대한산업공학회
2007년 추계학술발표대회

자원제약을 고려한 통합된 조립라인 밸런싱에 대한 연구

문일경, 성준

부산대학교 산업공학과

“이 논문은 2007년 정부(교육인적자원부)의 재원으로 한국학술진흥재단의 지원을 받아 수행된 연구임”
(지방연구중심대학육성사업/차세대물류IT기술연구사업단)

Contents

1. Introduction

Assembly Line Balancing (ALB) problem

- An assignment of the number of tasks to the workstations with restrictions
- Widely researched objectives :

  minimizing cycle time for a given number of workstations

  minimizing the number of workstations for a given cycle time
- The considered restrictions :

  precedence and incompatibility relations between tasks

  operating time at each workstation can not be greater than cycle time
- The ALB problem is generally known as an *NP-hard* problem.
  (Wee and Magazine, 1982, *Operations Research Letters*)

1. Introduction [Cont.]

Literature review

Author	Condition		Solution
	Condition	Objective	
Helgeson and Birnie (*JOIE*,1961)	Precedence tasks	Minimizing cycle time	Heuristic Method
Mansoor (*JOIE*,1964)			
Nars and Elsayed (*IJPR*,1990)	Precedence tasks with machines assignment	Minimizing cycle time	Heuristic Method
Graves and Holmes (*IJFM*, 1988)	Precedence tasks with equipments assignment	Minimizing the total cost (equipment usage and set-up costs)	Heuristic Method
Kim and Kim (*C&IE*, 1996)	Precedence tasks	(1) Minimizing the number of workstations (2) Minimizing cycle time (3) Maximizing workload smoothness (4) Maximizing work relatedness	Genetic Algorithm
Gregory (*EJOR*,2006)	Precedence tasks with machine assignment	Minimizing the cycle time	Genetic Algorithm
* Dimitriadis (*C&OR*, 2006)	Precedence tasks with workers' group operating	Maximizing line effectiveness (assumption: identical workers)	Heuristic Method

1. Introduction [Cont.]

Assembly Line Balancing problem with

- Task precedence constraints and multifunctional worker assignment

- **Motivation** : Multifunctional workers with different salaries depending on their skills
- **Objective** : Minimize total annual workstation cost & annual salary of the workers

▶ Integrated optimization to minimize total relevant costs

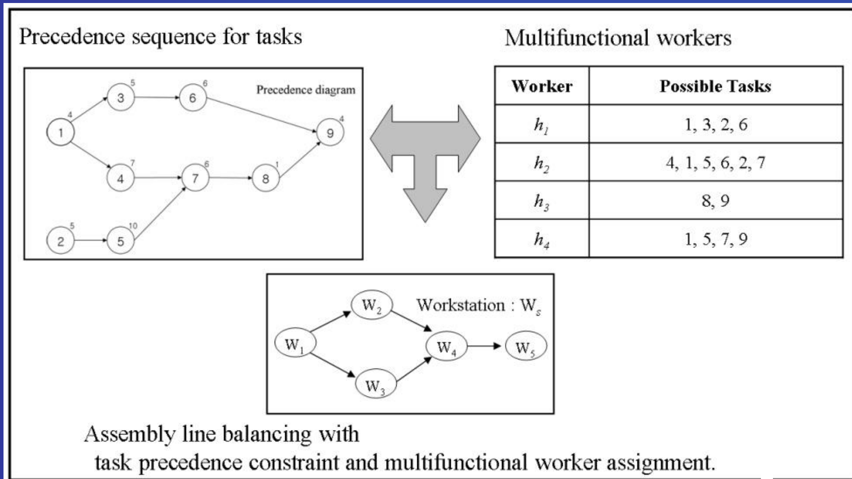


Figure 1. Optimization processes for integrated assembly line balancing problem

2. Mathematical Model

Assumptions

- The workers are multifunctional with different salaries.
- The workers can be assigned to only one workstation.
- The workers can be assigned to tasks depending on their skills.
- Multiple workers can be assigned to a single workstation.
- The precedence constraints determine the sequence.

