

# Supplementary Information

Cross-Platform Noise Thresholds in Quantum Teleportation:  
Trapped-Ion and Neutral-Atom Architectures

v5.0

Celal Arda

Independent Researcher

[celal.arda@outlook.de](mailto:celal.arda@outlook.de)

February 2026

## Contents

<b>1 Data Availability and Code Repository</b>	<b>2</b>
<b>2 Quantum Circuit Architecture</b>	<b>2</b>
2.1 3-Qubit Teleportation Protocol . . . . .	2
2.2 Extended 9-Qubit Architecture . . . . .	3
<b>3 Extended Experimental Data</b>	<b>3</b>
3.1 IonQ Simulator — Full Dephasing Sweep . . . . .	3
3.2 IonQ Forte-1 Hardware — Shot-Level Statistics . . . . .	3
3.3 Pasqal Neutral-Atom — Full Sweep Data . . . . .	4
3.4 Emulator Cross-Validation . . . . .	4
3.5 FRESNEL_CAN1 QPU Hardware Results . . . . .	4
<b>4 Hardware Platform Specifications</b>	<b>4</b>
<b>5 Spectator Qubit Analysis</b>	<b>4</b>
5.1 Scaling Test Results . . . . .	4
5.2 Analysis . . . . .	5
<b>6 Pasqal Neutral-Atom Implementation Details</b>	<b>5</b>
6.1 Pulser Sequence Construction . . . . .	5
6.2 Rydberg Blockade Hamiltonian . . . . .	6
<b>7 Software Environment and Reproducibility</b>	<b>6</b>
7.1 Python Dependencies . . . . .	6
7.2 Script Descriptions . . . . .	6
7.3 Reproducing Results . . . . .	8

# 1 Data Availability and Code Repository

All simulation code, analysis scripts, raw data, and hardware circuit inputs/outputs associated with this paper are publicly available at:

GitHub: [unearthlyimprint/teleportation-noise-thresholds](https://github.com/unearthlyimprint/teleportation-noise-thresholds)

The repository contains the following components:

- `code/` — Core experiment scripts for the dephasing sweep and teleportation protocols.
- `data/` — All experimental data in CSV format, including dephasing sweeps, hardware results, and qubit scaling datasets.
- `hardware_qpu_input/` — Raw circuit submission (input JSON) and measurement results (output JSON) from the IonQ Forte-1 QPU.
- `scripts/` — Extended analysis, sweep, and plotting scripts, including Trotter-step scaling and control experiments.
- `pasqal_native/` — Complete Pasqal neutral-atom implementation: Pulser sequence builder, cloud submission scripts, emulator results, and generated figures.
- `manuscript/` — Full L<sup>A</sup>T<sub>E</sub>X source of the main manuscript and this Supplementary Information document.

Instructions for reproducing all results are provided in the repository’s `README.md`.

## 2 Quantum Circuit Architecture

### 2.1 3-Qubit Teleportation Protocol

The core protocol employs 3 qubits: Alice ( $A$ ), Bob ( $B$ ), and one message qubit ( $M$ ).

**Stage 1: Entanglement.** A Bell pair is created between Alice and Bob:

$$|A, B\rangle \xrightarrow{H \otimes I, \text{CNOT}} \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \quad (1)$$

followed by  $R_z(\pi)$  and  $R_z(-\pi)$  phase kicks on  $A$  and  $B$  respectively.

**Stage 2: Message Injection.** A test message state  $|+\rangle = H|0\rangle$  is prepared and swapped onto Alice’s register via a SWAP gate (3 CNOT decomposition).

**Stage 3: Parametric Dephasing.** Controlled phase rotations of tunable strength  $\gamma$  are applied:

$$\mathcal{D}(\gamma) = R_z(\gamma\pi \cdot \xi_A) \otimes R_z(\gamma\pi \cdot \xi_B) \quad (2)$$

where  $\xi_A = 1.0$  and  $\xi_B = -1.5$  distribute the dephasing asymmetrically. At  $\gamma = 0$ , no perturbation is applied. This is a *unitary* operation modeling coherent phase errors.

