platform	
Atom species	$^{87}\mathrm{Rb}$
Interaction type	Rydberg blockade (van der Waals)
Emulator backends	`EMU_FREE`, `EMU_SV`, `EMU_FRESNEL`
QPU	FRESNEL_CAN1 (61 traps, 22 atoms used)
SDK	Pulser 1.7.0 + `pasqal-cloud` 0.20.8

Table S6: Hardware and emulator platform specifications.

5 Spectator Qubit Analysis

5.1 Scaling Test Results

A scaling experiment at $\gamma = 0$ tested fidelity as a function of nominal circuit size:

Nominal qubits	Entangling gates	Hardware F	Note
3	2	1.000	Baseline
5	6	1.000	Spectators removed
7	10	1.000	Spectators removed
9	14	1.000	Spectators removed

Table S7: Scaling test on IonQ Forte-1 at $\gamma = 0$. The hardware compiler identifies spectator qubits and optimizes them away. Note: at $\gamma = 0$, the parametric dephasing gates are identity operations, allowing the IonQ compiler to further optimize the nominal 10-CNOT baseline circuit to just 2 effective entangling gates.

5.2 Analysis

In the 9-qubit architecture, only the message qubit (M) and the first entangled pair (A_0, B_0) participate in information transfer. The remaining three pairs (A_1–A_3, B_1–B_3) form inter-boundary entanglement but never interact with the message path. IonQ's transpiler detects these as spectator qubits and removes them.

This has two consequences:

1. The $F \approx 0$ result at $\gamma = 0.535$ is driven by the *injected dephasing parameter* γ, not by circuit depth or qubit count.
2. Testing depth-dependent fidelity degradation requires circuits where *all gates lie on the message's critical path* (e.g., Trotter-step scaling).

6 Pasqal Neutral-Atom Implementation Details

6.1 Pulser Sequence Construction

The neutral-atom simulation uses the Pulser framework to define atom registers and laser pulse sequences. The atoms are arranged in a linear chain with inter-atomic spacing tuned to achieve Rydberg blockade:

- **Register:** 3 atoms at spacing $d = 6\,\mu\text{m}$ (within blockade radius $R_b \approx 9\,\mu\text{m}$). This minimal configuration was used for local validation and pedagogical demonstrations. Production emulator sweeps (Tables S3–S4) use 42-atom registers, and the FRESNEL_CAN1 QPU runs (Table S5) use 22-atom registers with 9 core qubits.
- **Channel:** Global Rydberg channel with $\Omega_{\text{max}} = 2\pi \times 4\,\text{MHz}$.
- **Dephasing injection:** Detuning ramp $\Delta(\gamma)$ applied after entangling pulse.
- **Measurement:** Final state sampled from the ground/Rydberg basis.

6.2 Rydberg Blockade Hamiltonian

The Rydberg blockade Hamiltonian is:

$$H = \frac{\Omega}{2} \sum_i \sigma_x^i - \Delta \sum_i n_i + \sum_{i<j} \frac{C_6}{|r_i - r_j|^6} n_i n_j \tag{S4}$$

where Ω is the Rabi frequency, Δ is the detuning, $n_i = |r\rangle\langle r|_i$ is the Rydberg number operator, and C_6 is the van der Waals coefficient.

The dephasing parameter γ maps to the detuning: increasing γ shifts the system from the resonant regime (where Rydberg excitations are favorable) to the off-resonant regime (ground state dominated).

7 Software Environment and Reproducibility

7.1 Python Dependencies

All code was developed and tested with Python ≥ 3.10. The full dependency list is available in the repository's `requirements.txt`:

Package	Version	Purpose
`numpy`	≥ 2.0	Numerical computation
`scipy`	≥ 1.12	Scientific computing
`matplotlib`	≥ 3.8	Visualization
`azure-quantum`	≥ 2.0	Azure Quantum SDK (IonQ)
`qsharp`	≥ 1.0	Q# integration
`pulser`	≥ 1.0	Pasqal sequence builder
`pasqal-cloud`	≥ 0.20	Pasqal Cloud SDK
`qutip`	≥ 5.0	Local quantum simulation
`python-dotenv`	≥ 1.0	Environment variable management

Table S8: Python package dependencies.

7.2 Script Descriptions

Table S9 provides a summary of all scripts in the repository.

Table S9: Repository script index with descriptions.

Script	Description
`code/experiment_1_phase_transition.py`	Dephasing sweep: varies $\gamma \in [0, 1]$ on IonQ simulator.
`code/teleportation_pulser_continuous.py`	Continuous-mode Pulser simulation of the Rydberg Hamiltonian.
`scripts/tier1_analysis.py`	Result aggregation and statistical analysis.
`scripts/tier1_depth_sweep.py`	Depth sweep: varies circuit size at fixed γ.
`scripts/tier1v3_trotter_sweep.py`	Trotter-step scaling: varies Trotter steps 1–8 on the 3-qubit architecture.
`scripts/teleportation_control_experiment.py`	Control: teleportation circuit without Bell pair.
`scripts/teleportation_hardware_correct.py`	Hardware-corrected protocol with classical post-processing.
`scripts/teleportation_local_test.py`	Local teleportation experiment with shot-level analysis.

Script	Description
`scripts/teleportation_sweep.py`	Parameter sweep of teleportation fidelity.
`scripts/plot_azure_data.py`	Plotting utilities for Azure Quantum results.
`scripts/trotter_noisy_corrected.py`	Local noisy Trotter sweep with IonQ Forte noise model.
`pasqal_native/scripts/run_teleportation_pasqal.py`	Pasqal Cloud submission: builds and submits sequences for γ sweep.
`pasqal_native/scripts/run_fine_sweep.py`	Fine-grained γ sweep near the threshold region.
`pasqal_native/scripts/run_emulator_comparison.py`	Cross-validation between `EMU_FREE` and `EMU_SV`.
`pasqal_native/scripts/analyze_results.py`	Analysis and figure generation from emulator results.
`pasqal_native/scripts/merge_results.py`	Merges multiple emulator result files.
`pasqal_native/scripts/run_fresnel_validation.py`	Fresnel validation of the neutral-atom layout.
`pasqal_native/scripts/fetch_fresnel_results.py`	Retrieves completed FRESNEL_CAN1 QPU batch results.
`pasqal_native/scripts/analyze_fresnel_can1.py`	Extracts core-qubit statistics from QPU data.

7.3 Reproducing Results

To reproduce all results presented in the main manuscript:

1. Clone the repository:

```
git clone https://github.com/unearthlyimprint/
   teleportation-noise-thresholds.git
cd teleportation-noise-thresholds
```

2. Install dependencies:

```
python -m venv venv && source venv/bin/
   activate
pip install -r requirements.txt
```

3. Configure Azure Quantum credentials (see `.env.example`).

4. Run the dephasing sweep:

```
python code/experiment_1_phase_transition.py
```

5. For Pasqal neutral-atom results, follow the instructions in `pasqal_native/README.md`.