Tasks	a_1	a_2	a_3	a_4	a_5
Time	3	1	3	1	4
Task Sequence	$a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow a_4 \rightarrow a_5$				

Workers	W_1	W_2	W_3	W_4
Possible Tasks	2	3, 4	1	5

cycle time =4

Workstation 1
- Task 1 – Worker 3
- Task 2 – Worker 1

Workstation 2
- Task 3
- Task 4

Worker 2

Workstation 3
- Task 5 – Worker 4

Figure 2. Example for assignment of 5 tasks and 4 workers

Model formulation

Notation

i	Index of tasks (i = 1,2,3,…,I)
s	Index of workstations (s = 1,2,3,…,S)
w	Worker index (w = 1,2,3,…, W)
C	cycle time
t_i	operating time for task i
FC	annual operating cost of a workstation
LC_w	annual salary of worker w
$P_{(ij)}$	set of pairs of tasks (i, j) such that there is an immediate precedence relation between them
M	Big M
A_w	set of available tasks that can be assigned to worker w

Decision Variables

F	Number of workstations to be used in the assembly line
X_{isw}	$\begin{cases} 1, & \text{if task } i \text{ is performed by worker } w \text{ at workstation } s \\ 0, & \text{otherwise} \end{cases}$
Y_{sw}	$\begin{cases} 1, & \text{if worker } w \text{ is assigned to workstation } s \\ 0, & \text{otherwise} \end{cases}$

2. Mathematical Model [Cont.]

Objective function

- relevant costs : total annual workstation cost and annual salary of workers
- objective function : minimizing the total relevant costs

$$Min\ FC \cdot F + \sum_{w=1}^{W} LC_w \left(\sum_{s=1}^{S} Y_{sw} \right)$$

Constraints

$$\sum_{s=1}^{S}\sum_{w=1}^{W} X_{isw} = 1 \quad \forall i, \qquad (2)$$

$$\sum_{s=1}^{S} Y_{sw} \le 1 \quad \forall w, \qquad (3)$$

$$\sum_{s=1}^{S}\sum_{w=1}^{W} (s \cdot X_{isw} - s \cdot X_{jsw}) \le 0 \quad \forall (i, j) \in P_{(ij)}, \qquad (4)$$

$$\sum_{i=1}^{I}\sum_{w=1}^{W} t_i \cdot X_{isw} \le C \quad \forall s, \qquad (5)$$

Constraints

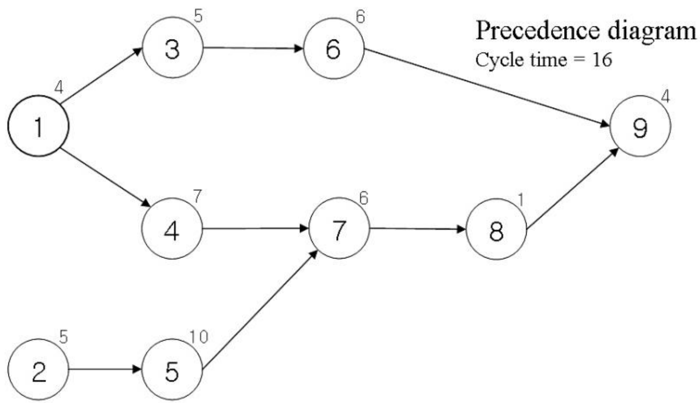
$$\sum_{i=1}^{I} X_{isw} \le M \cdot Y_{sw} \qquad \forall s, w, \qquad (6)$$

$$\sum_{i \notin A_w}^{I} \sum_{s=1}^{S} X_{isw} = 0 \qquad \forall w, \qquad (7)$$

$$\sum_{s=1}^{S} s \cdot X_{isw} \le F \qquad \forall i, w, \qquad (8)$$

$$X_{isw},\ Y_{sw} \in \{0,1\} \qquad (9)$$

■ Genetic algorithm (ex. number of tasks: 9, number of workers : 8)



Representation

	Workstation 1 Cycle time = 16			Workstation 2 Cycle time = 16		Workstation 3 Cycle time = 16			
Operating Sequence	1	2	3	4	5	6	7	8	9
Tasks	1	2	4	5	7	8	3	6	9
Workers	8	8	2	6	6	3	3	5	5

Figure 3. Representation for an example

Chromosomes

- The sequence chromosome for tasks and workstation assignment
- The assignment chromosome for multifunctional workers
- The length of each chromosome is equal to the total number of tasks.