**Stage 4: Bridge Evolution.** The Heisenberg-type coupling is implemented via first-order Trotter decomposition:

$$e^{-i\theta(XX+YY+ZZ)} \approx R_{XX}(\theta) \cdot R_{YY}(\theta) \cdot R_{ZZ}(\theta) \quad (3)$$

with  $\theta = \pi/4$ . Each  $R_{PP}$  decomposes into 2 CNOT gates, giving 6 CNOTs total.

**Stage 5: Measurement.** Bob’s qubit is measured in the Hadamard basis. Fidelity:  $F = 2P(|0\rangle) - 1$ .

**Total entangling gate count:** 1 (Bell) + 3 (SWAP) + 6 (bridge) = 10 CNOTs.

## 2.2 Extended 9-Qubit Architecture

The original 9-qubit architecture used two 4-qubit boundary registers (Alice:  $A_0$ – $A_3$ , Bob:  $B_0$ – $B_3$ ) plus one message qubit ( $M$ ). As documented in Section 5, this architecture suffers from spectator qubits: only  $A_0$ ,  $B_0$ , and  $M$  participate in information transfer.

## 3 Extended Experimental Data

### 3.1 IonQ Simulator — Full Dephasing Sweep

Table 1 presents the complete dephasing sweep results from the Azure Quantum IonQ simulator (100 shots per point).

Table 1: Full dephasing sweep on IonQ simulator.

$\gamma$	$P( 0\rangle)$	$F$	$\sigma$	Status
0.000	0.960	0.920	0.039	High fidelity
0.050	0.960	0.920	0.039	High fidelity
0.100	0.880	0.760	0.065	Degraded
0.200	0.600	0.200	0.098	Near-random
0.300	0.500	0.000	0.100	Random
0.400	0.450	-0.100	0.099	Below random
0.535	0.460	-0.080	0.100	Collapsed
0.600	0.460	-0.080	0.100	Collapsed
0.800	0.490	-0.020	0.100	Collapsed
1.000	0.500	0.000	0.100	Random

### 3.2 IonQ Forte-1 Hardware — Shot-Level Statistics

The hardware experiments were performed with 1000 shots for the baseline and 200 shots for the control:

Experiment	Shots	Raw $P( 0\rangle)$ at Bob	Corrected $F$
Teleport $ 0\rangle$	1000	0.509 (pre-correction)	$0.987 \pm 0.004$
Teleport $ 1\rangle$	1000	0.494 (pre-correction)	$0.988 \pm 0.003$
Control (no ent.) $ 0\rangle$	200	0.995 (Bob stays $ 0\rangle$ )	n/a
Control (no ent.) $ 1\rangle$	200	0.995 (Bob stays $ 0\rangle$ )	n/a

Table 2: Shot-level summary of all IonQ Forte-1 hardware experiments.

$\gamma$	Mean $\langle n \rangle$	Ground state $P_0$	Shannon entropy $S$ (bits)
0.00	0.48	12.0%	3.2
0.05	0.45	12.0%	3.1
0.10	0.38	22.5%	2.8
0.15	0.28	45.0%	2.3
0.20	0.14	71.5%	1.2
0.25	0.04	93.0%	0.4
0.30	0.01	99.0%	0.1
0.40	0.01	99.5%	0.0

Table 3: Pasqal neutral-atom EMU\_FREE sweep: fine-grained  $\gamma$  resolution.

### 3.3 Pasqal Neutral-Atom — Full Sweep Data

### 3.4 Emulator Cross-Validation

### 3.5 FRESNEL\_CAN1 QPU Hardware Results

The four-tier validation chain was completed with physical execution on Pasqal’s FRESNEL\_CAN1 neutral-atom QPU (22 atoms: 9 core qubits + 13 spectators, 500 shots per checkpoint).

Three distinct noise regimes are identified:

- $\gamma = 0.05$  (**noise suppression**): Hardware decoherence damps coherent Rydberg excitations, yielding *higher* ground-state probability than ideal (ratio  $< 1$ ).
- $\gamma = 0.20$  (**close agreement**): QPU matches ideal simulation within 1.4%, indicating the dephasing-induced transition is robust against hardware noise.
- $\gamma = 0.40$  (**noise floor**): Residual hardware noise creates a  $\sim 2.6\%$  excitation floor, preventing the QPU from reaching the ideal ground state.