Representation

	Workstation 1 Cycle time = 16			Workstation 2 Cycle time = 16		Workstation 3 Cycle time = 16			
Operating Sequence	1	2	3	4	5	6	7	8	9
Tasks	1	2	4	5	7	8	3	6	9
Workers	8	8	2	6	6	3	3	5	5

Figure 4. An example of chromosomes

- The operating sequence for tasks : 1-2-4-5-7-8-3-6-9
- The worker assignment for tasks : 8-8-2-6-6-3-3-5-5

3. Genetic Algorithm [Cont.]

▶ **Simple heuristic (Initialization for large-sized problem)**

Step 1. Arrange the tasks among the precedence constraints.

Step 2. To divide a workstation, calculate the operating time of the cumulative tasks by using the predetermined cycle time.

Step 3. Assign a worker to the task if the worker can be assigned.

The low skilled worker (who has a lower index for possible tasks) will be assigned first.

Step 4. If the index of a worker overlaps in different workstations, go to Step 3. Otherwise, return the result to the GA.

Parameters (decided after Pilot Test)

- For task arrangement considering precedence constraints
  - Crossover : PMX (Partially Matched Crossover)
  - Crossover rate: 0.5
  - Mutation rate: 0.3
- For multifunctional worker assignment
  - Crossover : one-cut-point crossover
  - Crossover rate: 0.5
  - Mutation rate: 0.4
- Terminating conditions
  - 200,000 generations (in the case of 9 tasks, 11 tasks, and 21 tasks)
  - when the best individual does not improve more than 0.01% for 4,000 generations

4. Numerical Examples

Input data (ex. number of tasks: 32, number of workers : 15)

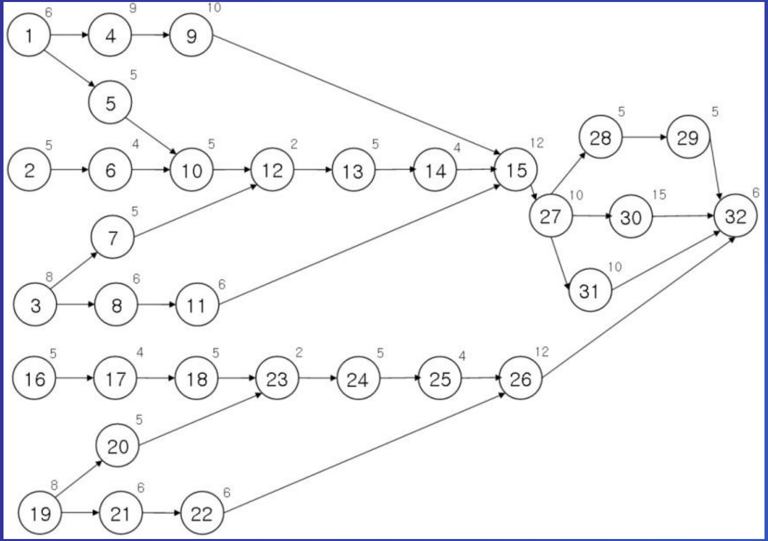


Figure 5. Precedence sequence diagram for 32 tasks

Table 1. multifunctional workers and salaries

Worker	Possible Tasks						Salary
1	11	14	16	19	20	23	$230
2	2	6	13	17	21	26	$230
3	4	9	16	19	24		$200
4	3	7	8	13	30		$200
5	10	16	20	27	28	32	$230
6	15	17	18	20	25		$200
7	1	6	13	21	28		$200
8	21	24	25	30	32		$200
9	1	11	14	21	27	30	$230
10	8	17	24	29	30	31	$230
11	3	12	14	16	17		$200
12	12	16	22	28	31		$200
13	3	5	10	18	24	32	$230
14	3	13	22	26	32		$200
15	5	10	12	13	22	26	$230

4. Numerical Examples [Cont.]

Data for example

- Number of tasks : 32
- Number of workers : 15
- Predetermined cycle time : 32 minutes
- Annual workstation operating cost : $120,000

Parameters

▶ For task arrangement
- Crossover : PMX
- Crossover rate: 0.5
- Mutation rate: 0.3

▶ For multifunctional worker assignment
- Crossover : one-cut-point crossover
- Crossover rate: 0.5
- Mutation rate: 0.4

▶ Terminating conditions
- 300,000 generations
- when the best individual does not improve more than 0.01% for 5,000 generations

4. Numerical Examples [Cont.]

Result of example

- Number of workstations : 7
- Assigned workers : 11
- Total relevant cost : $1,078,000

  total annual workstation operating cost : $840,000

  total annual salary of assigned worker : $238,000

Result of example

Table 2. Result of assignment of workers and tasks to the workstation

Operating Sequence	Index for Tasks	Index for Workstation	Operating Time	Cumulative Operating Time	Assigned Worker
1	2		5	5	2
2	6		4	9	2
3	3	1	8	17	4
4	7		5	22	4
5	8		6	28	4
6	1		6	6	9
7	11		6	12	9
8	5	2	5	17	15
9	10		5	22	15
10	12		2	24	15
11	13		5	29	15
12	4		9	9	3
13	9	3	10	19	3
14	16		5	24	3
15	19		8	32	3
16	14	4	4	4	1
17	20		5	9	1
18	23		2	11	1
19	15	4	12	23	6
20	17		4	27	6
21	18		5	32	6
22	27		10	10	5
23	28	5	5	15	5
24	29		5	20	10
25	31		10	30	10
26	21		6	6	8
27	24	6	5	11	8
28	25		4	15	8
29	30		15	30	8
30	22		6	6	14
31	26	7	12	18	14
32	32		6	24	14

Computational results

Table 3. Comparison result of mixed integer programming and GA

Examples	Mixed integer programming			Genetic algorithm		
	Computation time*	Objective function	Remark	Computation time*	Objective function	Remark
9 tasks & 8 workers	10 minutes	$4,160	Optimal	18.7 seconds	$4,160	Optimal
11 tasks & 8 workers	25 minutes	$5,600	Optimal	26.9 seconds	$5,600	Optimal
21 tasks & 15 workers	2 hours 17 minutes	$7,480	Optimal	10 minutes	$7,480	Optimal
32 tasks & 15 workers	18 hours 55 minutes	$10,780	Optimal	15 minutes	$10,780	Optimal

* : Average of 10 evaluation

Input data [ex. number of tasks: 61, number of workers : 20]