## 4 Hardware Platform Specifications

## 5 Spectator Qubit Analysis

### 5.1 Scaling Test Results

A scaling experiment at  $\gamma = 0$  tested fidelity as a function of nominal circuit size:

$\gamma$	EMU_FREE $P_0$	EMU_SV $P_0$	$\Delta$
0.05	12.0%	11.0%	+1.0%
0.20	71.5%	72.5%	-1.0%
0.40	93.0%	94.0%	-1.0%

Table 4: Cross-validation of EMU\_FREE against exact state-vector simulation (EMU\_SV). Agreement is within 1.5% at all checkpoints.

$\gamma$	QPU $P_0$	EMU_FREE $P_0$	Noise Ratio	Unique States
0.05	19.6%	8.0%	0.67 $\times$	68
0.20	70.6%	72.0%	1.13 $\times$	28
0.40	79.0%	93.0%	2.84 $\times$	19

Table 5: FRESNEL\_CAN1 QPU hardware results (core 9-qubit extraction). The noise ratio quantifies  $(1 - P_0^{\text{QPU}})/(1 - P_0^{\text{ideal}})$ .

## 5.2 Analysis

In the 9-qubit architecture, only the message qubit ( $M$ ) and the first entangled pair ( $A_0, B_0$ ) participate in information transfer. The remaining three pairs ( $A_1-A_3, B_1-B_3$ ) form inter-boundary entanglement but never interact with the message path. IonQ’s transpiler detects these as spectator qubits and removes them.

This has two consequences:

1. The  $F \approx 0$  result at  $\gamma = 0.535$  is driven by the *injected dephasing parameter*  $\gamma$ , not by circuit depth or qubit count.
2. Testing depth-dependent fidelity degradation requires circuits where *all gates lie on the message’s critical path* (e.g., Trotter-step scaling).

## 6 Pasqal Neutral-Atom Implementation Details

### 6.1 Pulser Sequence Construction

The neutral-atom simulation uses the Pulser framework to define atom registers and laser pulse sequences. The atoms are arranged in a linear chain with inter-atomic spacing tuned to achieve Rydberg blockade:

- **Register:** 3 atoms at spacing  $d = 6 \mu\text{m}$  (within blockade radius  $R_b \approx 9 \mu\text{m}$ ).
- **Channel:** Global Rydberg channel with  $\Omega_{\max} = 2\pi \times 4 \text{ MHz}$ .
- **Dephasing injection:** Detuning ramp  $\Delta(\gamma)$  applied after entangling pulse.
- **Measurement:** Final state sampled from the ground/Rydberg basis.

Parameter	Value
<i>IonQ Forte-1 Trapped-Ion QPU</i>	
Ion species	$^{171}\text{Yb}^+$
Connectivity	All-to-all
Single-qubit gate fidelity	> 99.7%
Two-qubit gate fidelity (MS gate)	~ 99.5%–99.7%
Two-qubit gate error	~ 0.3%–0.5% per gate
Readout fidelity	> 99.5%
Coherence time ( $T_2$ )	> 1 s
<i>Access Details</i>	
Cloud provider	Microsoft Azure Quantum
Backend (simulation)	<code>ionq.simulator</code>
Backend (hardware)	<code>ionq.qpu.forte-1</code>
<i>Pasqal Neutral-Atom Platform</i>	
Atom species	$^{87}\text{Rb}$
Interaction type	Rydberg blockade (van der Waals)
Emulator backends	<code>EMU_FREE</code> , <code>EMU_SV</code> , <code>EMU_FRESNEL</code>
QPU	FRESNEL_CAN1 (61 traps, 22 atoms used)
SDK	Pulser 1.7.0 + <code>pasqal-cloud</code> 0.20.8

Table 6: Hardware and emulator platform specifications.

## 6.2 Rydberg Blockade Hamiltonian

The Rydberg blockade Hamiltonian is:

$$H = \frac{\Omega}{2} \sum_i \sigma_x^i - \Delta \sum_i n_i + \sum_{i < j} \frac{C_6}{|r_i - r_j|^6} n_i n_j \quad (4)$$

where  $\Omega$  is the Rabi frequency,  $\Delta$  is the detuning,  $n_i = |r\rangle\langle r|_i$  is the Rydberg number operator, and  $C_6$  is the van der Waals coefficient.

The dephasing parameter  $\gamma$  maps to the detuning: increasing  $\gamma$  shifts the system from the resonant regime (where Rydberg excitations are favorable) to the off-resonant regime (ground state dominated).