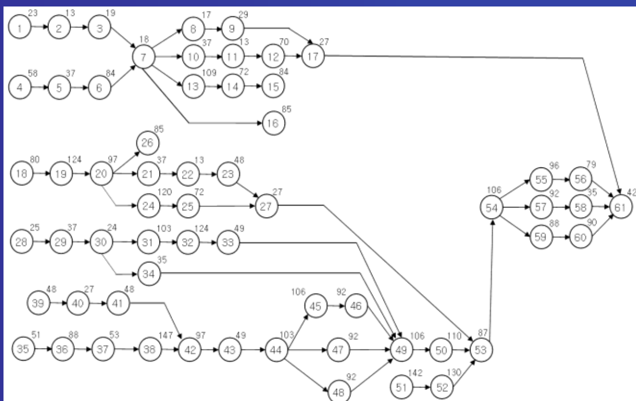


Figure 6. Assembly line for large-sized product

Kim *et al.* 1991,
Journal of the Korean Institute of Industrial Engineers

Table 4. multifunctional workers and salaries

Worker	Possible Tasks												Salary
1	1	3	5	6	9	14	19	26	33	54			$360
2	10	11	12	17	22	26	33	40	56	59			$360
3	1	5	19	23	35	40	45	52	61				$330
4	2	15	22	32	40	44	48	56	60				$330
5	7	20	23	29	36	37	53	56	59				$330
6	15	18	28	34	43	44	45	48	51	54			$360
7	2	5	9	14	20	21	42	56	61				$330
8	1	10	13	22	28	31	32	33	34	39	48		$390
9	4	15	26	39	40	41	42	46	50	54	59		$390
10	8	9	10	12	16	22	32	41	52				$330
11	2	24	25	28	39	44	48	52	54	55	57	58	$420
12	18	23	28	38	42	53	56	58	59	60			$390
13	12	23	29	30	34	38	44	51	55				$330
14	2	5	15	24	28	32	37	58	61				$330
15	4	11	18	20	28	36	40	46	57				$330
16	5	9	15	20	31	41	50	52	53	60			$390
17	4	7	13	35	47	51	54	55	57	58			$390
18	6	19	27	30	35	40	51	56	61				$330
19	17	25	34	36	44	46	47	49	52				$330
20	1	2	3	7	13	14	23	27	44	50	53		$330

4. Numerical Examples [Cont.]

Data for example

- Number of tasks : 61
- Number of workers : 20
- Predetermined cycle time : 350 minutes
- Annual workstation operating cost : $180,000

Parameters

- For task arrangement
  - Crossover : PMX
  - Crossover rate: 0.5
  - Mutation rate: 0.3
- For multifunctional worker assignment
  - Crossover : one-cut-point crossover
  - Crossover rate: 0.5
  - Mutation rate: 0.4
- Terminating conditions
  - 300,000 generations
  - when the best individual does not improve more than 0.01% for 5,000 generations

Result of example

- Number of workstations : 13
- Assigned workers : 19
- Total relevant cost : $3,020,000

  total annual workstation operating cost : $2,340,000

  total annual salary of assigned worker : $680,000
- Computation time (Average of 10 evaluations) : 18 minutes

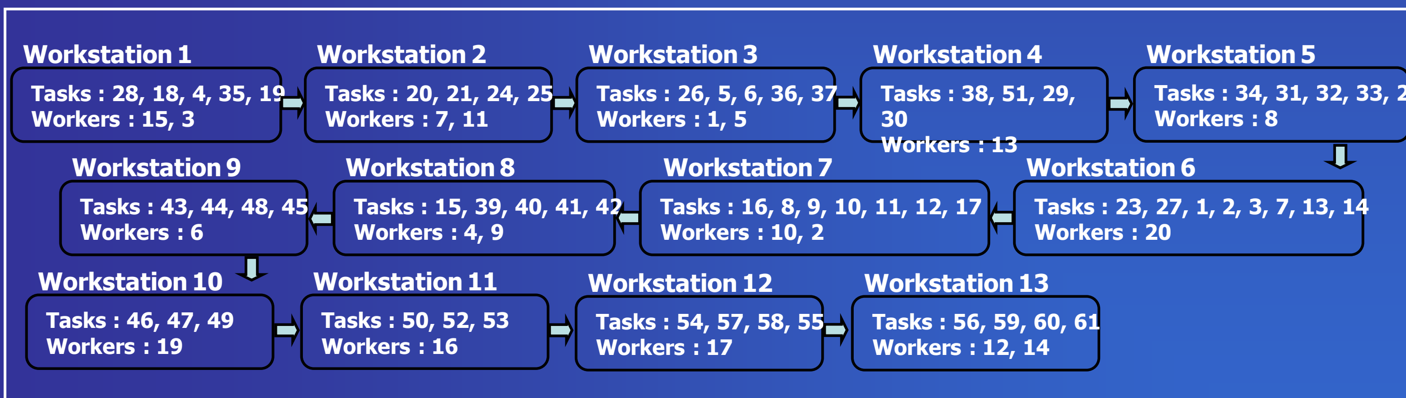


Figure 7. Result of assignment of workers and tasks to the workstations

5. Conclusions

- A mathematical model for the integrated ALB problem with precedence constraints and assignment of multifunctional workers has been constructed.
- We develop the GA to solve the realistic size of the ALB problem.
- The computational results demonstrate the efficiency of the developed GA.