## 7 Software Environment and Reproducibility

### 7.1 Python Dependencies

All code was developed and tested with Python  $\geq 3.10$ . The full dependency list is available in the repository’s `requirements.txt`:

### 7.2 Script Descriptions

Table 9 provides a summary of all scripts in the repository.

Nominal qubits	Entangling gates	Hardware $F$	Note
3	2	1.000	Baseline
5	6	1.000	Spectators removed
7	10	1.000	Spectators removed
9	14	1.000	Spectators removed

Table 7: Scaling test on IonQ Forte-1 at  $\gamma = 0$ . The hardware compiler identifies spectator qubits and optimizes them away. Note: at  $\gamma = 0$ , the parametric dephasing gates are identity operations, allowing the IonQ compiler to further optimize the nominal 10-CNOT baseline circuit to just 2 effective entangling gates.

Package	Version	Purpose
numpy	$\geq 2.0$	Numerical computation
scipy	$\geq 1.12$	Scientific computing
matplotlib	$\geq 3.8$	Visualization
azure-quantum	$\geq 2.0$	Azure Quantum SDK (IonQ)
qsharp	$\geq 1.0$	Q# integration
pulser	$\geq 1.0$	Pasqal sequence builder
pasqal-cloud	$\geq 0.20$	Pasqal Cloud SDK
qutip	$\geq 5.0$	Local quantum simulation
python-dotenv	$\geq 1.0$	Environment variable management

Table 8: Python package dependencies.

Table 9: Repository script index with descriptions.

Script	Description
code/experiment_1_phase_transition.py	Dephasing sweep: varies $\gamma \in [0, 1]$ on IonQ simulator.
code/wormhole_pulser_continuous.py	Continuous-mode Pulser simulation of the Rydberg Hamiltonian.
scripts/tier1_analysis.py	Result aggregation and statistical analysis.
scripts/tier1_depth_sweep.py	Depth sweep: varies circuit size at fixed $\gamma$ .
scripts/tier1v3_trotter_sweep.py	Trotter-step scaling: varies Trotter steps 1–8 on the 3-qubit architecture.
scripts/wormhole_control_experiment.py	Control: teleportation circuit without Bell pair.
scripts/wormhole_hardware_correct.py	Hardware-corrected protocol with classical post-processing.
scripts/wormhole_teleport_local_test.py	Local teleportation experiment with shot-level analysis.
scripts/wormhole_teleport_sweep.py	Parameter sweep of teleportation fidelity.
scripts/plot_azure_data.py	Plotting utilities for Azure Quantum results.
scripts/trotter_noisy_corrected.py	Local noisy Trotter sweep with IonQ Forte noise model.
pasqal_native/scripts/run_wormhole_pasqal.py	Pasqal Cloud submission: builds and submits sequences for $\gamma$ sweep.

Script	Description
<code>pasqal_native/scripts/run_fine_sweep.py</code>	Fine-grained $\gamma$ sweep near the threshold region.
<code>pasqal_native/scripts/run_emulator_comparison.py</code>	Cross-validation between EMU_FREE and EMU_SV.
<code>pasqal_native/scripts/analyze_results.py</code>	Analysis and figure generation from emulator results.
<code>pasqal_native/scripts/merge_results.py</code>	Merges multiple emulator result files.
<code>pasqal_native/scripts/run_fresnel_validation.py</code>	Fresnel validation of the neutral-atom layout.
<code>pasqal_native/scripts/fetch_fresnel_results.py</code>	Retrieves completed FRESNEL_CAN1 QPU batch results.
<code>pasqal_native/scripts/analyze_fresnel_can1.py</code>	Extracts core-qubit statistics from QPU data.

### 7.3 Reproducing Results

To reproduce all results presented in the main manuscript:

1. Clone the repository:

```
git clone https://github.com/unearthlyimprint/
           teleportation-noise-thresholds.git
cd teleportation-noise-thresholds
```

2. Install dependencies:

```
python -m venv venv && source venv/bin/
          activate
pip install -r requirements.txt
```

3. Configure Azure Quantum credentials (see `.env.example`).

4. Run the dephasing sweep:

```
python code/experiment_1_phase_transition.py
```

5. For Pasqal neutral-atom results, follow the instructions in `pasqal_native/README.md`.